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DAMPING TORQUE ANALYSIS OF POWER SYSTEMS WITH DFIG WIND TURBINE GENERATORS

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Abstract

Renewable energy resources are increasingly being used to meet electric power demands and are gradually replacing conventional generation. In particular, wind generation using variable speed wind turbines (VSWT) which include doubly-fed induction generator (DFIG) technology, can not only be adjusted by the amplitude and phase of rotor excitation current but also by active and reactive power independently. The purpose of the former adjustment is to maintain a constant frequency power output which can help to improve system operating efficiency and achieve higher quality power generation, while the latter adjustment can change the power angle in order to keep a stable operation of the generator. An important aspect of instability is the lack of damping to the low-frequency power oscillations in the system, which can be influenced by grid-connected DFIG wind turbine generators. In this paper, damping torque analysis (DTA) is applied to the established Phillips-Heffron model to examine the dynamic interaction between DFIG units and synchronous generators, in particular investigating how DFIGs may affect system oscillation modes. An analysis of the impact of the penetration of DFIG technology on power system stability is presented in the paper and this is exemplified by several interesting application studies and results.

1 Introduction

As a result of growing environmental concerns, more and more renewable energy resources are made to generate electricity, which are replacing the conventional electrical generation in the industry. Among various renewable resources, wind shows the greatest potential and development of wind power generations is fastest. Variable speed wind turbines (VSWT) utilizing doubly fed induction generator (DFIG) not only can be adjusted by the amplitude and phase of rotor excitation current to maintain a constant frequency power in order to improve the system operation efficiency and achieve high quality power generation, but also can be adjusted by the active and reactive power independently to change the power angle thereby keeping the stable operation

of the generator. It has become the most widely applied technology of wind power generation.

A DFIG is synchronized with a power system through power electronics converters. Its dynamic interactions with the power system are different from that of conventional synchronous generators. Compared with the conventional control, rotor currents of DFIG wind turbine generators are always controlled to extract maximum energy from wind. With the increasing usage of these wind turbines, the power system which is traditionally dominated by the synchronous machines will experience a change in terms of both dynamic and operational characteristics. In consequence, the effective inertia of the system will be reduced and system reliability when subjected to large disturbances could be significantly affected with the increasing usage of DFIG wind turbine generators in power systems. Grid connection of wind generators can either replace conventional synchronous machines or just meet the system load increase without replace any synchronous generators. An important issue of instability is insufficient damping for the low-frequency power oscillations in the system, which is affected by grid-connected DFIG wind turbine generators.

This paper proposes an approach to examine the effect of increasing usage of DFIG wind turbine generators on transient and small-signal stability of a single-machine infinite-bus power system. A Phillips-Heffron model of a single-machine infinite-bus power system integrated with a grid-connected DFIG wind turbine generator is established. This model is used to indicate that how the DFIG wind turbine generator will affect the system rotor angular stability. The damping torque analysis (DTA) is applied to the established Phillips-Heffron model to examine the dynamic interactions between the DFIG and synchronous generators on the aspect of changing in the system oscillation modes. Thus the analysis of the impact associated with the integration of DFIG on power system stability and how the grid-connected DFIG affects the power system rotor angular stability is presented.

2 Modelling of power system

2.1 A dynamic model of power system installed with a DFIG

Fig.1 shows the configuration of single-machine infinite-bus power system installed with DFIG wind turbine.

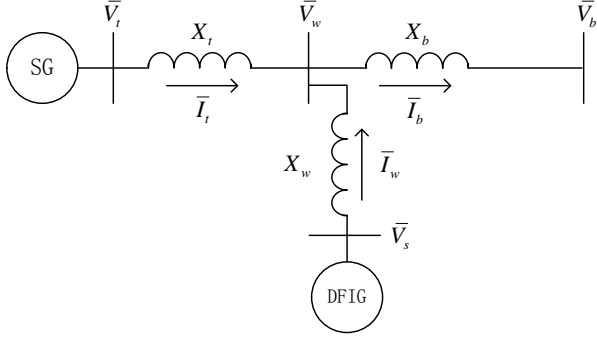


Figure 1: DFIG wind turbine connected to the SMIB power system

The third order equations of DFIG are:

$$\begin{aligned} \dot{E}'_{wd} &= \omega_0 \left(-\frac{R_r}{X_{rr}} E'_{wd} + s_w E'_{wq} + \frac{R_r X_m^2}{X_{rr}^2} I_{swq} - \frac{X_m}{X_{rr}} V_{rwq} \right) \\ \dot{E}'_{wq} &= \omega_0 \left(-s_w E'_{wd} - \frac{R_r}{X_{rr}} E'_{wq} - \frac{R_r X_m^2}{X_{rr}^2} I_{swd} + \frac{X_m}{X_{rr}} V_{rwd} \right) \\ \dot{s}_w &= \frac{1}{T_j} (P_{we} - P_{wm}) \end{aligned} \quad (1)$$

Where V_{rwd} , V_{rwq} , I_{swd} , I_{swq} , R_r are the rotor voltage, stator current and rotor resistance of wd or wq axis component respectively. And X_m is the excitation reactance, s_w is the slip ratio, X_s is the stator leakage reactance, X_r is the resistance value from rotor leakage to the stator side. $X_{ss} = X_m + X_s$, $X_{rr} = X_m + X_r$.

The other four order equations are derived for generator current transformer control system, as shown in Fig.1:

$$\begin{aligned} I_{swq}^{ref} &= K_{pvq}(s)(P_s^{ref} - P_s) \\ I_{swd}^{ref} &= K_{pvd}(s)(Q_s^{ref} - Q_s) \\ V_{rwd} &= K_{ivd}(s)(I_{rwd}^{ref} - I_{rwd}) + s_w \left(\frac{X_m^2}{X_{ss}} - X_{rr} \right) I_{rwq} \\ V_{rwq} &= K_{iwq}(s)(I_{rwq}^{ref} - I_{rwq}) - s_w \left(X_{rr} - \frac{X_m^2}{X_{ss}} \right) I_{rwd} + s_w \frac{X_m}{X_{ss}} V_s \end{aligned} \quad (2)$$

Where $K_{pvd}(s)$ and $K_{pvq}(s)$ are the transfer functions of outer ring PI controllers of active power and reactive power control system of generator side converter respectively. $K_{ivd}(s)$ and $K_{iwq}(s)$ are the transfer functions of inner ring PI controllers of wd axis and wq axis current control system of generator side converter.

The reference value of current control is:

$$\begin{aligned} I_{rwd}^{ref} &= -\frac{X_{ss}}{X_m} I_{swd}^{ref} - \frac{V_s}{X_m} \\ I_{rwq}^{ref} &= -\frac{X_{ss}}{X_m} I_{swq}^{ref} \end{aligned} \quad (3)$$

In the $wd-wq$ coordinate system, without considering the grid side converter control, rotor current can be obtained as follows:

$$\begin{aligned} I_{r3wd} &= I_{r3wd}^{ref} = 0 \\ I_{r3wq} &= \frac{s_w}{s_w - 1} I_{wwq} \end{aligned} \quad (4)$$

Based on the current direction specified in Fig.1, the following relationships can be derived:

$$\begin{aligned} I_{swd} &= I_{wwd} - I_{r3wd} = I_{wwd} \\ I_{swq} &= I_{wwq} - I_{r3wq} = \frac{1}{1 - s_w} I_{wwq} \end{aligned} \quad (5)$$

The relationship of stator voltage and current of DFIG is (with the stator winding resistance ignored):

$$\begin{aligned} V_{swd} &= E'_{wd} - X I_{swq} = 0 \\ V_{swq} &= V_s = E'_{wq} + X I_{swd} \end{aligned} \quad (6)$$

The relationship of rotor current and stator current is:

$$\begin{aligned} I_{rwd} &= -\frac{X_{ss}}{X_m} I_{swd} - \frac{1}{X_m} V_s \\ I_{rwq} &= -\frac{X_{ss}}{X_m} I_{swq} \end{aligned} \quad (7)$$

The output power of DFIG stator is:

$$\begin{aligned} P_s &= V_{swd} I_{swd} + V_{swq} I_{swq} = V_s I_{swq} \\ Q_s &= V_{swq} I_{swd} - V_{swd} I_{swq} = V_s I_{swd} \end{aligned} \quad (8)$$

The power of the DFIG is:

$$P_{we} = E'_{wd} I_{swd} + E'_{wq} I_{swq} \quad (9)$$

Fig.2 shows the phasor diagram of power system in fig.1.

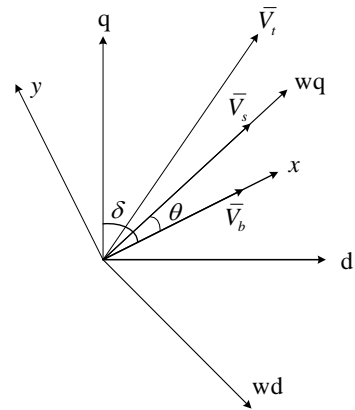


Figure 2: Phasor diagram of power system

From Fig.1, it can have:

$$\begin{aligned}\bar{V}_t &= \bar{V}_w + jX_t \bar{I}_t \\ \bar{V}_w &= \bar{V}_s - jX_w \bar{I}_w \\ \bar{V}_b &= \bar{V}_w - jX_b (\bar{I}_t + \bar{I}_w)\end{aligned}$$

From Eq.(10) and Fig.2, it can have:

$$\begin{aligned}\begin{bmatrix} I_{tq} \\ I_{wq} \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} V_b \sin \delta \\ V_s \sin(\delta - \theta) \end{bmatrix} \\ \begin{bmatrix} I_{td} \\ I_{wd} \end{bmatrix} &= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} V_s \cos(\delta - \theta) - V_b \cos \delta \\ E'_q - V_b \cos \delta \end{bmatrix}\end{aligned}$$

Where

$$\begin{aligned}\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} &= \begin{bmatrix} X_q + X_t + X_b & X_b \\ X_q + X_t & -X_w \end{bmatrix}^{-1} \\ \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} &= \begin{bmatrix} X_b & X_w + X_b \\ X'_d + X_t + X_b & X_b \end{bmatrix}^{-1}\end{aligned}$$

Eq.(11) is the interface electrical equations of SMIB power system with DFIG wind turbine.

From Eq.(2) and Fig.1, it can have the DFIG total input current relationship between $d-q$ and $wd-wq$ coordinate system:

$$\begin{aligned}I_{wvd} &= I_{wd} \cos(\delta - \theta) - I_{wq} \sin(\delta - \theta) \\ I_{wvq} &= I_{wd} \sin(\delta - \theta) + I_{wq} \cos(\delta - \theta)\end{aligned}\quad (12)$$

2.2 Linearization model

Linearization of Eq.(5) and Eq.(6), it can have:

$$\Delta I_{wvd} = \Delta I_{swd}\quad (13)$$

$$\Delta I_{wvq} = (1 - s_{w0}) \Delta I_{swq} - I_{swq0} \Delta s_w$$

$$\Delta E'_{wd} - X' \Delta I_{swq} = 0\quad (14)$$

$$\Delta V_s = \Delta E'_{wq} + X' \Delta I_{swd}$$

By substituting Eq.(14) into the linearization of Eq.(7), Eq.(8) and Eq.(9), it can have:

$$\Delta I_{rwd} = -\frac{X_{ss}}{X_m} \Delta I_{swd} - \frac{1}{X_m} \Delta V_s = \frac{X_{ss}}{X_m X'} \Delta E'_{wq} + \frac{-X' - X_{ss}}{X_m X'} \Delta V_s\quad (15)$$

$$\Delta I_{rvq} = -\frac{X_{ss}}{X_m} \Delta I_{swq} = -\frac{X_{ss}}{X_m X'} \Delta E'_{wd}$$

$$\Delta P_s = V_{s0} \Delta I_{swq} + I_{swq0} \Delta V_s = \frac{V_{s0}}{X'} \Delta E'_{wd} + I_{swq0} \Delta V_s\quad (16)$$

$$\Delta Q_s = V_{s0} \Delta I_{swd} + I_{swd0} \Delta V_s = -\frac{V_{s0}}{X'} \Delta E'_{wq} + \left(\frac{V_{s0}}{X'} + I_{swd0} \right) \Delta V_s$$

$$\Delta P_{we} = \left(I_{swd0} + \frac{E'_{wq0}}{X'} \right) \Delta E'_{wd} + \left(I_{swq0} - \frac{E'_{wd0}}{X'} \right) \Delta E'_{wq} + \frac{E'_{wd0}}{X'} \Delta V_s\quad (17)$$

By substituting Eq.(15), Eq.(16), Eq.(17) into the linearization of Eq.(1), it can have:

$$\begin{aligned}\Delta \dot{E}'_{wd} &= K_7 \Delta E'_{wd} + K_8 \Delta E'_{wq} + K_9 \Delta s_w + K_{dV} \Delta V_s \\ \Delta \dot{E}'_{wq} &= K_{10} \Delta E'_{wd} + K_{11} \Delta E'_{wq} + K_{12} \Delta s_w + K_{qV} \Delta V_s \\ \Delta \dot{s}_w &= K_{13} \Delta E'_{wd} + K_{14} \Delta E'_{wq} + K_{sV} \Delta V_s\end{aligned}\quad (18)$$

Linearization of Eq.(11), it can have:

$$\begin{aligned}(10) \quad \begin{bmatrix} \Delta I_{tq} \\ \Delta I_{wq} \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} V_b \cos \delta_0 \Delta \delta \\ V_{s0} \cos(\delta_0 - \theta_0) \Delta \delta - \\ V_{s0} \cos(\delta_0 - \theta_0) \Delta \theta + \sin(\delta_0 - \theta_0) \Delta V_s \\ [-V_{s0} \sin(\delta_0 - \theta_0) + V_b \sin \delta_0] \Delta \delta + \\ V_{s0} \sin(\delta_0 - \theta_0) \Delta \theta + \cos(\delta_0 - \theta_0) \Delta V_s \\ V_b \sin \delta_0 \Delta \delta + \Delta E'_q \end{bmatrix}\end{aligned}\quad (19)$$

(11) Linearization of Eq.(12), it can have:

$$\begin{aligned}\Delta I_{wvd} &= -\left[I_{wd0} \sin(\delta_0 - \theta_0) + I_{wq0} \cos(\delta_0 - \theta_0) \right] \Delta \delta \\ &\quad + \left[I_{wd0} \sin(\delta_0 - \theta_0) + I_{wq0} \cos(\delta_0 - \theta_0) \right] \Delta \theta \\ &\quad + \cos(\delta_0 - \theta_0) \Delta I_{wd} - \sin(\delta_0 - \theta_0) \Delta I_{wq} \\ \Delta I_{wvq} &= \left[I_{wd0} \cos(\delta_0 - \theta_0) - I_{wq0} \sin(\delta_0 - \theta_0) \right] \Delta \delta \\ &\quad - \left[I_{wd0} \cos(\delta_0 - \theta_0) - I_{wq0} \sin(\delta_0 - \theta_0) \right] \Delta \theta \\ &\quad + \sin(\delta_0 - \theta_0) \Delta I_{wd} + \cos(\delta_0 - \theta_0) \Delta I_{wq}\end{aligned}\quad (20)$$

By Eq.(19) substituting Eq.(20), it can have:

$$\Delta I_{wvd} = R_1 \Delta \delta + R_2 \Delta E'_q + R_3 \Delta \theta + R_4 \Delta V_s\quad (21)$$

$$\Delta I_{wvq} = R_5 \Delta \delta + R_6 \Delta E'_q + R_7 \Delta \theta + R_8 \Delta V_s$$

From Eq.(13) and Eq.(14), it can have:

$$\Delta I_{wvd} = R_9 \Delta E'_{wq} + R_{10} \Delta V_s\quad (22)$$

$$\Delta I_{wvq} = R_{11} \Delta E'_{wd} + R_{12} \Delta s_w$$

From Eq.(21) and Eq.(22), it can have:

$$\begin{aligned}\begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} &= \begin{bmatrix} K_{15} & K_{16} & K_{17} & K_{18} & K_{19} \\ K_{20} & K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix} \begin{bmatrix} \Delta E'_q \\ \Delta E'_{wd} \\ \Delta E'_{wq} \\ \Delta s_w \end{bmatrix}\end{aligned}\quad (23)$$

The linearization of four order dynamic equations of synchronous generator in [8] is:

$$\begin{aligned}\Delta \dot{\delta} &= \omega_0 \Delta \omega \\ \Delta \dot{\omega} &= \frac{1}{M} (-K_1 \Delta \delta - K_2 \Delta E'_q - K_{p\theta} \Delta \theta - K_{pV} \Delta V_s - D \Delta \omega) \\ \Delta \dot{E}'_q &= \frac{1}{T_{do}} (\Delta E'_{fd} - K_3 \Delta E'_q - K_4 \Delta \delta - K_{E\theta} \Delta \theta - K_{EV} \Delta V_s)\end{aligned}\quad (24)$$

$$\Delta \dot{E}'_{fd} = -\frac{1}{T_A} \Delta E'_{fd} - \frac{K_A}{T_A} (K_5 \Delta \delta + K_6 \Delta E'_q + K_{V\theta} \Delta \theta + K_{VV} \Delta V_s)$$

Thus, the linearization model of SMIB power system integrated with a DFIG wind turbine is:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{u}\quad (25)$$

$\mathbf{u} = \mathbf{C}\mathbf{X}$

Where

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_G \\ \mathbf{X}_W \end{bmatrix}, \mathbf{X}_G = \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E'_{fd} \end{bmatrix}, \mathbf{X}_W = \begin{bmatrix} \Delta E'_{wd} \\ \Delta E'_{wq} \\ \Delta s_w \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \Delta \theta \\ \Delta V_s \end{bmatrix}$$

$$A = \begin{bmatrix} A_G & \mathbf{0} \\ \mathbf{0} & A_W \end{bmatrix}, B = \begin{bmatrix} \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{0} & \mathbf{b}_{22} \end{bmatrix},$$

$$C = \begin{bmatrix} \mathbf{c}_{11} & \mathbf{c}_{12} \\ \mathbf{c}_{21} & \mathbf{c}_{22} \end{bmatrix} = \begin{bmatrix} K_{15} & 0 & K_{16} & 0 & K_{17} & K_{18} & K_{19} \\ K_{20} & 0 & K_{21} & 0 & K_{22} & K_{23} & K_{24} \end{bmatrix}.$$

Where

$$A_G = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ -\frac{K_1}{M} & -\frac{D}{M} & -\frac{K_2}{M} & 0 \\ -\frac{K_4}{T_{do}'} & 0 & -\frac{K_3}{T_{do}'} & \frac{1}{T_{do}'} \\ -\frac{K_A K_5}{T_A} & 0 & -\frac{K_A K_6}{T_A} & -\frac{1}{T_A} \end{bmatrix}, A_W = \begin{bmatrix} K_7 & K_8 & K_9 \\ K_{10} & K_{11} & K_{12} \\ K_{13} & K_{14} & 0 \end{bmatrix},$$

$$\mathbf{b}_{11} = \begin{bmatrix} 0 & -\frac{K_{p\theta}}{M} & -\frac{K_{E\theta}}{T_{do}'} & -\frac{K_A K_{V\theta}}{T_A} \end{bmatrix}^T,$$

$$\mathbf{b}_{12} = \begin{bmatrix} 0 & -\frac{K_{pV}}{M} & -\frac{K_{EV}}{T_{do}'} & -\frac{K_A K_{VV}}{T_A} \end{bmatrix}^T,$$

$$\mathbf{b}_{22} = \begin{bmatrix} K_{dV} & K_{qV} & K_{sV} \end{bmatrix}^T.$$

2.3 Damping torque analysis of power system

From Eq.(25), it can have the linearization model of power system as shown by Fig.3.

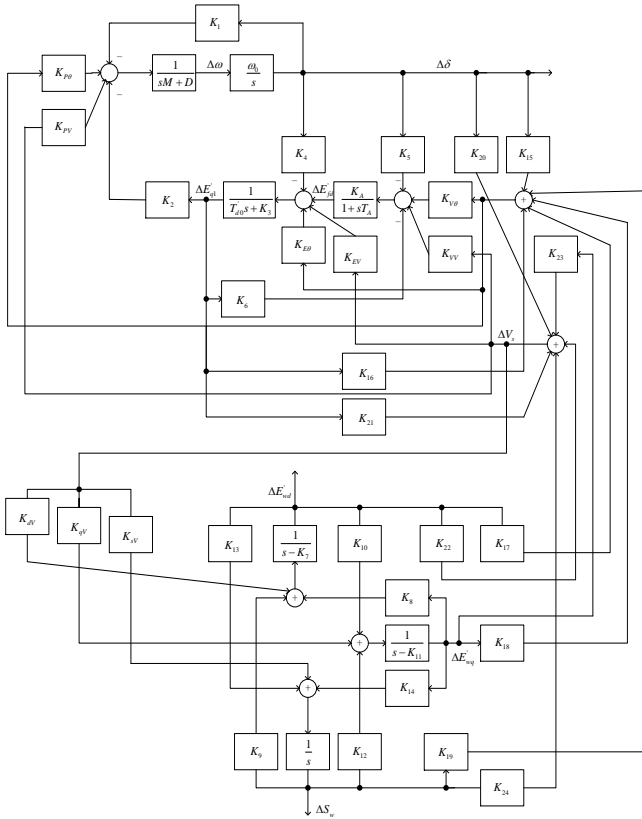


Figure 3: Linearization model of power system with DFIG wind turbine

Eq.(25) can be written as follows:

$$\dot{X}_G = A_G X_G + \mathbf{b}_{11} \Delta\theta + \mathbf{b}_{12} \Delta V_s \quad (26)$$

$$\dot{X}_W = A_W X_W + \mathbf{b}_{21} \Delta V_s$$

From Fig.3 and Eq.(26), it can have:

$$\Delta T_e = K_2 \Delta E_{qq}' + K_{p\theta} \Delta\theta' + K_{pV} \Delta V_s'$$

$$\Delta E_{qq}' = \frac{1}{K_3 + sT_{do}'} (\Delta E_{fd}' + K_{E\theta} \Delta\theta' + K_{EV} \Delta V_s')$$

$$\Delta E_{fd}' = \frac{K_A}{1 + sT_A} (K_6 \Delta E_{qq}' + K_{V\theta} \Delta\theta' + K_{VV} \Delta V_s') \quad (27)$$

$$\Delta\theta' = K_{16} \Delta E_{qq}' + u_\theta$$

$$\Delta V_s' = K_{21} \Delta E_{qq}' + u_V$$

From above equations, it can have the state equation of forward path is:

$$\begin{bmatrix} \Delta \dot{E}_{qq}' \\ \Delta \dot{E}_{fd}' \end{bmatrix} = \begin{bmatrix} \frac{K_{E\theta} K_{16} + K_{EV} K_{21} - K_3}{T_{do}'} & \frac{1}{T_{do}'} \\ \frac{K_A (K_6 + K_{V\theta} K_{16} + K_{VV} K_{21})}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta E_{qq}' \\ \Delta E_{fd}' \end{bmatrix} + \begin{bmatrix} \frac{K_{E\theta}}{T_{do}'} & \frac{K_{EV}}{T_{do}'} \\ \frac{K_{V\theta}}{T_A} & \frac{K_{VV}}{T_A} \end{bmatrix} \begin{bmatrix} u_\theta \\ u_V \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} \Delta E_{qq}' \\ \Delta E_{fd}' \end{bmatrix} + \mathbf{B}_2 \begin{bmatrix} u_\theta \\ u_V \end{bmatrix} \quad (28)$$

$$\Delta T_e = \begin{bmatrix} K_2 + K_{p\theta} K_{16} + K_{pV} K_{21} & 0 \end{bmatrix} \begin{bmatrix} \Delta E_{qq}' \\ \Delta E_{fd}' \end{bmatrix} +$$

$$\begin{bmatrix} K_{p\theta} & K_{pV} \end{bmatrix} \begin{bmatrix} u_\theta \\ u_V \end{bmatrix} = \mathbf{C}_2 \begin{bmatrix} \Delta E_{qq}' \\ \Delta E_{fd}' \end{bmatrix} + \mathbf{d}_2 \begin{bmatrix} u_\theta \\ u_V \end{bmatrix}$$

Thus, the damping torque contributed from DFIG wind turbine to the electromechanical oscillation loop of generator is:

$$\Delta T_e = [\mathbf{C}_2 (s\mathbf{I} - \mathbf{A}_2)^{-1} \mathbf{B}_2 + \mathbf{d}_2] \begin{bmatrix} u_\theta \\ u_V \end{bmatrix} \quad (29)$$

From Eq.(26), it can have:

$$u_\theta = \mathbf{c}_{12} X_W = \mathbf{c}_{12} (s\mathbf{I} - \mathbf{A}_W)^{-1} \mathbf{b}_{22} \Delta V_s \quad (30)$$

$$u_V = \mathbf{c}_{22} X_W = \mathbf{c}_{22} (s\mathbf{I} - \mathbf{A}_W)^{-1} \mathbf{b}_{22} \Delta V_s$$

From Eq.(26), it can have:

$$(K_3 + sT_{do}') \Delta E_q' = \Delta E_{fd}' - K_4 \Delta\delta - K_{E\theta} \Delta\theta - K_{EV} \Delta V_s$$

$$\Delta E_{fd}' = -\frac{K_A}{1 + sT_A} (K_5 \Delta\delta + K_6 \Delta E_q' + K_{V\theta} \Delta\theta + K_{VV} \Delta V_s) \quad (31)$$

From Eq.(31), it can have:

$$\Delta E_q' = G_1(s) \Delta\delta + G_2(s) \Delta\theta + G_3(s) \Delta V_s \quad (32)$$

Where

$$G_1(s) = -\frac{K_A K_5 + K_4 (1 + sT_A)}{(K_3 + sT_{do}') (1 + sT_A) + K_A K_6}$$

$$G_2(s) = -\frac{K_A K_{V\theta} + K_{E\theta} (1 + sT_A)}{(K_3 + sT_{do}') (1 + sT_A) + K_A K_6}$$

$$G_3(s) = -\frac{K_A K_{VV} + K_{EV}(1 + sT_A)}{(K_3 + sT_{do})(1 + sT_A) + K_A K_6}$$

From Eq.(30) and Eq.(31), it can have:

$$\Delta\theta = \mathbf{c}_{11}\mathbf{X}_I + \mathbf{c}_{12}\mathbf{X}_W = \mathbf{c}_{11}\mathbf{X}_I + \mathbf{c}_{12}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}\Delta V_s \quad (33)$$

$$\Delta V_s = \mathbf{c}_{21}\mathbf{X}_I + \mathbf{c}_{22}\mathbf{X}_W = \mathbf{c}_{21}\mathbf{X}_I + \mathbf{c}_{22}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}\Delta V_s$$

By substituting Eq.(32) into Eq.(33), it can have:

$$\begin{aligned} \Delta\theta &= K_{15}\Delta\delta + K_{16}\Delta E_q' + \mathbf{c}_{21}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}\Delta V_s \\ &= \frac{K_{15} + K_{16}G_1(s)}{1 - K_{16}G_2(s)}\Delta\delta + \frac{K_{16}G_3(s) + \mathbf{c}_{21}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}}{1 - K_{16}G_2(s)}\Delta V_s \quad (34) \\ &= G_4(s)\Delta\delta + G_5(s)\Delta V_s \end{aligned}$$

By substituting Eq.(34) into Eq.(32), it can have:

$$\begin{aligned} \Delta E_q' &= G_1(s)\Delta\delta + G_2(s)G_4(s)\Delta\delta + G_2(s)G_5(s)\Delta V_s + G_3(s)\Delta V_s \\ &= [G_1(s) + G_2(s)G_4(s)]\Delta\delta + [G_2(s)G_5(s) + G_3(s)]\Delta V_s \quad (35) \\ &= G_6(s)\Delta\delta + G_7(s)\Delta V_s \end{aligned}$$

From Eq.(33), it can have:

$$\begin{aligned} \Delta V_s &= \frac{\mathbf{c}_{21}\mathbf{X}_I}{1 - \mathbf{c}_{22}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}} \\ &= \frac{1}{1 - \mathbf{c}_{22}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22}}(K_{20}\Delta\delta + K_{21}\Delta E_q') \end{aligned} \quad (36)$$

By substituting Eq.(35) into Eq.(36), it can have:

$$\Delta V_s = \frac{K_{20} + K_{21}G_6(s)}{G_8(s)}\Delta\delta + \frac{K_{21}G_7(s)}{G_8(s)}\Delta V_s \quad (37)$$

From above equation, it can have:

$$\Delta V_s = \frac{K_{20} + K_{21}G_6(s)}{G_8(s) - K_{21}G_7(s)}\Delta\delta \quad (38)$$

By substituting Eq.(38) into Eq.(34), it can have:

$$\Delta\theta = [G_4(s) + G_5(s)\frac{K_{20} + K_{21}G_6(s)}{G_8(s) - K_{21}G_7(s)}]\Delta\delta \quad (39)$$

By substituting Eq.(37) into Eq.(29), the electric torque contributed from DFIG wind turbine to the electromechanical oscillation loop of synchronous generator is:

$$\begin{aligned} \Delta T_e &= [\mathbf{C}_2(s\mathbf{I} - \mathbf{A}_2)^{-1}\mathbf{B}_2 + \mathbf{d}_2] \\ & * \left[\begin{array}{c} \mathbf{c}_{12}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22} \\ \mathbf{c}_{22}(s\mathbf{I} - \mathbf{A}_W)^{-1}\mathbf{b}_{22} \end{array} \right] \frac{K_{20} + K_{21}G_6(s)}{G_8(s) - K_{21}G_7(s)}\Delta\delta \quad (40) \\ &= F_w(s)\Delta\delta \end{aligned}$$

Thus, if the rotor speed is ω_s , the damping torque is:

$$\Delta T_{ed} = -\frac{\omega_0}{\omega_s} \text{Im}[F_w(j\omega_s)]\Delta\omega \quad (41)$$

3 Case Study

The output active power of generator and DFIG wind turbine is 0.5 and 0.5 per unit in power system, respectively. From Fig.3 and above equations, the parameters of linearization model in Eq.(25) are:

$$\begin{aligned} K_1 &= 1.9418, K_2 = 0.9760, K_3 = 3.7273, K_4 = 0.7320, \\ K_5 &= 0.0527, K_6 = 0.8072, K_7 = -174.6969 + j26.2410, \\ K_8 &= 0, K_9 = 0, K_{10} = 0, K_{11} = -174.6969 + j26.2410, \\ K_{12} &= 0, K_{13} = 0, K_{14} = 0.4299, K_{p\theta} = -0.9795, \\ K_{pV} &= -0.1197, K_{E\theta} = -0.2224, K_{EV} = -1.3458, \\ K_{V\theta} &= -0.0377, K_{VV} = 0.0969, K_{dV} = 172.8919 - j26.2410, \\ K_{qV} &= -8.4650 + j2.1209, K_{sV} = 0.0696. \end{aligned}$$

From Eq.(28), it can have:

$$\mathbf{A}_2 = \begin{bmatrix} -0.8103 - j0.0000 & 0.2000 \\ 827.2244 - j0.0891 & -100 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -0.0444 & -0.2691 \\ -3.7700 & 9.6943 \end{bmatrix}$$

$$\mathbf{C}_2 = [0.8883 - j0.0019 \quad 0], \mathbf{d}_2 = [-0.9756 \quad -0.1197]$$

From Eq.(29), it can have the electric torque contributed from DFIG wind turbine to the electromechanical oscillation loop of synchronous generator is:

$$\Delta T_e = [-0.9742 + j0.0052 \quad -0.1131 + j0.0253] \begin{bmatrix} u_\theta \\ u_v \end{bmatrix}$$

From Eq.(30), the transfer function of control variables and voltage amplitude is:

$$\begin{aligned} u_\theta &= (-0.0214 - j0.0008)\Delta V_s \\ u_v &= (0.4474 - j0.0206)\Delta V_s \end{aligned}$$

And from above equations, it can have:

$$\begin{aligned} \Delta\theta &= (0.2538 + j0.0016)\Delta\delta + (-0.0212 + j0.0003)\Delta V_s \\ \Delta E_q' &= (-0.0027 + j0.0276)\Delta\delta + (0.0031 - j0.0085)\Delta V_s \\ \Delta V_s &= (-0.0069 + j0.0118)\Delta\delta \end{aligned}$$

Thus, it can have the electric torque contributed from DFIG wind turbine to the electromechanical oscillation loop of synchronous generator is: $\Delta T_e = (0.00005 - j0.0004)\Delta\delta$.

If $\omega_s = -1.3091 + j8.0555$, the damping torque of DFIG wind turbine is: $\Delta T_{ed} = 0.0169\Delta\omega$.

Table.1 gives the simulations results of damping torques contributed from DFIG to the electromechanical oscillation loop of synchronous generator of seven-order and thirteen-order differential model.

P_{SG}	P_{DFIG}	Damping torque	Oscillation mode (seven-order)	Oscillation mode (thirteen-order)
1.0	0.0	0.0151	-1.4165 + j8.5642	-1.4451 + j8.5571
0.9	0.1	0.0173	-1.3968 + j8.4509	-1.4244 + j8.4441
0.8	0.2	0.0187	-1.3751 + j8.3429	-1.4018 + j8.3366
0.7	0.3	0.0191	-1.3528 + j8.2390	-1.3786 + j8.2331
0.6	0.4	0.0185	-1.3305 + j8.1422	-1.3556 + j8.1367
0.5	0.5	0.0170	-1.3092 + j8.0555	-1.3337 + j8.0504

0.4	0.6	0.0144	-1.2899 + j7.9817	-1.3141 + j7.9770
0.3	0.7	0.0109	-1.2737 + j7.9233	-1.2976 + j7.9189
0.2	0.8	0.0064	-1.2613 + j7.8821	-1.2853 + j7.8780
0.1	0.9	0.0011	-1.2535 + j7.8594	-1.2778 + j7.8555
0.0	1.0	-0.0051	-1.2505 + j7.8556	-1.2754 + j7.8520

Table 1: Simulation results of damping torques for power system oscillation with seven-order and thirteen-order differential model ($P_{SG}+P_{DFIG}=1.0$)

4 Conclusion

The linear single-machine infinite-bus power system integrated with a grid-connected DFIG which could be modelled by either a seven order model or a thirteen order model can be used to calculate the eigenvalues and associated damping torques. The influence of the DFIG model on eigenvalues is small which confirms the rationality of simplified seven-order differential model. The damping torque contributed from DFIG to the electromechanical oscillation loop of synchronous generator is very small so that the system oscillation stability is not affected by the wind turbine. When the total power of DFIG and synchronous generator keeps constant, the damping torque contributed from DFIG to the electromechanical oscillation loop of synchronous generator will be changed with the variation of output power of wind turbine.

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Appendix

Parameters of example single-machine infinite-bus power system (in per unit except indicated):

Generator

$$X_d = 0.8, X_q = 0.4, X_d' = 0.05, M = 8, D = 20, T_{d0} = 5s$$

Transmission line

$$X_{r1} = 0.15, X_{r2} = 0.15, X_r = 0.15$$

AVR

$$T_A = 0.01s, K_A = 10$$

Steady point

$$V_{i0} = 1.05, V_{s0} = 1.0, V_{b0} = 1.0$$

DFIG wind turbine

$$T_j = 8, D = 0, S_w = 0.1, R_r = 0.0415, R_s = 0,$$

$$X_r = 0.1225, X_s = 0.1784, X_m = 2.4012.$$