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An assessment of commercial CFD turbulence models for near wake HAWT modelling

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Abstract

The simulation of the complex flow in a wind turbine wake is a challenging problem. To date, much of the research has been inhibited by both the time and computational costs associated with turbulence modelling. Additionally, the majority of numerical investigations focus on turbine performance and therefore neglect the near wake of a Horizontal Axis Wind Turbine (HAWT) entirely. This investigation focuses on experimentally and numerically quantifying the near wake structure of a model HAWT. The Shear Stress Transport (SST) $k - \omega$, Elliptical-Blending Reynolds Stress Model (EB-RSM) and the Reynolds Stress Transport (RST) turbulence models were used to model a turbine wake in the current study, with the results verified against experimental hot-wire data. Near wake velocity and turbulence characteristics were investigated to determine if low-order models can accurately predict the magnitude and distribution of velocity and turbulence values in the near wake of a model HAWT. The HAWT model was operated at two TSR values of 2.54 and 3.87. All models predicted velocity deficit values to within 2–4% and 4–7% of experimental results for TSR values of 2.54 and 3.87 respectively. Results showed that all models were able to accurately predict the mean velocity deficit generated in the near wake. All models were able to predict the fluctuating u and v velocity components in the near wake to the correct order of magnitude with the fluctuating velocity components having an inverse Laplace distribution in the wake. However, all models under-estimated the magnitude of these velocity values with predictions as low as -43% of experimental results.

Keywords: Wind turbine aerodynamics, SST $k-\omega$ turbulence model, Reynolds Stress Transport turbulence model, Computational Fluid Dynamics, Hot-wire Anemometry

1. Introduction

Up until recently, most aerodynamic modelling of wind turbines has been greatly simplified, with only a few researchers [1, 2, 3, 4] simulating the full turbine structure. The application

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4 of Computational Fluid Dynamics (CFD) models have been hindered due to the complex-
5 ity associated with modelling the relative motion between rotating and stationary turbine
6 components [5]. In addition, high performance computational resources are often not avail-
7 able, which results in CFD models being highly simplified. These simplifications include
8 neglecting the tower structure or just modelling one turbine blade, taking advantage of the
9 120° periodicity [6, 7, 8, 9]. Such models are not able to model the unsteady phenomena
10 associated with rotor tower/nacelle interaction, which has been shown to contribute very
11 high levels of turbulent kinetic energy and Reynolds stress in the wake [10, 11]. Removing
12 the tower from numerical simulations has a direct impact on the wake structure and velocity
13 deficit experienced behind a turbine. Not only does this corrupt the structure of the rotor
14 wake, but the increased turbulence leads to a faster reduction of the velocity deficit in the
15 lower half of the turbine wake. With respect to future wind turbine structural modelling
16 attempts, an understanding of the spatial distribution of stresses generated within the wake
17 is important. The fluctuating velocity components in the flow directly contribute to the
18 unsteady forces acting on turbine blades [5].

19
20 Noted in a review by O'Brien et al. [5], due to limited resources, most models are solved
21 using steady time [12, 13, 14, 15, 9, 7, 8]. The transient effects of tower interaction, dynamic
22 stall and wake meandering are difficult to model when using steady simulations [16, 17]. A
23 study by Gomez-Iradi et al. [1] simulated rotor/tower interference for an upwind configura-
24 tion turbine. This work simulated the displacement of the upwind stagnation point and
25 boundary layer separation points of the tower at a periodic frequency of three times per
26 revolution (for a three-bladed rotor). This resulted in periodic lateral loading of the tower.
27 This pulsating displacement of the stagnation point was not observed in steady simulations.

28
29 Primarily, near wake research (particularly numerical research) is focused on power produc-
30 tion and turbine performance [18]. However, this can be attributed to the fact that most
31 numerical models are validated against earlier experimental works, which, as noted by Ver-
32 meer et al. [18] focused on HAWT performance. Additionally, most numerical models are
33 validated against the NREL Phase VI measurement campaign of Hand et al. [19], which
34 only investigated aerodynamic rotor loads and pressure distributions over the blades [5]. The
35 current investigation focuses on the study of the near wake with a particular emphasis on
36 the near wake structure. The near wake is taken as the area just behind the rotor, where the
37 properties of the rotor can be discriminated [18]. The current investigation aims to validate
38 numerical models against detailed near wake measurements of a full model turbine structure.

39
40 Wake turbulence, especially in a wind farm setting, contributes to the unsteady loading on
41 wind turbine blades. As noted by Zhang et al. [20], limited information about the spatial
42 distribution of turbulence and vortex behaviour in the near wake hinders the capability of
43 the engineering community to predict wind turbine power production and fatigue loads in
44 wind farms. However, our understanding of such engineering quantities is further reduced
45 if the limitations of current turbulence modelling strategies with regards to predicting these
46 phenomena is not explored. However, keeping computational costs in mind, there are newly

47 released models available such as a new Elliptic Blending Reynolds Stress Model (EB-RSM)
48 released by STAR CCM+ as noted by O'Brien et al. [5]. This model was developed to meet
49 industrial needs whereby it is more detailed than two equation eddy-viscosity models, but
50 not as costly as a Reynolds Stress Transport model. This is accomplished by use of the
51 elliptic relaxation concept proposed by Durbin [21], whereby the redistributive terms in the
52 Reynolds stress equations are modelled by an elliptic relaxation equation. This model could
53 provide a compromise between cost and accuracy for future HAWT modelling attempts.
54 Detailed investigations of the near wake are valuable, especially for the validation of nu-
55 merical models as the near wake structure effectively provides the "building blocks" for far
56 wake analysis. However, to date, such measurements in the near wake are very rare [18, 22].
57 Finally, for future research regarding full scale modelling of HAWTs, the aerofoil data asso-
58 ciated with large scale blades is often not available due to commercial sensitivity. Therefore,
59 it is necessary to assess direct modelling techniques to ensure they accurately predict wake
60 characteristics, as opposed to ADM (Actuator Disk model) and ALM (Actuator Line model)
61 techniques. ADM and ALM were both neglected in the current study as they both rely on
62 the BEM method in order to compute body forces. BEM methods are highly reliant on the
63 aerofoil data chosen and dependant on empirical corrections to 2D aerofoil data.

64
65 In order to accurately model turbulence in any flow simulation, accurate modelling of the
66 boundary layer of a solid surface is important. Interaction between airflow and a solid sur-
67 face is in many engineering applications the origin of turbulence. In most commercial CFD
68 codes this is usually done by the allocation of pressure-strain relationships. However, many
69 pressure-strain relationships are dependant on y^+ wall treatments, which influence not only
70 the ability of the solver to resolve the boundary layer, but also how it applies wall functions
71 within the boundary layer to mimic turbulence dissipation rates and the two-component
72 turbulence limit. The review by O'Brien et al. [5] has noted that many numerical simula-
73 tions of HAWT wakes have been carried out, with mesh refinements in most cases taking y^+
74 values into consideration. The pressure-strain relationship and wall treatment used in these
75 studies is often not mentioned, which has a major impact on a turbulence model's ability
76 to solve for turbulence in the flow. No study has yet been carried out (to the knowledge
77 of the authors) to assess the ability of different pressure-strain relationships to accurately
78 model turbulence in HAWT wakes. This is essential for future FSI simulations, as noted
79 by Zhang et al. [20]. Additionally, modelling of flow/solid-surface interaction is the most
80 expensive and difficult part of any CFD simulation. Investigating different pressure-strain
81 relationships and how they impact the accuracy of a turbulence model could be used to
82 determine the least expensive approach to HAWT wake modelling.

83
84 This study aims to investigate the ability of low-order turbulence models to accurately
85 predict the turbulent characteristics of HAWT wakes. The ability of low-order turbulence
86 models to predict the turbulence characteristics of HAWT wakes, speaks to their suitability
87 for use in future FSI simulations of HAWTs.

88
89 The objectives of this study are as follows:

- 90 • To carry out an experimental measurement campaign in a wind tunnel of the near
91 wake of a model Horizontal Axis Wind Turbine (HAWT) structure in order to validate
92 numerical codes.
- 93 • To model, using advanced transient CFD the wake of the model HAWT to capture
94 the development of a HAWT near wake including the root vortex structure.
- 95 • To establish a meshing strategy for near wake analysis.
- 96 • To assess the ability of the SST $k-\omega$ model (coupled with the Vorticity Confinement
97 model), the EB-RSM and the RST model to accurately represent the mean and fluc-
98 tuating velocity characteristics of the near wake.

99 2. Experimental Setup

100 2.1. Wind Tunnel Facility

101 This investigation was carried out in a closed loop return wind tunnel at Queens University,
102 Belfast. The wind tunnel (see figure 1) has an enclosed test section of 0.85 m (height) x
103 1.15 m (width) x 3 m (length) with optically transparent side walls and can operate to a
104 maximum freestream velocity of 40 m/s. For the current investigation, the wind tunnel had
105 an average background freestream turbulence intensity of 0.19%. The turbine model was
mounted 0.75 m from the test section entrance.

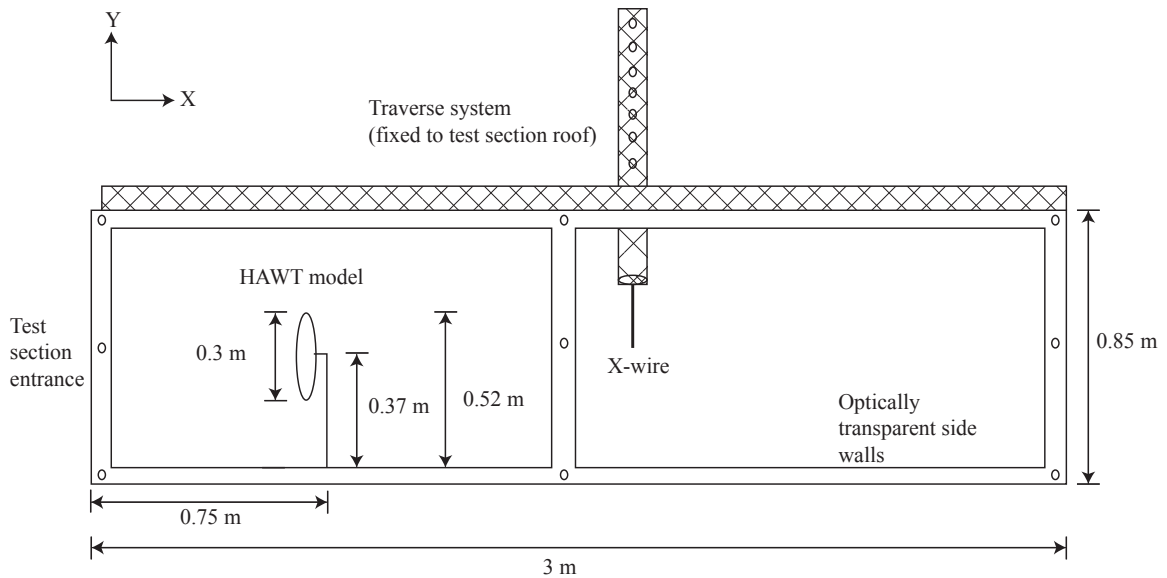


Figure 1: Wind tunnel schematic

107 *2.2. Wind Turbine Model*

108 The wind turbine model used for the present study was a three-bladed horizontal axis wind
 109 turbine. The turbine model was designed taking into account the Wind Atlas Analysis and
 110 Application Program (WAsP) wake model of [23], where the wind turbine wake is assumed
 111 to expand linearly with distance downstream. The wind turbine model has a rotor diameter
 112 of 0.3 m and a hub height of 0.37 m. Images of the turbine model and the main geometric
 113 parameters are given in figure 2. The blockage ratio for this study, defined as the ratio of
 114 the blade swept area to test section cross-sectional area is 7.22%. This was consistent with
 115 previous studies as the blockage ratio varies between 1-10% with 10% being the upper-most
 116 limit [24, 25, 26, 27, 28, 29]. The 10% upper limit criteria in this design was also based
 117 on a study carried out by McTavish et al. [30], which identified that the expansion of the
 118 near wake of a HAWT was not significantly modified if the blockage ratio remained between
 119 6–10%. Values greater than 14% caused the wake to narrow by 35% [30].

120
 121 The rotor blades for this study have a continuous FFA-W3-241 blade profile. The blades
 122 are twisted linearly with a pitch angle of 10° at the tip and 35° at the root. The chord
 123 of the blades also tapers linearly, with a tip chord of 10 mm and a root chord of 30 mm.
 124 The turbine model has a cone and tilt angle of 0° and the rotor plane was perpendicular
 125 to the free stream at all times. The turbine was rotated by a brush-less DC electric motor,
 126 which was controlled via a speed controller. During experimentation the turbine was run
 127 at two Tip Speed Ratios (TSR): 2.54 (1622 rpm) and 3.87 (2465 rpm). It should be noted
 128 that the turbine model freely rotated at a TSR of 2.54 with no input from the DC electric
 129 motor (optimum TSR). The rotational speed of the turbine was monitored using a high
 130 speed camera. A highly reflective tape was fixed to the rotor hub and was recorded using
 131 the high speed camera. All images were recorded at a frequency of 2 kHz which allowed for
 132 the rotational speed of the turbine model to be calculated and monitored during experiments.

133
 134 The inlet velocity profile was recorded one diameter upstream of the turbine model before
 135 wake velocity measurements were taken. The wind speed was found to be 10 m/s at hub
 136 height (i.e. $U_\infty=10$ m/s). This free stream velocity value was kept constant for all measure-
 137 ments. This resulted in a Reynolds number range of 21.7×10^3 to 20.4×10^3 and 20.8×10^3
 138 to 29.9×10^3 for TSR of 2.54 and 3.87 respectively. The Reynolds number was based on the
 139 tip chord length of 10 mm. The Reynolds number and relative velocity definition are given
 140 in equations 1 and 2,

$$Re = \frac{c_{tip} V_{rel}}{\nu} \quad (1)$$

$$V_{rel} = U_\infty \sqrt{(1 - a)^2 + \left(\frac{r \Omega_r}{U_\infty} (1 + a') \right)^2} \quad (2)$$

141

142

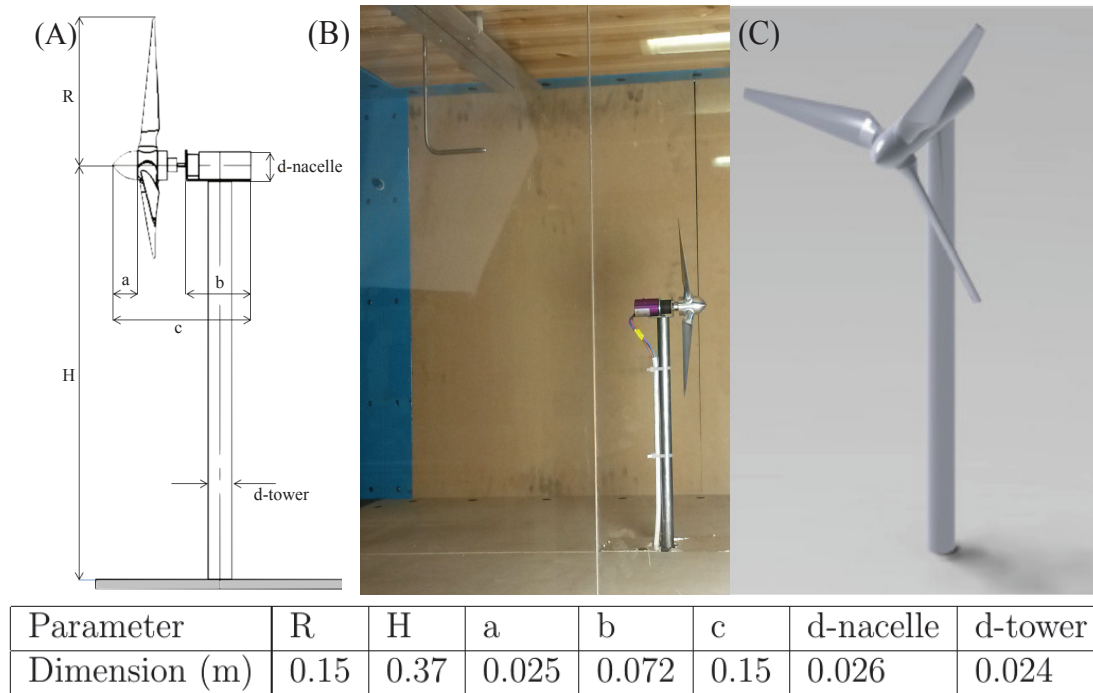


Figure 2: Schematic (A), physical (B) and numerical (C) images of the wind turbine model used in the current experimental investigation

143 where a and a' are the axial and tangential interference factors, ν is the kinematic viscosity
 144 of air, r is radius and Ω_r is rotational velocity.

145
 146 Few experiments regarding the near wake have been undertaken with most been performed
 147 at low Reynolds numbers (as related to blade chord and rotational speed). Despite these
 148 types of experiments not resembling a full-scale turbine, they can be used to verify numerical
 149 models [18]. The fundamental behaviour of the helical tip vortices and turbulent wake flow
 150 downstream of wind turbines is almost independent of the chord Reynolds number [31, 6, 11].
 151 Performance characteristics were not recorded for the current study as the main focus was
 152 to investigate mean and fluctuating velocity components as opposed to performance.

153 2.3. Velocity Measurement Techniques and Experimental Uncertainty

154 All velocity measurements were recorded using a two-component hot-wire x-probe. The
 155 Constant Temperature Anemometry (CTA) system used in this study was a TSI IFA 300
 156 system with a Dell Optiplex GX620 computer and THERMALPRO software. All measure-
 157 ments were recorded with a TSI 1240-T1.5 $5\ \mu\text{m}$ x-wire. In an attempt to capture the
 158 root vortex system, a measurement grid spacing of 1 cm across the entire rotor plane was
 159 adopted. This resulted in 30 measurement points across the rotor diameter and a total of
 160 1558 data points per plane. The probe was mounted to a two-axis traverse system, which
 161 was attached to the roof of the test section. The mean percentage errors associated with

162 the x-probe were less than 3% for velocities between 0 m/s and 3 m/s and less than 1% for
163 velocities greater than 3 m/s, as computed by the THERMALPRO software. The sampling
164 rate and time for the hot-wire probe were 1 kHz and 1 s respectively. Velocity measurements
165 were recorded in 3 separate planes ($0.66D$, $1D$ and $1.5D$) downstream from the rotor plane
166 (where D is defined as the rotor diameter). A numerical representation of the measurement
167 planes is shown in figure 3.

168
169 The recording time of 1 s at each recording location corresponded to measurements recorded
170 for 27 and 41 full rotor revolutions. The transient flow disturbances are the vortex shedding
171 from the tower and the rotational effects of the turbine. The vortex shedding from the
172 tower was estimated to be in the region of 83 Hz based on a Strouhal number of 0.2 (circular
173 cylinder) and the rotation of the turbine corresponded to 27-41 Hz, so in sampling at 1
174 kHz the likelihood of these effects dominating the signal is reduced. Given the large volume
175 of data that was collected in this study a 1 s sampling duration per grid point location in
176 the planes downstream of the turbine was considered adequate. This would eliminate any
177 transience in the recorded data and proved a statistically averaged data for comparison to
178 numerical data.

179 3. Numerical Simulation

180 3.1. Introduction

181 All simulations were carried out using the finite volume solver Star CCM+ and were
182 solved using a HPC cluster (Fionn) of the Irish Centre for High-End Computing (ICHEC).
183 Both the SST $k - \omega$ and RST models (EB-RSM model included) were run using 192 cores
184 respectively. All models simulated 2s of physical time with a timestep value that repre-
185 sented one degree of rotation. The timestep used in this study is similar to that used by
186 Li et al. [3] and Valiadis et al. [32]. Convergence criteria were enabled on the x, y and
187 z-momentum equations, turbulent dissipation and kinetic energy equations (only applied to
188 the SST $k - \omega$ model). These convergence criteria prevented the solution from advancing
189 to the next timestep until the residual values reduced below 1×10^{-4} . Residuals in Star
190 CCM+ are a measure of the imbalance of the conservation equations and the degree to which
191 their discretized form is satisfied. In effect, they represent the solution error of a particular
192 variable. However, in Star CCM+ the residual errors are auto-normalized by its maximum
193 recorded value in the first five iterations. The normalized residuals represent the order of
194 magnitude in which these values fall from their peak values.

195
196 The RST and EB-RSM models did not have any convergence criteria enabled on the x,y
197 and z-momentum criterion as the models proved to be numerically very robust. The RST
198 model was unable to meet the previously mentioned x,y,z criterion after 132,000 iterations
199 and therefore the models could not advance to the next timestep. This pointed to the solvers
200 requiring additional steps within each timestep in order to solve the physics to a satisfactory
201 level. For both RST and EB-RSM simulations, the number of inner iterations (defined as
202 the number of iterations computed by the solver for a single timestep) per timestep was

203 increased to 10 from the default value of 5. Residual values reduced below 1×10^{-6} for both
 204 SST $k - \omega$ simulations and below 1×10^{-5} for all RST and EB-RSM simulations. The SST
 205 $k - \omega$, EB-RSM and RST models required 50 hours, 119 hours and 144 hours to complete
 206 respectively. All models were run using a 2nd order central spatial discretization scheme
 207 with a double precision.

208
 209 For all simulations, velocity point data was recorded using presentation grids. A presentation
 210 grid samples data from regularly spaced intervals on a finite plane. These are illustrated in
 211 figure 3. Velocity data were recorded in the same locations for both the numerical solution
 212 and experimental investigation. All velocity measurements were recorded after the solution
 213 domain had experienced two flow throughs. One flow through is defined as the length of
 214 time required for a fluid particle to enter and exit the solution domain. At this point each
 215 simulation had built up to a statistically steady state.

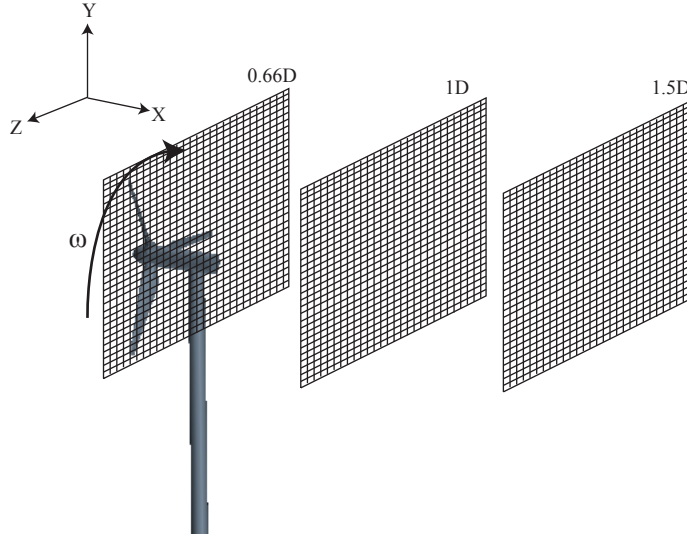


Figure 3: Positioning of presentation grids behind the wind turbine CFD model

216 3.2. SST $k-\omega$ Turbulence Model

217 Numerical simulations were firstly carried out using the implicit unsteady Shear Stress Trans-
 218 port (SST) $k - \omega$ turbulence model. SST is an aerospace industry standard turbulence model
 219 and is a good benchmark for higher fidelity models such as unsteady Reynolds Stress Trans-
 220 port (RST) and Large Eddy Simulation (LES) models. The governing equations for the SST
 221 $k-\omega$ model are the k and ω equations, given below as equations number 3 and 4, respec-
 222 tively, with the source terms omitted [33]. The reader is referred to Wilcox [33] for further
 223 information regarding the SST $k - \omega$ model.

$$\frac{\partial(\rho k)}{\partial t} + \nabla \cdot (\rho k \vec{U}) = \nabla \cdot \left[\left(\mu + \frac{\mu_t}{\rho_k} \right) \nabla k \right] + P_k - Y_k \quad (3)$$

$$\frac{\partial(\rho\omega)}{\partial t} + \nabla \bullet (\rho\omega\vec{U}) = \nabla \bullet \left[\left(\mu + \frac{\mu_t}{\rho\omega} \right) \nabla\omega \right] + P_\omega - Y_\omega + D_\omega \quad (4)$$

224

225

226 The shortcomings of eddy viscosity models representing highly complex rotational flows has
 227 been well documented [34, 35, 36]. However, attempts have been made to overcome the
 228 shortfalls of the $k - \omega$ model by adding correction terms. For the current study, the $k - \omega$
 229 model was coupled with curvature correction terms using second order discretization and the
 230 Vorticity Confinement Model of Steinhoff [37] and later refined by Löhner [38]. The curva-
 231 ture correction term supplies the effects of strong curvature and frame-rotation by altering
 232 the turbulent energy production term according to local rotation and vorticity rates. This
 233 term is applied as vortices tend to dissipate early with two equation models. The Vorticity
 234 Confinement Model adds a forcing term (f_ω) to the momentum equations in all directions
 235 in order to preserve the vortex – see equations 5 and 6, which highlight the addition of f_ω
 236 to the momentum equation in the x-direction.

237

238 An example of the modified x-direction momentum equation is defined as

$$\frac{\partial u}{\partial t} + \nabla \bullet (u\vec{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \bullet (\nabla u) + f_\omega \quad (5)$$

239 where the forcing term is defined as

$$f_\omega = -\epsilon\rho (\hat{n} \times \vec{\omega}) \quad (6)$$

240 where $\vec{\omega}$ is vorticity, ϵ is a user-defined constant (default value of 0.04 for three-dimensional
 241 cases) and \hat{n} is the unit vector. The flow was modelled as an unsteady turbulent gas,
 242 assuming constant density with a segregated flow model. An unsteady transient model was
 243 selected as the aerodynamic phenomena associated with wind turbine aerodynamics cannot
 244 be completely modelled when using steady time simulations.

245 3.3. Reynolds Stress Transport Turbulence Model

246 The RST turbulence model was used to investigate the accuracy of a Reynolds Stress Trans-
 247 port model coupled with the linear pressure-strain two-layer term modelled using the lin-
 248 ear approach of Gibson and Launder [39] to predict the characteristics of a turbine wake.
 249 By solving the Reynolds Stress Tensor, this model naturally accounts for effects such as
 250 anisotropy, which is associated with swirling motion, streamline curvature and rapid changes
 251 in strain rate. The model does not use Boussinesq’s eddy viscosity hypothesis to compute
 252 the Reynolds stresses, but instead solves transport equations for each of the six Reynolds
 253 stresses and a model equation for the isotropic turbulent dissipation ϵ . This is the same
 254 equation used in the standard k- ϵ turbulence model. The Reynolds stress transport equa-
 255 tion from Versteeg et al. [40] is given in equation 7 with the isotropic turbulent dissipation
 256 ϵ given by equation 8. In equation 8, S'_{ij} represents terms from the fluctuating strain rate

257 tensor.

258

$$\begin{aligned}
& \overbrace{\frac{\partial (\rho \overline{u'_i u'_j})}{\partial t}}^{\text{Unsteady Term}} + \overbrace{\frac{\partial}{\partial x_k} (\rho u_k \overline{u'_i u'_j})}^{C_{ij} \equiv \text{Convection}} = - \overbrace{\frac{\partial}{\partial x_k} \left[\overline{\rho u'_i u'_j u'_k} + p(\delta_{kj} u'_i + \delta_{ki} u'_j) \right]}^{D_{T,ij} \equiv \text{Turbulent Diffusion}} \\
& + \overbrace{\frac{\partial}{\partial x_k} \left[\mu \frac{\partial}{\partial x_k} (\overline{u'_i u'_j}) \right]}^{D_{L,ij} \equiv \text{Molecular Diffusion}} - \overbrace{\rho \left(\overline{u'_i u'_k} \frac{\partial u_j}{\partial x_k} + \overline{u'_j u'_k} \frac{\partial u_i}{\partial x_k} \right)}^{P_{ij} \equiv \text{Stress Production}} + \overbrace{p \left(\frac{\partial u'_i}{\partial x_k} + \frac{\partial u'_j}{\partial x_i} \right)}^{\Pi_{ij} \equiv \text{Pressure Strain-Interaction}} \quad (7) \\
& - \overbrace{2\rho \Omega_k (\overline{u'_j u'_m} e_{ikm} + \overline{u'_i u'_m} e_{jkm})}^{\Omega_{ij} \equiv \text{Rotation}} - \overbrace{2\mu \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k}}^{\epsilon_{ij} \equiv \text{Dissipation}} \\
& \epsilon = 2 \frac{\mu}{\rho} \overline{S'_{ij} S'_{ij}} \quad (8)
\end{aligned}$$

259 The RST model used the same physics continua at the SST k- ω model.

260 3.4. Pressure-Strain Term used in Reynolds Stress Transport Model

261 The pressure strain relationship is highly important regarding the subject of turbulence
262 modelling. The computational complexity and expense of Reynolds Stress Models (RSM)
263 models is often driven by the difficulty of modelling the effects of solid walls on adjacent
264 turbulent flows. These effects include pressure fluctuations due to eddies interacting with
265 each other with other areas of the freestream which have a different mean velocity.

266
267 The Linear Pressure-Strain Two Layer Term was investigated as the relationship is defined
268 as a two-layer formulation which is suitable for low-Reynolds number flows. Essentially, the
269 model was favoured as it allowed the RST model to use an all y^+ wall treatment which
270 would be more suitable for the mesh used in this investigation.

271
272 This model extends the linear model of Gibson et al. [39] so that it can be applicable to the
273 near-wall sub-layer where viscous effects are dominant. The extension allows the usual log-
274 law/local equilibrium matching to be discarded and enables boundary-layer problems to be
275 tackled where the flow structure in the inner region departs from what is usually termed the
276 “universal” wall law [41]. As noted by Launder et al. [41], there are many situations where
277 the application of local equilibrium conditions to turbulent stresses and energy dissipation
278 rates near the wall is not appropriate as this condition would be too complex to enforce.
279 For example, a study carried out by Launder et al. [42] highlighted that at high rotation
280 rates, turbulent mixing near the suction surface was annihilated. This feature cannot be
281 modelled if the wall-law approach is enforced. Additionally, streamwise pressure gradients,
282 body forces, strong secondary flows and separation can cause the flow to deviate from “uni-
283 versal” wall behaviour [41].

284

285 The linear pressure-strain model used in this case expresses the pressure-strain term as three
 286 components:

$$\Pi_{ij} = \Pi_{ij,1} + \Pi_{ij,2} + \Pi_{ij,3} \quad (9)$$

287
 288
 289 The so called “slow” pressure-strain or “return-to-isotropy” term $\Pi_{ij,1}$ represents a physi-
 290 cal process within the flow where there is a reduction of the anisotropic properties of the
 291 turbulent eddies due to their mutual interactions. The “rapid” pressure-strain term is de-
 292 fined as $\Pi_{ij,2}$. This term supposes that the rapid pressure partially counteracts the effect
 293 of production to increase the Reynolds-stress anisotropy [43]. The “wall reflection” term is
 294 defined as $\Pi_{ij,w}$. This term is responsible for the redistribution of normal stresses near the
 295 wall. It damps normal stresses perpendicular to the wall, while enhancing stresses that are
 296 parallel to the wall. The reader is referred to Gibson et al. [39] for further information.

297 *3.5. Elliptical Blending Reynolds Stress Turbulence Model*

298 A more robust and industry-friendly Reynolds Stress model was developed by Manceau et
 299 al. [44]. It will be referred to as the Elliptic Blending Reynolds Stress Model (EB-RSM).
 300 The model was developed to meet industrial needs and was noted by O’Brien et al. [5] as
 301 a possible substitute to a full RST model for HAWT analysis. While simple and robust,
 302 the model preserves the elliptic relaxation concept proposed by Durbin [21] whereby the
 303 redistributive terms in the Reynolds stress equations are modelled by an elliptic relaxation
 304 equation. This method avoids the need to use damping functions at the wall. However,
 305 the model proposed by Manceau et al. [44] uses only one scalar elliptic equation instead
 306 of six as proposed by Durbin [21]. Durbin’s original model consisted of six elliptic dif-
 307 ferential equations with boundary conditions to reproduce the near-wall behaviour of the
 308 redistributive term. The EB-RSM model is a low-Reynolds number model that is based on
 309 an inhomogeneous near-wall formulation of the quasi-linear Quadratic Pressure strain term.
 310 A blending function is used to blend the viscous sub-layer and the log-layer formulation
 311 of the pressure-strain term. This approach requires the solution of an elliptic equation for
 312 the blending parameter α . The EB-RSM model used in this investigation is based on the
 313 EB-RSM model of Manceau [44] and revised by Lardeau et al. [45]. The main objective of
 314 the Elliptic Blending approach is to account for the influence of wall blockage effects towards
 315 the wall-normal component of turbulence, which is required to produce the two-component
 316 limit of turbulence at the wall.

317
 318 The EB-RSM is based on a blending of near-wall and quadratic pressure-strain models for
 319 the pressure strain and dissipation, defined as follows:

$$\phi_{ij}^* - \epsilon_{ij} = (1 - \alpha^3)(\phi_{ij}^w - \epsilon_{ij}^w) + \alpha^3(\phi_{ij}^h - \epsilon_{ij}^h) \quad (10)$$

320
 321

322 where ϕ_{ij}^* is the pressure-strain tensor and ϵ_{ij} is the dissipation-rate tensor. The term α
 323 is a blending parameter. The blending parameter α is the solution of the following elliptic
 324 equation:

$$\alpha = L^2 \nabla^2 \alpha = 1 \quad (11)$$

325
 326
 327 whereby the solution of this equation goes to zero at the wall and close to unity far from
 328 the wall. The length-scale L defines the thickness of the region of influence of the near wall.
 329 The EB-RSM model used the same physics continua at the SST k- ω model.

331 *3.6. Mesh Generation and Boundary Conditions*

332 Figure 4 illustrates the computational domain used in this study. The flow direction is from
 333 left to right. The rotational domain refers to the area inside the outlined disc around the
 334 rotor, as illustrated in the magnified image in figure 4. The rotating domain includes the
 335 rotating blades, spinner and hub connections. The solution domain, turbine tower and na-
 336 celle combine to make the stationary region. The turbine nacelle was simplified in order to
 337 reduce the complexity of the mesh behind the rotating region of the turbine. The nacelle
 338 was modelled as a solid cylinder with a diameter of 0.026 m. Additionally, the computa-
 339 tional domain length was extended to 4 m for the CFD model. This was done to take into
 340 account the outlet pressure criteria set at the domain outlet. Near the domain outlet the
 341 flow can still be turbulent. This could cause an interaction to occur at the pressure out-
 342 let boundary, which could result in reversed flow occurring in cells near the exit. Reverse
 343 flow occurs when the pressure in the cell adjacent to the outlet boundary is lower than the
 344 pressure specified on the boundary itself and the adverse pressure gradient is sufficiently
 345 large to cause the flow at the outlet to reverse direction, ie. flow enters the domain from
 346 the outlet. This is commonly caused by the specification of a uniform outlet pressure when
 347 the flow near the boundary is highly non-uniform. In this case, the outlet boundary was
 348 too close to geometric features that cause flow non-uniformity. This error is specified in the
 349 simulation output window and is monitored by the user. To help mitigate this issue, the
 350 outlet boundary was positioned further from the obstruction to allow the flow to become
 351 more uniform prior to reaching the outlet boundary. In this case the outlet of the domain
 352 was extended from 3 m to 4 m, which resolved the issue. There was no data recorded in the
 353 current study to investigate if reversed flow errors would have resulted in decreased accu-
 354 racy of the simulation. The inlet was modelled as a velocity inlet with a free stream velocity
 355 value of 10 m/s and the freestream turbulence present was defined in the simulation physics.
 356 An Atmospheric Boundary Layer (ABL) was not modelled as it was deemed necessary to
 357 assess each models ability to predict wake characteristics with a uniform inlet velocity first.
 358 The introduction of an ABL would have complicated the simulation and make it difficult to
 359 determine if inaccuracies in the models were resulting from mesh quality, turbulence models
 360 and selected physics (vorticity confinement and pressure-strain relationships) or the addition

361 of turbulence in the freestream due to the ABL. In addition to this, using an ABL would
362 result in further refinement of the simulation mesh in front of the rotor to ensure the ABL
363 profile was resolved before entering the rotor. This would have increased the computational
364 expense of the simulation.

365
366 The computational domain (top, bottom and both sides of the numerical wind tunnel) and
367 the numerical HAWT were modelled as wall boundaries with a no-slip condition. The rota-
368 tion of the turbine was modelled using the sliding mesh approach. This method requires a
369 rotational velocity to be prescribed as a boundary condition on the solid rotor. The rotating
370 and stationary regions of the solution were also connected by internal interfaces. Interfaces
371 allow simulation quantities (such as mass, momentum and energy etc.) to pass between sta-
372 tionary and rotational regions. The RST and EB-RSM models were initially run in steady
373 state using moving reference frames. This provided the solver with an initial solution which
374 was then used in the unsteady case. Running the steady model beforehand reduced the risk
375 of divergence of the unsteady solution.

376

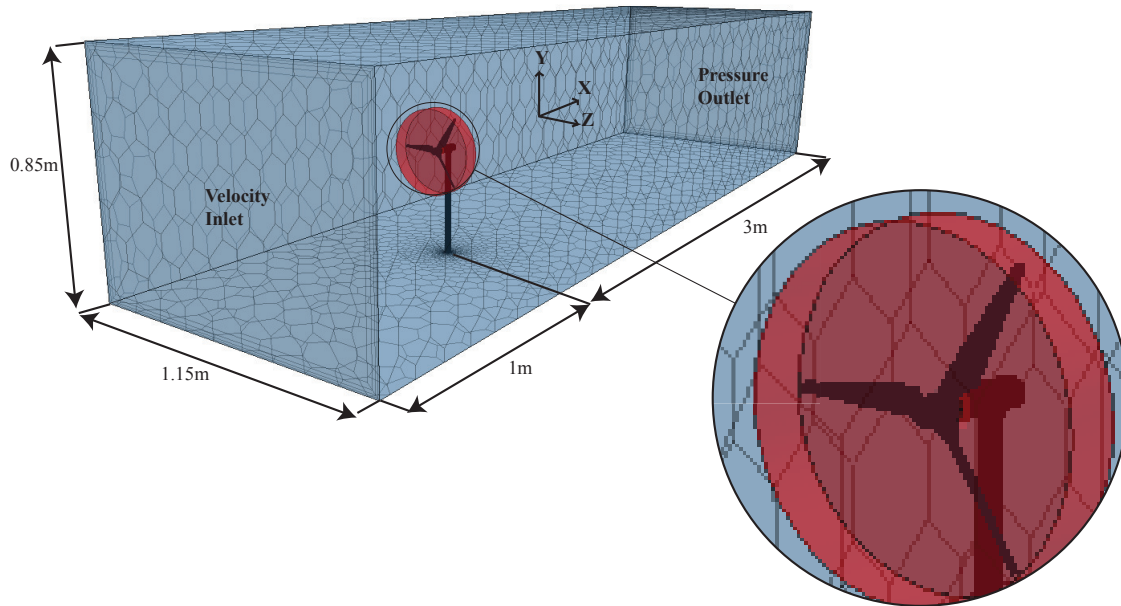


Figure 4: A schematic of the computational domain used in this study

377 For this simulation, an unstructured polyhedral mesh was used, as it is less computationally
378 demanding than a tetrahedral mesh. Polyhedral cells are orthogonal to the flow regardless
379 of flow direction. This makes them suitable for modelling highly rotational flows. Finally,
380 due to the large number of sides (12 for a polyhedral cell), polyhedral meshes are suitable
381 to mesh models that contain curved surfaces. Therefore, the polyhedral mesh was most
382 suitable to model the highly curved and twisted surfaces of the turbine blades. For the
383 purpose of this study, the mesh density for both the rotating and stationary regions were

384 treated separately. This can be accomplished by using the sliding mesh approach. This was
385 accomplished by subtracting a cylinder within the solution domain (stationary region). The
386 size of this cylinder is the same size as the rotor swept area. Within this area the rotor is
387 imprinted within the simulation. Therefore, this allows the rotor and the stationary region
388 to be meshed independently. This is advantageous as the turbine blades often need to be
389 meshed to a higher density than the rest of the solution domain. The mesh density of the
390 rotating region depended on a mesh sensitivity study, whereby an investigation was carried
391 out into the variation of average surface pressure on the blades and y^+ values. The wake
392 mesh was defined by investigating the maximum velocity deficit recorded behind the rotor at
393 $0.66D$ for different wake mesh densities. The results of the mesh sensitivity study highlight-
394 ing surface average pressure distribution over the rotor is presented in figure 5a. The red
395 circles in figures 5a and 5b highlight the point where increasing cell density resulted in no
396 change to the recorded engineering parameter observed. Following this study, the rotating
397 region contained 4×10^6 cells using a base size of 38 mm. Beyond this count there was
398 no appreciable change in the above-mentioned surface average pressure parameter. Curve
399 controls were then used on the turbine blades to refine the mesh further towards the leading
400 and trailing edge. This prevented the need for additional volumetric controls around the
401 turbine blades and improved the resolution of vortex shedding and rollup. A final cell count
402 of 4.28×10^6 was used for the rotating region.

403

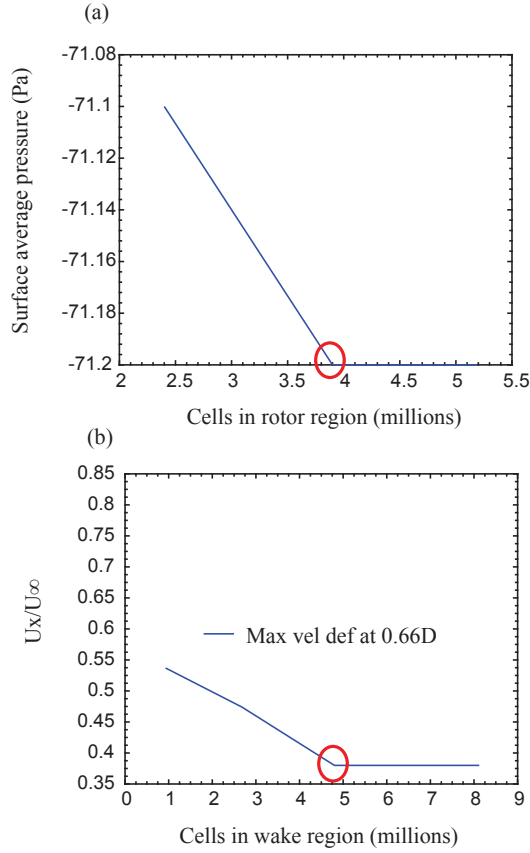


Figure 5: Mesh sensitivity results

404 The wake region was defined by a volumetric cone with a refined mesh density in order to
 405 capture the wake structure. The volumetric cone was sized, based on the WAsP wake model
 406 of Barthelmie et al. [23]. The maximum velocity deficit in the wake did not change after
 407 4.75×10^6 cells. All mesh sensitivity simulations for the wake mesh were carried out with
 408 a rotor mesh density of 4.28×10^6 cells. However, the mesh density of the wake region was
 409 increased to 5×10^6 cells in order to have a volume change between the rotating and wake
 410 region of 1, as shown in figure 6. This reduced any inaccuracies caused by large changes in
 411 cell sizes between the regions. The simulation had a combined cell count of 9.29×10^6 . A
 412 2D section of the meshed solution domain can be seen in figure 7. It can be noted that the
 413 wake volumetric control extends $0.5D$ in front of the turbine model. This allowed for the flow
 414 to be resolved to a high degree of accuracy before entering the rotor plane. Additionally, by
 415 extending the volumetric cone ahead of the turbine rotor, this created a favourable volume
 416 change across the mesh in the regions of interest. A large jump in volume from one cell to
 417 another can cause potential inaccuracies and instability in the solvers. Extending the wake
 418 cone made it possible to create a conformal mesh across the interface between the rotating
 419 and stationary regions. A conformal mesh produces a high-quality discretization for the
 420 analysis and eases the passage of information between regions.

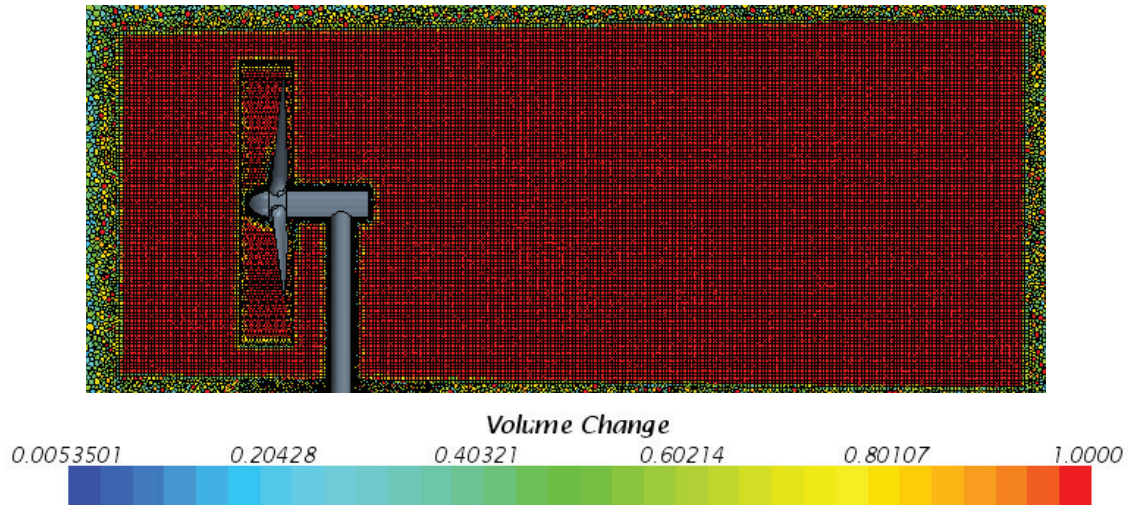


Figure 6: Volume change scalar scene

422 3.7. Wall Treatment

423 Within numerical simulations, walls are a source of vorticity and therefore, accurate pre-
 424 diction of flow and turbulence parameters across the wall boundary layer is essential. Star
 425 CCM+ uses a set of near-wall modelling assumptions known as “wall treatments”, for each
 426 turbulence model. Different y^+ wall treatments are used within Star CCM+ for different
 427 mesh resolutions near a wall boundary. Wall treatments are used to mimic the dimension-
 428 less velocity distribution inside the turbulent boundary layer.

429
 430 Wall treatments in Star CCM+ are divided into 3 categories. High y^+ wall treatment is the
 431 classic wall-function approach, where wall shear stress, turbulent production and turbulent
 432 dissipation are all derived from equilibrium turbulent boundary layer theory. It is assumed
 433 that the near-wall cell lies within the logarithmic region of the boundary layer, therefore the
 434 centroid of the cell attached to the wall should have $y^+ > 30$.

435
 436 The low y^+ wall treatment assumes that the viscous sublayer is well resolved by the mesh,
 437 and thus wall laws are not needed. It should only be used if the entire mesh is fine enough
 438 for y^+ to be approximately 1 or less. The all y^+ wall treatment is an additional hybrid wall
 439 treatment that attempts to combine the high y^+ wall treatment for coarse meshes and the
 440 low y^+ wall treatment for fine meshes. It is designed to give results similar to the low y^+
 441 treatment as $y^+ < 1$ and to the high y^+ treatment for $y^+ > 30$. It is also formulated to pro-
 442 duce reasonable answers for meshes of intermediate resolution, when the wall-cell centroid
 443 falls within the buffer region of the boundary layer, i.e. when $1 < y^+ < 30$.

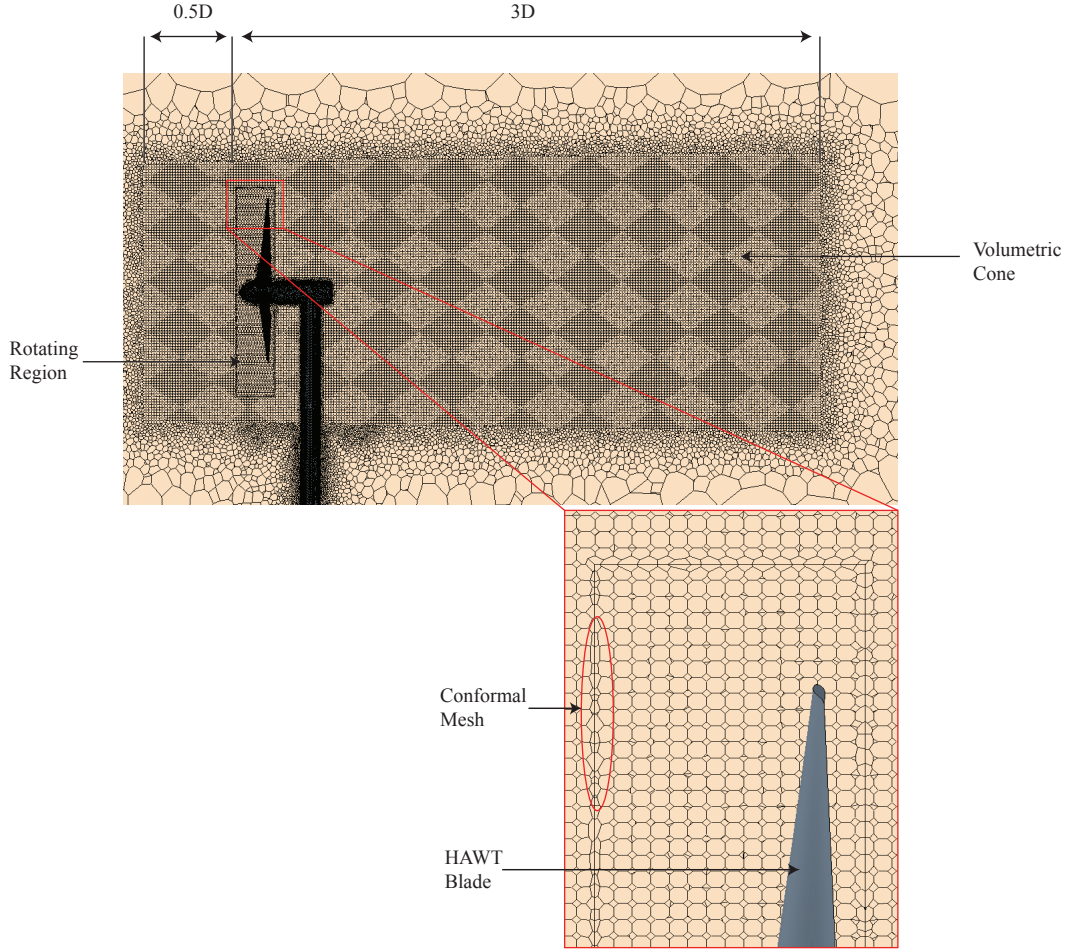


Figure 7: Meshed solution domain showing volumetric wake cone and conformal mesh across region interfaces

445 The use of these treatments is dependant on the y^+ values over the geometry where y^+ is a
 446 non-dimensional wall-normal distance, defined as:

447

$$y^+ = \frac{yu^*}{\nu} \quad (12)$$

448 where y is the normal distance from the wall to the wall-cell centroid. The term u^* is a
 449 reference velocity and ν is the kinematic viscosity. The reference velocity is related to the
 450 wall shear stress as follows:

$$u^* = \sqrt{\tau_w/\rho} \quad (13)$$

451 where τ_w is the wall shear stress and ρ is fluid density.

452

453 For this study, the y^+ value over the blades was kept below a value of 1 (maximum ≈ 0.67)
 454 during simulation initialization in order to accurately resolve the boundary layer flow (fig-
 455 ure 8) [46]. However, for a rotating component (the blades in this case) the y^+ values can
 456 change. The magnitude of this change has not been noted in previous CFD investigations
 457 reviewed by O'Brien et al. [5]. Changing y^+ values has a direct impact on the selected wall
 458 treatment used. In this case, the increasing tangential velocity value of the blade towards
 459 the blade tip altered the y^+ values. It is difficult to assess beforehand what the maximum
 460 y^+ over the blade will be during simulation. Blade y^+ values were monitored during initial
 461 simulations with the selected mesh density (outlined in section 3.6). Figures 9a and 9b show
 462 the average and maximum y^+ values of a blade after several rotor rotations for a TSR value
 463 of 2.54. It can be seen that peak y^+ values above 1 are recorded; therefore, an all y^+ wall
 464 treatment was used in the current study. Similar trends were monitored for the TSR equals
 465 3.87 case. Additionally, y^+ values over the tower varied in the range of $0.29 < y^+ < 8.87$,
 466 with a considerable amount of the tower structure falling into the buffer region. This again
 467 prompted the use of an all y^+ wall treatment. Correct selection of y^+ wall treatments is
 468 required to accurately model the boundary layer near a wall. An incorrectly selected wall
 469 treatment would result in an inaccurately predicted velocity gradient at the wall and there-
 470 fore inaccurate predictions of velocity and turbulent characteristics.

471

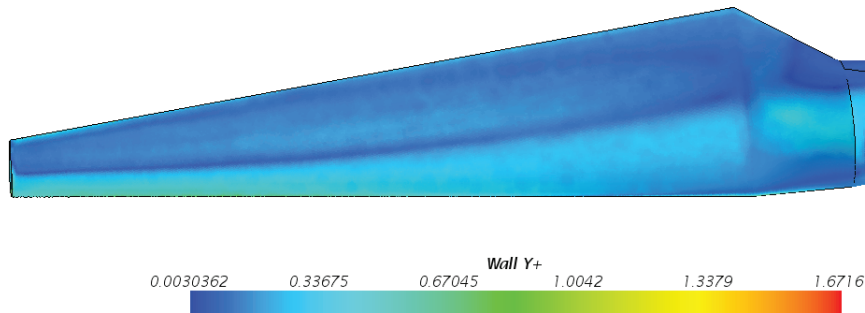


Figure 8: Y^+ values over numerical turbine blade (TSR 2.54)

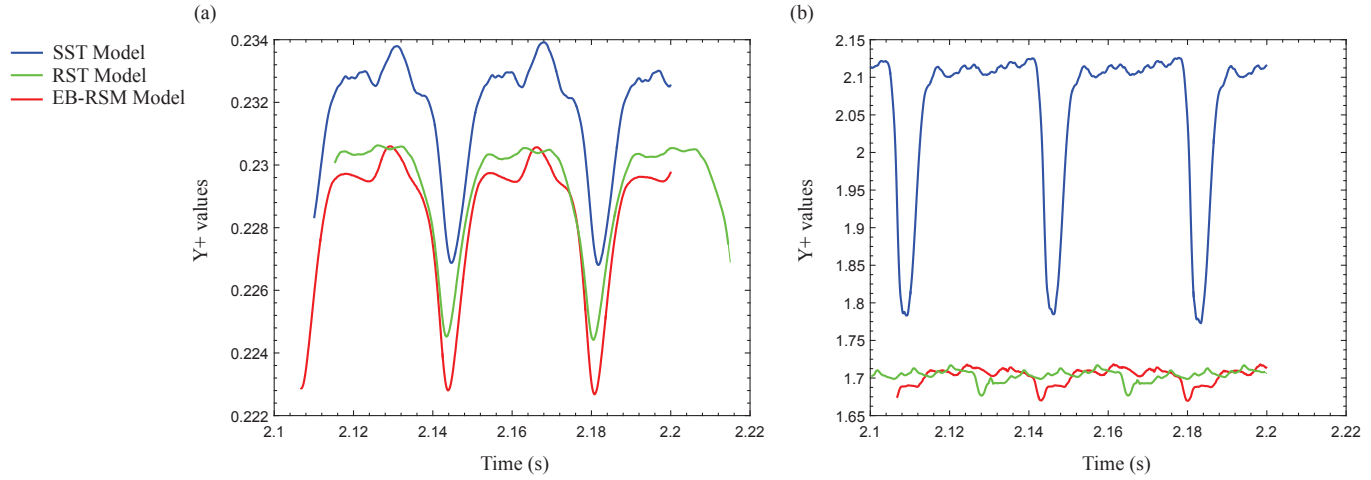


Figure 9: Varying Y^+ values over numerical turbine blade for average Y^+ (a) and maximum Y^+ (b) values (TSR 2.54)

472 4. Results and Discussion

473 4.1. Introduction

474 This section presents the results of the experimental investigation of the wake behind a
 475 model horizontal axis wind turbine in comparison to numerical predictions. The results are
 476 divided into three main sections: the mean velocity characteristics of the wake, turbulence
 477 characteristics of the wake and assessment of current modelling approaches. All results are
 478 presented from an upstream position, for a freestream velocity of 10 m/s. Two tip speed
 479 ratios (TSRs) were investigated (i.e. 2.54 and 3.87). TSR 2.54 represented the turbine op-
 480 erating at its optimum TSR. TSR 3.87 represented the turbine operating above its optimum
 481 TSR value. All graphs are normalized against the model turbine radius with Z/R equals 0
 482 representing the centre of the rotor. The axial velocity profile in the wake is presented in a
 483 non-dimensional format U_x/U_∞ , where U_x is the streamwise velocity at the plane and U_∞
 484 is the freestream velocity. Data is taken from a horizontal line through the middle of the rotor
 485 at three separate locations downstream (0.66D, 1D and 1.5D).

486
 487 A number of volume renders from CFD solutions will be presented. Because the flow is time
 488 dependant, the results discussed in this section represent a statistical average of the flow
 489 field for a large amount of rotor rotations.

490
 491 Firstly, the mean velocity deficit behind the rotor for both TSR cases is compared to nu-
 492 merical predictions. Accurate predictions of the wake velocity deficit would confirm that the
 493 numerical models were able to accurately model rotor loading and the momentum deficit
 494 created by the extraction of energy from the flow by the HAWT.

495

496 The ability of each model to predict turbulence characteristics of the wake is also investi-
497 gated. With respect to future wind turbine structural modelling attempts, an understanding
498 of the spatial distribution of stresses generated within the wake is important. Additionally,
499 understanding the limitations of turbulence models to predict stresses in the flow is impor-
500 tant as this would impact on the choice of modelling strategy used in future FSI simulations.
501 Turbulence plays a direct role on the unsteady forces and bending moments experienced by
502 turbine blades downstream. Additionally, a comprehensive understanding of the turbulent
503 characteristics of a turbine wake are necessary for validating and guiding the development
504 of sub-grid scale parameterizations in high fidelity numerical models such as LES [10]. The
505 $\overline{u'v'}$ Reynolds stress component is normalized by the square of the freestream velocity U_∞^2
506 and is presented against the non-dimensional distance Z/R .

507

508 *4.2. Mean Velocity Characteristics*

509 From the offset, it can be seen that there is good agreement between the experimental and
510 computational results. A strong correlation is observed between figures 10a, 10b and 10c in
511 terms of velocity deficit for a TSR value of 2.54. Both numerical and experimental results
512 show the velocity deficit takes the form of a Laplace distribution. Experimental data shows a
513 severe decrease in axial velocity, particularly around the region $Z/R = 0$, which corresponds
514 to the region directly behind the hub of the model wind turbine. All numerical models
515 predict similar results with average percentage errors between numerical and experimental
516 results ranging between 2–4%. The ability of each turbulence model to accurately capture
517 wake velocity deficit confirms that the numerical models were able to accurately model rotor
518 loading and the momentum deficit created by the extraction of kinetic energy from the flow
519 by the HAWT.

520

521 If the centre of the wake is defined as the point where the velocity deficit was maximum,
522 then as shown in figures 10a, 10b and 10c, the point of maximum recorded velocity deficit
523 in the wake is located at $Z/R = 0.1$. In figures 11a, 11b and 11c (which present contour
524 plots of experimentally recorded axial velocity), the centre of the wake tends to drift slightly
525 down to the right at $Z/R = 0.1$ and $Y/R = -0.12$ with minimum values of $0.33U_\infty$ at $0.66D$
526 and $0.64U_\infty$ at $1.5D$. This could be attributed to the pressure field around the turbine. For
527 explanation purposes, the root vortex system is compared to the tip vortex of a simple wing.
528 In flight, a pressure differential exists at the tip of a simple wing, which results in airflow
529 rotating around the wing tip from the high to low pressure region. Similarly, a low pressure
530 region exists behind the turbine, below the nacelle structure, due to the presence of the
531 tower structure. There is less obstruction to the wake in the upper region, which leads to
532 higher pressure values in the upper wake region, relative to the lower part of the wake. The
533 combination of high and low pressure regions may result in the centre of the wake drifting
534 downwards towards the low pressure region. Now, considering the simple wing case, the
535 movement of the airflow around the wing tip causes the tip vortex to move inboard [47].
536 Again, for the current study, the HAWT is rotating in an anti-clockwise direction. This
537 applies a torque to the wake, which results in a clockwise rotating wake and therefore a

538 clockwise rotating root vortex system. This, would result in the root vortex system drifting
539 outboard towards the right, which results in a wake centre that is off centre and to the right
540 of the nacelle/tower structure.

541
542 Velocity deficit values are seen to concentrate behind the hub structure (figures 11a, 11b and
543 11c). The velocity deficit extends $0.5R$, which would suggest that the root vortex system
544 and the turbine structure are the major contributors to the wake velocity deficit, as shown
545 in figure 11. However, beyond $0.5R$ the velocity deficit is seen to recover rapidly, which
546 contradicts previous studies. The wake is usually defined by a reduced velocity value where
547 the recovery of the wake to freestream values usually occurs near the edge of the rotor as
548 shown in a study carried out by Schüemann et al. [48]. This is possibly due to the aerofoil
549 design used. The rotor blades featured a FFA-W3-241 aerofoil (as indicated in section 2.2).
550 Originally designed at FFA (The Aeronautical Research Institute of Sweden); the aerofoil
551 has a relatively high thickness at 21% and is typically used on the inboard sections of turbine
552 blades [49], however, the aerofoil is not suitable for use at the outboard sections of the blade,
553 which would result in low rotor loading towards the rotor edge. However, the data taken
554 in this study allows us to see that the tower structure, nacelle and the central root vortex
555 system supply a constant velocity deficit to the wake. Additionally, the close comparison
556 of numerical and experimental data makes the current study suitable for turbulence model
557 validation as the characteristics of the FFA aerofoil are known. This is supported by Ver-
558 meer et al. [18], who stated that as long as the characteristics of the aerofoil are known, the
559 aerofoil is suitable for turbulence model validation.

560
561 The outer regions of the rotor blades do reduce the freestream velocity behind the rotor, but
562 it is noticeable only when data is recorded over a large period of time, as shown in figure
563 12 (where mean velocity deficits in the wake range predominantly from 66% of freestream
564 values for TSR equals 2.54). Once outside the influence of the turbine structure and thicker
565 blade roots, the velocity in the outer regions of the rotor is only periodically reduced as
566 opposed to continuously experiencing a constant velocity deficit. The influence of the blades
567 on the wake velocity deficit towards the rotor edge becomes smaller as the blade chord
568 length decreases. Thus, there is a return to freestream velocity values due to reduced rotor
569 solidity (and low blade loading), particularly in the other most region of the wake (as seen
570 for TSR 2.54 in figure 12). In addition, the large nose cone generates a considerable velocity
571 reduction (up to $0.7U_\infty$) in front of the rotor, as shown in figure 12, which would also aid the
572 wake velocity deficit in the region close to the wake centre. This aspect of model design has
573 not been mentioned in previous studies outlined in a review by O'Brien et al. [5] and should
574 be considered for future works on this topic. The nose cone design used has a great impact
575 on the formation of the wake, particularly the centre of the wake structure as flow over the
576 nose cone alters flow over the blade roots and therefore alters the structure of the central
577 vortex system of the HAWT wake. In the current study the wake structure is defined by a
578 velocity deficit generated by the central root vortex system and the tower/nacelle structure.
579 This central system is surrounded (seen on the upper half of the wake outside the influence
580 on the turbine structure in figure 12A) by a region of fluid moving at freestream velocity.

581 Outside this again, the presence of the tip vortex region can be identified as the region with
582 slightly reduced velocity values of $0.95U_\infty$.

583
584 Towards the blade tip region ($Z/R = \pm 1$), as shown in figure 10, all models tend to over
585 predict velocity values in this region. There are several possible reasons for this. Firstly,
586 when considering the SST $k - \omega$ model, the increase in velocity at $Z/R = \pm 1$ could be a
587 result of under-estimation of vortex diffusion and the prediction of a more tightly bound tip
588 vortex. This could be a result of the dense wake mesh used (outlined in section 3.6), com-
589 bined with the Vorticity Confinement Model. Typically the Vorticity Confinement model
590 is used to reduce the mesh density of a simulation and maintain a vortex structure by the
591 addition of a forcing term (outlined in section 3.2). However, the very dense mesh in the
592 wake region would already have minimized the dissipation of the tip vortices. This com-
593 bined with the Vorticity Confinement Model would have further reduced vortex dissipation,
594 which would explain the under-estimation of the diffusion of the tip vortices (resulting in
595 a stronger vortex) and the increase in axial velocity in the tip region. Noted by Anderson
596 et al. [50], an increase in vortex strength can result in an increase in axial velocity in the
597 core of a tip vortex. This relationship between core axial velocity and vortex strength was
598 also seen by O'Regan et al. [51]. Following this, a study by O'Regan et al. [47] recorded
599 that stronger vortices also have increased axial velocity values in the vicinity around them,
600 which would explain the increased axial velocity values at the tip region in the current study.

601
602 Regarding the RST model, the turbulent dissipation rate is obtained from a transport equa-
603 tion analogous to the $k - \epsilon$ model. As described by Menter [52], in the standard $k - \epsilon$ model,
604 eddy viscosity is determined from a single turbulence length scale; whereas, in reality all
605 scales of motion will contribute to the turbulent diffusion. The same process is used for the
606 EB-RSM model. This could have contributed to the under-estimation of the diffusion of the
607 tip vortices and lead to the same result as outlined above.

608

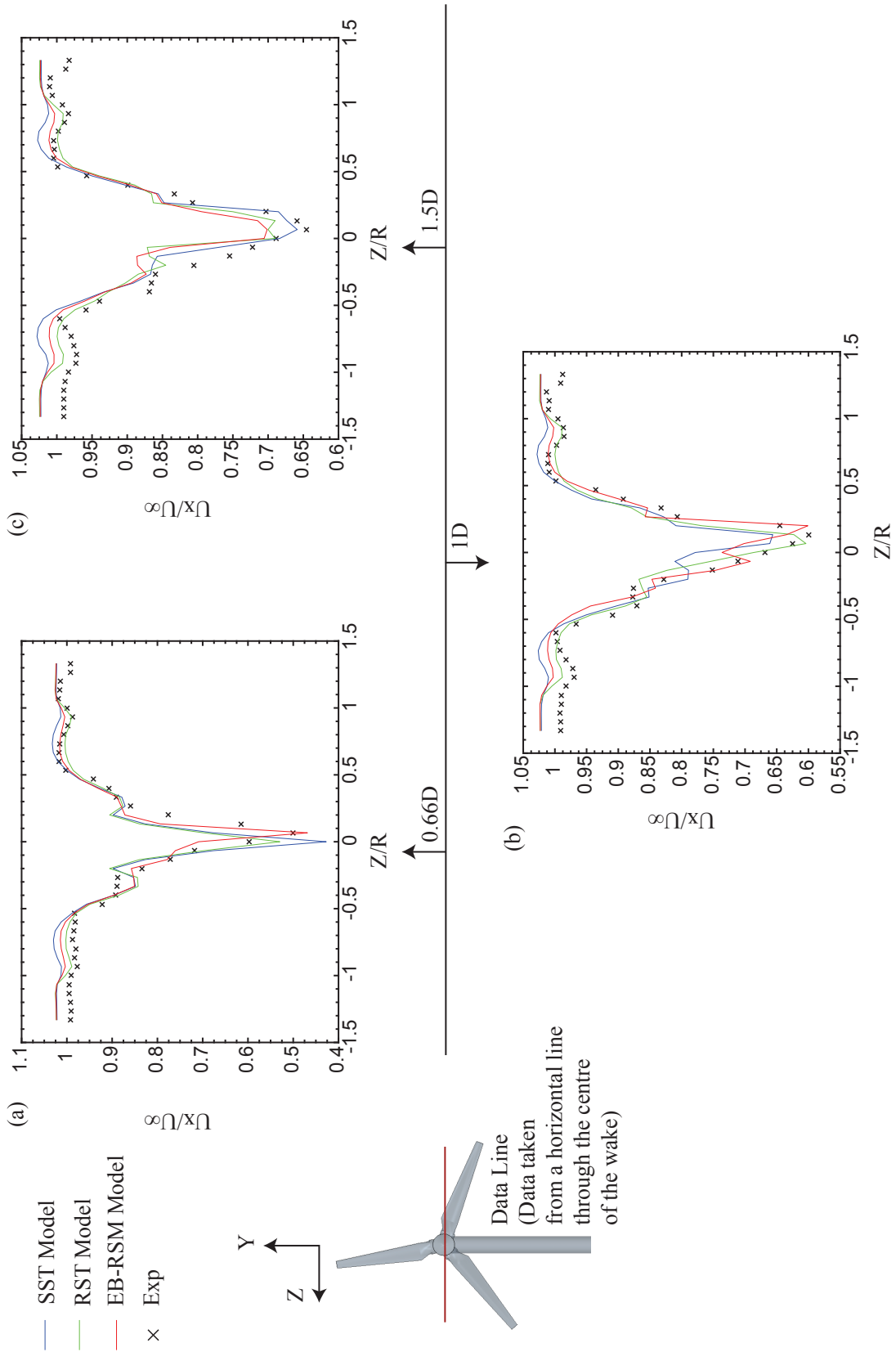
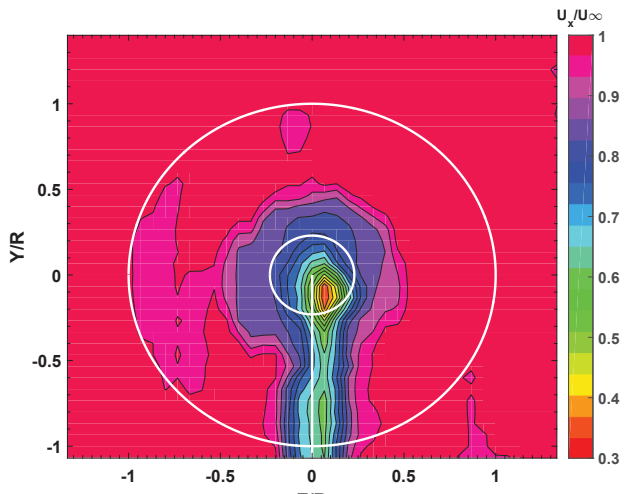
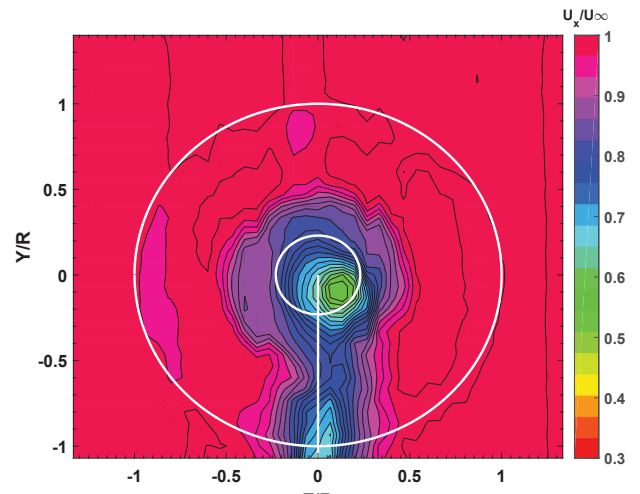


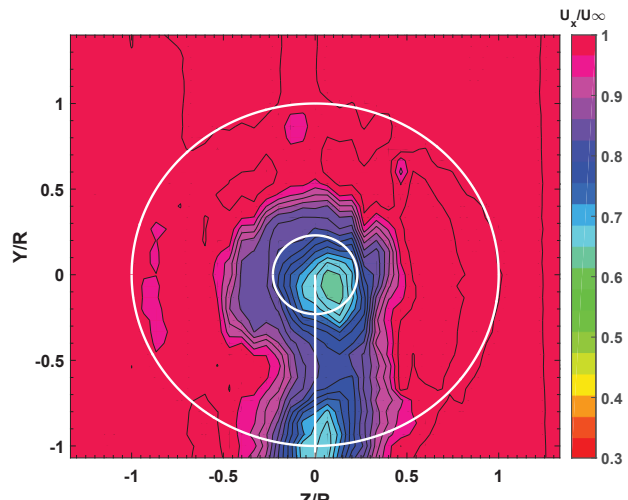
Figure 10: Axial velocity deficit values for a TSR value of 2.54



(a) Experimental 0.66D



(b) Experimental 1D



(c) Experimental 1.5D

Figure 11: Plots of axial velocity shown 0.66D, 1D and 1.5D downstream for $TSR = 2.54$

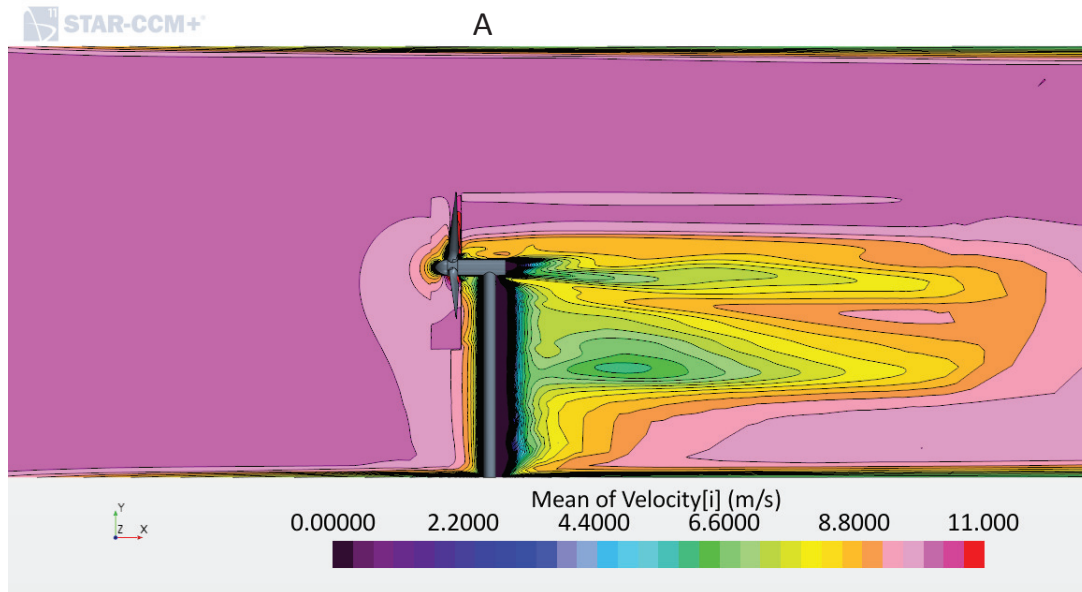


Figure 12: Mean axial velocity distribution within the HAWT wake for $TSR = 2.54$ predicted by RST simulation

609 From a qualitative point of view, the ability of the numerical models to capture the root
 610 vortex system and central velocity deficit in the wake has been attributed to the dense
 611 measurement grid and the thick aerofoil section used with a sharp taper towards the root
 612 section, creating distinct root vortices as shown in figure 13 . Due to their compact propaga-
 613 tion downstream and the strength of the root vortices, they appear to form a vortex sheet,
 614 with its formation attributed to the root vortices shedding from each blade. This formation
 615 has previously been described by Gómez-Elvira et al. [53] and Sanderse [17], but only in
 616 terms of the tip vortices. A volume render of the turbine wake is shown in figure 13, with
 617 the scaler range adjusted to promote viewing of the described central vortex. As seen in
 618 the volume render of figure 13, the central vortex does not fully develop until approximately
 619 $1.5D$ to $2D$ downstream. This was seen in all the numerical models. The individuality of
 620 each root vortex persisted longer downstream than experimental results. This again was
 621 attributed to the numerical models under-estimating vortex diffusion.

622

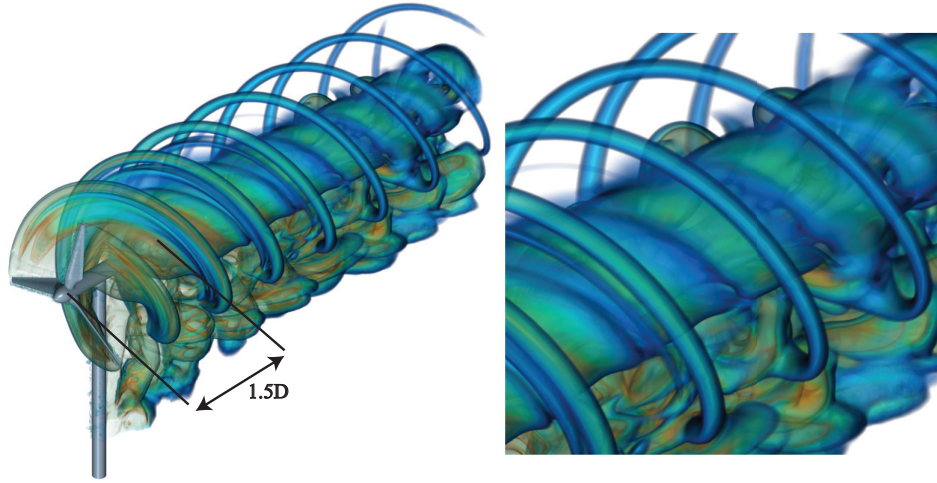


Figure 13: Volume render from RST simulation showing central vortex sheet. $TSR = 2.54$

623 Figure 14 shows the comparison between numerically predicted and experimentally recorded
 624 velocity deficit in the wake for a TSR value of 3.87. Similar to results for a TSR value of
 625 2.54, there is good agreement between numerical and experimental results. The velocity
 626 deficit takes the form of a Laplace distribution, which is typical of HAWT models. Experi-
 627 mental data shows the largest velocity deficit occurred at $Z/R = 0.1$, similar to TSR equals
 628 2.54. Maximum (experimentally recorded) axial velocity deficit behind the rotor is $0.51U_\infty$
 629 at $0.66D$ and recovers to $0.67U_\infty$ at $1.5D$. Average percentage errors between numerical and
 630 experimental results ranged between 4–7%.

631
 632 Similar to experimental results for TSR value of 2.54, the centre of the wake is slightly right
 633 of centre as shown in figures 15a, 15b and 15c. Here, minimum values of $0.4U_\infty$ at $0.66D$
 634 and $0.69U_\infty$ at $1.5D$ are recorded.

635
 636 It is noted that in experimental data shown in figures 15a, 15b and 15c that the deficit
 637 created by the tips almost completes 360° . This velocity deficit forms a complete circular
 638 pattern in experimental plots where TSR equals 3.87. This would suggest that the angle of
 639 the helical path of the wake, given by the flow angle at the blade tip, is inversely proportional
 640 to the tip speed ratio [11]. Similar to experimental results for TSR equals 2.54, the nacelle
 641 and tower structure had the greatest influence on the velocity deficit behind the turbine
 642 structure. A larger velocity deficit was noted when the turbine operated at a TSR value of
 643 2.54. This was the optimum operating condition for the HAWT model and therefore the
 644 maximum quantity of kinetic energy was extracted from the flow. The wake recovers slower
 645 for the TSR equals 2.54 case with a velocity deficit of $0.64U_\infty$ recorded in the centre of the
 646 wake at $1.5D$ downstream. At the same location for TSR equals 3.87, the velocity deficit
 647 value was $0.69U_\infty$. Common to both TSR cases, the velocity deficit recovers faster in the
 648 region behind the tower, which could be attributed to the enhanced mixing in the area due
 649 to a combination of both the tower and rotor wakes. There are two concentrated areas of

650 reduced velocity in figure 15c. These are defined by the authors as spurious data points and
651 should be ignored.

652
653 Considering the velocity deficit profile for the TSR equals 3.54 case, it is clear that there is
654 an increase in velocity at $Z/R = \pm 0.6$ (figure 14). This was attributed to the location of a
655 secondary vortex structure. The secondary vortex structure formed between 55% and 65%
656 of the blade span and resulted in an increase in the wake velocity to $1.07U_\infty$. The structure
657 merged with the tip vortex system and complete decay of the secondary structure occurred
658 at 2D downstream. A similar secondary structure was noted by Yang et al. [11] and Whale
659 et al. [6]. The structure formed between 50% and 60% of the blade span in the study carried
660 out by Yang et al. [11] and merged with the tip vortex system at 1D downstream. A similar
661 structure recorded by Whale et al. [6] merged with the tip vortex system at 1D downstream.
662 Figure 16 shows the flow over the suction surface of one of the turbine blades for both TSR
663 values. At the lowest TSR value of 2.54, it can be seen that the flow remains mostly attached
664 with minor separation beginning at the trailing edge. The flow is heavily influenced by cen-
665 trifugal effects with flow in the radial direction most dominant. At TSR equals 3.87, a large
666 area of separation occurs around 56% blade span, which coincides with the presence of the
667 secondary vortex structure. Again, all numerical models over predicted the axial velocity
668 in the wake at $Z/R = \pm 0.6$, which, as outlined earlier, could be a result of each turbulence
669 model over-estimating vortex strength and possibly under-estimating turbulent-diffusion.

670

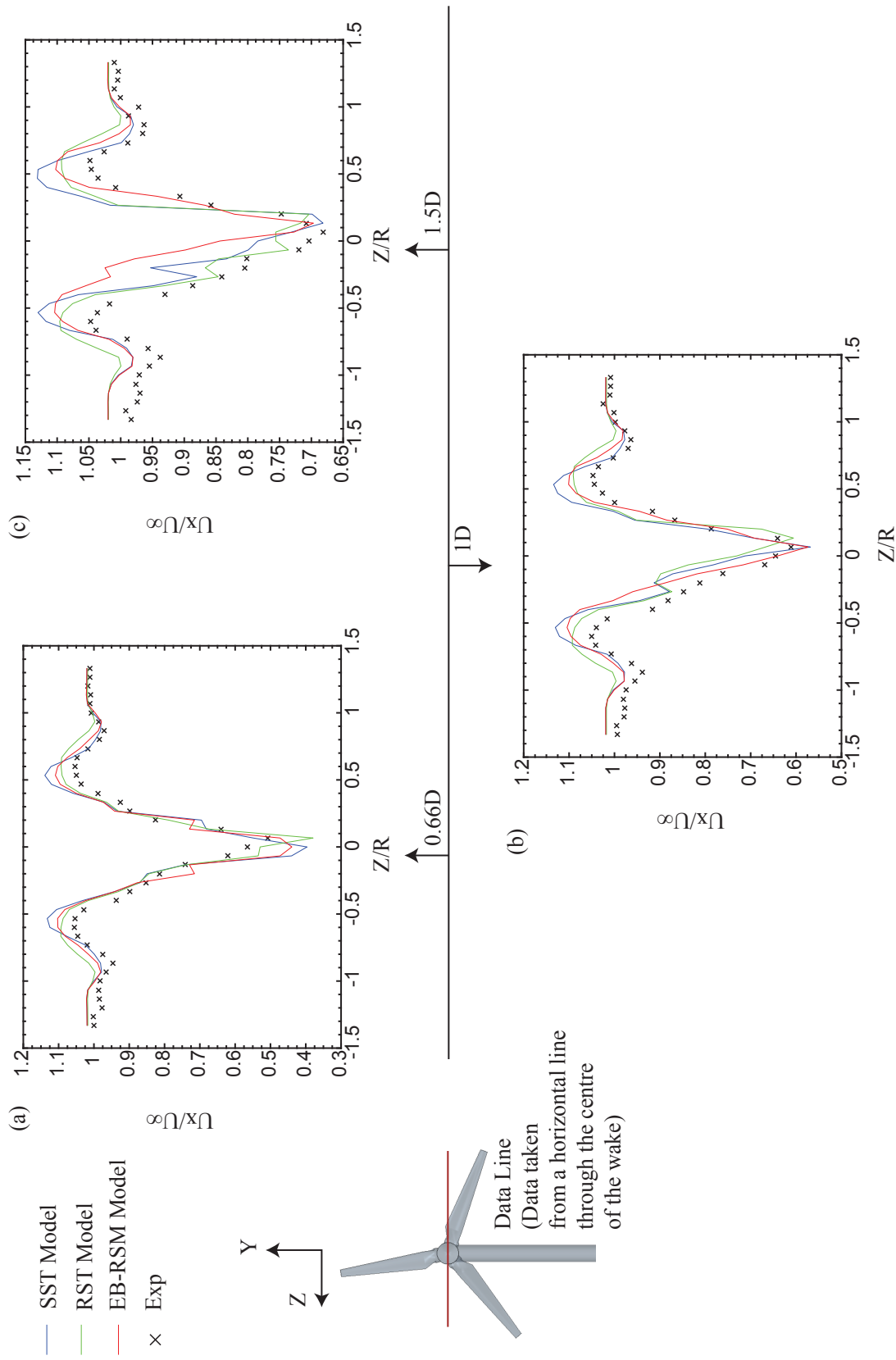


Figure 14: Axial velocity deficit values for a TSR value of 3.87

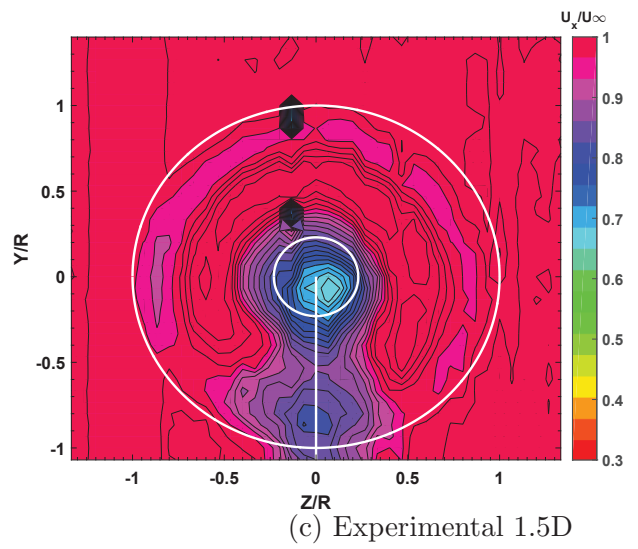
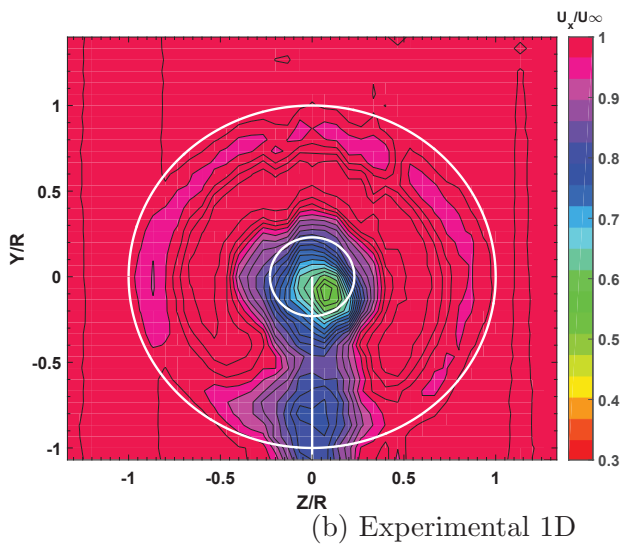
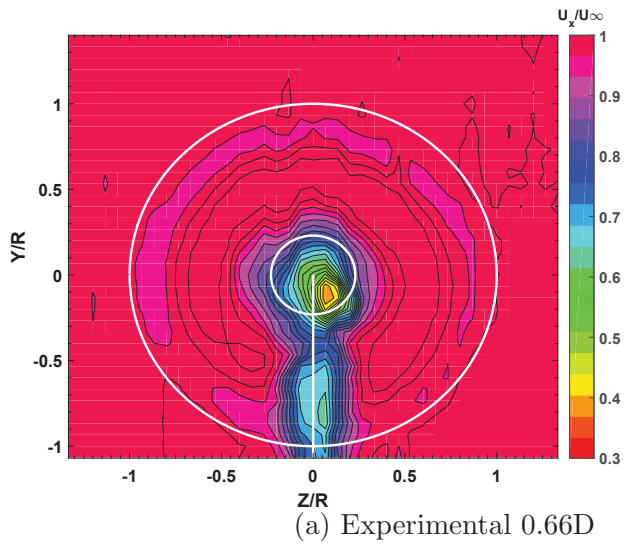


Figure 15: Plots of axial velocity shown 0.66D, 1D and 1D downstream for $TSR = 3.87$

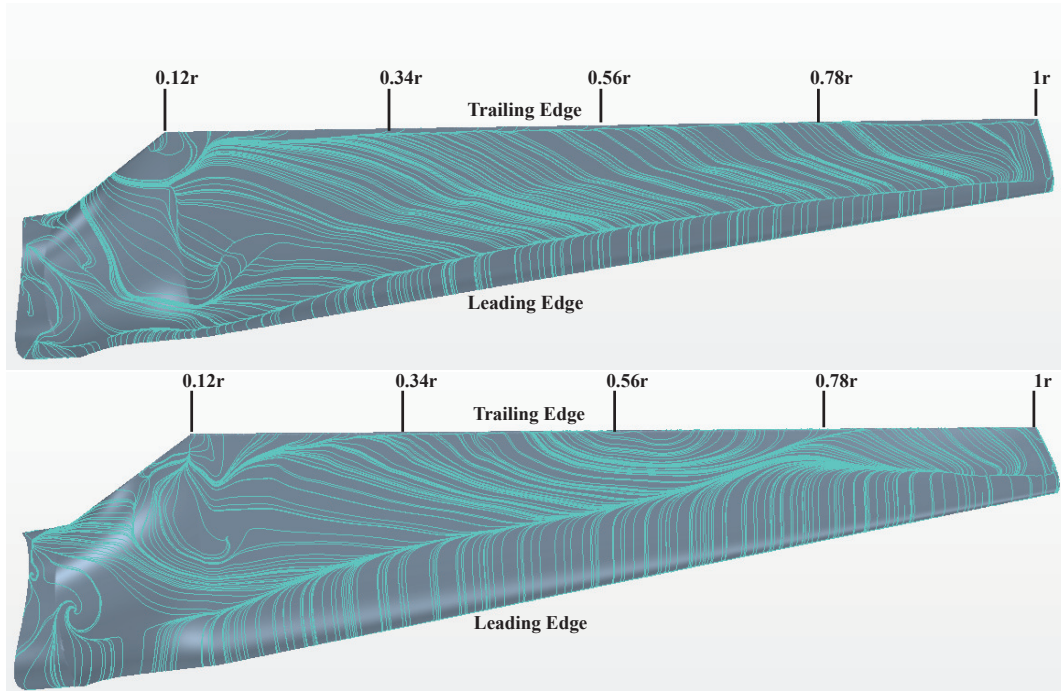


Figure 16: Streamlines on the surface of a computational blade at $TSR = 2.54$ (top) and 3.87 (bottom)

671 4.3. Turbulence Characteristics – Time averaged u' and v' velocity components

672 As outlined in section 4.2, peak velocities near the blade tip and root regions for TSR val-
 673 ues of 2.54 and 3.87 respectively have been attributed to turbulence models over-predicting
 674 vortex strength and under-estimating turbulent diffusion. Under-estimating turbulent dif-
 675 fusion, which would result in a more stable and stronger vortex system, arises from the
 676 under-prediction of fluctuating components in the flow. Under-predicting fluctuating ve-
 677 locity components and Reynolds stresses in the flow reduces the mixing rate within the
 678 wake and therefore, wake structures such as tip and root vortices can be preserved in the
 679 wake. Additionally, the wake itself is preserved further downstream. With respect to future
 680 wind turbine structural modelling attempts, an understanding of the spatial distribution of
 681 stresses generated within the wake is important. The fluctuating velocity components in the
 682 flow directly contribute to the unsteady forces acting on turbine blades.

683
 684 To understand the ability of each turbulence model to predict the turbulent characteristics
 685 of a HAWT wake, both the Reynolds stress values and the Root Mean Squared (rms) values
 686 of both the u' and v' fluctuating velocity components were investigated. This gave an appre-
 687 ciation of the ability of each turbulence model to predict the magnitude of the fluctuating
 688 velocity components in the wake.

689
 690 Figures 17 and 18 show the normalized rms velocity fluctuations for the u' velocity compo-
 691 nent at different locations downstream for both TSR values. Considering figure 17 (which

692 shows the rms u' values for TSR equals 2.54), it can be seen that there is good agreement
 693 between numerical and experiment results. All numerical models predicted the magnitude
 694 of the rms u' velocity component to the correct order of magnitude. However, at all down-
 695 stream locations, all numerical models under-predicted the rms u' component in the wake
 696 near the blade tip region $Z/R = \pm 1$ with the exception of the RST model at 0.66D. All
 697 models under-predicted the rms u' component in the centre of the wake around $Z/R = 0$,
 698 with the exception of the EB-RSM model at 1D downstream and both the SST $k - \omega$ and
 699 EB-RSM model at 1.5D downstream. Both experimental and numerical data show the rms
 700 u' velocity component take the form of an inverse Laplace distribution. For TSR value of
 701 2.54 (figure 17), experimentally recorded peak rms u' velocities consistently occur at Z/R
 702 $= 0.1$, which coincides with the centre of the wake as mentioned in section 4.2. Interaction
 703 between the central root vortex system and the turbine structure leads to enhanced mixing
 704 in this region. Maximum values of 0.12, 0.1 and 0.08 were recorded at 0.66D, 1D and 1.5D,
 705 respectively. The decreasing values of rms u' in the centre of the wake appear to descend
 706 linearly for a TSR value of 2.54 and illustrate the reduction of wake turbulence with distance
 707 downstream. Additional peaks are also present at $Z/R = \pm 1$ at each measurement location
 708 downstream, which coincides with the blade tip location. Maximum values of 0.025, 0.0225
 709 and 0.0212 were recorded at 0.66D, 1D and 1.5D, respectively. This again shows a near
 710 linear reduction of rms u' values in the blade tip region.

711
 712 Each numerical model appeared to have both advantages and disadvantages associated with
 713 them. Across all measurement planes in figure 17, the RST model tended to most accurately
 714 predict rms u' values in the blade tip region $Z/R = \pm 1$, with percentage errors ranging from
 715 -15% to +17% (the RST model under-predicted the rms u' values at every measurement
 716 plane with the exception at $Z/R = -1$ at 0.66D) of experimental results over 1.5D down-
 717 stream. Both the SST $k - \omega$ and EB-RSM model consistently under-predicted rms u' values
 718 in the blade tip region with percentage errors ranging from -3% to -34% of experimental
 719 results over 1.5D downstream. Regarding the blade tips at $Z/R = \pm 1$ for TSR value of 2.54,
 720 in this region each turbulence model consistently under-predicts the rms u' velocity compo-
 721 nent. The under-prediction of rms u' in this region would support the argument made in
 722 section 4.2 that all models under-predicted vortex diffusion, turbulent dissipation and overall
 723 turbulent instability in this region. All models were more consistent regarding the predic-
 724 tion of rms u' values in the region $Z/R = 0$. Here the EB-RSM model tended to predict the
 725 highest levels of rms u' values in the wake, with the RST model consistently predicting the
 726 lowest. At $Z/R = 0.1$, the percentage difference between numerical and experimental results
 727 were -8%, -43% and -7% for the SST $k - \omega$, RST and EB-RSM models respectively at 0.66D.
 728 With distance downstream, the RST model tended to more closely resemble experimental
 729 results at $Z/R = 0.1$ with the percentage difference between experimental results and the
 730 RST model being -26% at 1.5D. However, at this point both the SST $k - \omega$ and EB-RSM
 731 model over-predicted experimental results by 34% and 61%. Although experimental results
 732 show a linear reduction of peak rms u' values in the wake centre, none of the numerical
 733 models followed this trend.

734

735 Similar results are seen in figure 18, which shows the distribution of the rms u' velocity
736 component for a TSR value of 3.87. Peak rms u' velocities are located at $Z/R = 0.1$ (again
737 the location of the wake centre). Immediately, it can be seen in figure 18a that all numerical
738 models greatly under-predicted the rms u' velocity component. This trend continued with
739 distance downstream. Maximum values of 0.107, 0.1 and 0.08 were recorded at 0.66D,
740 1D and 1.5D, respectively for experimental results at $Z/R = 0.1$. The RST model greatly
741 under-predicted the magnitude of the rms u' velocity component. The RST model tended
742 to predict fluctuating rms u' velocities around 0.03 through most of the centre of the wake,
743 which suggests that the model predicted less shedding across the rotor. An increase in u'
744 velocities are located at $Z/R = \pm 0.8$, which are caused by the secondary vortex mentioned
745 in section 4.2. Here peak experimental rms u' values are 0.037, 0.035 and 0.032 at 0.66D, 1D
746 and 1.5D, respectively. Similar to peak rms u' velocities in the wake centre for TSR value
747 of 2.54, the rms u' velocities for the secondary vortical structure appear to decrease linearly
748 with distance downstream. Again, all turbulence models under-predicted rms u' values in
749 this region.

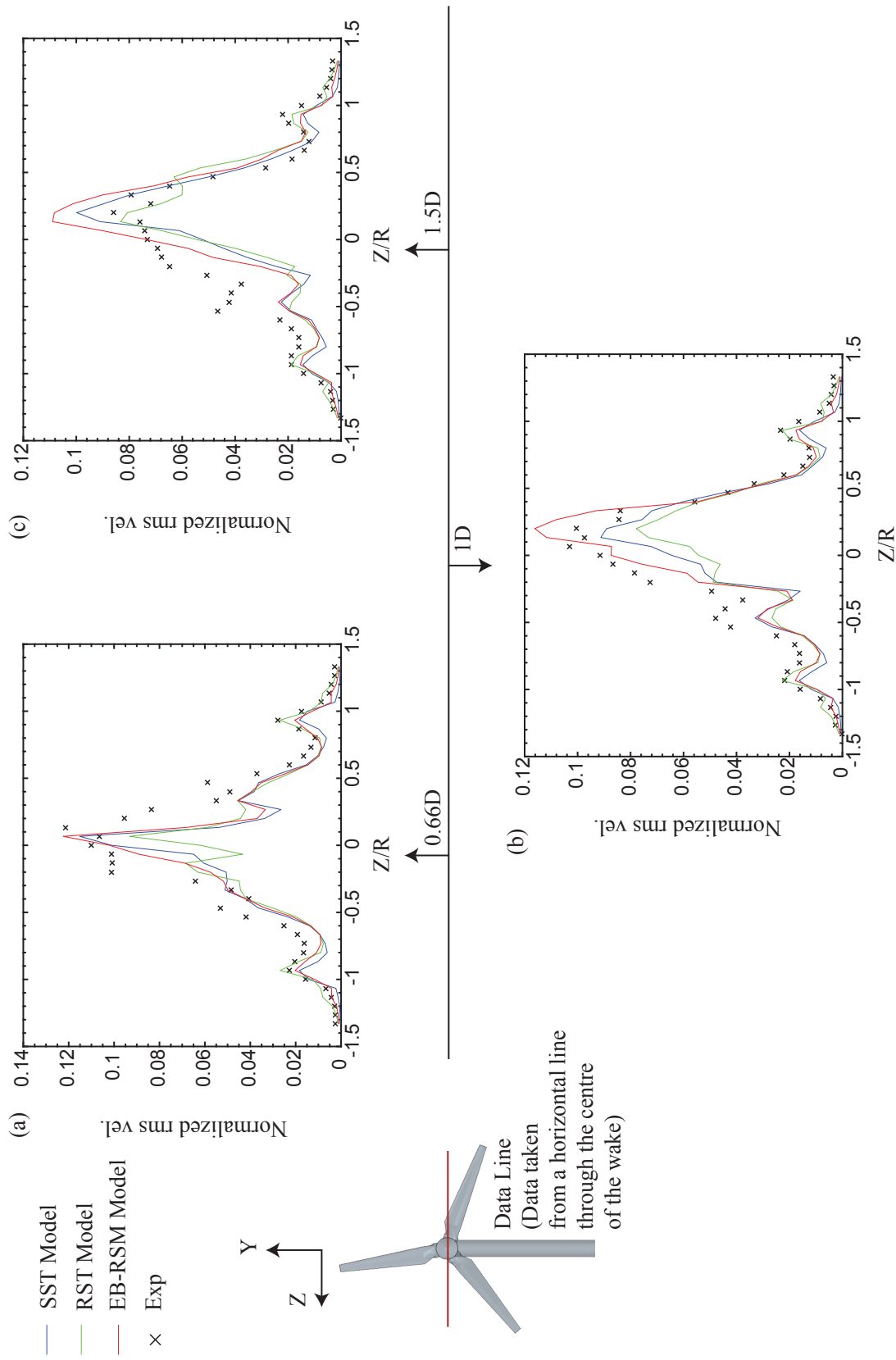


Figure 17: Normalized rms velocities $\frac{\sqrt{u'^2}}{U_\infty}$ at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 2.54

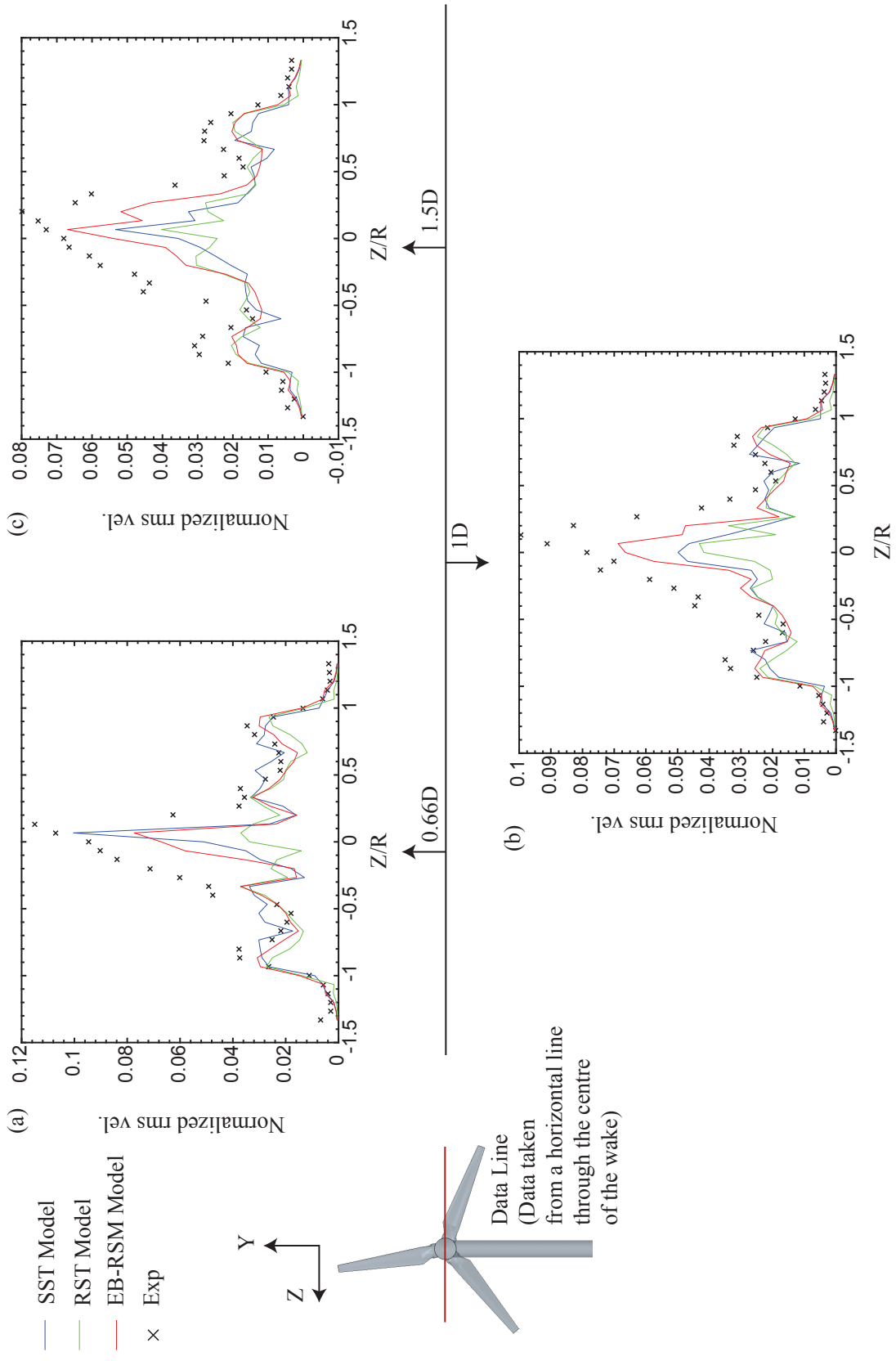


Figure 18: Normalized rms velocities $\frac{\sqrt{u'^2}}{U_\infty}$ at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 3.87

750 Figures 19 and 20 show the rms velocity fluctuations for the v' velocity component at dif-
751 ferent locations downstream for both TSR values. Again, the experimental rms v' values
752 in the wake take on an inverse Laplace distribution. Maximum experimentally recorded
753 rms v' values occur at $Z/R = 0.1$ for both TSR cases, similar to those recorded for the
754 TSR equals 2.54 case. However, unlike the TSR 2.54 case, peak experimental rms v' values
755 tend to stay the same with distance downstream with values of 0.101, 0.104 and 0.101 at
756 0.66D, 1D and 1.5D, respectively downstream for a TSR value of 2.54. Peak rms v' values
757 of 0.09, 0.09 and 0.075 were recorded at 0.66D, 1D and 1.5D, respectively downstream for
758 a TSR value of 3.87 (figure 20). Additionally, peak rms v' values occur at $Z/R = \pm 0.7$, the
759 location of the secondary vortex for a TSR value of 3.87. Here peak experimental values of
760 0.032, 0.032 and 0.029 are recorded at 0.66D, 1D and 1.5D downstream. Comparison of the
761 peak experimental rms u' and rms v' values for both TSR cases highlight that both velocity
762 components are similar in magnitude, with the rms v' magnitude on average being 95% of
763 the rms u' values.

764
765 Similar to the predicted values of rms u' , all turbulence models predicted the magnitude of
766 the rms v' components to the correct order of magnitude for both TSR cases. However, all
767 models tended to over-predict rms v' components from $Z/R = \pm 0.5$ and outward towards
768 the blade tip for both TSR equals 2.54 and 3.87 (figures 19 and 20).

769
770 The models for the most part under-predicted the rms of the fluctuating velocity components,
771 in particular for TSR equals 3.87 in the region $Z/R = 0$. This was attributed to varying
772 $y+$ distributions over the blades, which increased the difficulty for the models to resolve the
773 boundary layer. In addition to this, increased velocities at the high TSR value would have
774 altered the $y+$ values present over the nacelle even further. This would have further reduced
775 the resolution of the boundary layer over the nacelle for the TSR = 3.87 case. The larger
776 inaccuracies towards the $Z/R = 0$ region is attributed to the fact that the $y+$ values over
777 the nacelle are the same as the values specified for the tower structure. Therefore, shedding
778 from the nacelle structure itself would not have been resolved for to the same accuracy as
779 that of the blades. As noted by Sanderse [17] the root vortices are in close proximity to each
780 other. This, combined with the induced turbulence from the turbine structure, cause the
781 root vortices to be destroyed much earlier than the tip vortices. Sherry et al. [28] attributed
782 the interaction between the nacelle boundary layer and the root vortex as a component in
783 the root vortices early destruction. This is because the vorticity created within the nacelle
784 boundary layer is of the same order of magnitude and opposite in sign to the coherent root
785 vortices, resulting in cross-annihilation of vorticity between the root vortex and the nacelle
786 boundary layer.

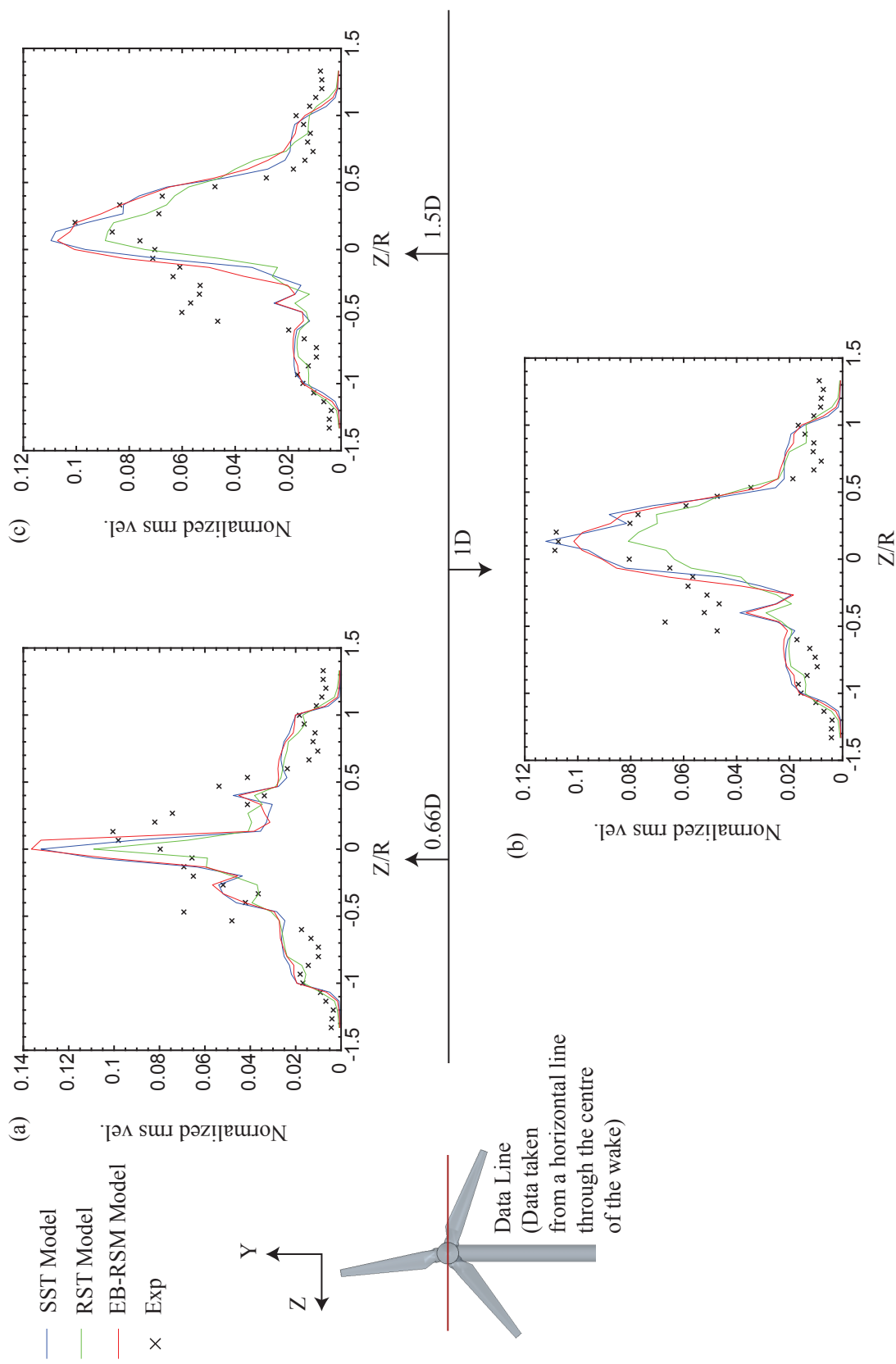


Figure 19: Normalized rms velocities $\frac{\sqrt{v'^2}}{U_\infty}$ at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 2.54

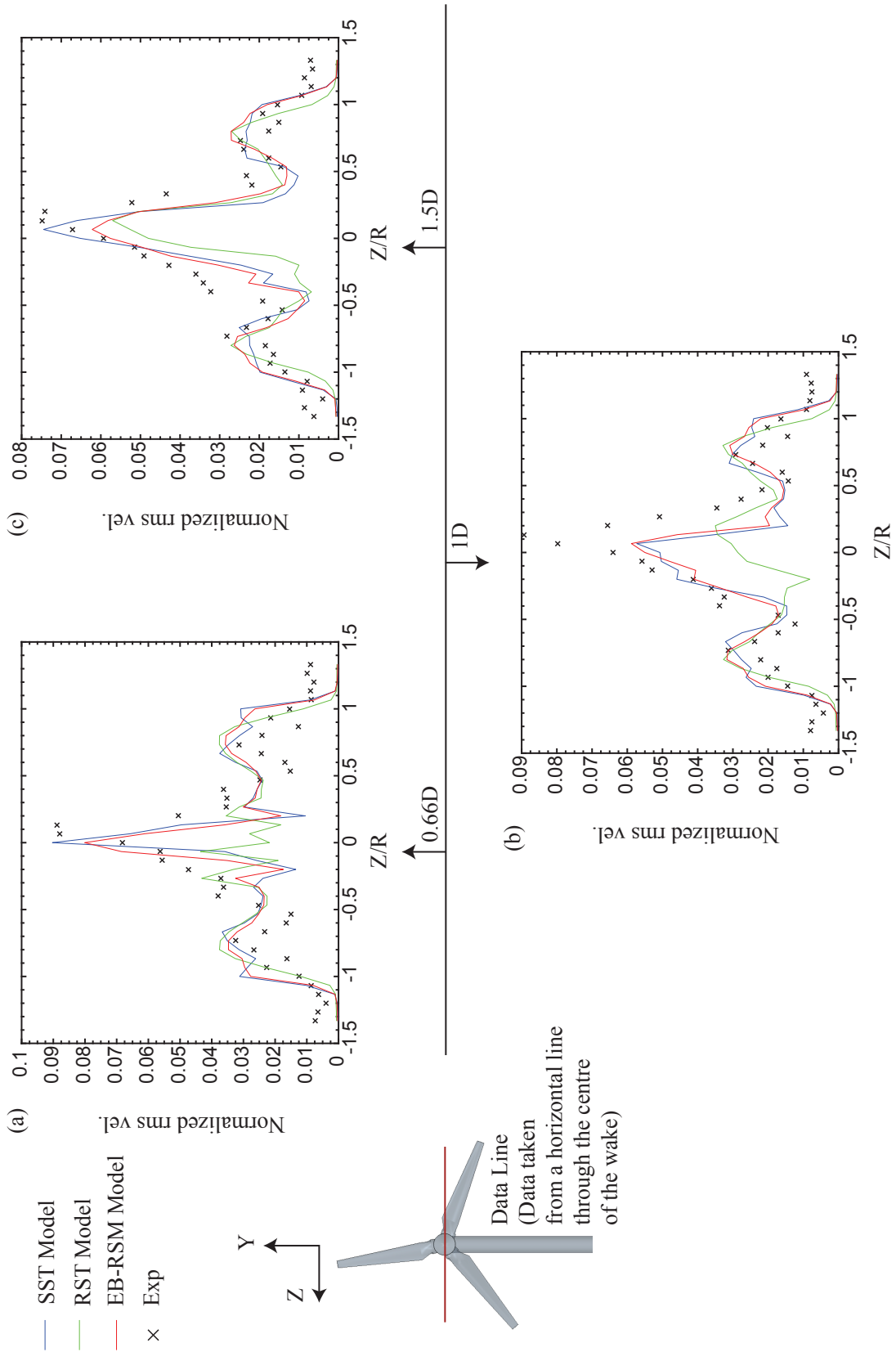


Figure 20: Normalized rms velocities $\frac{\sqrt{v'^2}}{U_\infty}$ at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 3.87

787 Figures 21 and 22 present the distribution of the $\overline{u'v'}/U_\infty^2$ Reynolds stress component for
 788 both TSR values in the z-direction. From the offset, it is clear that the tower and na-
 789 celle structure are the root source for the majority of the Reynolds stress within the flow.
 790 Reynolds stress values are seen to increase considerably when passing through $Z/R=0$ with
 791 interaction between the blade wake and shedding from the nacelle exciting the flow. For TSR
 792 equals 3.87 (figure 22), the experimental values of Reynolds stress begin to fluctuate around
 793 $Z/R \simeq \pm 0.7$. This coincided with the predicted location of the secondary vortical structure.
 794 Overall, the majority of the Reynolds stresses are localized behind the nacelle structure,
 795 suggesting that interaction between the wake and the nacelle structure contributes greatly
 796 to the stress levels within the flow. Stresses peak at $Z/R = \pm 0.1$ which is close to the root
 797 of the blade located at $r/R = 0.12$. It can be seen that the root vorticies for TSR equals
 798 3.87 (figure 22) are stronger than those at TSR equals 2.54 (figure 21). Initial Reynolds
 799 stress values associated with the root vorticies at $0.66D$ are 1.3×10^{-3} and 2.8×10^{-3} for
 800 TSR equals 2.54 and 3.87 respectively. However, despite having initially higher levels of
 801 shear stress, levels of stress reduce faster for TSR equals 3.87 in comparison to TSR 2.54.
 802 Again highlighting a faster degradation of the wake and return to freestream values. The
 803 transition from positive to negative values of stress at $Z/R = 0$ is due to the rotation of the
 804 root vortex system in the wake. Similar to both figures 21 and 22, the Reynolds stresses
 805 within the flow retreat towards 0 in the freestream. Reynolds stress values at TSR equals
 806 3.87 are larger than those for the TSR equals 2.54 case at the $Z/R = 0$ region. This could
 807 be attributed to the stronger root vortices present in this region due to the increased TSR.
 808
 809 Numerical predictions of the Reynolds stresses within the wake in the z-direction for TSR
 810 equals 2.54 and 3.87 are presented in figures 21 and 22. The location of maximum per-
 811 centage error between the computational and experimental results tends to be located at
 812 $-0.4 < Z/R < 0.4$ where the flow tends to be the most chaotic behind the nacelle struc-
 813 ture and in the root vortex system. The errors in this region are expected due to the over
 814 and under-prediction of the rms u' and v' components in this region mentioned earlier. All
 815 models do predict a reduction of the Reynolds stresses towards zero at $Z/R=\pm 1$, similar
 816 to experimental results. The values of Reynolds stress peak at $Z/R=\pm 0.1$, which coincides
 817 with experimental results. This is evident from $0.66D$ to $1.5D$. There is an overall trend of
 818 reduced Reynolds stress values with distance downstream, which indicates the decay of the
 819 vortical structures within the wake.

820

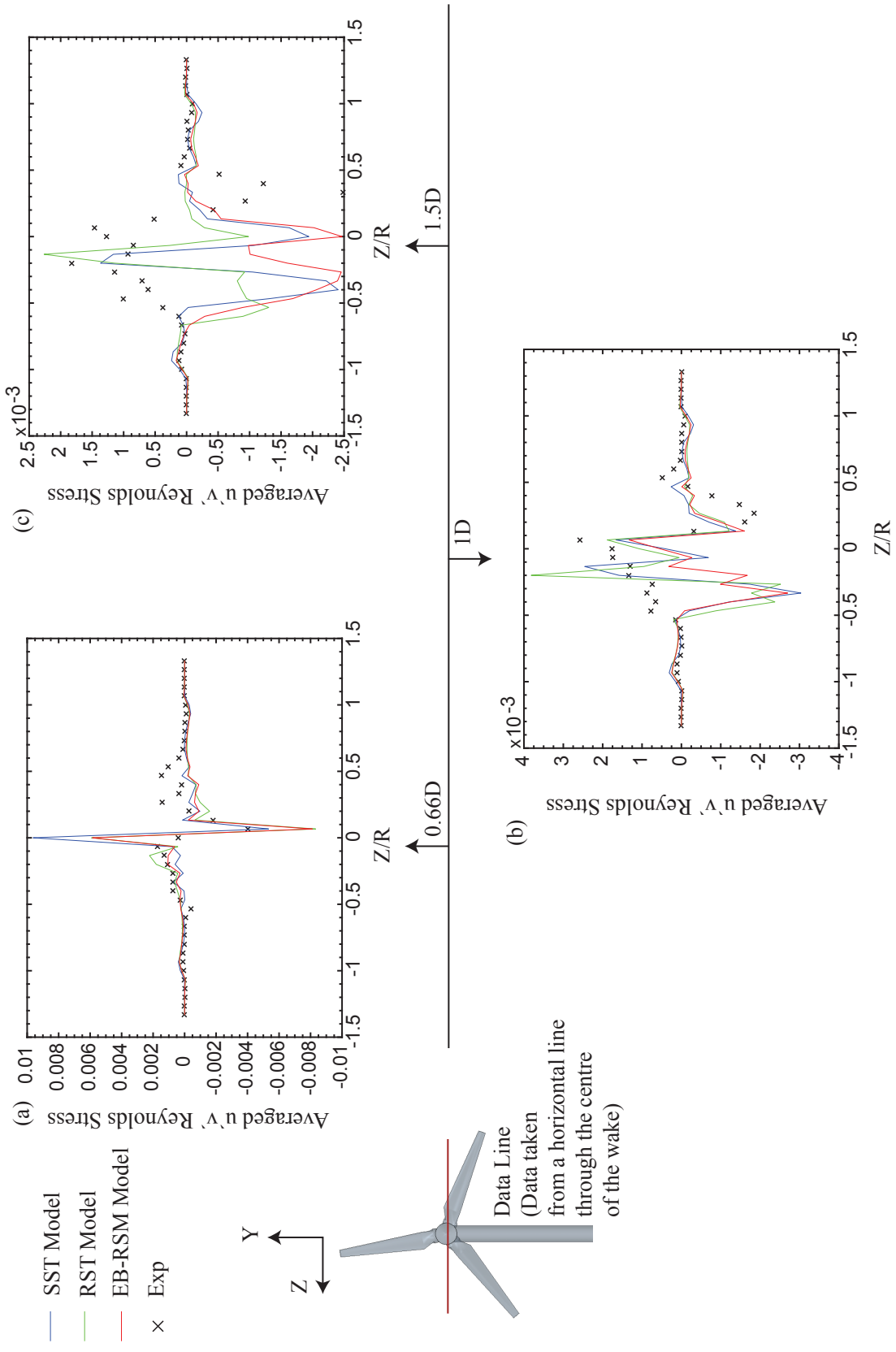


Figure 21: Averaged $u'v'$ Reynolds Stresses at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 2.54

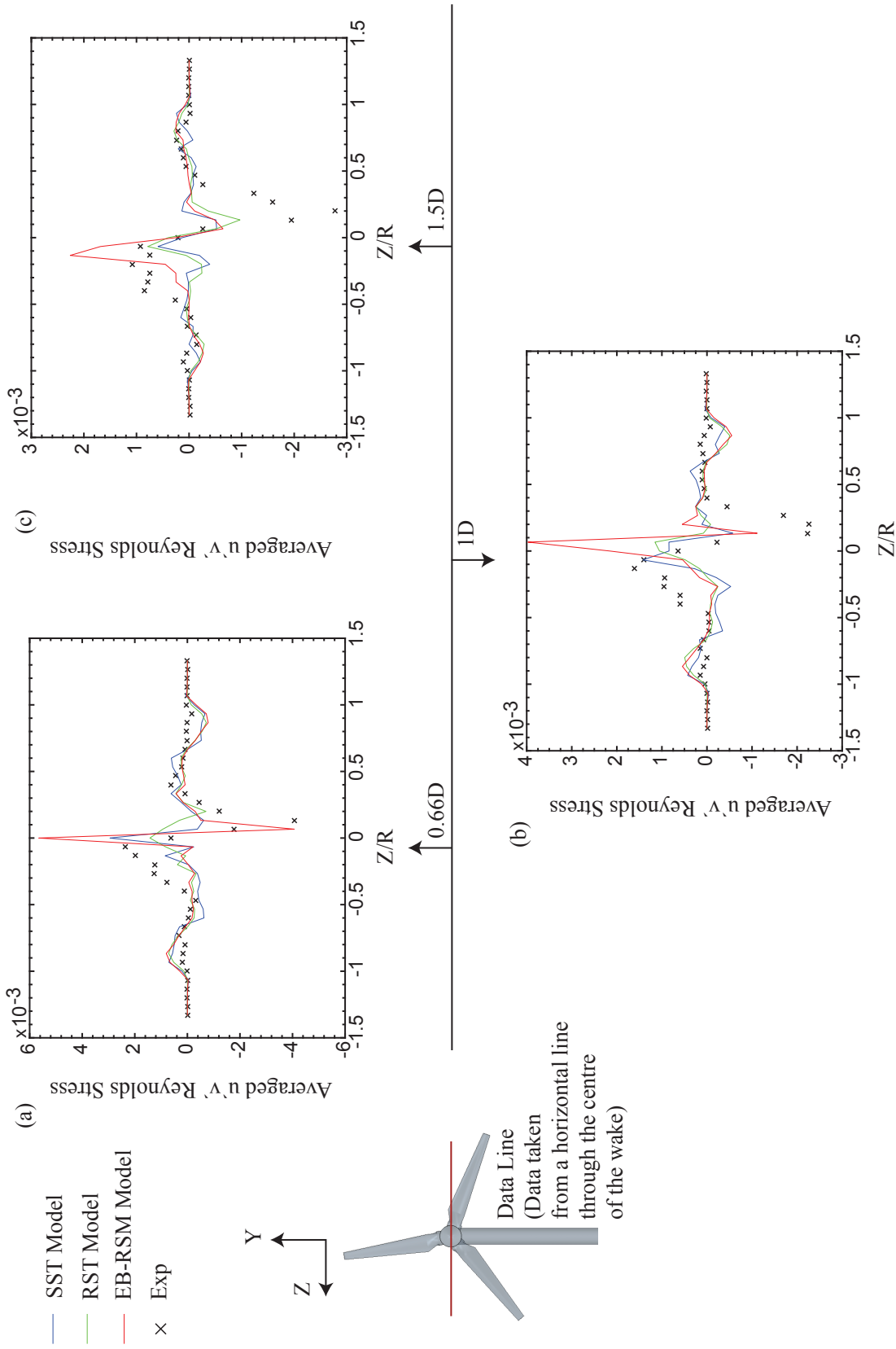


Figure 22: Averaged $u'v'$ Reynolds Stresses at 0.66D (a), 1D (b) and 1.5D (c) downstream for TSR equals 3.87

821 4.4. Assessment of Turbulence Modelling Strategies Used

822 As outlined in section 4.3, all models under-predicted the magnitude of the rms u' and v'
823 velocity components in the flow, which was highlighted again when comparing between ex-
824 perimentally recorded and numerically predicted $\overline{u'v'}/U_\infty^2$ Reynolds stress values. However,
825 by comparison of figures 17, 18, 19 and 20, all models tended to predict the inverse Laplace
826 distribution of both the rms u' and v' components (with the exception of the RST model,
827 which struggled to predict accurately the rms u' and v' components with accuracy for TSR
828 value of 3.87), which highlights that all models were able to model the physics of a HAWT
829 wake to varying degrees of accuracy.

830
831 Additionally, each turbulence modelling strategy employed, accurately predicted wake ve-
832 locity deficits with the largest discrepancy between the results of each turbulence modelling
833 strategy occurring when evaluating their ability to predict turbulence in the HAWT wake.
834 Their ability to model velocity fluctuations in the flow is where each model differs the most.
835 This is further supported when considering the RST and EB-RSM models. Both models
836 employed identical physics continua, mesh densities and y^+ wall treatments. The only dif-
837 ference between both models was the pressure strain relationship used by each. The RST
838 model used the linear pressure-strain two-layer relationship of Gibson et al. [39] and the
839 EB-RSM model used the elliptical-blending relationship of Manceau et al. [44]. As outlined
840 in section 4.3, the velocity fluctuations and essentially the origins of turbulence is often
841 developed from the interaction between the fluid flow and the wall.

842
843 Discrepancies between the models may highlight where the source of inaccuracy of each
844 model lies. For example, as outlined, each model had the same mesh distribution and y^+
845 wall treatments. The only difference between the models was the solvers themselves. How-
846 ever, despite this, as shown in section 3.7, each model predicted different maximum and
847 average y^+ values over the turbine blades. Considering that with a higher y^+ value there
848 is less resolution to capture the presence of the turbulent boundary layer. This would un-
849 doubtedly result in each blade predicting different velocity profiles at the boundary layer.
850 Additionally, having a combination of y^+ values with values greater and less than unity over
851 the blade could have made resolving the boundary layer difficult for each model. When y^+
852 values are below unity, the boundary layer is resolved. However, the boundary layer is mod-
853 elled using wall treatments when the y^+ values are above unity. The over and back between
854 both these methods could have increased the difficulty of solving the boundary layer for each
855 turbulence model. It is unknown if altering the blade mesh such that the y^+ values remain
856 above or below unity during the simulation would have improved the results.

857
858 The difference in modelled boundary layers can be highlighted when looking at the wall
859 shear stress values over the rotor. Figure 23 shows the distribution of wall shear stress
860 over the rotor, as predicted by the RST and SST $k - \omega$ models. The EB model predicted
861 similar wall shear stress values in comparison to the SST $k - \omega$ model and therefore, the
862 distribution of wall shear values predicted by the EB model are not shown. It can be seen
863 in figure 23(a) that the SST $k - \omega$ model predicted greater variation in wall shear stress

864 towards the trailing edge of the blades (highlighted in red). This swirl pattern at the trailing
 865 edge is associated with increased shear values and shedding along the length of the blade
 866 trailing edge. This is not captured in the RST model and might also explain why the model
 867 greatly under-predicted the rms velocity components for a TSR value of 3.

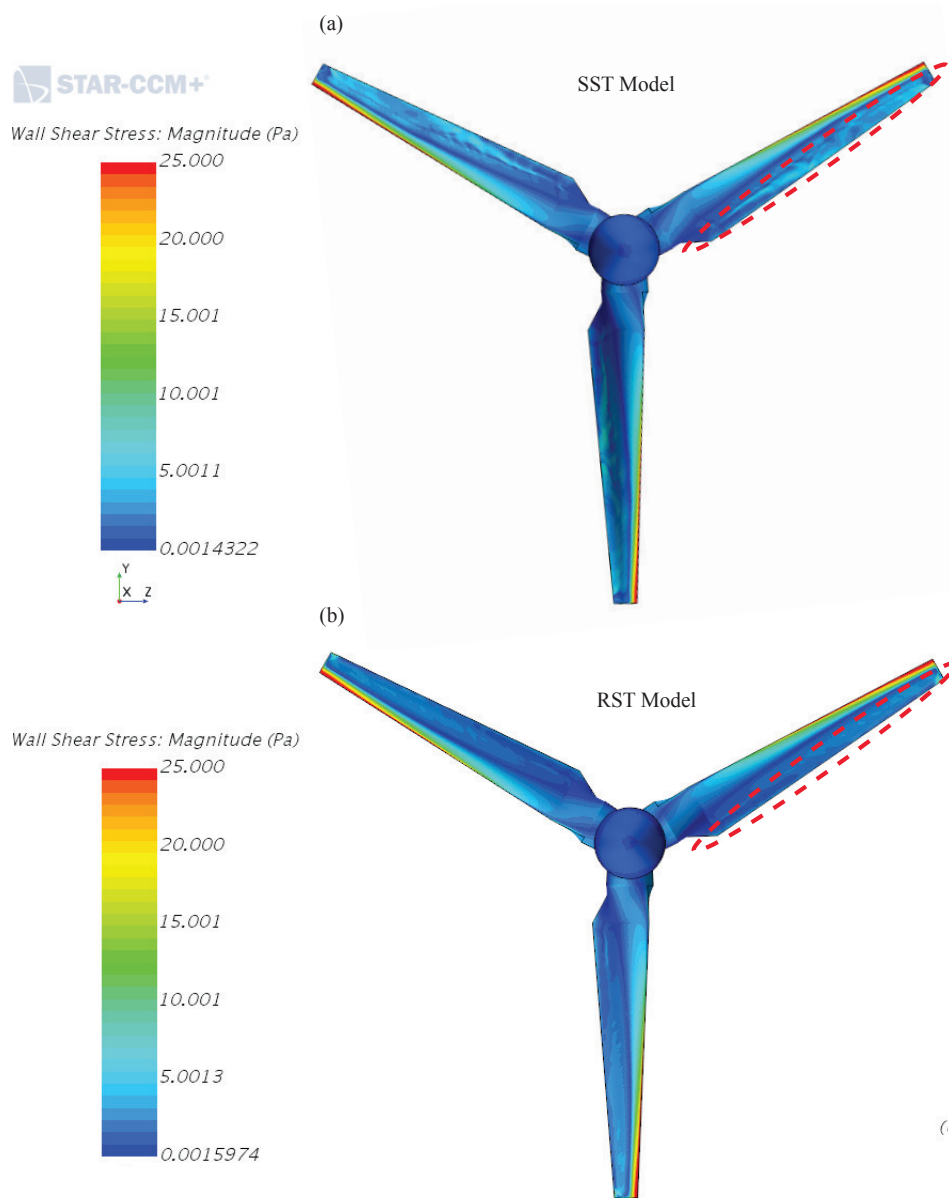


Figure 23: Wall shear stress values over the rotor predicted by SST $k - \omega$ model (a) and RST model (b).
 TSR = 3.87

868 A final consideration is the difference in results between the RST and EB-RSM models. The
 869 EB-RSM model was developed to be a less intensive version of the RST model as outlined

870 in section 3.5 by adapting the elliptical blending pressure-strain relationship of Manceau et
871 al. [44]. In the linear pressure-strain approach, the computation is divided into two layers.
872 In the layer adjacent to the wall, the turbulent dissipation rate ϵ and the turbulent viscosity
873 μ_t are specified as functions of wall distance. The formulation is the same as that used in
874 the $k - \epsilon$ model. The two-layer model prescribes values for ϵ algebraically based on distance
875 from the wall in the viscosity dominated near-wall flow regions. This might have resulted
876 in the solver being more sensitive to varying y^+ values in the current study. On the other
877 hand, the EB-RSM model used the standard elliptical blending method of Billard et al. [54],
878 which has been noted to be an improvement on the existing realizable $k - \epsilon$ model in terms
879 of accuracy, especially in the near-wall region. Additionally, as noted by Manceau et al.
880 [44], the elliptic blending strategy allowed for the integration of wall treatments down to the
881 wall without the use of damping functions with one elliptical equation instead of six.

882
883 However, the EB-RSM model itself was inaccurate. This could be attributed to the sim-
884 plification of the model. In order to simplify the model to reduce computational and time
885 costs the model suffered a reduction in accuracy in the prediction of anisotropies in the near
886 wall. Therefore, the model may be unable to accurately model the effects of turbulence in
887 the near wall region. The presence of a solid wall influences turbulent flow through two
888 mechanisms. The first is through viscous effects which require that velocity components in
889 all directions must equal zero at the wall. The second is the blocking effect whereby, due to
890 the impermeable nature of a solid wall, primarily fluctuations in the wall-normal direction
891 are suppressed. This creates highly anisotropic turbulent structures, as noted by Emory
892 et al. [55]. As noted by Manceau et al. [44], the formulation in the EB-RSM resulted in
893 anisotropic structures being poorly captured.

894
895 Generally, the numerical simulation results at a TSR value of 2.54 have better agreement
896 with experiment data than those at TSR=3.87. Reduced accuracy for predictions at TSR =
897 3.87 were attributed to reduced resolution of the boundary layer due to increased rotational
898 speed of the blades (which impacted y^+ values). This complicated the model and made it
899 more difficult for the turbulence models to resolve the turbulent boundary layer as show in
900 figure 23 (illustrating wall shear stress values).

901
902 Finally, a Root-Mean-Square-Error (RMSE) analysis was carried out to investigate the over-
903 all prediction capabilities of each individual model. The RMSE analysis is presented in tables
904 1, 2 and 3 for both TSR cases for both mean velocity deficit and rms fluctuating velocities.
905 Over a range of 0–1, the lower the RSME value, the better the behaviour of the numerical
906 model. The maximum RSME value for the mean velocity deficit was 0.083 (RST model
907 predictions at 1D for a TSR value of 3.87) The maximum RSME value for the fluctuating u
908 velocities was 0.028 (RST model predictions at 0.66D for a TSR value of 3.87). The maxi-
909 mum RSME value for the fluctuating v velocities was 0.026 (SST $k-\omega$ model predictions at
910 1D and 1.5D for a TSR value of 2.54).

Table 1: RMSE analysis for mean velocity deficit

Model	X/D	TSR 2.54	TSR 3.87
SST $k-\omega$	0.66	0.062	0.059
RST	0.66	0.058	0.059
EB-RSM	0.66	0.070	0.056
SST $k-\omega$	1	0.061	0.077
RST	1	0.065	0.083
EB-RSM	1	0.032	0.053
SST $k-\omega$	1.5	0.035	0.081
RST	1.5	0.040	0.070
EB-RSM	1.5	0.044	0.082

Table 2: RMSE analysis for fluctuating u velocities

Model	X/D	TSR 2.54	TSR 3.87
SST $k-\omega$	0.66	0.022	0.025
RST	0.66	0.022	0.028
EB-RSM	0.66	0.019	0.023
SST $k-\omega$	1	0.023	0.022
RST	1	0.023	0.024
EB-RSM	1	0.024	0.017
SST $k-\omega$	1.5	0.024	0.021
RST	1.5	0.023	0.021
EB-RSM	1.5	0.024	0.015

Table 3: RMSE analysis for fluctuating v velocities

Model	X/D	TSR 2.54	TSR 3.87
SST $k-\omega$	0.66	0.021	0.018
RST	0.66	0.018	0.019
EB-RSM	0.66	0.021	0.014
SST $k-\omega$	1	0.026	0.015
RST	1	0.024	0.020
EB-RSM	1	0.024	0.013
SST $k-\omega$	1.5	0.026	0.015
RST	1.5	0.025	0.018
EB-RSM	1.5	0.024	0.012

911 5. Conclusions

912 Experimental and numerical investigations were carried out to assess the ability of the SST
 913 $k-\omega$, RST and EB-RSM turbulence models to accurately model a HAWT wake. A compari-

914 son was made between the experimentally recorded and numerically predicted wake velocity
915 deficit and turbulence characteristics of the wake. It has been found that all models are
916 capable of predicting the mean flow characteristics within the wake structure of a HAWT.
917 All models predicted the generation and decay of root and tip vortices, and the formation
918 of a central vortex sheet. The values of axial velocity predicted by all models were compa-
919 rable with experimental results and coincide with previous studies, whereby increasing TSR
920 values reduced the wake velocity deficit. The greatest velocity deficit is consistently located
921 behind the nacelle and tower structure with the tower introducing considerable quantities of
922 recirculating flow to the wake. The tower and nacelle structures, combined with the central
923 vortex system supply a continuous velocity deficit to the wake with the blades only having
924 a periodic effect on the wake velocity deficit, noticeable only after monitoring the wake over
925 a long period of time. The study highlighted that all models could accurately predict the
926 mean flow characteristics of a HAWT wake and, therefore, could accurately predicted rotor
927 loading, also.

928
929 Investigation of the rms u' and v' velocities showed that again all turbulence modelling
930 strategies estimated these velocity components to the correct order of magnitude. All mod-
931 els (with the exception of the RST model at a TSR value of 3.87) predicted the rms velocity
932 values to have an inverse Laplace distribution in the wake, similar to experimental results.
933 However, all models under-estimated the magnitude of these velocity values with predictions
934 as low as -43% of experimental results. Peaks in fluctuating velocities at $Z/R = 0.8$ lead the
935 authors to conclude that a secondary vortex structure formed. Similar to Yang et al. [11]
936 and Whale et al. [6], the structure merged with the tip vortex further downstream (in this
937 case between 1.5D to 2D downstream).

938
939 Investigation of the $\overline{u'v'}$ Reynolds stress component showed increased stress levels behind
940 the nacelle. Again, similar to previous studies, stress levels in the wake reduced for higher
941 TSR values with the exception of stress levels in the root vortex region. Overall, all models
942 performed reasonably well, with the majority of errors located at the centre of the wake.

943
944 Inaccuracies in the models were attributed primarily to the fluctuating y^+ values over the
945 blades. This is thought to have increased the difficulty for each solver to accurately predict
946 the boundary layer and as shown for the TSR equals 3.87 case, despite using the same mesh
947 and wall treatments, both the SST $k - \omega$ and RST computed different levels of wall shear
948 stress at the trailing edge of the blades. This was concluded to be the reason why the RST
949 model greatly under-predicted rms u' and v' velocities in the wake for a TSR value of 3.87.

950
951 The current study was undertaken to investigate the ability of different turbulence models
952 to accurately predict the turbulence characteristics of a HAWT wake. This is important for
953 future FSI simulations as outlined in section 4.3. This study has shown that greater care
954 regarding y^+ values is required as y^+ values tend to fluctuate due to the movement of the
955 blades. The y^+ values should be maintained in one region (either above or below unity in
956 this case) to ensure only one method is used to resolve the boundary layer. This problem

957 could become increasingly difficult when considering deformable blades and the introduction
958 of atmospheric boundary layers in FSI simulations.

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