

Evolution of bridge frequencies and modes of vibration during truck passage

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17 Abstract

18 This paper reports an experimental campaign that aims at measuring the evolution of bridge 19 modal properties during the passage of a vehicle. It investigates not only frequency shifts due 20 to various vehicle positions, but also changes in the shape of the modes of vibration. Two 21 different bridges were instrumented and loaded by traversing trucks or trucks momentarily 22 stationed on the bridge. The measurements were analysed by means of an output-only 23 technique and a novel use of the continuous wavelet transform, which is presented here for 24 the first time. The analysis reveals the presence of additional frequencies, significant shifts in 25 frequencies and changes in the modes of vibration These phenomena are theoretically 26 investigated with the support of a simplified numerical model. This paper offers an 27 interpretation of vehicle-bridge interaction of two particular case studies. The results clearly 28 show that the modal properties of the vehicle and bridge do change with varying vehicle 29 position.

30

31 Keywords

32 vehicle-bridge interaction, modal analysis, nonstationary, wavelet

- 34 **1. Introduction**
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It is a well-known fact that the modal properties of two separate mechanical systems change when both systems interact. The coupled arrangement might have significantly different natural frequencies and modes of vibrations, compared to the uncoupled systems [1]. This is also acknowledged in bridge engineering to some extent, when investigating vehicles crossing the structure, i.e. it is understood that natural frequencies of a bridge change when heavy (massive) traffic traverses it.

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43 As pointed out by Frýba [2] the fundamental frequency of a loaded beam depends not only on 44 the magnitude of the mass on the deck but also on the position of the mass. A key factor in 45 the scale of frequency variation that occurs for different mass positions is the ratio between 46 the vehicle and bridge masses, with higher mass ratios producing larger shifts in the bridge 47 frequency. Despite the general acceptance that such frequency shifts will occur, this is a 48 problem not well studied in bridge engineering literature [3]. However, there have been some 49 recent studies, for example [4] describes changes in the fundamental frequency of a railway 50 bridge during passage of a train and provides an approximate formula to calculate changing 51 bridge frequency. Yang et al. [3] study the variation of both vehicle and bridge frequencies 52 and present a closed-form expression for a simply supported bridge considering only the first 53 mode of vibration. Cantero & OBrien [5] investigate numerically the effect of different mass 54 ratios and frequency ratios on the changes in system frequencies, where frequency ratio (FR) 55 = vehicle frequency / bridge frequency and mass ratio (MR) =vehicle mass / bridge mass. 56 The numerical analyses of coupled vehicle-bridge models in [5, 6] show that for certain mass 57 and frequency ratios it is possible to achieve positive frequency shifts in the fundamental 58 frequency of the bridge. There exist only a limited number of studies that investigate this 59 problem either experimentally, or in real operational bridges. For instance, in [7] the authors 60 use a variety of output-only techniques with the response of a scaled model and are able to 61 obtain clear frequency evolution diagrams for the case of large mass ratios. Also [6] performs 62 a controlled laboratory experiment obtaining frequency shifts that validate an approximate 63 closed-form solution of the frequency shift. The study in [8] investigates how a parked 64 vehicle on an operational bridge affects its fundamental frequency, reporting frequency 65 reductions of 5.4%. More recently, [9] explores the non-stationary nature of a 5-span bridge 66 traversed by a truck, using alternative time-frequency tools, with limited success. Frequency 67 is not the only modal property changing with load and its position; for instance [10] used 68 numerical simulation to show that damping of a pedestrian bridge also changes according to 69 number and location of pedestrians. That said, the majority of the limited papers available on 70 the topic focus only on tracking frequency changes and do not evaluate the effect of load on 71 the associated mode shapes.

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73 Although a small number of authors have used numerical models to study the problem of 74 frequency variation with load position, to date, no experimental investigation on full scale 75 bridges has been presented. Such a study is the main contribution of this paper. Two seperate 76 experiments were carried out, each using a different test truck on different instrumented 77 bridges. Bridge A is a three-span continuous structure monitored while a truck traverses it at 78 a constant speed. The measurements from Bridge A provide only weak evidence of the 79 evolution of the modal properties and hence it constitutes only a first attempt. A second 80 experiment is reported on Bridge B, which is a single span bridge. For the experiment on 81 Bridge B, a truck stops at certain locations on the bridge. The free vibration measurements of 82 the bridge accelerations, right after the vehicle stops, allows for the precise extraction of the 83 modal parameters of the coupled system. This is repeated for various vehicle stopping 84 positions to obtain the variation of the modal properties with respect to vehicle position. It is 85 important to note that the variation in modal properties reported here are specific to the two 86 case studies investigated; since these variations strongly depend on the particular vehicle and 87 bridge.

88

89 Over the course of the investigation, it is shown that a vehicle being present on the bridge 90 results in a coupled system, such that modal analysis results cannot be interpreted as two 91 separate systems (bridge and truck). The vehicle-bridge interaction is a non-stationary 92 problem where the modal parameters change with vehicle location. In general, the ideas and 93 results presented here are of interest to engineers and researchers involved in any vehicle-94 bridge interaction study. However, the findings reported here have particular consequences 95 for the current research thread on extracting bridge modal properties from passing instrumented vehicles, e.g. [11-13]. In general, these publications acknowledge that there is 96 97 vehicle-bridge coupling, but fail to consider the changes in modal properties with vehicle 98 position. In these papers modal analysis techniques are often applied to the full length of the 99 signal obtained during vehicle passage. However, attempting to analyse what is in effect a 100 non-stationary signal with conventional modal analysis techniques developed for stationary 101 signals will necessarily result in unreliable modal properties.

103 As well as demonstrating that the bridge acceleration signal recorded during the passage of a 104 truck is non-stationary, this paper provides advice and insight on a number of related issues. 105 First, a modified and novel approach for performing the Continuous Wavelet Transform 106 (CWT) is presented, and is shown to be an effective signal processing technique to visualise 107 variations in system frequencies. Next, the source of the additional frequency peak in the 108 spectra of the forced (i.e. loaded) bridge acceleration signal is investigated. This is carried out 109 using a relatively simple but insightful numerical model, and experimental data from Bridges A and B. Moreover, this paper shows for the first time that not only do the natural 110 111 frequencies evolve during traffic passage, but that the shapes of the associated modes of 112 vibration also evolve. For every vehicle location, the vehicle-bridge system features distinctly 113 different modes. This is supported by a theoretical analysis of the problem, and carefully 114 extracted experimental results. However, it should be noted that this paper only reports 115 findings on the first longitudinal mode of the bridge, no torsional or higher modes are investigated. 116

117

118 The remainder of this paper has four primary sections. Section 2 provides a theoretical 119 background on the numerical model, modal analysis, and signal processing techniques used 120 in this study. Section 3 describes an experimental test where a truck was driven across a 3-121 span bridge. Additional frequencies were observed in the spectra of the recorded bridge 122 response. A numerical model is used to postulate the origin of the additional frequency peak. 123 However, to experimentally confirm the validity of the model predictions it was necessary to 124 redo the experiment using a revised procedure where the truck would stop at a series of 125 discrete locations on a bridge. The outcome of the revised experiment is reported in Section 126 4.

127

128 **2. Methods**

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This section provides the reader a brief overview of the tools used throughout this study. Section 2.1 describes the numerical model that helps explain non-intuitive changes in modal properties observed in the experiments. Section 2.2 provides references on the modal analysis procedures employed to analyse the measured acceleration signals. Finally, Section 2.3 describes a modified form of wavelet analysis that is used to visualise variations in the system frequencies for the non-stationary acceleration signals recorded on site.

137 **2.1 Numerical model**

138

139 The coupled vehicle-bridge model was programmed in Matlab [14] and a pictorial 140 representation of the numerical model is shown in Fig. 1. The truck is simulated as a sprung 141 mass m supported on a spring k, where the spring represents the suspension of the vehicle. 142 The bridge is simulated using a finite element beam model where each beam element has 4 143 degrees of freedom, namely a rotation and a vertical translation at each end of the element. 144 Elemental matrices for this kind of element can be found in the literature, e.g. [15]. The beam 145 is defined by its span L, section area A, modulus of elasticity E, second moment of area I and 146 mass per unit length ρ . The location of the vehicle is defined by the distance from the left 147 support (x) and in the simulations the vehicle can be positioned anywhere on the beam $(0 \le x)$ 148 \leq L). The coupling between both systems, i.e. bridge and vehicle, can be written in terms of 149 the beam element shape functions and the relative position of the vehicle within that element 150 [16]. However, defining a sufficiently dense mesh that has a node exactly at the location of 151 the vehicle reduces the complexity of the procedure. In that case the matrices of both systems 152 are assembled diagonally, and the coupling terms are off-diagonal negative stiffness values 153 that link together the appropriate degrees of freedom. As two different bridges will be 154 modelled, (each with different boundary conditions), for now the boundary conditions of the 155 model are indicated with question marks in Fig. 1. Models of this type have previously been 156 presented in the literature [17].

157



158 159

Fig. 1: Coupled Vehicle-Bridge finite element model

Fundamentally, the purpose of this model is to allow the vehicle to be moved incrementally across the bridge and to track how the bridge frequency changes with the position of the vehicle. For a given vehicle position, the bridge frequencies and associated modes of vibration can be determined using an eigenvalue analysis. Simulating a multi-axle truck as a

165 single degree of freedom sprung mass is a simplification, and for some applications it would 166 be an over simplification. However, it is shown later that for the purpose of this study, where 167 the primary interest is in explaining the evolution of frequency with respect to truck position, 168 the model is effective. Initially values for area (A), second moment of area (I) and mass per 169 unit length (ρ) were determined from the available bridge drawings. For the Young's Modulus (E), standard values for steel and concrete of $2x10^{11}$ N/m² and $2x10^{10}$ N/m² 170 respectively were used. After getting an initial estimate of bridge frequencies from the model, 171 172 the bridge properties (in the model) are revised so that the fundamental bridge frequency of the model matches the free vibration frequency observed on site, this is further described in 173 174 Sections 3 and 4. For the vehicle, the spring stiffness (k) is adjusted so that the vehicle 175 frequency in the model matches the vehicle frequency inferred from the acceleration 176 measurements recorded experimentally when the truck was traversing the real bridge. Table 1 177 gives a summary of relevant information about the vehicle and bridge properties used in this 178 paper. It can be seen in Table 1 that the vehicle properties postulated for the test vehicles give body bounce frequencies that are in accordance with typical values for heavy vehicles 179 180 (1 Hz to 4 Hz) as shown in [18].

181

	-	Test on Bridge A	Test on Bridge B
Bridge	Туре	3-span continuous	1-span
	Spans (m)	18+31+18	36
	f _b (Hz)	3.50	3.13
	Mass (kg)	26 000	32 000
Vehicle	f _v (Hz)	2.80	2.60
	Number of axles	3	4
	Axle distances (m)	1.4 + 4.1	2.0+3.5+1.4
	Velocity (m/s)	3.63	-

182 Table 1: Vehicle and bridge properties

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184 **2.2 Bridge modal analysis**

185

The Introduction provided an overview of literature dealing with variation in bridge frequency with respect to variation in mass distribution. It was also highlighted that previous studies have not looked at how the mode shapes associated with these frequencies change with respect to variation in mass distribution. To address this limitation this study attempts to experimentally capture the mode shape associated with a particular truck position. This is achieved using output-only modal analysis methods, i.e. no information on the excitation is measured. Due to the size/mass of road bridges, output-only methods are often the only logistically feasible approach to extract modal parameters, because using shakers or impact hammers to excite the structure is often not practical. Specific details on the theory/ mathematics underlying output-only modal analysis are not provided here as the topic has been extensively covered in other publications such as [19]. The particular method used in this paper is Frequency Domain Decomposition (FDD) and details on this method are given in [20].

199

200 **2.3 Wavelets**

201

202 To be able to accurately visualise the variation in frequency with respect to time, some time-203 frequency representation of the recorded signals is necessary. There are a number of time-204 frequency analysis methods available, e.g. Short Time Fourier Transform, Hilbert-Huang 205 transform and Wavelet transform. Within each of these methods, different options in their 206 implementation can significantly change the time-frequency plots that are output. All time-207 frequency analysis methods involve a trade-off in resolution, i.e. high resolution in the 208 frequency domain typically means poor resolution in the time domain, and vice versa. 209 Ultimately, it is up to the analyst to identify which method best achieves their objective. In 210 this paper, the objective of the time-frequency analysis is to visualise how the bridge 211 frequency changes as a truck traverses the bridge.

In essence, the CWT compares the wavelet bases (a wave-form of finite length) to the 212 213 analysed signal and gives a wavelet coefficient, so that the better the match, the larger the 214 coefficient. This wavelet is then shifted in time to cover the whole length of the signal, 215 resulting in a vector of wavelet coefficients. The wavelet is then scaled (i.e. stretched) and the 216 process is repeated. For each scale used in the analysis a vector of wavelet coefficients 217 results. Scale can be regarded as inversely proportional to frequency and thus can be 218 transformed approximately to frequency, or more specifically pseudo-frequency. The result 219 of CWT analysis is a plot of wavelet coefficients in the time-frequency plane that are 220 proportional to the energy of the signal. For additional information on wavelets and to find a 221 full mathematical description further details are provided by other authors [21,22].

222

When using the CWT, several wavelet basis functions are available, e.g. Morlet, Gaussian, Mexican hat. The results from the CWT are significantly affected by the wavelet basis used in the analysis so it is paramount to choose an appropriate basis. Knowing which wavelet basis will give the best results for a given application is not always obvious, and often there is a degree of trial and error involved. However, [23] showed that the Modified Littlewood-Paley (MLP) wavelet basis was effective when analysing the acceleration signals of bridges subject to vehicle loading, and therefore this is the wavelet basis used in this study.

230

231 In addition, this paper proposes a non-conventional normalisation step that proves very 232 effective when analysing bridge signals that contain a mixture of free and forced vibration. 233 Using a conventional CWT to analyse a bridge signal that has both free and forced vibration 234 can be difficult. The forced vibration part of the signal has the largest amplitude, and as a 235 result this will dominate the resulting CWT plot. This makes it very difficult to track the 236 frequency evolution between the free and forced parts of the signal because the frequency 237 from the free vibration part will be practically invisible. The novel procedure adopted here 238 gets around this problem by normalising the wavelet coefficients at each time instant and is 239 presented schematically in Fig. 2.

240

241 A signal with linearly increasing frequency and linearly decreasing amplitude is analysed 242 with a conventional CWT and the result is shown in Fig. 2(a). The plot represents a 3D 243 wavelet surface as a 2D 'contour' plot where the magnitude of the wavelet coefficients are 244 conveyed using colour, with darker colours implying large values of wavelet coefficient. The 245 non-stationarity property and decreasing amplitude of this numerically generated signal can 246 clearly be appreciated in the plot. Unfortunately, from the point of view of frequency 247 tracking, the large amplitudes in the early part of the signal are resulting in high wavelet 248 coefficients that are in a sense dominating the plot and making it difficult to see the frequency 249 content in the latter part of the signal. However, if one is prepared to sacrifice information 250 relating to amplitude, which for the purpose of this paper we are not concerned with, then this 251 representation can be improved. The first step is to fit an envelope to the wavelet coefficients 252 for a given scale and to accept this curve as the representative result from the CWT. An 253 example of this curve fitting is shown in Fig. 2(b). The blue plot in Fig. 2(b) shows the 254 wavelet coefficients at a particular scale, the red curve has been fitted to the blue plot. If a 255 similar curve is fitted at every scale, and then if all the 'fitted' curves are plotted in 2D, the 256 plot shown in Fig. 2(c) results. The second step is to normalise each wavelet coefficient at a 257 given time instant by the total energy content for that time instant. The result of applying this 258 normalisation is shown in Fig. 2(d). The consequence of this normalization is that it gives the 259 same importance to the frequency of small amplitude vibrations as it does to the frequency of 260 large amplitude vibrations. The usefulness of this normalization will become clear when 261 studying the measured accelerations in Sections 3 and 4 below. Obviously, the substitution by 262 the envelope curve and then later application of normalization comes with a cost. The final 263 map of wavelet coefficients cannot be used for signal reconstruction. However, for 264 visualization purposes these two operations greatly improve the final result from the CWT.





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Fig. 2: Enhancement of energy map from CWT analysis

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269 3. Experimental study of Bridge A and moving truck

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271 This section describes the first experimental investigation carried out on a 3-span road bridge. 272 A truck is driven over the bridge and the bridge acceleration is recorded at a number of 273 locations. This acceleration data is subsequently analysed to examine how the modal 274 parameters of the bridge change as the truck crosses the bridge. Section 3.1 describes the 275 bridge and experiment setup used. Section 3.2 presents the results of modal analysis carried 276 out on free and forced vibration data. Finally, Section 3.3 puts forward a theoretical model to

explain the behaviour observed in Section 3.2. Note that this experiment on Bridge A is only the first attempt to study the evolution of modal properties during vehicle passage and a plausible explanation is provided based only on weak evidence. A second experiment that provides stronger evidences is performed on a different bridge and is reported in Section 4.

281

282 **3.1 Bridge and instrumentation description**

283

284 The bridge used in the experiment is shown in Fig. 3(a). It is a 3-span bridge carrying a minor 285 road (4 m wide) over a dual carriageway. The deck consists of 2 steel girders supporting a 286 concrete deck. The centre span is 31 m and each of the side spans are 18 m. There were two 287 primary reasons for selecting this bridge. Firstly, the bridge deck is relatively light, narrow 288 carriageway and primary members are steel. This is advantageous because a high (vehicle-289 bridge) mass ratio should lead to larger changes in modal properties. The second reason for 290 selecting this bridge is that the traffic volumes on the bridge are very light, which made it 291 logistically feasible to carry out the test. The vehicle used in the test is a 3-axle truck with a 292 total mass of 26 tonnes, shown in Fig. 3(b). The truck crossed the bridge twice (once in each 293 direction) at a crawling speed of approximately 13 km/h (3.63 m/s). Such a low speed 294 effectively reduces the dynamic effects associated with (i) road profile unevenness, (ii) 295 loading frequencies due to the vehicle's axle spacing and (iii) shifting of bridge frequencies 296 [24]. Despite the low speed the truck still provides sufficient excitation to the system.



298

Fig. 3: (a) Bridge A elevation (3-span bridge); (b) Truck used in experiment

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Fig. 4 shows a plan view of the bridge deck. The position of the piers is indicated using dashed lines and for convenience the spans are labelled as spans 1-3. The bridge has a 4 m wide carriageway with 0.5 m wide footways on either side. Due to the impossibility of road closure, the instrumentation had to be installed on the footway and it was installed as close as possible to centre of the main beams. The location of the six accelerometers (A-F) used in the test are indicated in Fig. 4. One accelerometer was placed at mid-span of each of the three
spans on both sides of the bridge. The accelerometers used were tri-axial Micro-Electro
Mechanical System (MEMS) accelerometers scanning at 128 Hz.



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Fig. 4: Plan view and accelerometer layout on Bridge A.

311

312 **3.2 Modal analysis of free and forced vibration data**

313

314 The first step in analysing the data is to perform modal analysis on the free vibration data, i.e. no truck on the bridge. The FDD modal analysis approach described in Section 2.2 is used to 315 316 analyse the free vibration data. Singular Value Decomposition (SVD) of the Power Spectral 317 Density matrix is plotted in Fig. 5(a) where a clear peak is visible at 3.5 Hz indicating the 318 likely presence of a mode. Note that the poor frequency resolution is due to the short duration 319 of analysed signal. The associated mode of vibration is extracted and presented in Fig. 5(b). 320 The square data markers represent the bridge supports, i.e. the modal amplitude at these 321 locations is assumed zero. The circular data markers (from left to right) indicate the modal 322 amplitudes at sensor locations A, B and C, see Fig. 4. If the modal ordinates for sensor 323 locations D, E and F are plotted the same mode shape is apparent. Thus it is clear that the 324 mode at 3.5 Hz is the first bending mode. This result is consistently obtained for various 325 different free vibration measurements.



Fig. 5: Modal analysis of signals during free vibration of Bridge A; (a) Singular Value
Decomposition magnitude; (b) Extracted fundamental mode

330 Once the free vibration data was analysed the next step was to analyse the forced vibration 331 response, i.e. the acceleration recorded while the truck was on the bridge. The results of 332 analysing the forced vibration data is presented in Fig. 6. The analysis procedures used are 333 the same as those used to generate the plots in Fig. 5. However, there are in this case, some 334 noticeable differences in the results. The SVD analysis in Fig. 6(a) identifies the presence of two distinct peaks at 2.63 Hz and 3.63 Hz respectively, but the fundamental bridge mode at 335 336 3.5 Hz identified in Fig. 5 is no longer evident. The mode shapes associated with the two 337 frequency peaks are shown in Fig. 6(b).

338

339 Starting with the mode shape for the 3.63 Hz mode, it is noticeable that it is very similar in 340 shape to the mode shown in Fig. 5(b), so it is reasonable to assume that this is the same mode. 341 However, the presence of the truck has changed the frequency of the mode slightly. It is 342 interesting to note that the fundamental frequency of the bridge has increased. Intuitively one 343 would expect a slight reduction in the frequency because the truck is adding mass to the deck. 344 Moving on to the mode identified at 2.63 Hz, its origins are less clear. One possibility is that 345 perhaps the loading frequency produced an excitation in the region of 2.63 Hz. For this truck 346 three possible axle spacings need to be considered, namely 1.4 m, 4.1 m and 5.5 m, which are 347 the distances from axle-1 to axle-2, axle-2 to axle-3, and axle-1 to axle-3 respectively. For a 348 traversing speed 3.63 m/s the possible loading frequencies are 0.38 Hz, 1.13 Hz and 1.52 Hz. 349 Another possibility is that the shift in bridge frequency is due to the driving velocity of the 350 vehicle, as discussed in Yang et al. [24]. This shift in frequency is directly proportional to the 351 vehicle speed and inversely proportional to double the bridge span. Due to the low speed of the traversing vehicle, only shifts of ± 0.03 Hz in the bridge fundamental frequency can be expected. Therefore, neither the vehicle loading frequency nor the frequency shift due to driving velocity explain the frequency peak at 2.63 Hz.

355

356 Obviously, the origins of the 2.63 Hz frequency is likely to be related to the vehicle's 357 presence, and it is reasonable to consider that the 2.63 Hz may be the vehicle frequency 358 however, it is difficult to be definitive just on the evidence of Fig. 6. Interestingly the mode 359 shape associated with the 2.63 Hz peak is practically a duplicate of the fundamental bridge 360 mode identified in Fig. 5(b). Therefore, to get a better theoretical understanding of why the 361 presence of a truck is; (i) causing a slight increase in the frequency of the fundamental mode 362 and (ii) resulting in the appearance of a new mode, the vehicle-bridge model described in 363 Section 2.1 is used in the next section to calculate the system frequencies for a series of 364 different vehicle positions.

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369 **3.3 Theoretical model of observed behaviour**

370

In an effort to better understand the frequencies observed in Fig. 6 the vehicle-bridge model described in Section 2.1 is used here to position the vehicle model at a series of discrete points along the length of the beam and to examine how the frequencies of the system (vehicle and bridge) are affected. The bridge is modelled as a 3-span continuous beam with restrained vertical displacements at the ends and intermediate locations, which represent the 376 support conditions at the abutments and over the piers. The bridge properties in the model are 377 revised so that the fundamental frequency in the model is 3.5 Hz and the properties of the 378 vehicle model have been adjusted to get a vehicle frequency of 2.8 Hz. The total mass of the 379 vehicle in the model is 26000 kg. Although the exact frequency of the vehicle was not 380 measured on site, based on the experimental observations in the previous section, and the 381 information in the literature [18], a vehicle frequency of 2.8 Hz seems reasonable. It should 382 be noted that the purpose of this model is not to exactly simulate the vehicle crossing event 383 recorded experimentally. Instead, the purpose is to examine what happens to the bridge and 384 vehicle frequencies if the sprung mass is placed at a series of discrete points along the length 385 of the beam. This is achieved by positioning the sprung mass at a given point on the bridge 386 and performing an eigenvalue analysis the system matrices of the coupled model system to 387 identify the system frequencies for that vehicle position. Then the vehicle is consecutively 388 moved to the next point on the bridge and the system frequencies for each new position are 389 calculated. As the vehicle-bridge system is coupled, technically these frequencies should be 390 termed the 'first system frequency', 'second system frequency', etc. However, for convention 391 in the following discussion they are also referred to as 'vehicle' and 'bridge' frequencies.

392

393 The evolution of the system frequencies for various vehicle positions is presented in Fig. 7. 394 The horizontal axis in Fig. 7 shows the position of the vehicle relative to the left support as a 395 percentage of the total bridge length L. So when the vehicle is exactly over the left support its 396 position is 0% of L, when it is half way across its position is 50% of L, and when it is exactly 397 over the right support its position is 100% of L. The two dashed vertical lines in the figure at 398 26% and 73% indicate the position of the two piers. The ordinates in Fig. 7 are frequency 399 values. The two horizontal lines at 3.5 Hz and 2.8 Hz represent the vehicle and bridge 400 frequencies in isolation, i.e. in the absence of any interaction between them.

401

402 The lower solid line in Fig. 7 shows the variation in the vehicle frequency as the vehicle is at 403 various positions along the length of the bridge. Tracing this plot from left to right, it can be 404 seen that when the vehicle is positioned over the left support its frequency (2.8 Hz) remains 405 unchanged. However, when the vehicle is positioned toward the centre of span 1 ($x \approx 13\%$) 406 the vehicle frequency drops below 2.8 Hz. Then, as the vehicle is positioned at the first pier 407 $(x \approx 26\%)$, the vehicle frequency goes back up to 2.8 Hz. As the vehicle is incrementally moved toward the centre of span 2 the vehicle frequency shows a steady reduction in 408 409 frequency to a minimum value of approximately 2.4 Hz at the mid-span of span 2 ($x \approx 50\%$).

410 As the position of the vehicle continues toward pier 2 the vehicle frequency shows a gradual 411 increase and it recovers completely to 2.8 Hz when the vehicle is over pier 2. A similar 412 reduction in vehicle frequency is evident when the vehicle is positioned in the centre of span 413 3. If the vehicle is thought of in isolation, i.e. if it is visualised as a mass supported on a 414 spring, this pattern is difficult to understand. However, if, for the crossing event, the vehicle 415 is thought of as a mass on two vertical springs, (one on top of the other) it is easier to 416 understand. The upper spring being the vehicle suspension and the lower spring being the 417 bridge, i.e. it is now a 2 degree of freedom system. The stiffness of the upper spring (the 418 vehicle suspension) is constant. The stiffness of the lower spring (the bridge) is not constant 419 since it depends on where the vehicle is positioned on the bridge. When the vehicle is over a 420 bridge support the lower spring could be regarded as infinitely stiff so the vehicle behaves as 421 an uncoupled single DOF system and the frequency remains 2.8 Hz. However, when the 422 vehicle is at the mid-span of the bridge the lower spring is no longer infinitely stiff, as the 423 system of springs supporting the mass is more flexible than it was before (when the vehicle 424 was over a support) so the frequency of the system drops. Note that the 2 degree of freedom 425 model/visualisation constitutes only an analogy that encapsulates the frequency evolution 426 phenomena. Similar models have been reported in [25, 26] to study the dynamics of vehicle-427 bridge interaction systems.

428

429 Turning our attention to the upper solid line in Fig. 7, the result shows how the bridge 430 frequency changes with respect to the position of the vehicle on the bridge. The most relevant 431 thing about this plot is that for certain truck positions the bridge frequency is actually 432 predicted to increase. This is counterintuitive because one would expect the bridge frequency 433 to reduce slightly if a concentrated un-sprung mass was placed on the bridge deck. (This is 434 indeed what would happen and this is demonstrated later in Fig. 12). However, it appears that 435 when the moving mass is sprung, there are situations where the bridge frequency can actually 436 increase slightly. It is conceivable that the sprung mass (truck body) adds a kind of inertial 437 resistance to bridge's motion. In other words, the vehicle mass is providing some restraint to 438 the upper end of the truck suspension (spring), which is touching the bridge deck. This can be 439 interpreted as if the truck provides an extra spring support at the location the truck is located 440 at. Obviously, from a static point of view, the number of bridge supports remains unchanged. 441 For convenience in this paper we will term this apparent localised stiffening of the beam where the truck is parked an 'inertial spring support'. It can be seen in the upper solid line in 442 443 Fig. 7 that when the truck is at either of the 2 short side spans the addition of this inertial 444 spring support makes very little difference to the bridge frequency, indicating that it is adding 445 relatively little stiffness to the system. However, when the truck is on the longer central span, 446 the addition of an 'inertial spring support' does result in a significant increase in frequency.

447

448 Conceptualising the body of the vehicle as described above is helpful for initial visualisation 449 as it allows the bridge to be idealised in a conventional static structural arrangement. 450 However, in reality the vehicle-bridge system is a dynamic system so the behaviour is more 451 complex and insight on the behaviour is provided by [5]. Using a simple numerical model of 452 a sprung mass on a single span beam, they investigated how the system frequencies changed 453 as the sprung mass was positioned at different points on the beam. The results of [5] showed 454 frequency variation patterns similar to those shown in Fig. 7. Moreover, they found that the 455 increase and decrease in bridge and vehicle frequencies respectively was sensitive to the 456 frequency ratio (FR), where FR= vehicle frequency / bridge frequency. For systems where 457 the vehicle frequency was less than the bridge frequency (which is the situation here) and 458 when FR was close to one (e.g. 0.95), their model shows that large shifts in bridge and 459 vehicle frequencies would occur. However, when FR was not close to one (e.g. 0.5) the 460 frequency shifts predicted by the model were significantly smaller. The difference in the 461 magnitude of the frequency shift with respect to FR shows that it is not as simple as thinking 462 of the truck mass as a restraint. It appears that the closer the vehicle frequency is to the bridge 463 frequency the more pronounced this restraint is, which demonstrates the dynamic nature of 464 the restraint. It was also shown in [5] that the frequency shifts predicted by the model were 465 larger for higher mass ratios (MR) where MR=vehicle mass / bridge mass.



467

Fig. 7: Numerical frequency evolution of uncoupled system (dashed lines) and coupled
 system (solid lines). Vertical dotted lines indicate intermediate bridge supports.

471 Although the numerical model used to generate Fig. 7 is only an approximation of the real 472 bridge, it does clearly show that the frequency content during a vehicle passage is likely to 473 change. This variation in frequency with respect to vehicle position makes the problem non-474 stationary and the acceleration signals recorded during the passage of the vehicle should 475 reflect the non-stationary nature of the process, i.e. a change in frequency should be evident. 476 To examine if this frequency change is evident, the acceleration response from centre of span 477 3 (sensor C in Fig. 4) is analysed using the wavelet approach described in Section 2.3. 478 Fig. 8(a) shows the acceleration time series recorded at sensor C during a truck passing event. 479 For this crossing event the first axle of the truck enters the bridge at 6 s and the last axle exits 480 the bridge at 26 s. The truck entering and leaving the bridge is indicated in the figure by 481 dotted vertical lines. Thus, the signal between these two lines corresponds to forced vibration 482 data, whereas the acceleration after the truck leaves is the free vibration data. Fig. 8(b) shows 483 the conventional wavelet transform of the complete time series shown in Fig. 8(a) and 484 Fig. 8(c) shows the wavelet coefficients after calculating the envelope along scales and 485 normalizing by instantaneous energy (see Section 2.3). In Fig. 8(b) & (c) the truck entering 486 and leaving the bridge is again indicated using dotted vertical lines. Parts (b) and (c) of the 487 figure also have dashed horizontal lines at 3.5 Hz and 2.8 Hz. The dashed horizontal line at 488 3.5 Hz is the uncoupled bridge frequency and the dashed horizontal line at 2.8 Hz is believed 489 to be the approximate uncoupled vehicle frequency. In the absence of a modal test on the 490 vehicle, one cannot say definitively that 2.8 Hz is the vehicle frequency, but based on the 491 numerical model and the available experimental data the authors believe this is a reasonable 492 supposition. The conventional CWT result (Fig. 8(b)) shows only some high energy 493 concentration within the studied frequency range when the vehicle is traversing the middle 494 span. On the other hand, the processed wavelet coefficients (Fig. 8(c)) provide a better 495 picture of the relative energy distribution in the time-frequency plane. The frequency 496 evolution is not entirely clear in the CWT plot in Fig. 8(c). However, it is apparent that 497 during free vibration the bridge is vibrating only at its fundamental frequency (3.5 Hz) as all 498 the energy is concentrated there. On the other hand when the truck is on the bridge (forced 499 vibration) there is also a significant amount of energy near what the authors believe to be the 500 vehicle's first frequency (2.8 Hz). Furthermore, a trend seems to be evident in Fig. 8(c) 501 similar to the one predicted Fig. 7. During the period 12-20 s when the vehicle is crossing the 502 central span of the bridge the vehicle frequency seems to go down and the bridge frequency 503 seems to go up.



Fig. 8: Acceleration and frequency content for truck passage on Bridge A (a) Acceleration
signal; (b) Raw CWT result; (c) Processed CWT; Vertical lines = start/end of forced
vibration; Horizontal dashed lines = uncoupled system frequencies

Although Fig. 8 partially supports the theoretical construct presented in Fig. 7, it is difficult to draw any firm conclusions about the validity of the suggested explanations. This is because the frequencies presented in Fig. 7 are calculated for the vehicle model being situated at a

512 series of discrete locations on the beam. Unfortunately, the experimental data in this section 513 is for a moving truck and it could justifiably be argued that it is not correct to apply FDD to a 514 non-stationary process to extract the modal properties. Therefore, it is not possible to reliably 515 extract the modes of the coupled system while the vehicle is moving. This means that the 516 frequency peaks shown in Fig. 6 are likely to be a good approximation of the real frequencies 517 but will not be totally accurate. To overcome these issues a new experiment, where a truck is 518 parked at a series of discrete locations on a bridge, is undertaken and this work is reported in 519 the next section.

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- 522

521 **4. Experimental study of Bridge B and stationary truck**

- 523 As explained at the end of the previous section the experimental results from Bridge A cannot 524 really be used to check the validity of the concept presented in Fig. 7. In the previous 525 experiment the truck was moving, but in the numerical model the truck was parked at a series 526 of discrete locations. To resolve this issue a second experimental campaign was undertaken 527 where a truck was actually parked at a number of discrete locations on the bridge and the 528 results are described herein. To make sure that the bridge behaviour observed in Section 3 529 was not specifically related to Bridge A or the test truck shown in Fig. 3(b), in this next 530 experiment a different bridge and truck are used. It is important to note that when a vehicle is 531 parked on the bridge the system is coupled but stationary, i.e. the modal parameters will 532 remain constant. Therefore, using output-only modal analysis techniques such as FDD to 533 extract the modal properties is appropriate.
- 534

535 4.1 Bridge and instrumentation description

536

537 A photo of the bridge used in this experiment is shown in Fig. 9(a) and a plan view in 538 Fig. 10(a). The bridge is a half through steel girder bridge, it spans 36 m and the deck is 539 simply supported. The 7.6 m wide, and 200 mm deep concrete deck is supported on a series 540 of 450 mm deep steel beams, which span transversely between the main girders which are 541 approximately 2 m deep. As explained in Section 3.1, for experiments of this type, a high 542 vehicle-bridge mass ratio is desirable, so a light bridge deck is advantageous. The reason for 543 choosing this bridge is that the deck is light compared to other bridges of the same span, i.e. 544 the primary members are steel and the deck is relatively narrow. Again with the objective of 545 having a high (vehicle-bridge) mass ratio, the truck selected for this test had a total weight of 546 32 tonnes, which is heavier than the 26 tonnes truck used in the previous test. The test truck 547 used has four axles and is shown in Fig. 9(b). While the bridge was chosen for its technical 548 advantages described above, logistically the disadvantage of the bridge was that it was in an 549 urban area and frequently trafficked, which made finding a quiet time to carry out the test 550 challenging.

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552

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Fig. 9: (a) Bridge B elevation; (b) Test truck

554 The instrumentation used in this experiment consisted of four accelerometers attached to the 555 main girders. The position of the four accelerometers (A-D) is shown in Fig. 10(a). The 556 accelerometers used in this test were Honeywell QA750 force balance accelerometers and the 557 scanning frequency used was 128 Hz. Fig. 10(b) shows accelerometer B attached to the 558 underside of the top flange of the main girder via a magnet. The vehicle was parked for short 559 durations at ¹/₄-span, mid-span and ³/₄-span. A full bridge closure was not permitted so the test 560 was carried out early in the morning when there was little traffic. Ideally, the truck would 561 stay parked at a given location for as long as possible, because the longer the time series the 562 more accurate the subsequent modal analysis is likely to be. However, the fact there was no 563 bridge closure meant that the stops had to be kept relatively short. Only stop durations of 10-564 12 s were feasible. However, signals of this length are sufficiently long to allow the modal 565 properties to be determined accurately.



Fig. 10: Dimensions and instrumentation details for Bridge B (a) Plan view of bridge deck
and sensor locations; (b) Accelerometer attached to underside of the girder top flange.



570 **4.2 Evolution of Vehicle-Bridge system**

571

572 Analysing the ambient vibration data, the fundamental (first bending) frequency of the bridge 573 was identified as 3.13 Hz. Fig. 11(a) shows the time series recorded at accelerometer B for a 574 full set of truck movements, namely; truck coming on to the bridge, parking at ¹/₄-span, 575 moving on and parking at mid-span, then finally moving to ³/₄-span and parking briefly before 576 exiting the bridge. The different portions of the signal are demarcated using vertical dotted 577 lines and the parts of the signal corresponding to the truck being parked at particular locations 578 on the bridge can be identified using the annotations on the bottom of the figure. The 579 annotations on the top of the figure have been added to allow the reader visualise what the 580 truck is doing for each section of the signal. For the first 25 seconds the bridge is in ambient 581 vibration (A). Then the truck moves (TM) on to the bridge arriving at the 1/4-span at 582 approximately 35 s. On arrival at ¹/₄-span the truck stops and remains there for approximately 583 12 seconds and this section of the signal is termed 'loaded free vibration (LF)'. TM and LF 584 are repeated in sequence so that the truck can be parked for a short duration at mid-span and 585 ³/₄-span. When the truck leaves the bridge, the bridge is in free vibration (F). For the data 586 presented in Fig. 11 the only vehicle on the bridge was the test truck, i.e. there was no other 587 traffic crossing the bridge. Much of the bridge vibration evident in the figure is believed to be 588 due to the energy input into the bridge during the four truck movements.

589

590 To observe how the bridge frequency evolves over the course of the truck movements, the 591 time series in Fig. 11(a) is analysed using CWT, and the results are presented in Fig. 11(b). 592 Again, the vertical dotted lines demarcate the different parts of the signal (i.e. the lines 593 correspond to those shown in part (a) of the figure) and it can be seen that during ambient 594 vibration at the start of the signal the bridge vibrates predominantly at its unloaded 595 fundamental frequency (3.13 Hz) with no significant energy at any other frequency. The 596 same is true for the free vibration at the end of the signal. During the four truck movement 597 phases (TM) there is no clear pattern of the energy distribution in the time-frequency domain. 598 However, during the loaded free vibration events (LF), the energy is concentrated along clear 599 frequency bands. For example, when the truck is parked at mid-span (65-81 s) the energy is 600 concentrated in two distinct bands at approximately 2.5 Hz and 3.5 Hz. Similarly, when the truck is at the $\frac{3}{4}$ -point (95-105 s) it can be seen that there is significant energy at these bands 601 602 with almost no energy at the fundamental frequency, indicated by the horizontal dashed line 603 in the figure.



Fig. 11: Experimental data from Bridge B; (a) acceleration signal recorded at mid-span
during a series of truck movements; (b) CWT of acceleration signal; Vertical lines = start/end
of forcing regime; Horizontal dashed lines = bridge's fundamental frequencies

609 While the CWT plot shown in Fig. 11(b) is useful to visualise the frequency shift for the 610 different truck positions, its frequency resolution is limited. To identify the frequencies more 611 accurately the LF portions of the signal when the truck is at ¹/₄-span, mid-span and ³/₄-span are 612 analysed using FDD and the identified frequencies are plotted as circular data markers at 613 25%, 50% and 75% of L respectively, in Fig. 12. The experimental results indicate that the 614 bridge and vehicle frequencies increase and decrease respectively when the truck is on the 615 bridge with the largest changes occurring when the truck is in the centre of the bridge. The 616 upper and lower (solid) lines in Fig. 12 respectively show the bridge and vehicle frequencies predicted by the numerical model described in Section 2.1, for a simply supported single span 617 618 beam. In line with the modelling philosophy described in Section 3.3, the bridge properties in 619 the model were revised so that the uncoupled bridge frequency in the model matches the 620 experimentally observed fundamental bridge frequency (3.13 Hz). A similar approach is also 621 used to revise vehicle properties. Based on the extracted values in Fig. 12 an uncoupled 622 vehicle frequency in the region of 2.6 Hz seems sensible. Therefore the suspension property 623 of the vehicle model (i.e. the spring stiffness) has been amended such that for a sprung mass 624 of 32,000 kg the uncoupled vehicle frequency is 2.6 Hz. As the numerical model is a 625 relatively simple, the frequencies predicted by the model do not exactly match the 626 frequencies observed experimentally. However, the comparison highlights that the trends are 627 the same. This is important because it demonstrates that the evolution of the system frequencies (bridge and vehicle) predicted by the model are credible. Moreover, it shows that 628 629 the hypothesis put forward in Section 2.3 to explain the behaviour observed in Bridge A is 630 also credible.





Fig. 12: Frequency evolution during vehicle passage. Solid line = Coupled system; Dashed
line = uncoupled system; Dotted line = Moving mass case; Red dots = experimental values

Finally, the dotted plot in Fig. 12 shows the bridge frequency predicted by the numerical model if an un-sprung mass of 32,000 kg is placed at a series of discrete locations along the length of the bridge. The model predicts that for an un-sprung mass the bridge frequency will be reduced, with the largest reduction occurring when the mass is at the centre of the bridge. This reduction in frequency with the addition of mass is in line with what one might intuitively expect for a (sprung) truck but this is clearly not what actually occurs.

642

643 **4.3 Modes of vibration**

644

645 So far previous sections have focused on studying how different truck positions affect the 646 frequencies of the vehicle-bridge system. In this section, changes in the associated mode 647 shapes of the vehicle-bridge system are reported. To make sense of the theoretical frequency 648 predictions presented in Fig. 7 the reader was prompted to visualise the body mass of the 649 vehicle as supported on two springs, the upper spring representing the vehicle suspension and 650 lower spring representing the bridge stiffness. While this is a useful analogy to visualise what 651 is happening it is technically incorrect because the lower spring is in fact a beam. The 652 significance of this is that when the sprung mass is on the bridge, the frequency that we have 653 been referring to up to now as the vehicle frequency will have a mode associated with it that 654 includes the deformed shape of the beam.

Up to now this paper has talked about 'vehicle' frequency and 'bridge' frequency because 656 657 based on conventional thinking it is the most straightforward way to explain the experimental 658 results that have been reported so far. However, to understand the modes associated with the 659 observed frequencies it is important to appreciate that as soon as the vehicle is on the bridge, 660 the vehicle and the bridge behave as one system, not two independent systems. Therefore, 661 technically it is not appropriate to talk about vehicle and bridge modes, it would be more 662 correct to talk about the coupled system's first and second mode. However, for simplicity and 663 convention, when presenting the relevant modes below they will still be referred to as 664 'vehicle mode' and 'bridge mode' even though it is not totally correct.

665

666 The easiest way to appreciate the mode of vibration of the coupled system is to examine the 667 modes predicted by the numerical model. In particular, Fig. 13 shows the modes of vibration 668 for three different vehicle locations; (i) over the left support, (ii) ¹/₄-span and (iii) mid-span. 669 The eigenvalue analysis of the coupled system is carried out and modal ordinates of the 670 degrees of freedom of the vehicle and bridge can easily be computed. When the vehicle is at 671 the bridge's left support, both systems are effectively uncoupled and the familiar 672 (independent) modes for the vehicle (Fig. 13(a)) and bridge (Fig. 13(b)) are observed. In particular note how the bridge part of the 'vehicle mode' (Fig. 13(a)) remains straight. 673 674 However, when the vehicle is at ¹/₄-span the bridge clearly plays a role in the 'vehicle mode' 675 as the bridge is now in a curved shape (see Fig. 13(c)). Interestingly when the vehicle is at ¹/₄-676 span the deformed shape of the bridge is approximately similar for both the 'vehicle mode' 677 (Fig. 13(c)), and the 'bridge mode' (Fig. 13(d)). A similar pattern is observed when the 678 vehicle is at mid-span Figs. 13 (e) and (f).



680

Fig. 13: Numerical mode evolution for coupled system

It should be noted that the modes of vibration plotted in Fig. 13 are schematic in nature. Their 682 683 primary purpose is to demonstrate that when the vehicle is on the bridge the system is coupled. The resulting modes can be more usefully thought of as the system's 1st and 2nd 684 modes. To examine in more detail how the bridge part of the full system modes of vibration 685 vary with truck position, just the bridge part of the 1st and 2nd system modes are plotted in 686 687 Fig. 14. Since no acceleration was measured on the vehicle, only the bridge part of the mode 688 can be examined in detail. Parts (a), (b) and (c), (d) of Fig. 14 are generated using the 689 numeric model and experimental data respectively. The bridge part of the system 1st mode 690 ('vehicle mode') predicted by the numerical model for three different truck positions (1/4-691 span, mid-span, and ³/₄-span) are plotted in Fig. 14(a). In the figure it can be seen that the 692 bridge part of the 'vehicle mode' has three distinct shapes for the three different truck 693 locations considered. When the truck is at ¹/₄-span the bridge part of the mode is slightly skewed to the left, for the ³/₄-span position it is skewed to the right and when the vehicle is at mid-span it is symmetric. Fig. 14(c) shows the equivalent modal ordinates obtained experimentally and for the three test points. Admittedly as the experiment only provides three modal ordinates it is not possible to make definitive comment on whether the mode shapes are skewed or not. However, for the three modal ordinates available, we can observe that they are behaving in a manner consistent with the equivalent location of the theoretical mode shapes shown in Fig. 14(a).

701

Fig. 14(b) shows the bridge part of the system 2nd mode ('bridge mode') predicted by the 702 703 numerical model for three different truck positions. It can be seen in the figure that the bridge 704 part of the system 2nd mode does not change significantly with vehicle position but there is some small variation. Essentially, the numerical model indicates that the bridge part of the 705 706 mode is slightly skewed to the opposite side of where the vehicle is located. The equivalent 707 experimental modal ordinates are plotted in Fig. 14(d). Similar to Fig. 14(c), in Fig. 14(d) 708 only three modal ordinates are available and therefore there is insufficient evidence to 709 determine if the subtle skewing of modes evident in Fig. 14(b) is also present experimentally. 710 However, it can be said that the magnitude of the modal ordinates at a given location are 711 quite similar for all three truck positions. This is consistent with the theoretical modes 712 presented in Fig. 14(b) which as mentioned previously appear relatively insensitive to vehicle 713 position. Note that all the plots in Fig. 14 have been normalized to have a minimum value of -714 1 at mid-span for ease of comparison.



Fig. 14: Bridge part of system 1st and 2nd modes for different truck positions (a) 1st mode
calculated theoretically, (b) 2nd mode calculated theoretically, (c) 1st mode measured
experimentally, (d) 2nd mode measured experimentally.

720 **5. Conclusions**

721

This paper investigated the changes in frequencies and modes of vibration of a vehicle-bridge system. Two different bridges A and B were studied. Initial experimental results observed on bridge A included some unexpected behaviour. In particular when the truck was on the bridge the fundamental bridge frequency seemed to increase and a frequency peak not present in free vibration appeared on the spectrum. This prompted the development of a numerical model to try and provide a theoretical explanation for the observed behaviour. The model provided a theoretical framework which seemed to explain the observed behaviour. However, to further investigate the phenomena a second experiment was carried out where the truck parked at a series of discrete locations on the bridge. This experiment was carried out on Bridge B and, by using time-frequency analysis and output-only modal analysis, the unexpected behaviour was further clarified.

733

734 Furthermore, in the course of the investigation a number of interesting observations were 735 made. For example, a coupled vehicle-bridge system might feature significant changes in 736 natural frequencies depending on the vehicle's position. Also when analysing forced 737 vibration signals the presence of additional frequencies on the spectrum proves system 738 coupling. Moreover, it is shown numerically and experimentally, that the modes of vibration 739 of the coupled system do change with the location of the vehicle. However, the amount of 740 change differs for the 'vehicle' and the 'bridge' modes. In particular, it is shown that when 741 the vehicle is on the bridge the 'vehicle' mode has a significant 'bridge part' associated with 742 it and the shape of this part is very similar to the bridge's fundamental mode of vibration.

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Numeric models indicate the magnitude of the changes in modal parameters will be more pronounced for situations with high vehicle-bridge mass ratios. However, this paper shows that it is a reality for conventional heavy vehicles and relatively light standard bridges.

747

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757 References

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[1] E.B. Magrab, Vibrations of Elastic Systems – With Applications to MEMS and NEMS,
Springer, Dordrecht, 2012.

762	[2] L. Frýba, Vibration of Solids and Structures under Moving Loads, third ed., Thomas
763	Telford, Prague, 1999.
764	
765	[3] Y.B. Yang, M.C. Cheng, K.C. Chang, Frequency variation in vehicle-bridge interaction
766	systems, Int. J. Struct. Stab. Dyn. 13 (2013) 1-22.
767	
768	[4] J. Li; M. Su; and L. Fan, Natural frequency of railway girder bridges under vehicle loads,
769	J. Bridge Eng. 8 (2003) 199-203.
770	
771	[5] D. Cantero, E.J. OBrien, The non-stationarity of apparent bridge natural frequencies
772	during vehicle crossing events, FME Trans. 41 (2013) 279-284.
773	
774	[6] K.C. Chang, C.W. Kim, S. Borjigin, Variability in bridge frequency induced by a parked
775	vehicle, Smart Struct. Syst. 13 (2014) 755-773.
776	
777	[7] M.D. Spiridonakos, S.D. Fassois, Parametric identification of time-varying structure
778	based on vector vibration response measurements, Mech. Syst. Sig. Process. 23 (2009) 2029-
779	2048.
780	
781	[8] C.Y. Kim, D.S. Jung, N.S. Kim, S.D. Kwon, M.Q. Feng, Effect of vehicle weight on
782	natural frequencies of bridges measured from traffic-induced vibration. Earthquake Eng. Eng.
783	Vibr. 2 (2003) 109-115.
784	
785	[9] F. Xiao, G.S. Chen, J.Leroy Hulsey, W. Zatar, Characterization of non-stationary
786	properties of vehicle-bridge response for structural health monitoring, Adv. Mech. Eng. 9
787	(2017) 1-6.
788	
789	[10] C.C. Caprani, E. Ahmadi, Formulation of human-structure interaction system models for
790	vertical vibration, J. Sound Vib. 377 (2016) 346-367.
791	
792	[11] X. Kong, C.S. Cai, B. Kong, Numerically extracting bridge modal properties from
793	dynamic responses of moving vehicles, ASCE J. Eng. Mech. 142 (2016) 1-12
794	

795	[12] E.J. OBrien, A. Malekjafarian. A mode shape-based damage detection approach using
796	laser measurement from a vehicle crossing a simply supported bridge, Struct. Control Health
797	Monit. 23 (2016) 1273-1286.
798	
799	[13] D.M. Siringoringo, Y. Fujino, Estimating bridge fundamental frequency from vibration
800	response of instrumented passing vehicle: analytical and experimental study, Adv. Struct.
801	Eng. 15 (2012) 417-433.
802	
803	[14] Matlab, MathWorks, 2013.
804	
805	[15] O.C. Zienkiewicz, R.L. Taylor, The Finite Element Method - Volume 1: The Basis, fifth
806	ed., Butterworth Heinemann, Oxford, 2000.
807	
808	[16] P. Lou. Finite element analysis of train-track-bridge interaction system, Arch. Appl.
809	Mech. 77 (2007) 707-728.
810	
811	[17] Y.B. Yang, J.D. Yau, Y.S. Wu, Vehicle-Bridge Interaction Dynamics – With
812	Applications to High-Speed Railways, World Scientific, Singapore, 2004.
813	
814	[18] L. Gyenes, C.G.B. Mitchell, S.D. Phillips, Dynamic pavement loads and tests of road-
815	friendliness for heavy vehicle suspensions, in: D. Cebon, C.G.B. Mitchell (Eds.), Heavy
816	Vehicles and Roads: Technology, Safety and Policy, Thomas Telford, London, 1992, pp.
817	243-251.
818	
819	[19] D.J. Ewins, Modal Testing. Theory, Practice and Application. second ed., Research
820	Studies Press LTD, Baldock, 2000.
821	
822	[20] C. Rainieri, G. Fabbrocino, Operational Modal Analysis of Civil Engineering Structures.
823	An Introduction and Guide for Applications, Springer, New York, 2014.
824	
825	[21] A. Cohen, R.D. Ryan, Wavelets and Multiscale Signal Processing, Chapman & Hall,
826	London, 1995.
827	

828	[22] S.G. Mallat, A Wavelet Tour of Signal Processing, second ed., Academic Press, London,
829	1999.
830	
831	[23] D. Cantero, M. Ülker-Kaustell, R. Karoumi. Time-frequency analysis of railway bridge
832	response in forced vibration, Mech. Syst. Sig. Process. 76-77 (2016) 518-530.
833	
834	[24] Y.B. Yang, C.W. Lin, J.D. Yau, Extracting bridge frequencies from the dynamic
835	response of passing vehicle. J. Sound Vib. 2072 (2004) 471-493.
836	
837	[25] R. Cantieni, Investigation of vehicle-bridge interaction for highway bridges. In: Heavy
838	Vehicles and Roads: Technology, Safety and Policy, Thomas Telford, London, 1992.
839	
840	[26] H. Ludescher, E. Brühwiler, Dynamic amplification of traffic loads on road bridges,
841	Struct. Eng. Int. 19 (2009) 190-197.