Feedback stabilization of Networked Control Systems

Feedback Stabilization of Networked Control Systems

George Bitsoris, Nikolaos Athanasopoulos and Leonidas Dritsas

Abstract— **In this paper the stability analysis and control synthesis problems for Networked Control Systems (NCS) with bounded transmission delays (constant and unknown or time-varying) are investigated. First, stability conditions for NCS described by ARMA models are established and a method for the determination of admissible delay range is developed. Then, a linear programming method for the design of linear state-feedback controllers guaranteeing the stability of the system for any delay belonging to a prespecified range is developed. Contrary to the usual approaches based on the use of quadratic Lyapunov functions, a polyhedral Lyapunov approach is adopted for both analysis and synthesis. A control synthesis numerical example is given to illustrate the reduction of conservatism of the tolerable delay range when compared to former results .**

I. INTRODUCTION

It is well known that the major control challenge in analysis and synthesis of Networked Controlled System (NCS) is to face the problems due to the presence of uncertain network-induced delays stemming from the very fact of utilizing a common communication channel for closing the loop [1]. These delays stem from the information flow between: a) the sensor and the controller, and b) the controller and the actuator. They have in general different characteristics depending primarily on the utilized network protocol, the scheduling methods and the communication overhead (packet collisions/retransmissions/losses) used in NCS, while their presence imposes strict limitations on the achievable feedback performance [1], [2], [3], [4]. For the case of a discrete static feedback implemented with a period h, these delays can be lumped into a single term τ^k, where k refers to the sampling instant kh [2].

Significant effort has recently been invested in developing control methodologies to handle the network delay effect in NCSs (see surveys [1], [5]). In [3], LMIs are used for robust stability analysis and controller synthesis for networked systems subject to uncertain time-varying delays upper bounded by a sampling period; the case of NCS with delays longer than one sampling period is presented in [4], [6]. By treating the uncertain NCS delay as a time-varying parameter uncertainty, sufficient conditions, expressed as LMIs, for the existence of a static stabilizing state feedback controller appear in [7], [8], [9]. Switched system approaches that explicitly take into account both the delay uncertainty and the controller "asynchronicity" have recently developed [10], [11].

Most of these control approaches use as analysis and synthesis tools either Lyapunov-Krasovskii functionals or quadratic Lyapunov functions. In the best of the authors' knowledge, the use of polyhedral Lyapunov functions has not yet been investigated in the context of NCS, although it has been shown to be a powerful tool in many interesting control problems (including the case of robust and constrained control) where it yields generic and less conservative results compared to quadratic Lyapunov approaches [12], [13], [14], [15]. In this paper NCSs are described by ARMA models. Thus the results concerning the positive invariance of polyhedral sets for ARMA models, established in [21] are used.

The paper is organized as follows: Section II refers to the modelling aspects of NCS with varying transmission delays. In Section III, stability conditions for NCS systems described by ARMA models are established. These conditions lead to the development of a method for the determination of the admissible delay range for a systems controlled by linear state-feedback. In Section IV the design problem of linear state-feedback controllers guaranteeing the stability of the system for any delay belonging to a prespecified range is investigated. An illustrative control synthesis numerical example is given in Section V.

II. NCS-DYNAMICS

Throughout this paper, capital letters denote real matrices and lower case letters denote column vectors or scalars. For two real vectors $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$ and $y = \begin{bmatrix} y_1 & y_2 & \dots & y_n \end{bmatrix}^T$ $x < y \quad (x \leq y)$ is equivalent to $x_i < y_i \quad (x_i \leq y_i)$ for $i = 1, 2, ..., n$. Similar notation holds for matrices. Given a real matrix $H = (h_{ij})$, $|H|$ denotes the matrix $|H| = (|h_{ij}|)$. Finally, $\mathbb{O}_n$ denotes the $n \times n$ null matrix.

The dynamics of the NCS under investigation is described by the combination of a continuous–time linear time–invariant plant with a discrete–time controller and its configuration is shown in Figure 1. This configuration corresponds to the case of a remote controller, non-collocated with the sensor and actuator [16], [17], [18].

The sampling period h is assumed to be constant and known, whereas both controller and actuator (including the zero-order-hold – ZOH) are event-driven devices in the sense that they update their outputs as soon as they receive a new sample. The state vector x is sampled periodically, transmitted through the network, fed to the discrete–time controller which computes the control action and transmits

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it to the actuator after an uncertain delay. The plant receives this command after an uncertain delay τ^k. Inhere, the case

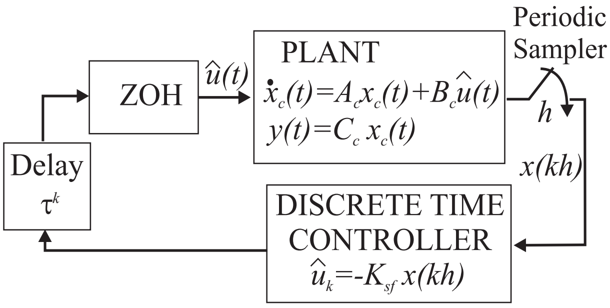

Fig. 1. NCS structural framework

of SISO systems with less than one sampling period delay, ($\tau^k < h$), is examined. For the control architecture shown in Figure 1, the system dynamics is described below, for $t \in \left[kh + \tau^k, kh + h + \tau^{k+1}\right)$:

$$\dot{x}(t) = A_c x(t) + B_c \hat{u}(t), \quad y(t) = C_c x(t), \tag{1}$$

$$\hat{u}(t) = \begin{cases} u(k-1), & t \in \left[kh - h + \tau^{k-1},\ kh + \tau^k\right) \\ u(k), & t \in \left[kh + \tau^k,\ kh + h + \tau^{k+1}\right) \end{cases}. \tag{2}$$

The total delay within the *k*th sampling period, that is the time from the instant when the sampling node samples sensor data from the plant to the instant when actuators exert a control action (whose computation was based on this sample) to the plant is denoted by $\tau^k = \tau^k_{sc} + \tau^k_{ca}$. Moreover this total delay is assumed upper bounded as $0 \le \tau_{\min} < \tau^k \le \tau_{\max} = h$ and in general it is a time–varying and uncertain quantity, reflecting the nature of the network involved, the network load, etc. In (2), $\hat{u}(t)$ is the "most recent" control action presented to the event–driven actuator at the time instance t within a sampling period (i.e. within the time interval $[kh,\ kh+h)$), and can take either one of the two values $u(k-1)$ or $u(k)$. Certain part of the material in this section can be traced in recent publications [16], [17], [18] hence the presentation will be brief.

The important modeling issue arising from (2) is that the actuation time instances are not equidistant because the piecewise constant control action $\hat{u}(t)$ experiences a "jump" at the uncertain time instance $kh+\tau^k$ when the control action coming out of the event–driven ZOH device is updated from value $u(k-1)$ into $u(k)$. Hence, unless τ^k is constant, it is not in general possible to treat the ensuing NCS in a standard sampled-data or "time–delayed" setting and a "hybrid" setup should be used [1], [16],[19], [20].

Despite the "jump" nature of $\hat{u}(t)$, the discretization of (2) between consecutive sampling instances is straightforward and the ensuing exact discretization is given by [17], [18]:

$$x(k+1) = \Phi x(k) + \Gamma_0(\tau^k)\hat{u}(k) + \Gamma_1(\tau^k)u(k-1) \tag{3}$$

where $\Phi = \exp(A_c h)$ and

$$\begin{aligned} \Gamma_0(\tau^k) &= \int_0^{h-\tau^k} \exp(A_c\lambda)B_c d\lambda\ , \\ \Gamma_1(\tau^k) &= -\Gamma_0(\tau^k) + \int_0^{h} \exp(A_c\lambda)B_c d\lambda \end{aligned} \tag{4}$$

The uncertain delay can always be decomposed as $\tau^k = \tau^\circ + \tau^k_\Delta$ with τ° denoting the selected nominal value, $\tau^o \in [\tau_{min}, \tau_{max}]$. In this paper the nominal value τ° of the uncertain delay is chosen to be $\tau^\circ = \tau_{\min}$. System variables with $(^\circ)$ as superscript will denote the corresponding nominal value.

The matrices $\Gamma_0(\tau^k),\ \Gamma_1(\tau^k)$ can then be decomposed into constant and known nominal parts $\Gamma_0(\tau^\circ),\ \Gamma_1(\tau^\circ)$ and uncertain though bounded parts $\Delta\Gamma_0,\ \Delta\Gamma_1$, that is

$$\Gamma_i(\tau^k) \triangleq \Gamma_i(\tau^\circ) + \Delta\Gamma_i(\tau^k, \tau^\circ) \qquad i = 0, 1$$

where

$$\Gamma_0(\tau^\circ) = \int_0^{h-\tau^\circ} \exp(A_c\lambda)B_c d\lambda$$

$$\Gamma_1(\tau^\circ) = \int_{h-\tau^\circ}^{h} \exp(A_c\lambda)B_c d\lambda$$

$$\Delta\Gamma_1(\tau^k, \tau^\circ) = \int_{h-\tau^k}^{h-\tau^\circ} \exp(A_c\tau)B_c d\tau = -\Delta\Gamma_0(\tau^k, \tau^\circ) \tag{5}$$

System (3) can thus be equivalently written in the form

$$\begin{aligned} x(k+1) \quad &= \quad \Phi x(k) + (\Gamma_0(\tau^0) + \Delta\Gamma_0(\tau^k, \tau^\circ))u(k) + \\ &+(\Gamma_1(\tau^0) + \Delta\Gamma_1(\tau^k, \tau^\circ))u(k-1) \end{aligned} \tag{6}$$

Using a discrete–time linear state feedback law $u(k) = K_{sf}x(k)$, the closed–loop dynamics becomes

$$x(k+1) = \left[\Phi + \Gamma_0(\tau^k)K_{sf}\right]x(k) + \left[\Gamma_1(\tau^k)K_{sf}\right]x(k-1)\ . \tag{7}$$

III. Stability analysis of NCS

System (3) can be written in the form

$$A^*(q^{-1})x(k) = 0 \tag{8}$$

where q^{-1} is the backward shift operator and $A^*(q^{-1})$ is a real polynomial matrix of the form

$$A^*(q^{-1}) = I_n + A^*_1 q^{-1} + A^*_2 q^{-2}$$

The stability of this of class systems via polyhedral Lyapynov functions has been investigated in [21]. The authors of this paper have established necessary and sufficient conditions for a scalar function

$$v(x) \triangleq \max_{1\le i\le n}\left\{\frac{|(Gx)_i|}{w_i}\right\}$$

to be a Lyapunov function for system (8). These conditions are stated in the following theorem:

Theorem 1:[21] If there exist matrices $G \in \mathbb{R}^{p\times p}, p \geq n$, $rankG = n$, $H_0 \in \mathbb{R}^{p\times p}, H_1 \in \mathbb{R}^{p\times n}$, a vector $w \in R^p$ with positive components and a $\varepsilon > 0$ such that

$$GA_i^* = H_i^* G, \quad i = 1, 2$$

$$(|H_1^*| + |H_2^*|)w \leq \varepsilon w$$

$$\varepsilon < 1$$

then the equilibrium $x = 0$ of system (8) is asymptotically stable.

By applying this result to system (3) we establish conditions guaranteeing the asymptotic stability of NCS with fixed delay τ^k:

Theorem 2: If there exist matrices $G \in \mathbb{R}^{p\times n}, p \geq n$, $rankG = n$, $H^0(\tau^k) \in \mathbb{R}^{p\times p}, H^1(\tau^k) \in \mathbb{R}^{p\times p}$, a vector $w \in R^p$ with positive components and a $\varepsilon(\tau^k) > 0$ such that

$$G[\Phi + \Gamma_0(\tau^k)K_{sf}] = H^0(\tau^k)G \quad (9)$$

$$G\Gamma_1(\tau^k)K_{sf} = H^1(\tau^k)G \quad (10)$$

$$(|H_0(\tau^k)| + |H_1(\tau^k)|)w \leq \varepsilon(\tau^k)w \quad (11)$$

$$\varepsilon(\tau^k) < 1 \quad (12)$$

then the equilibrium $x = 0$ of system (3) is asymptotically stable.

It can be easily seen that if for a fixed matrix $G \in \mathbb{R}^{p\times n}, p \geq n$, $rankG = n$ and a vector $w \in R^p$ with positive components conditions (9)-(12) of Theorem 2 are satisfied for all τ^k belonging to a delay range $[\tau_{min}, \tau_{max}]$, the equilibrium $x = 0$ of system (3) is asymptotically stable for any time varying delay $\tau^k \in [\tau_{min}, \tau_{max}]$. Conditions (9)-(12) in Theorem 2 can be used to determine the range of admissible delay time for which the stability of the closed-loop NCS is guaranteed. This is illustrated in the following example.

We consider the open-loop stable continuous time linear system (1) with

$$A_c = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The sampling period is $h = 1.333$ sec while the uncertain input delay can vary between zero and one full sampling period, i.e. $\tau^k \in [0, h)$. We assume a state-feedback gain matrix K_{sf} of the form

$$K_{sf} = \begin{bmatrix} 0 & K_{of} \end{bmatrix}$$

which in fact corresponds to an output feedback control $u(k) = K_{of}y(k)$. In Fig. 2, the stability margins are drawn for two different choices of matrix G: The smaller margins are computed when the nonsingular matrix $G \in \mathbb{R}^{2\times 2}$ and the positive vector w are chosen randomly, while the larger delay bounds for which stability is preserved are computed by setting $w = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ and selecting matrix G as follows: For each value of K_{of}, G is composed of the left eigenvectors of matrix $(\Phi + B_d K_{sf})$ where $B_d = \int_0^h \exp(A_c\lambda)B_c d\lambda$. Comparing to [18], where the stability of the same NCS has been studied, it can be clearly seen that using Theorem 2 larger delay bounds are computed even when matrix G is chosen randomly.

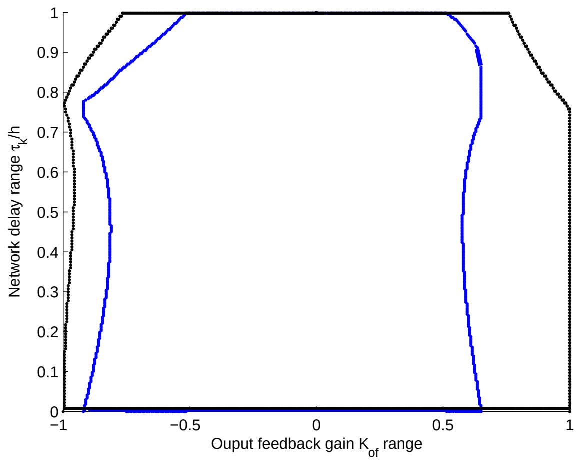

Fig. 2. Stability margins for the closed-loop system when output feedback gain varies from -1 to 1.

IV. A Design approach for fixed and uncertain delays

The design problem is formulated as follows: Given the continuous-time system (1), the bounds $\tau_{\min}$ and $\tau_{\max}$ of the uncertain input delay τ^k and a sampling period h, determine a state-feedback control law $u(k) = K_{sf}x(k)$ such that the resulting closed-loop NCS is asymptotically stable for any time varying delay $\tau^k \in [\tau_{\min}, \tau_{\max}]$.

In order to establish an approach to this problem we consider the perturbed description of the NCS:

$$\begin{aligned} x(k+1) &= \Phi + \Gamma_0(\tau^0)K_{sf} + \Delta\Gamma_0(\tau^k, \tau^\circ)K_{sf})x(k) + \\ &\quad +(\Gamma_1(\tau^0) + \Delta\Gamma_1(\tau^k, \tau^\circ)K_{sf})x(k-1) \quad (13) \end{aligned}$$

Theorem 3: If for a nonsingular matrix $G \in \mathbb{R}^{n\times n}$,there exist $n \times n$ matrices , H^0, H^1, $\Delta H(\tau^k), \overline{H}_0^+(\tau^k), \overline{H}_0^-(\tau^k), \overline{H}_1^+(\tau^k), \overline{H}_1^-(\tau^k)$, a vector $w \in R^n$ with positive components and a positive scalar $\varepsilon < 1$ such that

$$G[\Phi + \Gamma_0(\tau^0)K_{sf}] = H^0 G \quad (14)$$

$$G\Gamma_1(\tau^0)K_{sf} = H^1 G \quad (15)$$

$$G\Delta\Gamma_1(\tau^k)K_{sf} = \Delta H(\tau^k)G \quad (16)$$

$$H^0 - \Delta H(\tau^k) = \overline{H}_0^+(\tau^k) - \overline{H}_0^-(\tau^k) \quad (17)$$

$$H^1 + \Delta H(\tau^k) = \overline{H}_1^+(\tau^k) - \overline{H}_1^-(\tau^k) \quad (18)$$

$$(\overline{H}_0^+(\tau^k)+\overline{H}_0^-(\tau^k)+\overline{H}_1^+(\tau^k)+\overline{H}_1^-(\tau^k))w\le\varepsilon w \quad (19)$$

$$\overline{H}_0^+(\tau^k)\ge\mathbb{O}_n, \overline{H}_0^-(\tau^k)\ge\mathbb{O}_n$$

$$\overline{H}_1^+(\tau^k)\ge\mathbb{O}_n, \overline{H}_1^-(\tau^k)\ge\mathbb{O}_n$$

for all $\tau^k\in[\tau_{\min},\tau_{\max}]$ then the equilibrium $x=0$ of system (13) is asymptotically stable for any time varying delay time $\tau^k\in[\tau_{\min},\tau_{\max}]$.

Proof: Taking into account that $\Delta\Gamma_0(\tau^k)=-\Delta\Gamma_1(\tau^k)$, from (14)- (19) it follows that

$$G[\Phi+\Gamma_0(\tau^k)K_{sf}]=G[\Phi+\Gamma_0(\tau^0)K_{sf}+\Delta\Gamma_0(\tau^k)K_{sf}]=$$

$$=(H^0-\Delta H(\tau^k))G$$

$$G[\Gamma_1(\tau^k)K_{sf}]=G[\Gamma_1(\tau^0)K_{sf}+\Delta\Gamma_1(\tau^k)K_{sf}]=$$

$$=(H^1+\Delta H(\tau^k))G$$

$$(|H^0-\Delta H(\tau^k)|+|H^1+\Delta H(\tau^k)|)w=$$

$$(|\overline{H}_0^+(\tau^k)-\overline{H}_0^-(\tau^k)|+|\overline{H}_1^+(\tau^k)-\overline{H}_1^-(\tau^k)|)w\le$$

$$\le(|\overline{H}_0^+(\tau^k)|+|\overline{H}_0^-(\tau^k)|+|\overline{H}_1^+(\tau^k)|+|\overline{H}_1^-(\tau^k)|w=$$

$$=(\overline{H}_0^+(\tau^k)+\overline{H}_0^-(\tau^k)+\overline{H}_1^+(\tau^k)+\overline{H}_1^-(\tau^k))w\le\varepsilon w$$

because $\overline{H}_0^+(\tau^k)\ge\mathbb{O}_n$, $\overline{H}_0^-(\tau^k)\ge\mathbb{O}_n$, $\overline{H}_1^+(\tau^k)\ge\mathbb{O}_n$, $\overline{H}_1^-(\tau^k)\ge\mathbb{O}_n$. Thus , setting

$$H_0(\tau^k)=H^0-\Delta H(\tau^k)$$

$$H_1(\tau^k)=H^1+\Delta H(\tau^k)$$

we conclude that all hypotheses of Theorem 2 are satisfied. Therefore the equilibrium $x=0$ of system (13) is asymptotically stable for any time varying delay time $\tau^k\in[\tau_{\min},\tau_{\max}]$.

A direct application of this result to the design of state-feedback controllers is not possible because unknown matrices $\overline{H}_0^+(\tau^k),\overline{H}_0^-(\tau^k),\overline{H}_1^+(\tau^k),\overline{H}_1^-(\tau^k)$ depend on τ^k.In order to overcome these difficulties we next establish stability conditions independent of τ^k.Due to space limitations the analysis is restricted to systems with real open-loop eigenvalues.

The exponential $exp(A_c\tau)$ of a matrix can always be written in the form

$$exp(A_c\tau)=a_1(\tau)Z_1+a_2(\tau)Z_2+...+a_n(\tau)Z_n \quad (20)$$

where $Z_i\in\mathbb{R}^{n\times n}$ $i=1,2,...,n$ are real matrices (constituent matrices) and $a_i(\tau)$ $i=1,2,...,n$ are real functions of the form $a_i(\tau)=\tau^q e^{\lambda_i\tau}$.

Therefore,

$$\Delta\Gamma_1(\tau^k,\tau^\circ)=\int_{h-\tau^k}^{h-\tau^\circ}\exp(A_c\tau)B_c d\tau=$$

$$=\sum_{i=1}^{n}\int_{h-\tau^k}^{h-\tau^\circ}a_i(\tau)d\tau Z_iB_c=\sum_{i=1}^{n}c_i(\tau^k)Z_iB_c \quad (21)$$

where $c_i(\tau^k)$ are integrals of the form

$$c_i(\tau^k)=\int_{h-\tau^k}^{h-\tau^\circ}e^{\lambda_i\tau}d\tau$$

in the case where Z_i corresponds to a simple real eigenvalue or

$$c_i(\tau^k)=\int_{h-\tau^k}^{h-\tau^\circ}\tau^q e^{\lambda_i\tau}d\tau$$

in the case where Z_i corresponds to a multiple real eigenvalue.

Let us define

$$c_{\max}\triangleq c_{1\max}+c_{2\max}+...+c_{n\max}$$

with

$$c_{i\max}\triangleq\max_{\tau_{\min}\le\tau^k\le\tau_{\max}}|c_i(\tau^k)|$$

In the case where $\tau^\circ=\tau_{\min}$, $c_i(\tau^k)$ are positive for any $\tau^k\in[\tau_{\min},\tau_{\max}]$, it follows that $c_{i\max}\triangleq c_i(\tau_{\max})$. We can now establish the following result**:**

Theorem 4: If for a nonsingular matrix $G\in\mathbb{R}^{n\times n}$, there exist $n\times n$ matrices , H^0,H^1, $H_Z^1,H_Z^2,...,H_Z^r$, a vector $w\in R^n$ with positive components and a $\varepsilon>0$ such that

$$G[\Phi+\Gamma_0(\tau^0)K_{sf}]=H^0G \quad (22)$$

$$G\Gamma_1(\tau^0)K_{sf}=H^1G \quad (23)$$

$$(|H^0|+|H^1|)w\le\varepsilon w \quad (24)$$

$$c_{\max}GZ_iB_cK_{sf}=H_Z^jG \quad j=1,2,...,n \quad (25)$$

$$(|H^0-H_Z^j|+|H^1+H_Z^j|)w\le\varepsilon w \quad j=1,2,...,n \quad (26)$$

$$\varepsilon<1 \quad (27)$$

then the equilibrium $x=0$ of system (13) is asymptotically stable for any time varying delay $\tau^k\in[\tau_{\min},\tau_{\max}]$.

Proof: From (21) it follows that

$$G\Delta\Gamma_1(\tau^k)K_{sf}=c_1(\tau^k)GZ_1B_cK_{sf}+c_2(\tau^k)GZ_2B_cK_{sf}+...$$

$$+...+c_r(\tau^k)GZ_nB_cK_{sf}$$

for any $\tau^k\in[\tau_{\min},\tau_{\max}]$.and ,by virtue of (25),

$$G\Delta\Gamma_1(\tau^k)K_{sf} = \frac{c_1(\tau^k)}{c_{\max}}H_Z^1G+...+\frac{c_2(\tau^k)}{c_{\max}}H_Z^nG=$$

$$= \left(\sum_{i=1}^{n}\frac{c_i(\tau^k)}{c_{\max}}H_Z^i\right)G=\Delta H(\tau^k)G \quad (28)$$

where

$$\Delta H(\tau^k)=\sum_{i=1}^{n}\frac{c_i(\tau^k)}{c_{\max}}H_Z^i$$

Taking into account that $\Delta\Gamma_0(\tau^k)=-\Delta\Gamma_1(\tau^k)$, from (22), (23) and (29) it follows that

$$G[\Phi+\Gamma_0(\tau^k)K_{sf}]=G[\Phi+\Gamma_0(\tau^0)K_{sf}+\Delta\Gamma_0(\tau^k)K_{sf}]=$$

$$=(H^0-\Delta H(\tau^k))G$$

$$G[\Gamma_1(\tau^k)K_{sf}] = G[\Gamma_1(\tau^0)K_{sf} + \Delta\Gamma_1(\tau^k)K_{sf}] =$$

$$= (H^1 + \Delta H(\tau^k))G$$

Thus conditions (9) and (10) of Theorem 2 are satisfied with

$$H_0 = H^0 - \Delta H(\tau^k)$$

$$H_1 = H^1 + \Delta H(\tau^k)$$

Furthermore,

$$(|H_0(\tau^k)| + |H_1(\tau^k)|)w =$$

$$= (|H^0 - \Delta H(\tau^k)| + |H^1 + \Delta H(\tau^k)|)w =$$

$$= \left(\left|H^0 - \left(\sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} H_Z^i\right)\right| + \left|H^1 + \left(\sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} H_Z^i\right)\right|\right) w =$$

$$= \left|\left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) H^0 + \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} (H^0 - H_Z^i)\right| w +$$

$$+ \left|\left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) H^1 + \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} (H^1 + H_Z^i)\right| w \leq$$

$$\leq \left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) |H^0| w + \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} |H^0 - H_Z^i| w +$$

$$+ \left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) |H^1| w + \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} |H^1 + H_Z^i| w \leq$$

$$\leq \left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) (|H^0| + |H^1|) w +$$

$$+ \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} (|H^0 - H_Z^i| + |H^1 + H_Z^i|) w$$

$$\leq \left(1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}}\right) \varepsilon w + \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} \varepsilon w = \varepsilon w$$

because $1 - \sum_{i=1}^{n} \frac{c_i(\tau^k)}{c_{\max}} \geq 0$. Therefore conditions (11) and (12) of Theorem 2 are also satisfied for any delay τ^k belonging to the time interval $[\tau_{\min}, \tau_{\max}]$ Consequently the equilibrium $x = 0$ of system (13) is asymptotically stable for any time varying delay $\tau^k \in [\tau_{\min}, \tau_{\max}]$.

According to this result, a solution to the mentioned problem is obtained by first selecting a pair (G, w) and then by solving relations (22)-(27) with respect to the unknown matrices $K_{sf}, H^1, H^0, \Delta H(\tau^k), H_Z^i \quad i = 1, 2, ..., n$ and parameter ε. Since these conditions imply the positive invariance of the polyhedral set $R(G, w) = \{x \in \mathbb{R}^n : |Gx| \leq w\}$ with respect to the system described by the equation $x(k+1) = \Phi x(k) + \Gamma_0(\tau^0)u(k)$,a pair (G, w) which is $(\Phi, \Gamma_0(\tau^0))-$positively invariant [22] must be selected. These relations can be formulated as linear algebraic equalities and inequalities by setting

$$H^0 - H_Z^j = \overline{H}_{0Z}^{j+} - \overline{H}_{0Z}^{j+}$$

$$H^1 + H_Z^j = \overline{H}_{1Z}^{j+} - \overline{H}_{1Z}^{j+} \qquad i = 1, 2, ..., n$$

with $\overline{H}_{0Z}^{j+} \geq 0, \overline{H}_{0Z}^{j-} \geq 0, \overline{H}_{1Z}^{j+} \geq 0, \overline{H}_{1Z}^{j-} \geq 0 \quad i = 1, 2, ..., n.$

A solution of relations (22)-(27) can be obtained by defining an optimization problem having these relations as linear constraints. Thus, a state-feedback control law $u(k) = K_{sf}x(k)$ that stabilizes the NCS for any delay time $\tau^k \in [\tau_{\min}, \tau_{\max}]\tau^k$ can be determined by solving the linear programming problem

$$\min_{K_{sf}, H^1, H^0, H_Z^j, \overline{H}_{0Z}^{j+}, \overline{H}_{0Z}^{j-}, \overline{H}_{1Z}^{j+}, \overline{H}_{1Z}^{j-}, \varepsilon} \{\varepsilon\} \tag{29}$$

under linear constraints

$$G[\Phi + \Gamma_0(\tau^0)K_{sf}] = H^0 G \tag{30}$$

$$G\Gamma_1(\tau^0)K_{sf} = H^1 G \tag{31}$$

$$(|H^0| + |H^1|)w \leq \varepsilon w \tag{32}$$

$$c_{\max} G Z_i B_c K_{sf} = H_Z^j G \quad j = 1, 2, ..., n \tag{33}$$

$$H^0 - H_Z^j = \overline{H}_{0Z}^{j+} - \overline{H}_{0Z}^{j-} \qquad i = 1, 2, ..., n \tag{34}$$

$$H^1 + H_Z^j = \overline{H}_{1Z}^{j+} - \overline{H}_{1Z}^{j+} \qquad i = 1, 2, ..., n \tag{35}$$

$$(\overline{H}_{0Z}^{j+} + \overline{H}_{0Z}^{j+} + \overline{H}_{1Z}^{j+} + \overline{H}_{1Z}^{j+})w \leq \varepsilon w \quad j = 1, 2, ..., n \tag{36}$$

$$\overline{H}_{0Z}^{j+} \geq 0, \overline{H}_{0Z}^{j-} \geq 0, \overline{H}_{1Z}^{j+} \geq 0, \overline{H}_{1Z}^{j-} \geq 0 \quad j = 1, 2, ..., n \tag{37}$$

If the optimal value of parameter ε satisfies inequality $\varepsilon < 1$ the corresponding control law $u(k) = K_{sf}x(k)$ is a solution to the problem under consideration.

It should be emphasized that minimization of parameter ε results to improved transient behavior, because parameter ε is a measure of the exponential convergence of the state to the equilibrium of the delayed system. Indeed, it can be proven that under conditions (29)-(36) the positive definite function

$$v(x) \triangleq \max_{1 \leq i \leq n} \left\{ \frac{|(Gx)_i|}{w_i} \right\}$$

is a Lyapunov function for system (13) which satisfies inequality $v(x(k+1)) \leq \varepsilon v(x(k))$ with $\epsilon < 1$.

V. Numerical Example

We consider an unstable continuous-time linear system (1) with matrices

$$A_c = \begin{bmatrix} 1.7208 & 2.9184 \\ -1.1396 & -2.0408 \end{bmatrix}, \quad B_c = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The sampling period is $h = 1.1$ second, the bounds of the uncertain input delay are $\tau_{min} = 0$, $\tau_{max} = 0.7$ and the nominal discrete-time dynamics are computed for $\tau_0 = 0$. The feedback gain $K_{sf} = \begin{bmatrix} -0.1419 & -0.1771 \end{bmatrix}$ was computed by solving the linear programming problem (29)-(36) setting

$$G = \begin{bmatrix} 0.6468 & 0.7626 \\ 0.5313 & 0.8472 \end{bmatrix}, \quad w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The optimal value of ε is $0.98 < 1$. In Fig.3 and Fig.4 the state response of the discrete-time system and the control effort for initial state $x_0 = [0.5947\ \ 0.8070]^T$ are shown respectively.

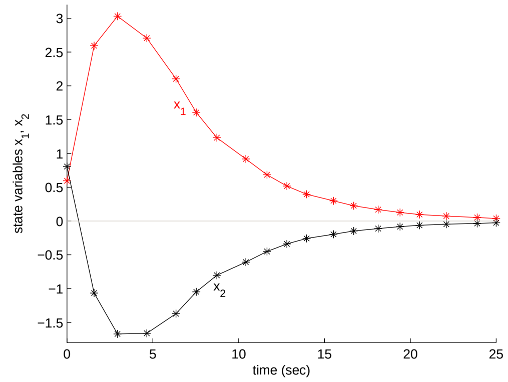

Fig. 3. State response of the closed-loop system for initial state $x_0 = [0.5947\ 0.8070]^T$.

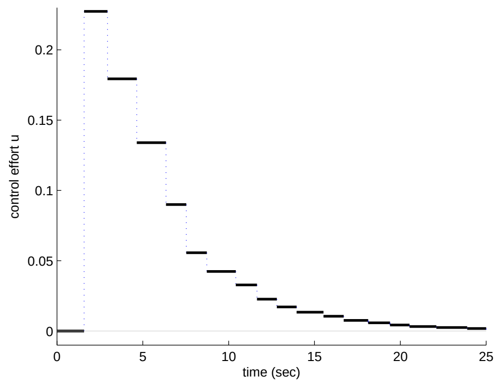

Fig. 4. Control strategy for initial state $x_0 = [0.5947\ 0.8070]^T$.

VI. Conclusions.

A novel approach for both the stability analysis and state feedback controller design for linear Networked Control Systems has been presented. The NCS dynamics is described by ARMA models. Contrary to common quadratic Lyapunov functions or Lyapunov-Krasovskii functionals used in previous papers, asymptotic stability is proved using polyhedral Lyapunov functions. A benefit of this approach is the reduction of conservativeness in the stability analysis when compared to other existing results. It has also been shown that the controller design problem can be reduced to a simple LP optimization problem having as objective the minimization of a parameter closely related to the transient behavior of the NCS. An example of an unstable networked system is given to illustrate the performance of this approach.

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