

# Generalized Selection Combining for Cognitive Relay Networks over Nakagami-m Fading

Deng, Y., Wang, L., Elkashlan, M., Kim, K. J., & Duong, T. Q. (2015). Generalized Selection Combining for Cognitive Relay Networks over Nakagami-m Fading. *IEEE Transactions on Signal Processing*, *63*(8), 1993-2006. https://doi.org/10.1109/TSP.2015.2405497

## Published in:

**IEEE Transactions on Signal Processing** 

**Document Version:** Peer reviewed version

## Queen's University Belfast - Research Portal:

Link to publication record in Queen's University Belfast Research Portal

## Publisher rights

(c) 2015 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other users, including reprinting/ republishing this material for advertising or promotional purposes, creating new collective works for resale or redistribution to servers or lists, or reuse of any copyrighted components of this work in other works.

## General rights

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

## Take down policy

The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.

#### **Open Access**

This research has been made openly available by Queen's academics and its Open Research team. We would love to hear how access to this research benefits you. – Share your feedback with us: http://go.qub.ac.uk/oa-feedback

## Generalized Selection Combining for Cognitive Relay Networks over Nakagami-m Fading

Yansha Deng, Student Member, IEEE, Lifeng Wang, Student Member, IEEE, Maged Elkashlan, Member, IEEE, Kyeong Jin Kim, Senior Member, IEEE, and Trung Q. Duong, Senior Member, IEEE

Abstract—We consider transmit antenna selection with receive generalized selection combining (TAS/GSC) for cognitive decodeand-forward (DF) relaying in Nakagami-m fading channels. In an effort to assess the performance, the probability density function and the cumulative distribution function of the endto-end SNR are derived using the moment generating function, from which new exact closed-form expressions for the outage probability and the symbol error rate are derived. We then derive a new closed-form expression for the ergodic capacity. More importantly, by deriving the asymptotic expressions for the outage probability and the symbol error rate, as well as the high SNR approximations of the ergodic capacity, we establish new design insights under the two distinct constraint scenarios: 1) proportional interference power constraint, and 2) fixed interference power constraint. Several pivotal conclusions are reached. For the first scenario, the full diversity order of the outage probability and the symbol error rate is achieved, and the high SNR slope of the ergodic capacity is 1/2. For the second scenario, the diversity order of the outage probability and the symbol error rate is zero with error floors, and the high SNR slope of the ergodic capacity is zero with capacity ceiling.

Index Terms—Cognitive relay network, generalized selection combining, Nakagami-m fading.

## I. INTRODUCTION

The conflict between the stringent demand for high data rate and data service on the one hand, and the unbalanced spectrum occupation in time and geographic domains on the other hand, has become a challenge for future wireless systems [1]. To cope with this, cognitive radio, first coined by Mitola, has rekindled increasing interest in the efficient use of radio spectrum. In the underlay paradigm, the secondary users (SUs) are allowed to access the spectrum allocated to primary users (PUs) as long as the interference generated by the secondary transmission is restricted below a certain threshold, namely, interference temperature [2]. The constrained transmit power at SU typically results in unstable transmission and restricted coverage, which drives the demand for robust transmission techniques suited for networks that are subject to power and interference constraints [3]. Relaying is regarded as a cost-effective approach for supporting high speed and long distance networks [4, 5].

The majority of the studies on cognitive relay networks have focused on single antenna protocols [6–8]. Multipleinput multiple-output (MIMO) techniques, well-known for their many benefits including enhanced reliability [9], spectral efficiency [10], and co-channel interference suppression [11], open up new dimensions for cognitive radio. For example, as shown in [12], multi-antennas are utilized at SU to achieve spatial multiplexing. In [13], the novel distributed antenna selection is proposed in relaying system. In [14], the effect of transmit antenna selection with receive maximal ratio combining (TAS/MRC) on the the ergodic capacity was analyzed. In [15], the outage performance of TAS/MRC and TAS/SC are examined over Nakagami-*m* fading channel. It is shown in [16] that the diversity order is independent of the number of PUs and the selected number of receive antennas at SU.

Different from the aforementioned works, in this paper, we consider cognitive relay networks from the viewpoint of TAS/GSC as an effective design to enhance the reliability of the secondary network and to mitigate interference to the primary network. From a power perspective, cognitive spectrum sharing with network cooperation addresses fundamental constraints on the transmit power at the SUs, while keeping the interference temperature at the PUs to a minimum [17]. On the one hand, TAS is acknowledged as a core component for uplink 4G long term evolution (LTE) and LTE Advanced systems because of its low feedback requirement compared with closed-loop transmit diversity [18]. On the other hand, with the merits of low power demand and RF cost, GSC offers a performance/implementation tradeoff between MRC and selection combining (SC) for the secondary network  $[19, 20]^1$ . Additionally, by excluding the antenna chains with weak channel powers, GSC can be more robust to channel estimation errors than MRC [21]. In [22], it is shown that GSC outperforms MRC in a non-identically distributed noise scenario.

The objective of this paper is to examine the impact of TAS/GSC in underlay cognitive relay networks over

Copyright (c) 2015 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org. Manuscript received Jan.31, 2014; revised Sep. 22, 2014; accepted Jan. 26, 2015. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Zhengdao Wang. This work of Y. Deng was supported by China Scholarship Council. This paper has been presented in part at IEEE International Conference on Communications (ICC), Sydney, 2014.

Y. Deng, L. Wang, and M. Elkashlan are with Queen Mary University of London, London E1 4NS, UK (email: {y.deng, lifeng.wang, maged.elkashlan}@qmul.ac.uk).

K. J. Kim is with Mitsubishi Electric Research Laboratories (MERL), 201 Broadway, Cambridge, MA, USA (email: kkim@merl.com).

T. Q. Duong is with Queen's University Belfast, Belfast BT7 1NN, UK (email: trung.q.duong@qub.ac.uk).

<sup>&</sup>lt;sup>1</sup>GSC is well applied to commercial wireless networks where the receiver is subject to resource constraints, such as limited RF chains due to size and complexity limitations [26].

Nakagami-m fading. The Nakagami-m fading environment is considered due to its versatility in providing a good match to various empirically obtained measurement data [23]. In the secondary network, a single antenna which maximizes the signal-to-noise ratio (SNR) is selected at the secondary transmitter, while a subset of receive antennas with highest SNRs are combined at the secondary receiver. For coverage and reliability enhancement, a decode-and-forward (DF) relay is used in the secondary network to assist the secondary transmission. Note that the transmit powers at the secondary source (S) and the secondary relay (R) are limited by two constraints: 1) the peak interference constraint at the primary receiver, and 2) the peak transmit power constraint at S and R. It is also important to note that the performance of underlay spectrum sharing is typically restricted due to these two strict power constraints. With the help of TAS/GSC relaying, less transmit power is required at S and R, which in turn reduces the interference at the PU, allowing for high speed data services over wide area coverage. The main contributions of this paper are summarized as follows.

- We derive new exact closed-form expressions for the cumulative distribution function (CDF) of the SNR with TAS/GSC. Although the CDF expressions were presented in [19, 24] with the aid of the trapezoidal rule, they are not in closed-form and cannot be used to derive the CDF of the SNR with TAS/GSC.
- We derive new exact closed-form expressions for the outage probability and the symbol error rate (SER) to accurately assess the joint impact of antenna configuration and channel fading. We further derive the asymptotic expressions for the outage probability and the SER under the two cases: 1) proportional interference power constraint, and 2) fixed interference power constraint. We confirm that the full diversity order is achieved for the proportional interference power constraint. For the fixed interference power constraint, the diversity order is zero with error floors in the high SNR regime.
- We derive an exact closed-form expression for the ergodic capacity. Notably, this is the first closed-form expression for cognitive relay networks with TAS/GSC in Nakagami*m* fading channels. More importantly, we obtain a tight high SNR approximation of the ergodic capacity for the two cases: 1) proportional interference power constraint, and 2) fixed interference power constraint. Interesting conclusions are reached. On the one hand, the high SNR slope is independent of the antenna configuration and the fading parameters, but on the other hand, the high SNR power offset is fully described by the antenna configuration and the fading parameters in the primary and secondary networks. The high SNR slope is 1/2 for the proportional interference power constraint, and is equal to zero for the fixed interference power constraint.

## **II. SYSTEM AND CHANNEL DESCRIPTION**

We consider a dual-hop cognitive DF relay network consisting of S with  $N_S$  antennas, R with  $N_R$  antennas, D with  $N_D$  antennas, and PU with a single antenna. We assume that the PU transmitter is located far away from the secondary network. This assumption is typical in large scale networks where the interference from the PU transmitter is negligible [6, 25, 26]. We also assume there is no direct link between S and D due to long distance and deep fades [27]. Both the primary channel and the secondary channel are assumed to undergo quasi-static fading with independent and identically distributed (i.i.d.) Nakagami-m distribution. We assume perfect channel state information (CSI) between the secondary transmitter and the PU can be obtained through direct feedback from the PU [28], indirect feedback from a third party, and periodic sensing of pilot signal from the PU [29]. In the secondary network, a single transmit antenna among  $N_S$  antennas which maximizes the GSC output SNR at R is selected at S, while the  $L_R$  $(1 \le L_R \le N_R)$  strongest receive antennas are combined at R. The signal transmitted by R is decoded and forwarded using a single transmit antenna among  $N_R$  antennas which maximizes the GSC output SNR at D, and then combined at D with the  $L_D$   $(1 \le L_D \le N_D)$  strongest receive antennas. Let  $\{g_{1ij}\}$ denote the channel coefficients of the  $N_{\rm S} \times N_{\rm R}$  channels from **S** to **R** with  $i \in \{1, ..., N_S\}, j \in \{1, ..., N_R\}$ , and  $\{g_{2jk}\}$ denote the channel coefficients of the  $N_{\rm R} \times N_{\rm D}$  channels from **R** to **D** with  $k \in \{1, \ldots, N_D\}$ . Also,  $\{h_{1i}\}$  denote the channel coefficients of the  $N_S \times 1$  channels from S to PU, and  $\{h_{2i}\}$ denote the channel coefficients of the  $N_R \times 1$  channels from R to PU. The channel coefficients follow the Nakagami-mdistribution with fading parameters  $m_{q1}$ ,  $m_{q2}$ ,  $m_{h1}$ , and  $m_{h2}$ , and average channel power gains  $\Omega_{q1}$ ,  $\Omega_{q2}$ ,  $\Omega_{h1}$ , and  $\Omega_{h2}$ . In the following,  $\|\cdot\|$  is the Euclidean norm,  $|\cdot|$  is the absolute value, and  $\mathbb{E}[\cdot]$  is the expectation.

The pilot symbol block  $P_i$ ,  $(1 \le i \le N_S)$ , are transmitted from each transmit antenna at different time slots. Based on these pilot symbols, R perfectly estimates CSI, then arranges  $\{|g_{1ij}|^2\}_{j=1}^{N_R}$  in descending order as  $|g_{1i(1)}|^2 \ge |g_{1i(2)}|^2 \ge$  $\dots \ge |g_{1i(N_R)}|^2 \ge 0$  for the each transmit antenna *i* at S. Note that before the transmission process, the selected number of antenna chains  $L_R$  and  $L_D$  at the receivers are determined by the limited number of radio frequency (RF) chains due to size and complexity limitations. According to the rule of GSC, the first  $L_R$   $(1 \le L_R \le N_R)$  received signal power(s) are comibined at R to obtain  $\theta_i = \sum_{j=1}^{L_R} |g_{1i(j)}|^2$ . The selected transmit antenna  $i^*$  is determined at R by

$$i^* = \arg\max_{1 \le i \le N_S} \left\{ \theta_i = \sum_{j=1}^{L_R} |g_{1i(j)}|^2 \right\},$$
 (1)

which maximizes the total received signal power. To this end, the index of the selected transmit antenna is sent back to S through the feedback channel, so that only  $\lceil \log_2(N_S) \rceil$  bits needs to be sent to S. As such, the selected channel vector is denoted as  $\mathbf{g}_{1i^*\theta_{i^*}} = [g_{1i^*(1)}, \cdots, g_{1i^*(L_R)}]$ . Similarly, in the second hop, the index of the selected transmit antenna at R is determined by

$$j^* = \arg\max_{1 \le j \le N_R} \left\{ \theta_j = \sum_{k=1}^{L_D} \left| g_{2j(k)} \right|^2 \right\}.$$
 (2)

As such, we denote the selected channel vector as  $\mathbf{g}_{2j^*\theta_{j^*}} =$  $[g_{2j^*(1)}, \cdots, g_{2j^*(L_D)}].$ 

According to underlay cognitive relay networks, the transmit powers at S and R are constrained as

$$P_{S} = \min\left(P, \frac{Q}{|h_{1i^{*}}|^{2}}\right) \text{ and } P_{R} = \min\left(P, \frac{Q}{|h_{2j^{*}}|^{2}}\right),$$
(3)

respectively, where P is the maximum transmit power constraint at S and R, and Q is the peak interference power constraint at PU.

The instantaneous end-to-end SNR of the spectrum sharing network with TAS/GSC and DF relaying is defined as  $\gamma =$  $\min\{\gamma_1, \gamma_2\}$ , where the instantaneous SNR of S  $\rightarrow$  R link is

$$\gamma_{1} = \min\left( \|\mathbf{g}_{1i^{*}\theta_{i^{*}}}\|^{2} \bar{\gamma}_{P}, \frac{\|\mathbf{g}_{1i^{*}\theta_{i^{*}}}\|^{2} \bar{\gamma}_{Q}}{|h_{1i^{*}}|^{2}} \right)$$
(4)

and the instantaneous SNR of  $R \rightarrow D$  link is

$$\gamma_{2} = \min\left(\left\|\mathbf{g}_{2j^{*}\theta_{j^{*}}}\right\|^{2} \bar{\gamma}_{P}, \frac{\left\|\mathbf{g}_{2j^{*}\theta_{j^{*}}}\right\|^{2} \bar{\gamma}_{Q}}{\left|h_{2j^{*}}\right|^{2}}\right).$$
(5)

In (4) and (5), we define  $\overline{\gamma}_P = \frac{P}{N_0}$  and  $\overline{\gamma}_Q = \frac{Q}{N_0}$ , where  $N_0$  is the noise power of an additive white Gaussian noise (AWGN).

## **III. NEW STATISTICAL PROPERTIES**

In this section, we derive new statistical properties of the end-to-end SNR, which is a challenging problem due to the complex nature of TAS/GSC in Nakagami-m fading. Based on these statistical characteristics, we present the exact and asymptotic outage probability, SER, and ergodic capacity. Without loss of generality, these new statistics can be easily applied to other wireless networks with TAS/GSC.

Based on the expressions of  $\gamma_1$  and  $\gamma_2$  in (4) and (5), respectively, we first derive the CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  in the following lemmas.

A. Expressions for CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  in the Secondary Channel

**Lemma 1.** The expressions for the CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  are derived as

$$F_{\parallel \mathbf{g}_{1i^{*}\theta_{i^{*}}}\parallel^{2}}(x) = \left(\frac{L_{R}}{(m_{g1}-1)!} \binom{N_{R}}{L_{R}}\right)^{N_{S}} N_{S}!$$
$$\widetilde{\sum_{S_{R}^{|\mathcal{S}_{K}|}}} \hbar_{k} x^{\theta_{k}} e^{-\eta_{k}x}, \tag{6}$$

$$\{(n_{\tau,1},\ldots,n_{\tau,|\mathcal{S}_K|}) | \sum_{k=1}^{|\mathcal{S}_K|} n_{\tau,k} = N_S \} \text{ with } \{n_{\tau,k}\} \in \mathbb{Z}^+,$$

 $|S_K|$  is the cardinality of the set  $S_K$ , and  $S_K$  denotes a set of  $(2m_{g1}+1)$ -tuples satisfying the following condition

$$S_{K} = \left\{ \left( n_{k,0}^{\Phi}, \dots, n_{k,m_{g_{1}}-1}^{\Phi}, n_{k,0}^{F}, \dots, n_{k,m_{g_{1}}}^{F} \right) \right|$$
$$\sum_{i=0}^{m_{g_{1}}-1} n_{k,i}^{\Phi} = L_{R} - 1; \sum_{j=0}^{m_{g_{1}}} n_{k,j}^{F} = N_{R} - L_{R} \right\},$$

thereby  $|\mathcal{S}_{K}| = \binom{m_{g_{1}}+L_{R}-2}{m_{g_{1}}-1}\binom{m_{g_{1}}+N_{R}-L_{R}}{m_{g_{1}}}$ , and  $\mathcal{S}_{R}^{k} = \left\{ \left(n_{\rho_{k},0},\ldots,n_{\rho_{k},m_{g_{1}}L_{R}+b_{k}^{F}}\right) \Big| \sum_{n=0}^{m_{g_{1}}L_{R}+b_{k}^{F}} n_{\rho_{k},n} = n_{\tau,k} \right\}$ ,  $k = 1, \cdots, |\mathcal{S}_{K}|$ , with  $\{n_{k,i}^{\Phi}\}, \{n_{k,i}^{\Phi}\}, \{n_{k,j}^{F}\}, and \{n_{\rho_{k},n}\} \in \mathbb{Z}^{+}$ . In (6),  $\hbar_{k}$ ,  $\theta_{k}$ , and  $\eta_{k}$  are respectively given by  $m_{o_{1}}L_{R}+b_{k}^{F}$ 

$$\hbar_{k} = \prod_{k=1}^{|\mathcal{S}_{K}|} \left( a_{k}^{\Phi} a_{k}^{F} \frac{(n_{2}-1)!}{L_{R}^{n_{2}}} \right)^{n_{\tau,k}} \left( \frac{\prod_{n=0}^{\infty} \ell_{k}(n)^{n_{\rho_{k},n}}}{\prod_{n=0}^{m_{g_{1}}L_{R}+b_{k}^{F}} n_{\rho_{k},n}!} \right),$$
(7)

$$\theta_k = \sum_{k=1}^{|\mathcal{S}_K|} \sum_{n=0}^{m_{g1}L_R + b_k^F} \mu_k(n) n_{\rho_k, n}, \tag{8}$$

and

$$\eta_k = \sum_{k=1}^{|\mathcal{S}_K|} \sum_{n=0}^{m_{g1}L_R + b_k^F} \nu_k(n) n_{\rho_k, n}, \tag{9}$$

where  $n_2$ ,  $\mu_k(n)$ ,  $\nu_k(n)$ ,  $\ell_k(n)$   $a_k^{\Phi}$ ,  $a_k^F$ ,  $b_k^{\Phi}$ ,  $b_k^F$ ,  $\Upsilon_{k1}$ ,  $\Upsilon_{k2}$ ,  $\Upsilon_{k3}$ , and  $\Upsilon_{k4}$  are defined in Appendix A.

B. Expressions for the CDF of  $\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2$  in the Secondary Channel

The CDF of  $\|\mathbf{g}_{2j^*\theta_{j^*}}\|^2$  follow from (6) by interchanging the parameters  $m_{g1} \rightarrow m_{g2}$ ,  $m_{h1} \rightarrow m_{h2}$ ,  $L_R \rightarrow L_D$ ,  $N_R \rightarrow N_D$ ,  $N_S \rightarrow N_R$ ,  $S_R \rightarrow S_D$ ,  $|S_K| \rightarrow |S_T|$ ,  $S_R^{|S_K|} \rightarrow S_D^{|S_T|}$ ,  $\hbar_k \rightarrow \hbar_t$ ,  $\theta_k \rightarrow \theta_t$ , and  $\eta_k \rightarrow \eta_t$ , where  $\sum_{\substack{S_D \\ D_L}} \triangleq \sum_{\substack{S_D \\ S_D}} \sum_{\substack{S_D \\ S_D}} \cdots \sum_{\substack{S_D \\ S_D}} \sum_{\substack{S_D \\ S_D}}$ 

 $\left\{\left(n_{\tau,1},\ldots,n_{\tau,|\mathcal{S}_T|}\right)\Big|\sum_{t=1}^{|\mathcal{S}_T|} n_{\tau,t} = N_R\right\} \text{ with } \{n_{\tau,t}\} \in \mathbb{Z}^+, |\mathcal{S}_T|$ is the cardinality of the set  $S_T$ , and  $S_T$  denotes a set of  $(2m_{a2}+1)$ -tuples satisfying the following condition

$$S_T = \left\{ \left( n_{t,0}^{\Phi}, \dots, n_{t,m_{g^2}-1}^{\Phi}, n_{t,0}^{F}, \dots, n_{t,m_{g^2}}^{F} \right) \right| \\ \sum_{i=0}^{m_{g^2}-1} n_{t,i}^{\Phi} = L_D - 1; \sum_{j=0}^{m_{g^2}} n_{t,j}^{F} = N_D - L_D \right\},$$

 $\text{where} \quad \underbrace{\sum_{\mathcal{S}_{R}^{|\mathcal{S}_{K}|}}}_{\mathcal{S}_{R}^{|\mathcal{S}_{K}|}} \stackrel{\text{(6)}}{=} \underbrace{ \text{thereby} } |\mathcal{S}_{T}| = \binom{m_{g2}+L_{D}-2}{m_{g2}-1} \binom{m_{g2}+N_{D}-L_{D}}{m_{g2}}, \text{ and} \\ \mathcal{S}_{D}^{t} = \left\{ \left(n_{\rho_{t},0}, \dots, n_{\rho_{t},m_{g2}L_{D}+b_{t}^{F}}\right) \middle| \begin{array}{c} \sum_{n=0}^{m_{g2}+L_{D}+b_{t}^{F}} \\ \sum_{n=0} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \cdots \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S}_{R}} \\ \sum_{\mathcal{S}_{R}} \sum_{\mathcal{S$ changing the parameters  $\mu_k(n) \to \mu_t(n)$ ,  $\nu_k(n) \to \nu_t(n)$ ,  $\ell_k(n) \to \ell_t(n) \ a_k^{\Phi} \to a_t^{\Phi}, \ b_k^{\Phi} \to b_t^{\Phi}, \ c_k^{\Phi} \to c_t^{\Phi}, \ a_k^F \to a_t^F$ ,

$$\begin{split} b_k^F &\rightarrow b_t^F, \, c_k^F \rightarrow c_t^F, \, \Upsilon_{k1} \rightarrow \Upsilon_{t1}, \, \Upsilon_{k2} \rightarrow \Upsilon_{t2}, \, \Upsilon_{k3} \rightarrow \Upsilon_{t3}, \\ \text{and} \, \Upsilon_{k4} \rightarrow \Upsilon_{t4}. \, \text{Here}, \, \mu_t(n), \, \nu_t(n), \, \ell_t(n) \, a_t^\Phi, b_t^\Phi, c_t^\Phi, a_t^F, b_t^F, \\ c_t^F, \, \Upsilon_{t1}, \, \Upsilon_{t2}, \, \Upsilon_{t3}, \, \text{and} \, \Upsilon_{t4} \, \text{follow from (A.15), (A.16), (A.14), } \\ (A.6), (A.7), \, (A.9), \, (A.10), \, (A.17), \, (A.18), \, (A.19), \, \text{and} \, (A.20) \\ \text{by interchanging the parameters} \, m_{g1} \rightarrow m_{g2}, \, m_{h1} \rightarrow m_{h2}, \\ \Omega_{h1} \rightarrow \Omega_{h2}, \, L_R \rightarrow L_D, \, N_R \rightarrow N_D, \, N_S \rightarrow N_R, \, \mathcal{S}_R \rightarrow \mathcal{S}_D, \\ |\mathcal{S}_K| \rightarrow \, |\mathcal{S}_T|, \, \mathcal{S}_R^{|\mathcal{S}_K|} \rightarrow \, \mathcal{S}_D^{|\mathcal{S}_T|}, \, n_{\tau,k} \rightarrow n_{\tau,t}, \, n_{k,i}^\Phi \rightarrow n_{t,i}^\Phi, \\ n_{k,i}^\Phi \rightarrow n_{t,i}^\Phi, \, n_{k,j}^F \rightarrow n_{t,j}^F, \, \text{and} \, n_{\rho_k,n} \rightarrow n_{\rho_t,n}. \end{split}$$

## C. Expressions for the CDF of $\gamma_1$

With the help of the CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  and  $|h_{1i^*}|^2$ , the closed-form CDF of  $\gamma_1$  is evaluated in the following lemma.

**Lemma 2.** The expression for the CDF of  $\gamma_1$  is represented as

$$F_{\gamma_1}(x) = \left(\frac{L_R}{(m_{g1}-1)!} \binom{N_R}{L_R}\right)^{N_S} N_S! \underbrace{\sum}_{\mathcal{S}_R^{|\mathcal{S}_K|}} \hbar_k \Xi_k(x), \quad (10)$$

where

$$\Xi_{k}(x) = \left(1 - \frac{\Gamma\left(m_{h1}, \frac{m_{h1}Q}{\Omega_{h1}P}\right)}{\Gamma\left(m_{h1}\right)} \left(\frac{x}{\bar{\gamma}_{P}}\right)^{\theta_{k}} e^{-\eta_{k}\frac{x}{\bar{\gamma}_{P}}} + \left(\frac{m_{h1}}{\Omega_{h1}}\right)^{m_{h1}} \left(\frac{x}{\bar{\gamma}_{Q}}\right)^{\theta_{k}} \frac{\Gamma\left(\theta_{k} + m_{h1}, \left(\frac{m_{h1}}{\Omega_{h1}} + \frac{\eta_{k}x}{\bar{\gamma}_{Q}}\right)\frac{Q}{P}\right)}{\left(m_{h1} - 1\right)! \left(\frac{m_{h1}}{\Omega_{h1}} + \frac{\eta_{k}x}{\bar{\gamma}_{Q}}\right)^{\theta_{k} + m_{h1}}}.$$
 (11)

Proof. See Appendix B.

## D. Expressions for the CDF of $\gamma_2$

Similarly, the CDF of  $\gamma_2$  follows from (10) and (11) by interchanging the parameters  $m_{g1} \rightarrow m_{g2}$ ,  $m_{h1} \rightarrow m_{h2}$ ,  $\Omega_{h1} \rightarrow \Omega_{h2}$ ,  $\eta_k \rightarrow \eta_t$ , and  $\theta_k \rightarrow \theta_t$ . Note that our expressions are valid for arbitrary fading severity parameters in all the links.

## IV. OUTAGE PROBABILITY

In this section, we concentrate on the outage probability. We derive a new closed-form expression for the exact outage probability. In order to assess the performance at high SNRs, we derive the asymptotic outage probabilities with the proportional interference power constraint and the fixed interference power constraint.

## A. Exact Analysis

In DF relaying, the end-to-end outage probability is determined by the worst link between  $S \rightarrow R$  and  $R \rightarrow D$  links, which is given by [30]

$$P_{out}(\gamma_{th}) = \Pr\left(\min(\gamma_1, \gamma_2) \le \gamma_{th}\right)$$
$$= F_{\gamma_1}(\gamma_{th}) + F_{\gamma_2}(\gamma_{th}) - F_{\gamma_1}(\gamma_{th}) F_{\gamma_2}(\gamma_{th}). \quad (12)$$

By substituting (10) and the CDF of  $\gamma_2$  into (12), the outage probability is finally derived in the following theorem.

**Theorem 1.** The closed-form expression for the outage probability of spectrum sharing networks with TAS/GSC and DF relaying in Nakagami-m fading is derived as

$$P_{out}(\gamma_{th}) = 1 - \left(1 - \left(\frac{L_R}{(m_{g1} - 1)!} \binom{N_R}{L_R}\right)^{N_S} N_S! \widetilde{\sum_{\mathcal{S}_R^{|\mathcal{S}_K|}}} \hbar_k \Xi_k(\gamma_{th})\right) \\ \left(1 - \left(\frac{L_D}{(m_{g2} - 1)!} \binom{N_D}{L_D}\right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \hbar_t \Xi_t(\gamma_{th})\right).$$
(13)

Our new closed-form expression for the outage probability is valid for an arbitrary number of antennas of the secondary network and arbitrary fading severity parameters in all the links.

## B. Asymptotic Analysis

1) Proportional Interference Power Constraint:

We first examine the asymptotic behavior with the proportional interference power constraint. As such, we assume that both P and Q grow large in the high SNR regime. This applies to the scenario where the PU is able to tolerate a high amount of interference from S and R. With this in mind, we take into account and study the effect of the so-called *power scaling* on the outage probability. Similar to [7, 25], we consider  $Q = \mu P$ , where  $\mu$  is the power scaling factor and is a positive constant.

**Theorem 2.** When Q scales with P, the asymptotic outage probability of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as

$$P_{out}^{\infty}\left(\gamma_{th}\right) = \left(G_c \overline{\gamma}_P\right)^{-G_d} + o\left(\overline{\gamma}_P^{-G_d}\right),\tag{14}$$

where the diversity order is

$$G_d = N_R \times \min\{m_{g1}N_S, m_{g2}N_D\}$$
(15)

and the SNR gain is

$$G_{c} = \begin{cases} \frac{\Delta_{1}(L_{R})}{\gamma_{th}} & m_{g1}N_{\rm S} < m_{g2}N_{\rm D} \\ \frac{\Delta_{2}(L_{R})}{\gamma_{th}} & m_{g1}N_{\rm S} > m_{g2}N_{\rm D} \\ \frac{\Delta_{1}(L_{R})}{\gamma_{th}} + \frac{\Delta_{2}(L_{R})}{\gamma_{th}} & m_{g1}N_{\rm S} = m_{g2}N_{\rm D} , \end{cases}$$
(16)

with

$$\Delta_{1}(L_{R}) = \frac{\Omega_{g1}}{m_{g1}} \left[ \frac{K(S_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi})}{(m_{g1}N_{R})!} \right]^{-\frac{1}{m_{g1}N_{R}}} \left[ \Phi(m_{h1}, \Omega_{h1}) + \frac{\Xi(m_{g1}, m_{h1}, \Omega_{h1}, N_{S})}{\mu^{m_{g1}N_{R}N_{S}}} \right]^{-\frac{1}{m_{g1}N_{R}N_{S}}}, (17)$$

and

$$\Delta_{2}(L_{R}) = \frac{\Omega_{g2}}{m_{g2}} \left[ \frac{\mathrm{K}(\mathcal{S}_{T}^{\Phi}, N_{D}, L_{D}, m_{g2}, a_{t}^{\Phi}, b_{t}^{\Phi})}{(m_{g2}N_{D})!} \right]^{-\frac{1}{m_{g2}N_{D}}} \left[ \Phi(m_{h2}, \Omega_{h2}) + \frac{\Xi(m_{g2}, m_{h2}, \Omega_{h2}, N_{D})}{\mu^{m_{g2}N_{R}N_{D}}} \right]^{-\frac{1}{m_{g2}N_{R}N_{D}}}.$$
 (18)

In (17) and (18), we have

$$K(\mathcal{S}^{\Phi}, N, L, m_g, a^{\Phi}, b^{\Phi}) = \frac{L\binom{N}{L}}{(m_g - 1)!(m_g!)^{N-L}}$$
$$\sum_{\mathcal{S}^{\Phi}} a^{\Phi} \frac{(b^{\Phi} + m_g(N - L + 1) - 1)!}{(L)^{b^{\Phi} + m_g(N - L + 1)}},$$
(19)

$$\Phi(m_h, \Omega_h) = 1 - e^{-\mu \frac{m_h}{\Omega_h}} \sum_{j=0}^{m_h-1} \frac{\left(\mu \frac{m_h}{\Omega_h}\right)^j}{j!},$$
(20)

$$\Xi(m_g, m_h, \Omega_h, N) = \frac{\Gamma(m_g N_R N + m_h, \mu \frac{m_h}{\Omega_h})}{(m_h - 1)! (\frac{m_h}{\Omega_h})^{m_g N_R N}}.$$
 (21)

Proof. See Appendix C.

Based on (15), we see that the diversity order is dominated by the fading severity parameter of the two hops and the total number of antennas at S, R, and D. Interestingly, it is independent of the fading severity parameters of the interference channel, and the selected number of antennas at R and D. The negative impact of the peak interference power constraint is reflected in the SNR gain.

**Corollary 1.** The SNR gap between GSC and SC is derived as

$$G_{c} = \begin{cases} -\frac{10}{m_{g_{1}}N_{R}}\log\left(\mathrm{T}_{1}\right) & m_{g_{1}}N_{\mathrm{S}} < m_{g_{2}}N_{\mathrm{D}} \\ -\frac{10}{m_{g_{2}}N_{D}}\log\left(\mathrm{T}_{2}\right) & m_{g_{1}}N_{\mathrm{S}} > m_{g_{2}}N_{\mathrm{D}} \\ 10\log\left(\frac{\Delta_{1}(L_{R}) + \Delta_{2}(L_{R})}{\Delta_{1}(1) + \Delta_{2}(1)}\right) & m_{g_{1}}N_{\mathrm{S}} = m_{g_{2}}N_{\mathrm{D}} , \end{cases}$$

$$(22)$$

where

$$T_{1} = \frac{(m_{g1}!)^{N_{R}-1} (m_{g1}-1)!}{N_{R} (m_{g1}N_{R}-1)!} K(S_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi}) (23)$$

and

$$T_{2} = \frac{(m_{g2}!)^{N_{D}-1} (m_{g2}-1)!}{N_{D} (m_{g2}N_{D}-1)!} K(S_{T}^{\Phi}, N_{D}, L_{D}, m_{g2}, a_{t}^{\Phi}, b_{t}^{\Phi}).$$
(24)

## 2) Fixed Interference Power Constraint:

Different from the proportional interference power constraint which can tolerate an extremely high peak interference power constraint and may potentially violate and harm the PU transmission [6], in this subsection, we focus on a stricter constraint where the peak interference power constraint is fixed [31]. We present the asymptotic outage probability with the fixed interference power constraint in the following theorem.

**Theorem 3.** Under the fixed interference power constraint, the asymptotic outage probability of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as

$$P_{out}^{\infty}(\gamma_{th}) = \begin{cases} H_1\left(\Phi_1\left(\frac{\gamma_{th}}{\bar{\gamma}_P}\right)^{m_{g1}N_RN_S} + \Xi_1\left(\frac{\gamma_{th}}{\bar{\gamma}_Q}\right)^{m_{g1}N_RN_S}\right) \\ H_2\left(\Phi_2\left(\frac{\gamma_{th}}{\bar{\gamma}_P}\right)^{m_{g2}N_RN_D} + \Xi_2\left(\frac{\gamma_{th}}{\bar{\gamma}_Q}\right)^{m_{g2}N_RN_D}\right) \\ H_2\left(\Phi_2\left(\frac{\gamma_{th}}{\bar{\gamma}_P}\right)^{m_{g2}N_RN_D} + \Xi_2\left(\frac{\gamma_{th}}{\bar{\gamma}_Q}\right)^{m_{g2}N_RN_D}\right) \\ (H_1\Phi_1 + H_2\Phi_2)\left(\frac{\gamma_{th}}{\bar{\gamma}_P}\right)^{m_{g1}N_RN_S} \\ + \left(H_1\Xi_1 + H_2\Xi_2\right)\left(\frac{\gamma_{th}}{\bar{\gamma}_Q}\right)^{m_{g1}N_RN_S} \\ m_{g1}N_S = m_{g2}N_D , \end{cases}$$
(25)

where

$$H_{1} = \left[\frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}N_{R}} K\left(\mathcal{S}_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi}\right)}{(m_{g1}N_{R})!}\right]^{N_{S}} (26)$$

$$H_{2} = \left[\frac{\left(\frac{m_{g2}}{\Omega_{g2}}\right)^{m_{g2}N_{D}} K\left(\mathcal{S}_{T}^{\Phi}, N_{D}, L_{D}, m_{g2}, a_{t}^{\Phi}, b_{t}^{\Phi}\right)}{(N_{S})!}\right]^{N_{R}} (27)$$

$$\Phi_{1} = \Phi(m_{h1}, \Omega_{h1}), \quad \Phi_{2} = \Phi(m_{h2}, \Omega_{h2}), \quad (28)$$

$$\Xi_1 = \Xi \left( m_{g1}, m_{h1}, \Omega_{h1}, N_S \right), \tag{29}$$

and

$$\Xi_2 = \Xi (m_{g2}, m_{h2}, \Omega_{h2}, N_D).$$
(30)

*Proof.* The proof can be done in the same way as the proof of Theorem 2.  $\Box$ 

From (25), we see that the diversity order of the outage probability tends to zero under the fixed interference power constraint.

#### V. SYMBOL ERROR RATE

In this section, we focus on the SER as another important performance evaluation metric. For most modulation schemes, the SER of a conventional wireless communication system can be expressed as [32]

$$P_e = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_\gamma(\gamma) d\gamma, \qquad (31)$$

where a and b are modulation specific constants. For example, a = 1, b = 1 for BPSK (binary phase shift keying), a = 2(M - 1)/M,  $b = 3/(M^2 - 1)$  for M-PAM (M-ary pulse amplitude modulation), and a = 2,  $b = \sin^2(\pi/M)$  for M-PSK (M-ary phase shift keying).

## A. Exact Analysis

Substituting (10) into (31), the SER of  $S \rightarrow R$  link can be derived by utilizing [33, eq.8.310.1], [33, eq.8.352.2], [33, eq.9.211.4.8] and the polynomial expansion. Using the same method,  $P_{e2}$ , which is the SER of  $R \rightarrow D$  link can be easily computed. Substituting the derived expressions of  $P_{e1}$  and  $P_{e2}$ into

$$P_e = 1 - (1 - P_{e1}) \times (1 - P_{e2}), \tag{32}$$

yields the SER of cognitive relay networks with TAS/GSC and DF relaying in the following theorem.

**Theorem 4.** The closed-form expression for the SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading is derived as

$$P_{e} = 1 - \left(1 - \frac{a}{2}\sqrt{\frac{b}{\pi}} \left(\frac{L_{R}}{(m_{g1} - 1)!} \binom{N_{R}}{L_{R}}\right)\right)^{N_{S}} N_{S}!$$

$$\widetilde{\sum_{\mathcal{S}_{R}^{|\mathcal{S}_{K}|}}} \hbar_{k} \Pi\left(m_{h1}, \Omega_{h1}, \theta_{k}, \eta_{k}\right)\right)$$

$$\left(1 - \frac{a}{2}\sqrt{\frac{b}{\pi}} \left(\frac{L_{D}}{(m_{g2} - 1)!} \binom{N_{D}}{L_{D}}\right)^{N_{R}} N_{R}!$$

$$\widetilde{\sum_{\mathcal{S}_{D}^{|\mathcal{S}_{T}|}}} \hbar_{t} \Pi\left(m_{h2}, \Omega_{h2}, \theta_{t}, \eta_{t}\right)\right), \qquad (33)$$

where

$$\Pi(m_h, \Omega_h, \theta, \eta) = \left[ \left( 1 - \frac{\Gamma\left(m_h, \frac{m_h Q}{\Omega_h P}\right)}{\Gamma(m_h)} \right) \frac{\Gamma\left(\theta + \frac{1}{2}\right)}{\left(b + \frac{\eta}{\bar{\gamma}_P}\right)^{\theta + \frac{1}{2}}} \left(\frac{1}{\bar{\gamma}_P}\right)^{\theta} + \frac{1}{\left(m_h - 1\right)!} \left(\theta + m_h - 1\right)! e^{-\frac{m_h Q}{\Omega_h P}} \left(\frac{1}{\eta}\right)^{\theta + \frac{1}{2}} \Gamma\left(\theta + \frac{1}{2}\right) \sum_{m=0}^{\theta + m_h - 1} \frac{1}{m!} \left(\frac{Q}{P}\right)^m \left(\frac{m_h}{\Omega_h}\right)^{m + \frac{1}{2}} \bar{\gamma}_Q^{\frac{1}{2}} + \Psi\left(\theta + \frac{1}{2}, m - m_h + \frac{3}{2}; \left(b + \frac{\eta Q}{\bar{\gamma}_Q P}\right) \frac{\bar{\gamma}_Q m_h}{\eta \Omega_h}\right) \right].$$
(34)

## B. Asymptotic Analysis

## 1) Proportional Interference Power Constraint:

Substituting (14) into (31), together with the help of [33, eq. (3.310)], we derive the asymptotic SER under the proportional interference power constraint in the following theorem.

**Theorem 5.** When Q is proportional to P, the asymptotic SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as

$$P_e^{\infty} = \left(G_c \overline{\gamma}_P\right)^{-G_d} + o\left(\overline{\gamma}_P^{-G_d}\right), \qquad (35)$$

where the diversity order is

$$G_d = N_R \times \min\{m_{g1}N_S, m_{g2}N_D\}$$
(36)

and the SNR gain is

$$G_{c} = \begin{cases} \Lambda_{1}\Delta_{1} & m_{g1}N_{\rm S} < m_{g2}N_{\rm D} \\ \Lambda_{2}\Delta_{2} & m_{g1}N_{\rm S} > m_{g2}N_{\rm D} \\ \Lambda_{1}\Delta_{1} + \Lambda_{2}\Delta_{2} & m_{g1}N_{\rm S} = m_{g2}N_{\rm D} , \end{cases}$$
(37)

where

$$\Lambda_1 = \left[\frac{a}{2}\sqrt{\frac{1}{\pi}}\Gamma\left(m_{g1}N_R N_S + \frac{1}{2}\right)\right]^{-\frac{1}{m_{g1}N_R N_S}}b,\qquad(38)$$

$$\Lambda_2 = \left[\frac{a}{2}\sqrt{\frac{1}{\pi}}\Gamma\left(m_{g2}N_R N_D + \frac{1}{2}\right)\right]^{-\frac{1}{m_{g2}N_R N_D}}b,\qquad(39)$$

## and $\Delta_1$ and $\Delta_2$ are given in (17) and (18), respectively.

Based on (35), we find that the diversity order is independent of the modulation scheme and the peak interference power constraint Q. The fading severity parameters of each hop and the antenna configuration have a direct impact on the diversity order while the interference power constraint at PU has a direct impact on the SNR gain.

2) Fixed Interference Power Constraint:

Substituting (25) into (31), we derive the asymptotic SER under the fixed interference power constraint in the following theorem.

**Theorem 6.** Under the fixed interference power constraint, the asymptotic SER of cognitive spectrum sharing with TAS/GSC and DF relaying in Nakagami-m fading at high SNRs is derived as

 $P_e^{\infty}$ 

=

$$\begin{cases}
\Theta_{1}\left(\Phi_{1}\left(\frac{1}{\bar{\gamma}_{P}}\right)^{m_{g}N_{R}N_{S}}+\Xi_{1}\left(\frac{1}{\bar{\gamma}_{Q}}\right)^{m_{g}N_{R}N_{S}}\right) \\
\Theta_{2}\left(\Phi_{2}\left(\frac{1}{\bar{\gamma}_{P}}\right)^{m_{g1}N_{R}N_{D}}+\Xi_{2}\left(\frac{1}{\bar{\gamma}_{Q}}\right)^{m_{g2}N_{R}N_{D}}\right) \\
\Theta_{2}\left(\Phi_{2}\left(\frac{1}{\bar{\gamma}_{P}}\right)^{m_{g1}N_{R}N_{D}}+\Xi_{2}\left(\frac{1}{\bar{\gamma}_{Q}}\right)^{m_{g2}N_{R}N_{D}}\right) \\
\left(\Theta_{1}\Phi_{1}+\Theta_{2}\Phi_{2}\right)\left(\frac{1}{\bar{\gamma}_{P}}\right)^{m_{g}N_{R}N_{S}} \\
+\left(\Theta_{1}\Xi_{1}+\Theta_{2}\Xi_{2}\right)\left(\frac{1}{\bar{\gamma}_{Q}}\right)^{m_{g}N_{R}N_{S}} \\
m_{g1}N_{S}=m_{g2}N_{D}, \\
(40)
\end{cases}$$

where

$$\Theta_1 = \frac{a\Gamma(m_{g1}N_RN_S + \frac{1}{2})}{2\sqrt{\pi}b^{m_{g1}N_RN_S}}H_1,$$
(41)

$$\Theta_2 = \frac{a\Gamma(m_{g2}N_RN_D + \frac{1}{2})}{2\sqrt{\pi}b^{m_{g2}N_RN_D}}H_2,$$
(42)

and  $H_1$ ,  $H_2$ ,  $\Phi_1$ ,  $\Phi_2$ ,  $\Xi_1$ , and  $\Xi_2$  are given by (26), (27), (28), (28), (29), (30), respectively.

From (40), we find that the diversity order of the SER goes to zero under the fixed interference power constraint.

## VI. ERGODIC CAPACITY

The ergodic capacity is an important performance indicator for cognitive underlay spectrum sharing. It is defined as the maximum achievable long-term rate, where no delay limit is taken into account. Under these assumptions, the ergodic capacity is expressed as

$$C_{erg} = \frac{1}{2} \int_{0}^{\infty} \log_2 \left(1+x\right) f_{\gamma}\left(x\right) dx = \frac{1}{2\ln 2} \int_{0}^{\infty} \frac{1-F_{\gamma}\left(x\right)}{1+x} dx.$$
(43)

To simplify (43), we define  $F_{\gamma_1}(x) = 1 + \tilde{F}_{\gamma_1}(x)$  and  $F_{\gamma_2}(x) = 1 + \tilde{F}_{\gamma_2}(x)$ , and rewrite (43) as

$$C_{erg} = \frac{1}{2\ln 2} \int_{0}^{\infty} \frac{\tilde{F}_{\gamma_{1}}(x) \tilde{F}_{\gamma_{2}}(x)}{1+x} dx,$$
 (44)



Fig. 1. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S}=2$ ,  $N_{\rm R}=3$ ,  $N_{\rm D}=3$ ,  $m_{g1}=1$ ,  $m_{g2}=2$ ,  $m_{h1}=m_{h2}=2$ , and  $\overline{\gamma}_Q=2\overline{\gamma}_P$ .

where

$$\tilde{F}_{\gamma_{1}}(x) = \left(\frac{L_{R}}{(m_{g1}-1)!} \binom{N_{R}}{L_{R}}\right)^{N_{S}} N_{S}! \widetilde{\sum_{\mathcal{S}_{R}^{|\mathcal{S}_{K}|}}} \hbar_{k} \operatorname{sgn}\left(\eta_{k}\right) \Xi_{k}(x)$$
(45)

and

$$\tilde{F}_{\gamma_2}(x) = \left(\frac{L_D}{\left(m_{g2} - 1\right)!} \binom{N_D}{L_D}\right)^{N_R} N_R! \widetilde{\sum_{\mathcal{S}_D^{|\mathcal{S}_T|}}} \hbar_t \operatorname{sgn}\left(\eta_t\right) \Xi_t(x).$$
(46)

In the following, we assume  $m_{h1} = m_{h2} = m_h$  and  $\Omega_{h1} = \Omega_{h2} = \Omega_h$ .

## A. Exact Analysis

Substituting (45) and (46) into (44), and with the help of [33, eq.8.352.2], [33, eq.9.211.4.8], and the partial fraction expression [33, eq.2.102], we obtain a general closed-form expression for the ergodic capacity in the following theorem.

**Theorem 7.** Our new closed-form expression for the ergodic capacity of cognitive TAS/GSC relaying in Nakagami-m fading is given in (47) at the top of the next page. In (47), we have defined the following terms

$$\nabla\left(\theta\right) \stackrel{\triangle}{=} \left(1 - \Gamma\left(m_{h}, \frac{Q}{P} \frac{m_{h}}{\Omega_{h}}\right) \middle/ \Gamma\left(m_{h}\right)\right) \left(\frac{1}{\bar{\gamma}_{P}}\right)^{\theta}, \quad (48)$$

$$\Delta(\theta,\eta,j,k) \stackrel{\triangle}{=} \frac{(\theta+m_h-1)!}{(m_h-1)!} \left(\frac{1}{\bar{\gamma}_Q}\right)^{\theta} e^{-\frac{m_h Q}{\Omega_h P}} \sum_{j=0}^{\theta+m_h-1} \frac{1}{j!}$$

$$\left(\frac{Q}{P}\right)^{j} \sum_{k=0}^{j} {\binom{j}{k}} \left(\frac{m_{h}}{\Omega_{h}}\right)^{m_{h}+j-k} \left(\frac{\eta}{\bar{\gamma}_{Q}}\right)^{k},\tag{49}$$
(50)



Fig. 2. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S}=2$ ,  $N_{\rm R}=3$ ,  $N_{\rm D}=3$ ,  $m_{g1}=1$ ,  $m_{g2}=2$ ,  $m_{h1}=m_{h2}=2$ , and  $\overline{\gamma}_Q=20$  dB.

$$\nu\left(\eta, l, k_{1}, k_{2}\right) \stackrel{\triangle}{=} \Gamma\left(\tau\right) \left(\bar{\gamma}_{Q} m_{h} / \eta \Omega_{h}\right)^{\tau - l} \Psi\left(\tau, \tau + 1 - l; \right. \\ \left(\eta_{t} + \eta_{k}\right) \bar{\gamma}_{Q} m_{h} / \bar{\gamma}_{p} \eta \Omega_{h}\right),$$
(51)

$$\partial(\eta, l) \stackrel{\triangle}{=} \frac{(\bar{\gamma}_Q m_h / \eta \Omega_h - 1)^{l-1}}{(\bar{\gamma}_Q m_h / \eta_t \Omega_h - 1)^{\theta_t + m_h} (\bar{\gamma}_Q m_h / \eta_k \Omega_h - 1)^{\theta_k + m_h}},$$
  

$$\kappa(\theta, \eta, l, j) \stackrel{\triangle}{=} \frac{(-1)^{\theta + m_h - l+1} \binom{j+\theta + m_h - l-1}{j-1}}{(\bar{\gamma}_Q m_h / \Omega_h)^{j+\theta + m_h - l} (1/\eta_t - 1/\eta_k)^{j+\theta + m_h - l}},$$
(52)

with  $\tau = \theta_k + k_1 + \theta_t + k_2 + 1$ .

Our result can be applied and simplified to the special cases of TAS/MRC and TAS/SC in Nakagami-*m* fading channel, as well as TAS/GSC in Rayleigh fading channels.

## B. High SNR Capacity analysis

To examine the capacity performance in the high SNR regime with  $\overline{\gamma}_P \rightarrow \infty$ , we derive the high SNR approximation o the ergodic capacity in closed-form. With the aid of the Jensen's inequality, a tight upper bound on the ergodic capacity is given by [34]

$$C_{erg} = \frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \gamma \right) \right] \le \frac{1}{2} \log_2 \mathbb{E} \left( 1 + \gamma \right).$$
 (53)

Thus, the tight high SNR approximation of the ergodic capacity is presented as [34, 35]

$$C_{erg}^{\infty} \approx \frac{1}{2} \log_2 \mathbb{E} \left( 1 + \gamma \right) \approx \frac{1}{2} \log_2 \mathbb{E} \left( \gamma \right).$$
 (54)

$$\begin{split} C_{erg} &= \frac{1}{2\ln 2} \Big( \frac{L_R}{(m_{g1} - 1)!} \binom{N_R}{L_R} \Big)^{N_S} N_S! \underbrace{\sum_{S_R^{|S_K|}}} \hbar_k \mathrm{sgn} \left( \eta_k \right) \Big( \frac{L_D}{(m_{g2} - 1)!} \binom{N_D}{L_D} \Big)^{N_R} N_R! \underbrace{\sum_{S_R^{|S_K|}}} \hbar_t \mathrm{sgn} \left( \eta_t \right) \Big[ \nabla \left( \theta_k \right) \\ \nabla \left( \theta_t \right) \nu \left( \bar{\gamma}_Q m_h / \Omega_h, 1, 0, 0 \right) + \Delta \left( \theta_t, \eta_t, j_2, k_2 \right) \nabla \left( \theta_k \right) \frac{\left( \nu \left( \bar{\gamma}_Q m_h / \Omega_h, 1, 0, k_2 \right) - \frac{\theta_t + m_h}{l_2 = 1} \frac{\nu (\eta_t, l_2, 0, k_2)}{(\bar{\gamma}_Q m_h / \eta_t \Omega_h - 1)^{1 - l_2}} \right)}{(m_h / \Omega_h - \eta_t / \bar{\gamma}_Q \right)^{\theta_t + m_h}} \\ &+ \frac{\left( \nu \left( \bar{\gamma}_Q m_h / \Omega_h, 1, k_1, 0 \right) - \frac{\theta_k + m_h}{l_1 = 1} \frac{\nu (\eta_k, l_1, k_1, 0)}{(\bar{\gamma}_Q m_h / \eta_k \Omega_h - 1)^{1 - l_1}} \right)}{(m_h / \Omega_h - \eta_k / \bar{\gamma}_Q \right)^{\theta_k + m_h}} \nabla \left( \theta_t \right) \Delta \left( \theta_k, \eta_k, j_1, k_1 \right) + \Delta \left( \theta_k, \eta_k, j_1, k_1 \right) \\ \Delta \left( \theta_t, \eta_t, j_2, k_2 \right) \left( \frac{\bar{\gamma}_Q}{\eta_k} \right)^{\theta_t + m_h} \left( \frac{\bar{\gamma}_Q}{\eta_t} \right)^{\theta_t + m_h} \left[ \partial \left( \eta, 1 \right) \nu \left( \bar{\gamma}_Q m_h / \Omega_h, 1, k_1, k_2 \right) + \sum_{l_1 = 1}^{\theta_k + m_h} \left( -\partial \left( \eta_k, l_1 \right) + \right) \\ \sum_{j = 1}^{\theta_k + m_h} \frac{\kappa \left( \theta_k, \eta_k, l_1, j \right)}{(\bar{\gamma}_Q m_h / \eta_t \Omega_h - 1)^{\theta_t + m_h - j + 1}} \right) \nu \left( \eta_k, l_1, k_1, k_2 \right) + \mathrm{sgn} \left( |\eta_k - \eta_t| \right) \nu \left( \eta_t, l_2, k_1, k_2 \right) \sum_{l_2 = 1}^{\xi_t - m_h} \left( -\partial \left( \eta_t, l_2 \right) \right) \\ + \sum_{i = 1}^{\theta_k + m_h} \frac{\left( -1 \right)^{i + \theta_t + m_h - i_2}}{(\bar{\gamma}_Q m_h / \eta_k \Omega_h - 1)^{\theta_k + m_h - i_1 + 1}} \kappa \left( \theta_t, \eta_t, l_2, i \right) \right) - \sum_{l_3 = \theta_k + m_h + 1}^{\theta_k + \theta_t + 2m_h} \frac{\left( 1 - \mathrm{sgn} \left( |\eta_k - \eta_t| \right) \right) \nu \left( \eta_k, l_3, k_1, k_2 \right)}{(\bar{\gamma}_Q m_h / \eta_k \Omega_h - 1)^{\theta_k + \theta_t + 2m_h - m_t + 1}}} \right] \right]. \tag{47}$$

Therefore, we can rewrite (54) as

$$C_{erg}^{\infty} \approx \frac{1}{2} \log_2 \left( \int_0^\infty x f_{\gamma}(x) \, dx \right) = \frac{1}{2} \log_2 \left( \int_0^\infty (1 - F_{\gamma}(x)) \, dx \right)$$
$$= \frac{1}{2} \log_2 \int_0^\infty \tilde{F}_{\gamma_1}(x) \, \tilde{F}_{\gamma_2}(x) \, dx.$$
(55)

## 1) Proportional Interference Power Constraint:

Based on (55), the high SNR approximation for the ergodic capacity with the proportional interference power constraint is written as

$$C_{erg}^{\infty} = \frac{1}{2} \left[ \log_2(\bar{\gamma}_P) + \log_2(\int_0^{\infty} \tilde{F}_{\gamma_1}(x) \tilde{F}_{\gamma_2}(x) d(\frac{x}{\bar{\gamma}_P})) \right].$$
(56)

Substituting (45) and (46) into (56), and with the help of [33, eq.8.352.2], [33, eq.9.211.4.8] and the binomial expansion, the high SNR approximation for ergodic capacity with the proportional interference power constraint is derived in the following theorem.

**Theorem 8.** When Q is proportional to P, the high SNR approximation of the ergodic capacity is derived as

$$C_{erg}^{\infty} \approx \frac{1}{2} \log_2\left(\overline{\gamma}_P\right) + \frac{1}{2} \log_2\left(\Upsilon\right), \tag{57}$$

where

$$\Upsilon = \left(\frac{L_R}{(m_{g1}-1)!} \binom{N_R}{L_R}\right)^{N_S} N_S! \underbrace{\sum_{S_R^{|S_K|}}}_{S_R^{|S_K|}} \hbar_k \operatorname{sgn}(\eta_k) \\
\left(\frac{L_D}{(m_{g2}-1)!} \binom{N_D}{L_D}\right)^{N_R} N_R! \underbrace{\sum_{S_D^{|S_K|}}}_{S_D^{|S_T|}} \hbar_t \operatorname{sgn}(\eta_t) \\
\left[\lambda^2 \frac{\Gamma\left(\theta_k + \theta_t + 1\right)}{(\eta_k + \eta_t)^{\theta_k + \theta_t + 1}} + \lambda \sum_{j_2=0}^{\theta_t + m_h - 1} \sum_{k_2=0}^{j_2} \Delta_s\left(\theta_t, j_2, k_2\right)\right] \\
\frac{\nu_s\left(\eta_t, \theta_t, 0, k_2\right)}{(1/\mu)^{\theta_k + j_2 + 1}} + \lambda \sum_{j_1=0}^{\theta_k + m_h - 1} \sum_{k_1=0}^{j_1} \frac{\Delta_s\left(\theta_k, j_1, k_1\right)}{(1/\mu)^{\theta_t + j_1 + 1}} \tag{58}$$

$$\nu_s\left(\eta_k, \theta_k, k_1, 0\right) + \sum_{j_1=0}^{\theta_k + m_h - 1} \sum_{j_2=0}^{j_1} \sum_{k_2=0}^{\theta_t + m_h - 1} \sum_{j_2=0}^{j_2} \Omega \\
- \frac{\Delta_s\left(\theta_k, j_1, k_1\right) \Delta_s\left(\theta_t, j_2, k_2\right)}{(m_h/\Omega_h)^{\theta_k + \theta_t + 1}(1/\mu)^{j_1 + j_2 + 1}} \right].$$
(59)

In (58), we have defined

$$\Delta_s(\theta, j, k) \stackrel{\triangle}{=} \frac{(\theta + m_h - 1)!}{(m_h - 1)! j!} e^{-\mu \frac{m_h}{\Omega_h}} \binom{j}{k} \left(\frac{m_h}{\Omega_h}\right)^{\theta_k + \theta_t + j + 1 - \theta}$$
(60)

$$\kappa_s\left(\theta,\eta,l\right) \stackrel{\triangle}{=} \frac{(-1)^{\left(\theta+m_h-l\right)} \left(\frac{\theta_k+\theta_t+2m_h-l-1}{\theta+m_h-l}\right)}{\left(1/\eta_t-1/\eta_k\right)^{\theta_k+\theta_t+2m_h-l}\eta^{k_1+k_2-l}},\tag{61}$$

$$\nu_{s}\left(\eta,\varepsilon,k_{1},k_{2}\right) \stackrel{\triangle}{=} \Gamma\left(\theta_{k}+k_{1}+\theta_{t}+k_{2}+1\right)\left(1/\eta\right)^{\theta_{k}+\theta_{t}+1} \Psi\left(\theta_{k}+k_{1}+\theta_{t}+k_{2}+1,\ \theta_{k}+k_{1}+\theta_{t}+k_{2}+2-m_{h}-\varepsilon;\right) \left(\frac{\eta_{t}+\eta_{k}}{\Omega_{h}\eta}\right),$$
(62)

and



Fig. 3. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S} = 2$ ,  $N_{\rm R} = 3$ ,  $N_{\rm D} = 3$ ,  $m_{g1} = 1$ ,  $m_{g2} = 2$ , and  $\overline{\gamma}_Q = 2\overline{\gamma}_P$ .

$$\Omega \stackrel{\Delta}{=} (1 - \operatorname{sgn}(|\eta_k - \eta_t|)) \nu_s(\eta_k, \theta_k + \theta_t + m_h, k_1, k_2),$$

$$+ \frac{\operatorname{sgn}(|\eta_k - \eta_t|)}{\eta_k^{\theta_k + m_h - k_1} \eta_t^{\theta_t + m_h - k_2}} \left[ \sum_{l_1=1}^{\theta_k + m_h} \kappa_s(\theta_k, \eta_k, l_1) \right]$$

$$\nu_s(\eta_k, l_1 - m_h, k_1, k_2) + \sum_{l_2=1}^{\theta_t + m_h} (-1)^{\theta_k + \theta_t + 2m_h - l_2} \left[ \kappa_s(\theta_t, \eta_t, l_2) \nu_s(\eta_t, l_2 - m_h, k_1, k_2) \right].$$
(63)

Note that similar as the asymptotic ergodic capacity, the tight high SNR approximations can well predict the performance behaviours in the high SNR regime. Thus, we deduce the high SNR scaling law from the high SNR approximations similar to the approach in [36] and [37]. Based on (57), we characterize two key parameters determining the affine approximation of the ergodic capacity in the high SNR regime, namely the high SNR slope and the high SNR power offset [38]. The high SNR slope is also known as the degrees of freedom or the multiplexing gain [39]. The high SNR power offset captures the joint effects of the fading model, the number of antennas at each terminal, and the interference power constraint. We represent the high SNR approximation of the ergodic capacity as [38]

$$C_{erg}^{\infty} \approx S_{\infty} \left( \log_2 \left( \overline{\gamma}_P \right) - \mathcal{L}_{\infty} \right), \tag{64}$$

where  $S_{\infty}$  is the high SNR slope in bits/s/Hz/(3 dB)

$$S_{\infty} = \lim_{\overline{\gamma}_P \to \infty} \frac{C_{erg}^{\infty}}{\log_2\left(\overline{\gamma}_P\right)} = \frac{1}{2}$$
(65)

and  $\mathcal{L}_\infty$  is the high SNR power offset in 3 dB units

$$\mathcal{L}_{\infty} = \lim_{\overline{\gamma}_P \to \infty} \left( \log_2\left(\overline{\gamma}_P\right) - \frac{C_{erg}^{\infty}}{S_{\infty}} \right) = -\log_2\left(\Upsilon\right).$$
(66)



Fig. 4. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S}=2$ ,  $N_{\rm R}=3$ ,  $N_{\rm D}=3$ ,  $m_{g1}=1$ ,  $m_{g2}=2$ , and  $\overline{\gamma}_Q=25$  dB.

From (65), we see that the high SNR slope  $S_{\infty}$  is independent of the interference power constraint, the selected number of antennas at the receiver, and the primary network. We also see that the high SNR power offset  $\mathcal{L}_{\infty}$  is independent of  $\overline{\gamma}_{P}$  from (66).

2) Fixed Interference Power Constraint:

Substituting (45) and (46) into (55), we obtain the high SNR approximation of the ergodic capacity under the fixed interference power constraint.

**Theorem 9.** When Q is fixed, the high SNR approximation of the ergodic capacity is given in (67) at the top of the next page.

From (67), we find that for the fixed interference power constraint, the high SNR slope collapses to zero.

## VII. NUMERICAL RESULTS

In this section, we present numerical results to verify our new analytical derivations for cognitive TAS/GSC relaying in Nakagami-*m* fading channels. We set the threshold SNR as  $\gamma_{th} = 5$  dB. All the figures clearly show that the exact curves are in precise agreement with the Monte Carlo simulations. Importantly, the asymptotic lines accurately predict the exact behaviour in the high SNR regime.

Fig. 1 plots the outage probability with the proportional interference constraint as we vary  $\mu$ ,  $L_R$  and  $L_D$ . The exact and asymptotic curves are plotted by using (13) and (14), respectively. For the same  $\mu$ , we observe that the outage probability decreases with increasing  $L_R$  and  $L_D$ , due to an increase in the SNR gain (16). We also confirm that the diversity order is independent of  $L_R$  and  $L_D$  as reflected by the parallel slope. Another observation is that the outage probability decreases with increasing  $\mu$ , which is due to the relaxed interference power constraint at the PU receiver.

$$C_{erg}^{\infty} = \frac{1}{2} \log_2 \left[ \left( \frac{L_R}{(m_{g1} - 1)!} \binom{N_R}{L_R} \right) \right)^{N_S} N_S! \widetilde{\sum_{S_R^{|S_K|}}} \hbar_k \operatorname{sgn}(\eta_k) \left( \frac{L_D}{(m_{g2} - 1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{S_D^{|S_T|}}} \hbar_t \operatorname{sgn}(\eta_t) \left[ \lambda^2 \bar{\gamma}_P \right] \right]^{N_S} N_S! \widetilde{\sum_{S_R^{|S_K|}}} \hbar_k \operatorname{sgn}(\eta_k) \left( \frac{L_D}{(m_{g2} - 1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{S_D^{|S_T|}}} \hbar_t \operatorname{sgn}(\eta_t) \left[ \lambda^2 \bar{\gamma}_P \right]^{N_S} N_S! \widetilde{\sum_{S_R^{|S_K|}}} h_k \operatorname{sgn}(\eta_k) \left( \frac{1}{(m_{g2} - 1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{S_D^{|S_T|}}} \hbar_t \operatorname{sgn}(\eta_t) \left[ \lambda^2 \bar{\gamma}_P \right]^{N_S} N_S! \widetilde{\sum_{S_R^{|S_K|}}} h_k \operatorname{sgn}(\eta_k) \left( \frac{1}{(m_{g2} - 1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{S_D^{|S_T|}}} h_t \operatorname{sgn}(\eta_t) \left[ \lambda^2 \bar{\gamma}_P \right]^{N_S} N_S! \widetilde{\sum_{S_R^{|S_T|}}} h_k \operatorname{sgn}(\eta_k) \left( \frac{1}{(m_{g2} - 1)!} \binom{N_D}{L_D} \right)^{N_R} N_R! \widetilde{\sum_{S_D^{|S_T|}}} h_t \operatorname{sgn}(\eta_t) \left[ \lambda^2 \bar{\gamma}_P \right]^{N_S} N_S! \widetilde{\sum_{S_D^{|S_T|}}} h_t$$



Fig. 5. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S}=2$ ,  $N_{\rm R}=3$ ,  $N_{\rm D}=3$ ,  $m_{g1}=1$ ,  $m_{g2}=2$ ,  $m_{h1}=m_{h2}=2$ , and  $\overline{\gamma}_Q=2\overline{\gamma}_P$ .



Fig. 3 plots the exact and asymptotic SER with the proportional interference power constraint from (33) and (35), respectively. The plot confirms that the diversity order is independent of the modulation scheme,  $L_R$ , and  $L_D$ . We see that the SER decreases as  $L_R$  and  $L_D$  increase. We also see that BPSK outperforms QPSK, which is predicted from the SNR gain (16).

Fig. 4 plots the exact and asymptotic SER with the fixed interference power constraint from (33) and (40), respectively.



Fig. 6. Cognitive spectrum sharing with TAS/GSC and DF relaying:  $N_{\rm S}=2$ ,  $N_{\rm R}=3$ ,  $N_{\rm D}=3$ ,  $m_{g1}=1$ ,  $m_{g2}=2$ ,  $m_{h1}=m_{h2}=2$ , and  $\overline{\gamma}_Q=25$  dB.

We see that the SER decreases as  $L_R$  and  $L_D$  increase, and BPSK outperforms QPSK. Similar to Fig. 2, the SER becomes saturated for  $\overline{\gamma}_P > 22$  dB, which confirms that the diversity order goes to zero.

Fig. 5 plots the exact ergodic capacity and its high SNR approximation with the proportional interference power constraint from (47) and (57), respectively. We see that the high SNR approximations of the ergodic capacity are tight and well predict the behavior of the ergodic capacity at high SNRs. It is obvious that the ergodic capacity can be improved by increasing  $L_R$  and  $L_D$ . The parallel curves confirm that the high SNR slope is independent of  $L_R$  and  $L_D$ .

Fig. 6 examines the impact of the fixed interference power constraint on the ergodic capacity. The exact ergodic capacity and its high SNR approximation are from (47) and (67), respectively. Interestingly, we find that the capacity ceiling occurs for  $\overline{\gamma}_P > 30$  dB. This is due to the fact that when  $\gamma_P \to \infty$ ,  $\min(P, Q/|h_{1i^*}|^2) \approx (Q/|h_{1i^*}|^2)$  and  $\min(P, Q/|h_{2j^*}|^2) \approx (Q/|h_{2j^*}|^2)$ . Once again, the fixed interference power constraint becomes the dominant factor.

By setting  $L_R = L_D = 1$  and  $L_R = L_D = 3$ , we see that TAS/MRC outperforms TAS/GSC and TAS/GSC outperforms TAS/SC.

## VIII. CONCLUSIONS

We have taken into account the cognitive DF relay network with TAS/GSC over Nakagami-m fading. This framework is well suited for the reliability enhancement of the secondary network and interference alleviation of the primary network. We derived new statistical properties of the end-to-end SNR. Based on these, we have derived closed-form expressions for the exact and asymptotic outage probability, symbol error rate, and ergodic capacity with the proportional and the fixed interference power constraints. Our results are valid for Nakagamim fading and arbitrary number of antennas in the secondary network. Based on the relationship of the maximum transmit power constraint and peak interference power constraint, we conclude that: 1) under the proportional interference power constraint, the diversity order is determined by the fading parameter and the antenna configuration of the secondary network, and the high SNR slope is 1/2; and 2) under the fixed interference power constraint, the diversity order is zero with error floor, and the high SNR slope is zero with capacity ceiling.

## APPENDIX A A Proof of Lemma 1

We first present the probability density function (PDF) and CDF for the channel power gain of a single branch of the secondary network channel with the Nakagami-m fading as [40]

$$f(x) = \frac{x^{m_{g1}-1}}{(m_{g1}-1)!} \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}} e^{-\frac{m_{g1}}{\Omega_{g1}}x}$$
(A.1)

and

$$F(x) = 1 - \frac{\Gamma\left(m_{g1}, x \frac{m_{g1}}{\Omega_{g1}}\right)}{\Gamma(m_{g1})},$$
(A.2)

respectively. The marginal moment generating function (MGF) of (A.1) is given by [19]

$$\Phi(s,x) = \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}} \sum_{i=0}^{m_{g1}-1} \frac{x^i e^{-\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)x}}{i! \left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}-i}}.$$
 (A.3)

As shown in [19, 24], the MGF expression for the channel power gain with GSC is expressed as

$$\Phi_{GSC}(s) = L_R \binom{N_R}{L_R} \int_0^\infty e^{-sx} f(x) \left(\Phi(s,x)\right)^{L_R-1} (F(x))^{N_R-L_R} dx.$$
(A.4)

Here the MGF is defined as  $\Phi_{\gamma}(s) = E[e^{-\gamma s}].$ 

Based on (A.3), and using the multinomial theorem [41], we rewrite  $(\Phi(s, x))^{L_R-1}$  as

$$(\Phi(s,x))^{L_{R}-1} = \left(\frac{m_{g}}{\Omega_{g}}\right)^{m_{g}\left(L_{R}-1\right)} \sum_{\mathcal{S}_{K}^{\Phi}} a_{k}^{\Phi} x^{b_{k}^{\Phi}} e^{-c_{k}^{\Phi} x}$$

$$\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_{k}^{\Phi} - m_{g1}\left(L_{R}-1\right)},$$
(A.5)

where  $S_K^{\Phi} = \left\{ \left( n_{k,0}^{\Phi}, \dots, n_{k,m_{g_1}-1}^{\Phi} \right) \Big| \sum_{i=0}^{m_{g_1}-1} n_{k,i}^{\Phi} = L_R - 1 \right\}$ with  $\left\{ n_{k,i}^{\Phi} \right\} \in \mathbb{Z}^+, a_k^{\Phi}, b_k^{\Phi}$ , and  $c_k^{\Phi}$  are, respectively, given by

$$a_{k}^{\Phi} = \frac{\left(L_{R}-1\right)!}{\prod_{i=0}^{m_{g_{1}}-1} n_{k,i}^{\Phi}!} \prod_{i=0}^{m_{g_{1}}-1} \left(\frac{1}{i!}\right)^{n_{k,i}^{\Phi}}, b_{k}^{\Phi} = \sum_{i=0}^{m_{g_{1}}-1} n_{k,i}^{\Phi}i, \quad (A.6)$$

and 
$$c_k^{\Phi} = (L_R - 1) \left( s + \frac{m_{g1}}{\Omega_{g1}} \right).$$
 (A.7)

Based on (A.2), we proceed to employ the multinomial theorem to express  $(F(x))^{N_R-L_R}$  as

$$(F(x))^{N_R - L_R} = \sum_{\mathcal{S}_K^F} a_k^F x^{b_k^F} e^{-c_k^F x},$$
 (A.8)

where  $\mathcal{S}_{K}^{F} = \left\{ \left( n_{k,0}^{F}, \dots, n_{k,m_{g_{1}}}^{F} \right) \Big| \sum_{j=0}^{m_{g}} n_{k,j}^{F} = N_{R} - L_{R} \right\}$ with  $\left\{ n_{k,j}^{F} \right\} \in \mathbb{Z}^{+}, a_{k}^{F}, b_{k}^{F}$ , and  $c_{k}^{F}$  are, respectively, given by

$$a_{k}^{F} = \frac{\left(N_{R} - L_{R}\right)!}{\prod_{j=0}^{m_{g1}} n_{k,j}^{F}!} \prod_{j=0}^{m_{g1}-1} \left(\frac{-1}{j!}\right)^{n_{k,j+1}^{F}} \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{b_{k}^{F}}, \quad (A.9)$$
$$b_{k}^{F} = \sum_{j=0}^{m_{g1}-1} j n_{k,j+1}^{F}, \text{and} \quad c_{k}^{F} = \frac{m_{g1}}{\Omega_{g1}} \sum_{j=1}^{m_{g1}} n_{k,j}^{F}. \quad (A.10)$$

Substituting (A.1), (A.5) and (A.8) into (A.4), and applying [33, eq. (3.351.3)],  $\Phi_{GSC}(s)$  is derived as

$$\Phi_{GSC}(s) = \frac{L_R}{(m_{g1}-1)!} {N_R \choose L_R} \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}L_R} \sum_{S_k^{\Phi} \in \mathcal{S}_K^{\Phi}} \sum_{S_k^{F} \in \mathcal{S}_K^{F}} a_k^{\Phi} a_k^{F} \\ \frac{\left(b_k^{\Phi} + b_k^{F} + m_{g1} - 1\right)! \left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi} - m_{g1}} \left(L_R - 1\right)}{\left(s + c_k^{\Phi} + c_k^{F} + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi} + b_k^{F} + m_{g1}}}.$$
(A.11)

Let  $F_{GSC}(x)$  denote the CDF of the channel power gain of the secondary network with GSC. The Laplace transform of  $F_{GSC}(x)$  is given by  $\mathcal{L}[F_{GSC}(x)] = \Phi_{GSC}(s)/s$  [20]. Therefore, the Laplace transform for  $F_{GSC}(x)$  is

$$\mathcal{L}\left[F_{GSC}\left(x\right)\right] = \frac{L_R}{(m_{g1} - 1)!} \binom{N_R}{L_R} \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}L_R} \\ \sum_{\mathcal{S}_K^{\Phi}} \sum_{\mathcal{S}_K^{F}} a_k^{\Phi} a_k^F \frac{\left(b_k^{\Phi} + b_k^F + m_{g1} - 1\right)!}{(L_R)^{b_k^{\Phi} + b_k^F + m_{g1}}} \\ \frac{\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi} - m_{g1}}(L_R - 1)}{s\left(s + \frac{c_k^F}{L_R} + \frac{m_{g1}}{\Omega_{g1}}\right)^{b_k^{\Phi} + b_k^F + m_{g1}}}.$$
 (A.12)

Using the partial fraction expansion [33, eq. (2.102)], we can rewrite (A.12) in an equivalent form. Then, taking the inverse Laplace transform of  $\mathcal{L}[F_{GSC}(x)]$ , we obtain

$$F_{GSC}(x) = \frac{L_R}{(m_{g1} - 1)!} \binom{N_R}{L_R} \sum_{\mathcal{S}_K} \sum_{n=0}^{m_{g1}L_R + b_k^F} a_k^{\Phi} a_k^F \frac{(b_k^{\Phi} + b_k^F + m_{g1} - 1)!}{L_R^{b_k^{\Phi} + b_k^F + m_{g1}}} \ell_k(n) x^{\mu_k(n)} e^{-\nu_k(n)x},$$
(A.13)

where the set  $S_K$  has been defined in *Lemma 1*,  $\ell_k(n)$ ,  $\mu_k(n)$  and  $\nu_k(n)$  are, respectively, given by

$$\ell_{k}(n) = \begin{cases} \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_{k}(n)-b_{k}^{F}} \left(\frac{1}{L_{R}}\sum_{j=1}^{m_{g1}}n_{k,j}^{F}+1\right)^{-n_{2}} \\ & n = 0 \\ \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_{k}(n)-b_{k}^{F}} \left(\Upsilon_{k1}+\Upsilon_{k2}-\frac{1-\operatorname{sgn}\left(c_{k}^{F}\right)}{(n-1)!}\right) \\ & 1 \le n \le m_{g1}\left(L_{R}-1\right)-b_{k}^{\Phi} \\ \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{\mu_{k}(n)-b_{k}^{F}} \left(\Upsilon_{k3}+\Upsilon_{k4}-\frac{1-\operatorname{sgn}\left(c_{k}^{F}\right)}{(n-1)!}\right) \\ & m_{g1}\left(L_{R}-1\right)-b_{k}^{\Phi} < n \le m_{g1}L_{R}+b_{k}^{F} \end{cases}$$
(A.14)

$$\begin{aligned} & \mu_k(n) \\ & = \begin{cases} 0, & n = 0 \\ n - 1, & 1 \le n \le m_{g1} \left( L_R - 1 \right) - b_k^{\Phi} \\ n - \operatorname{sgn} \left( c_k^F \right) \left( m_{g1} \left( L_R - 1 \right) - b_k^{\Phi} \right) - 1, \\ & m_{g1} \left( L_R - 1 \right) - b_k^{\Phi} < n \le m_{g1} L_R + b_k^F \\ & (A.15) \end{aligned}$$

and

$$\nu_{k}(n) = \begin{cases} 0, & n = 0\\ \frac{m_{g1}}{\Omega_{g1}}, & 1 \le n \le m_{g1} (L_{R} - 1) - b_{k}^{\Phi}\\ \frac{c_{k}}{L_{R}} + \frac{m_{g1}}{\Omega_{g1}}, & m_{g1} (L_{R} - 1) - b_{k}^{\Phi} < n \le m_{g1} L_{R} + b_{k}^{F}, \end{cases}$$
(A.16)

with  $n_2 = b_k^{\Phi} + b_k^F + m_{g1}$ . In (A.14),  $\Upsilon_{k1}$ ,  $\Upsilon_{k2}$ ,  $\Upsilon_{k3}$ , and  $\Upsilon_{k4}$ 

are given by

$$\Upsilon_{k1} = -\frac{\operatorname{sgn}(c_k^F)}{(n-1)!} \left(\frac{1}{L_R} \sum_{j=1}^{m_{g1}} n_{k,j}^F + 1\right)^{-n_2},$$
(A.17)

$$\Upsilon_{k2} = (-1)^{1-n_1} \frac{\operatorname{sgn}(c_k^F)}{(n-1)!} \sum_{l=1}^{n_2} \left(\frac{1}{L_R} \sum_{n=1}^{m_{g1}} n_{k,n}^F + 1\right)^{-(n_2-l+1)} \\ {\binom{l-n_1-1}{l-1}} \left(\frac{1}{L_R} \sum_{n=1}^{m_{g1}} n_{k,n}^F\right)^{n_1-l},$$
(A.18)

$$\Upsilon_{k3} = -\frac{\operatorname{sgn}(c_k^F)}{(n_1 - 1)!} \left(\frac{1}{L_R} \sum_{j=1}^{m_{g_1}} n_{k,j}^F + 1\right)^{-(n_2 - n_1 + 1)}, \quad (A.19)$$

and

$$\Upsilon_{k4} = \frac{\operatorname{sgn}(c_k^F)}{(n_1 - 1)!} \sum_{l=1}^{m_{g_1}(L_R - 1) - b_k^{\Phi}} (-1)^{l+1} \\ \binom{n_2 - n_1 + l - 1}{l - 1} (\frac{1}{L_R} \sum_{j=1}^{m_{g_1}} n_{k,j}^F)^{-(n_2 - n_1 + l)},$$
(A.20)

where  $n_1 = n - m_{g1} (L_R - 1) + b_k^{\Phi}$ . The CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  is given by  $F_{\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2}(x) = (F_{GSC}(x))^{N_S}$ . Based on (A.13), and employing the multinomial theorem, we can derive the CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  as (6).

## APPENDIX B A Proof of Lemma 2

According to (4), the CDF of  $\gamma_1$  can be written as

$$F_{\gamma_{1}}(x) = \Pr\left\{ \left\| \mathbf{g}_{1i^{*}\theta_{i^{*}}} \right\|^{2} \leq \frac{x}{\overline{\gamma}_{P}}, \left| h_{1i^{*}} \right|^{2} \leq \frac{Q}{P} \right\} + \Pr\left\{ \frac{\left\| \mathbf{g}_{1i^{*}\theta_{i^{*}}} \right\|^{2}}{\left| h_{1i^{*}} \right|^{2}} \leq \frac{x}{\overline{\gamma}_{Q}}, \left| h_{1i^{*}} \right|^{2} \geq \frac{Q}{P} \right\}.$$
 (B.1)

The CDF of  $\left|h_{_{1i^{*}}}\right|^{2}$  is expressed as

$$F_{|h_{1i^*}|}(x) = 1 - \frac{\Gamma\left(m_{h1}, x\frac{m_{h1}}{\Omega_{h1}}\right)}{\Gamma\left(m_{h1}\right)}$$
(B.2)

By substituting (B.2) and (6) into (B.1), we derive the closed-form expression of  $F_{\gamma_1}(x)$  as (10).

## APPENDIX C A Proof of Theorem 2

Based on (A.8), we consider transmission in the high SNR regime with  $\bar{\gamma}_P \to \infty$ . Applying the Taylor series expansion truncated to the *k*th order given by  $e^x = \sum_{j=0}^k \frac{x^j}{j!} + o(x^k)$  in (A.8), the asymptotic expression for (A.8) is written as

$$(F(x))^{N_R-L_R} = \frac{\left(\frac{m_{g_1}}{\Omega_{g_1}}\right)^{m_{g_1}\left(N_R-L_R\right)} x^{m_{g_1}\left(N_R-L_R\right)}}{\left(m_{g_1}!\right)^{N_R-L_R}}.$$
 (C.1)

Substituting (A.1), (A.5) and (C.1) into (A.4) yields

$$\Phi_{GSC}(s) = \left(\frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}N_R} \frac{\mathrm{K}\left(\mathcal{S}_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi}\right)}{\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{m_{g1}N_R}}.$$
(C.2)

Note that  $\mathcal{L}[F_{GSC}(x)] = \Phi_{GSC}(s) / s$  [20], we derive

$$L[F_{GSC}(x)] = K\left(\mathcal{S}_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi}\right)$$
$$\left(\frac{1}{s} - \sum_{r=1}^{m_{g1}N_{R}} \frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{r-1}}{\left(s + \frac{m_{g1}}{\Omega_{g1}}\right)^{r}}\right).$$
(C.3)

Taking the inverse Laplace transform of (C.3), we obtain

$$F_{GSC}(x) = K \left( S_K^{\Phi}, N_R, L_R, m_{g1}, a_k^{\Phi}, b_k^{\Phi} \right) \\ \left( 1 - \sum_{r=1}^{m_{g1}N_R} \frac{\left(\frac{m_{g1}}{\Omega_{g1}}\right)^{r-1}}{(r-1)!} x^{r-1} e^{-\frac{m_{g1}}{\Omega_{g1}}x} \right).$$
(C.4)

Again, employing the Taylor series expansion truncated to the kth order given by  $e^x = \sum_{j=0}^k \frac{x^j}{j!} + o(x^k)$  in (C.4), (C.4) can be rewritten as

$$F_{GSC}(x) = \left(\frac{m_{g1}x}{\Omega_{g1}}\right)^{m_{g1}N_R} \mathcal{K}\left(\mathcal{S}_K^{\Phi}, N_R, L_R, m_{g1}, a_k^{\Phi}, b_k^{\Phi}\right).$$
(C.5)

Based on (C.5), the asymptotic expression for the CDF of  $\|\mathbf{g}_{1i^*\theta_{i^*}}\|^2$  is given by

$$F_{\left\|\mathbf{g}_{1i^{*}\theta_{i^{*}}}\right\|^{2}}\left(x\right) = \left[\left(\frac{m_{g1}x}{\Omega_{g1}}\right)^{m_{g1}N_{R}} \mathrm{K}\left(\mathcal{S}_{K}^{\Phi}, N_{R}, L_{R}, m_{g1}, a_{k}^{\Phi}, b_{k}^{\Phi}\right)\right]^{N_{S}}.$$
 (C.6)

By substituting (C.6) into (B.1), the first non-zero order expansion of the CDF of  $\gamma_1$  is attained and yields the asymptotic outage probability of cognitive relay network with the proportional interference power constraint as (14).

#### REFERENCES

- [1] P. Kolodzy and I. Avoidance, "Spectrum policy task force," Federal Commun. Comm., Washington, DC, Rep. ET Docket, no. 02-135, 2002.
- [2] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [3] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1811–1822, May 2010.
- [4] K. J. Kim, T. Q. Duong, and X.-N. Tran, "Performance analysis of cognitive spectrum-sharing single-carrier systems with relay selection," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6435–6449, Dec. 2012.
- [5] T. Xu, J. Ge, and H. Ding, "An efficient distributed link selection scheme for AF-based cognitive selection relaying networks," *IEEE Commun. Lett.*, vol. 18, no. 2, pp. 253–256, Feb. 2014.
- [6] C. Zhong, T. Ratnarajah, and K.-K. Wong, "Outage analysis of decodeand-forward cognitive dual-hop systems with the interference constraint in Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. 60, no. 6, pp. 2875–2879, Jul. 2011.
- [7] T. Q. Duong, P. L. Yeoh, V. N. Q. Bao, M. Elkashlan, and N. Yang, "Cognitive relay networks with multiple primary transceivers under spectrum-sharing," *IEEE Signal Process. Lett.*, vol. 19, no. 11, pp. 741– 744, Nov. 2012.

- [8] C. K. Datsikas, N. C. Sagias, F. I. Lazarakis, and G. S. Tombras, "Outage analysis of decode-and-forward relaying over Nakagami-m fading channels," *IEEE Signal Process. Lett.*, vol. 15, pp. 41–44, Jan. 2008.
- [9] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.
- [10] H. Ding, J. Ge, D. B. da Costa, and Z. Jiang, "Link selection schemes for selection relaying systems with transmit beamforming: New and efficient proposals from a distributed concept," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 533–552, Feb. 2012.
- [11] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
- [12] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics Signal. Process.*, 2008.
- [13] H. Ding, J. Ge, D. B. da Costa, and T. Tsiftsis, "A novel distributed antenna selection scheme for fixed-gain amplify-and-forward relaying systems," *IEEE Trans. Veh. Technol.*, vol. 61, no. 6, pp. 2836–2842, Jul. 2012.
- [14] V. Blagojevic and P. Ivanis, "Ergodic capacity for TAS/MRC spectrum sharing cognitive radio," *IEEE Commun. Lett.*, vol. 16, no. 3, pp. 321– 323, Mar. 2012.
- [15] P. L. Yeoh, M. Elkashlan, T. Q. Duong, N. Yang, and D. B. da Costa, "Transmit antenna selection for interference management in cognitive relay networks," *IEEE Trans. Veh. Technol.*, 2014.
- [16] Y. Deng, M. Elkashlan, P. Yeoh, N. Yang, and R. Mallik, "Cognitive MIMO relay networks with generalized selection combining," *IEEE Trans. Wireless Commun.*, vol. PP, no. 99, pp. 1–1, May. 2014.
- [17] K. Letaief and W. Zhang, "Cooperative communications for cognitive radio networks," *Proc. IEEE*, vol. 97, no. 5, pp. 878–893, May. 2009.
- [18] N. B. Mehta, S. Kashyap, and A. F. Molisch, "Antenna selection in LTE: From motivation to specification," *IEEE Commun. Mag.*, vol. 50, no. 10, pp. 144–150, Oct. 2012.
- [19] A. Annamalai and C. Tellambura, "A new approach to performance evaluation of generalized selection diversity receivers in wireless channels," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Fall 2001, pp. 2309–2313.
- [20] X. Cai and G. B. Giannakis, "Performance analysis of combined transmit selection diversity and receive generalized selection combining in Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1980–1983, Nov. 2004.
- [21] M.-S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalized selection combining over rayleigh fading channels," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 401–415, Mar. 2000.
- [22] I. Ahmed, A. Nasri, R. Schober, and R. K. Mallik, "Asymptotic performance of generalized selection combining in generic noise and fading," *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 916–922, Apr. 2012.
- [23] A. Lodhi, F. Said, M. Dohler, and A. H. Aghvami, "Closed-form symbol error probabilities of STBC and CDD MC-CDMA with frequencycorrelated subcarriers over Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. 57, no. 2, pp. 962–973, Mar. 2008.
- [24] Y. Ma and S. Pasupathy, "Efficient performance evaluation for generalized selection combining on generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 29–34, Jan. 2004.
- [25] J. Lee, H. Wang, J. G. Andrews, and D. Hong, "Outage probability of cognitive relay networks with interference constraints," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 390–395, Feb. 2011.
- [26] H. Ding, J. Ge, D. B. da Costa, and Z. Jiang, "Asymptotic analysis of cooperative diversity systems with relay selection in a spectrum-sharing scenario," *IEEE Trans. Veh. Technol.*, vol. 60, no. 2, pp. 457–472, Feb. 2011.
- [27] I. Krikidis, H. Suraweera, P. Smith, and C. Yuen, "Full-duplex relay selection for amplify-and-forward cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4381–4393, Dec. 2012.
- [28] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 649–658, Feb. 2007.
- [29] J. M. Peha, "Approaches to spectrum sharing," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 10–12, Feb. 2005.
- [30] K. J. R. Liu, A. K. Sadek, and W. Su, et al., Cooperative Communications and Networking. Cambridge University Press, 2009.
- [31] T. C. Clancy, "Formalizing the interference temperature model," Wireless Commun. Mobile Comput., vol. 7, no. 9, pp. 1077–1086, May 2007.

- [32] M. K. Simon and M.-S. Alouini, *Digital communication over fading channels*. New York: Wiley, 2005.
- [33] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. San Diego, C.A.: Academic Press, 2007.
- [34] H. Shin and J. H. Lee, "Capacity of multiple-antenna fading channels: spatial fading correlation, double scattering, and keyhole," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2636–2647, Oct. 2003.
- [35] S. S. Ikki and S. Aissa, "Multihop wireless relaying systems in the presence of cochannel interferences: Performance analysis and design optimization," *IEEE Trans. Veh. Technol.*, vol. 61, no. 2, pp. 566–573, Feb. 2012.
- [36] V. Gopal, M. Matthaiou, and C. Zhong, "Performance analysis of distributed MIMO systems in Rayleigh/Inverse-Gaussian fading channels," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2012, pp. 2468–2474.
- [37] M. Matthaiou, N. D. Chatzidiamantis, G. K. Karagiannidis, and J. A. Nossek, "On the capacity of generalized-K fading MIMO channels," *IEEE Trans. Signal Process.*, vol. 58, no. 11, pp. 5939–5944, Nov. 2010.
- [38] S. Jin, M. R. McKay, C. Zhong, and K.-K. Wong, "Ergodic capacity analysis of amplify-and-forward MIMO dual-hop systems," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2204–2224, May 2010.
- [39] E. Bjornson, P. Zetterberg, M. Bengtsson, and B. Ottersten, "Capacity limits and multiplexing gains of MIMO channels with transceiver impairments," *IEEE Commun. Lett.*, vol. 17, no. 1, pp. 91–94, Jan. 2013.
- [40] D. B. da Costa and S. Aissa, "Cooperative dual-hop relaying systems with beamforming over Nakagami-m fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 3950–3954, Aug. 2009.
- [41] R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics*. New York: Addison-Wesley, 1989.



Maged Elkashlan (M'06) received the Ph.D. degree in Electrical Engineering from the University of British Columbia, Canada, 2006. From 2006 to 2007, he was with the Laboratory for Advanced Networking at University of British Columbia. From 2007 to 2011, he was with the Wireless and Networking Technologies Laboratory at Commonwealth Scientific and Industrial Research Organization (C-SIRO), Australia. During this time, he held an adjunct appointment at University of Technology Sydney, Australia. In 2011, he joined the School

of Electronic Engineering and Computer Science at Queen Mary University of London, UK, as an Assistant Professor. He also holds visiting faculty appointments at the University of New South Wales, Australia, and Beijing University of Posts and Telecommunications, China. His research interests fall into the broad areas of communication theory, wireless communications, and statistical signal processing for distributed data processing, millimeter wave communications, cognitive radio, and wireless security.



Kyeong Jin Kim (SM'11) received the M.S. degree from the Korea Advanced Institute of Science and Technology in 1991, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of California at Santa Barbara, Santa Barbara, CA, USA, in 2000. From 1991 to 1995, he was a Research Engineer with the Video Research Center, Daewoo Electronics, Ltd., Seoul, Korea. In 1997, he joined the Data Transmission and Networking Laboratory at the University of California at Santa Barbara. After receiving his degrees, he joined the

Nokia Research Center and Nokia Inc., Dallas, TX, USA, as a Senior Research Engineer, where he was an L1 Specialist from 2005 to 2009. From 2010 to 2011, he was an Invited Professor with Inha University, Incheon, Korea. Since 2012, he has been a Senior Principal Research Staff with Mitsubishi Electric Research Laboratories, Cambridge, MA, USA. His research has been focused on the transceiver design, resource management, scheduling in the cooperative wireless communications systems, cooperative spectrum sharing systems, device-to-device communications, secrecy systems, and GPS systems.



**Yansha Deng** (S'13) is currently working toward the M.S. degree at Central South University, Changsha, China. She is also currently working toward the Ph.D. degree in electronic engineering at Queen Mary University of London, London, U.K.

Her research interests include multiple-antenna systems, cognitive radio, cooperative networks, molecular communication, and physical layer security.



**Lifeng Wang** (S'12) is working towards his Ph.D. degree in Electronic Engineering at Queen Mary University of London. Before that, he received the M.S. degree in Electronic Engineering from the University of Electronic Science and Technology of China, in 2012.

His research interests include physical layer security, massive MIMO, millimeter-wave communications and 5G HetNets.



**Trung Q. Duong** (S'05, M'12, SM'13) received his Ph.D. degree in Telecommunications Systems from Blekinge Institute of Technology (BTH), Sweden in 2012, and then continued working at BTH as a project manager. Since 2013, he has joined Queen's University Belfast, UK as a Lecturer (Assistant Professor). He held a visiting position at Polytechnic Institute of New York University and Singapore University of Technology and Design in 2009 and 2011, respectively. His current research interests include cooperative communications, cognitive radio

networks, physical layer security, massive MIMO, cross-layer design, mmwaves communications, and localization for radios and networks.