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***Abstract*—Orthogonal frequency division multiplexing index modulation (OFDM-IM) has been emerging as a promising solution to increase the reliability at low complexity. However, the transmit diversity of the OFDM-IM schemes has been limited to two, due to unbalanced transmit diversity between index bits and complex data bits. In this work, we propose a new precoded OFDM-IM employing several spread matrices, named as (S-OFDM-IM). This aims to increase the transmit diversity, exploiting not only a subset of active sub-carrier indices, but also spreading the active indices over all the sub-carriers available. In this context, the transmit diversity in the detection of both index bits and data bits will be increased, properly taking into account both the multipath and index diversities. As for low-complexity detection, we propose two linear-complexity detections that exploit the minimum mean square error and the zero-forcing, along with the likelihood ratio. To analyse the performance, the bit error probability is derived and the transmit diversity analysis is provided. Through simulation results, it is clearly presented that the proposed scheme can outperform the benchmarks in terms of the transmit diversity, being higher than two. The enhanced transmit diversity is desired to the low-complexity machine type applications with high reliability.**

***Index Terms*—OFDM-IM, index modulation (IM), precoding, minimum mean squared error (MMSE), zero-forcing (ZF).**

I. Introduction

Index modulation (IM) is an emerging concept which refers to modulation techniques that exploit the indices of active channels, through which non-zero data symbols are transmitted, as an additional resource. One application of IM in the orthogonal frequency division multiplexing (OFDM) framework is orthogonal frequency division multiplexing with index modulation (OFDM-IM) [1], [2]. In OFDM-IM, only a subset of sub-carriers are activated to convey data bits via both M-ary complex data symbols and the indices of active sub-carriers. There have been various designs of OFDM-IM. For example, in [3], the number of active sub-carriers is not fixed to increase the spectral efficiency. In [4], OFDM-IM is applied to multiple input multiple output (MIMO) systems. An overview of various IM techniques is presented in [5] and a performance analysis is carried out when a maximum likelihood (ML) detector is employed in [6].

To improve the performance of OFDM-IM, a variety of OFDM-IM concepts have been developed, mainly improving the error performance and the data rate. To increase the spectral efficiency, in [7], the IM concept is applied to inphase and quadrature components to double the number of index bits. In [8], dual-mode OFDM-IM is proposed to exploit inactive sub-carriers conveying additional data symbols in different constellation set. As for the reliability, interleaving in the level of sub-carriers was proposed to the OFDM-IM framework in [9]. This approach can provide the bit error probability (BEP) better than the classical schemes, but not increase the diversity order. Coordinate interleaved OFDM-IM, named as CI-OFDM-IM, was presented in [10] in order to obtain the transmit diversity, taking into account both inphase and quadrature components of complex symbols across different sub-carriers. In [11], the OFDM-IM with transmit diversity (OFDM-IM-TD) is proposed to employ multiple constellations for the same data bits at both active and inactive sub-carriers. As for the analysis of transmit diversity, the novel frameworks for the symbol error probability (SEP) and BEP of OFDM-IM have been recently studied under uncertain channel state information in [12], [13], where it has been clearly shown that the transmit diversity of the OFDM-IM is dominated mainly in the detection of the M-ary symbols. This important observation suggests to properly balance the transmit diversities of the index symbols and the M-ary symbols, which results in the overall transmit diversity of the OFDM-IM. As for the use of coding for OFDM-IM, for example, the coded OFDM-IM in [14] is proposed to provide the coding gain for M-ary symbol detection and the transmit diversity for index detection. In [15], the trellis coded modulation for OFDM-IM is proposed to further increase the transmit diversity of index detection, begin higher than two. However, the overall transmit diversity of OFDM-IM systems in the single-antenna framework has been limited to two, due to unbalanced diversity between index and M-ary symbols. To the best of our knowledge, potentials of a precoded OFDM-IM employing both multipath and index diversities have been overlooked in this field.

In this paper, we propose a precoded OFDM-IM scheme that has not only a simple design, but also enhances the transmit diversity in the bit error performance. For this, we employ a precoding matrix to be combined with the OFDM-IM transmission and detection designs. Particularly, employing the spreading concept to the OFDM-IM, we select only K out of N sub-carriers available for every transmission such that a precoding matrix enables to not only spread the K non-zero data symbols over all the sub-carriers available, but also to compress both the K data symbols and active indices of sub-carriers that correspond to the K data symbols into each sub-carrier. This will ensure to exploit both multipath and index diversities, in the detection of both index and data bits. As for low-complexity receiver design, we propose two simple detection schemes, taking into account the minimum mean square error and the zero-forcing receivers. For the performance analysis, we derive the bit error probability in the detection of both index bits and the data symbol bits. The performance analysis and simulation results will clearly present that the proposed spread OFDM-IM can outperform the benchmarks in terms of the error probability. This will be promising for low-complexity, highly reliable wireless communications applications.

The rest of the paper is organized as follows. Section II presents the system model for the proposed spread OFDM-IM. The low-complexity detection schemes are addressed in Section III, while the performance analysis in terms of the bit error probability is derived in Section IV. In Section V, the simulation results are provided, and Section VI concludes this work.

Notation: Upper-case bold and lower-case bold letters and are used for matrices and vectors, respectively. $(.)^*$, $(.)^T$ and $(.)^H$ stand for the complex conjugation, the transpose operation and the Hermitian operation, respectively. $\|.\|$ denotes the Frobenious norm. The binomial coefficient is presented by $C(,)$, while the floor function is denoted by $\lfloor . \rfloor$. $\mathcal{CN}\left(0,\sigma^2\right)$ represents the complex Gaussian distribution with zero mean and variance σ^2. j is the unit imaginary number $j=\sqrt{-1}$.

II. System Model of Spread OFDM-IM

Consider OFDM-IM single-antenna system with N_c sub-carriers which are divided into G clusters of N sub-carriers, i.e., $N_c = GN$. Suppose that an OFDM-IM method is independently operated in each cluster. Moreover, the number of clusters does not influence on the system performance. Therefore, for simplicity and without loss of generality, we describe only one cluster hereinafter.

We propose new precoded OFDM-IM to spread active indices and then compress them to all N sub-carriers, namely spread OFDM-IM (S-OFDM-IM). Particularly, the block diagram of one cluster of the proposed scheme is illustrated in Fig. 1. For every S-OFDM-IM transmission, K out of N sub-carriers are activated to convey data bits via both K complex M-ary symbols and the indices of active sub-carriers. For this, the transmitter splits p incoming bits into two streams of p_1 and p_2 bits. The first stream of p_1 bits are mapped to a set θ of K active sub-carriers, i.e., $\theta=\{\alpha_1,...,\alpha_K\}$, where $\alpha_k \in \{1,...,N\}$. The remaining p_2 bits are mapped into K complex M-ary symbols, which are denoted by $\mathbf{s}=[s_1,...,s_K]^T$. As a result, we obtain $p_1=\lfloor \log_2 C(N,K) \rfloor$, $p_2 = K\log_2 M$ and the number of bits per sub-carrier (or data rate) given by

$$R=\frac{\lfloor \log_2 C(N,K) \rfloor + K\log_2 M}{N}. \quad (1)$$

Unlike the classical OFDM-IM, the proposed scheme intends to compress both θ and $\mathbf{s}$ into N sub-carriers, employing a precode matrix. Particularly, the IM cluster creator uses θ and $\mathbf{s}$ to create the data symbol vector $\mathbf{x}=[x_1,...,x_N]^T$, where $x_\alpha = 0$ for $\alpha \notin \theta$ and $x_{\alpha_k}=s_k$ for $\alpha_k \in \theta$. Prior to the inverse fast Fourier transform (IFFT) operator, $\mathbf{x}$ is multiplied with the spread matrix $\mathbf{G}$ of size $N\times N$. Note that the details in designing $\mathbf{G}$ will be discussed subsequently. The precoded vector for spread OFDM-IM is $\mathbf{z}=\mathbf{Gx}$ to transmit, in the frequency domain, over N flat Rayleigh fading channels. Denote by $\mathbf{H}$, a diagonal channel matrix of $\mathbf{H}=\mathrm{diag}\{h_1,...,h_N\}$, where h_α is independently distributed complex Gaussian random variables with zero mean and unit variance, i.e., $h_\alpha \sim \mathcal{CN}(0,1)$. Consequently, the received signal vector in the frequency-domain is given by

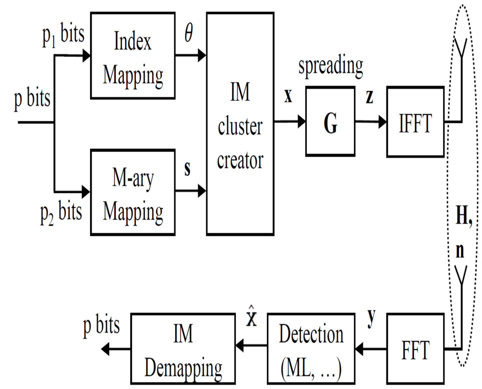

Fig. 1. Block diagram of one cluster of the spread OFDM-IM.

$$\mathbf{y}=\mathbf{HGx}+\mathbf{n}, \quad (2)$$

where $\mathbf{n}=[n_1,...,n_N]^T$ is the frequency-domain noise vector with $n(\alpha)\sim\mathcal{CN}(0,N_0)$. Denote by φE_s the average power per non-zero M-ary symbol, where $\varphi = N/K$ is the power allocation ratio. Hence, the average signal-to-noise ratio (SNR) per active sub-carriers can be given by $\bar{\gamma}=\varphi E_s/N_0$.

At the receiver, the frequency-domain signal vector $\mathbf{y}$ is then processed by a detector to recover the data symbol vector $\hat{\mathbf{x}}$, or, equivalently, the active indices $\hat{\theta}$ and the data symbols $\hat{\mathbf{s}}$. For example, to achieve the optimal error performance, the maximum likelihood (ML) detector can be used through making a joint decision on the M-ary symbols and active indices as

$$\left(\hat{\theta},\hat{\mathbf{s}}\right)=\arg\min_{\theta,\mathbf{s}}\|\mathbf{y}-\mathbf{HGx}\|^2. \quad (3)$$

Here, it is assumed that the channel matrix $\mathbf{H}$ is perfectly known at the receiver. Utilizing $\hat{\theta}$ and $\hat{\mathbf{s}}$, the data bits can be recovered by the IM demapper as shown in Fig. 1.

The computational complexity of the ML in terms of complex multiplications is $\sim\mathcal{O}\left(CM^K\right)$, where $C=2^{p_1}$, which exponentially grows. This makes the ML impractical when K and M increase. This prompts us to propose the low-complexity detectors in the next section.

A. Precoding Matrices for Spread OFDM-IM

We adopt two well-known square precoding matrices, namely the Walsh-Hadamard (WH) and Zadoff-Chu (ZF) matrices, which are applied in the classical OFDM. The WH matrix is most common and recursively determined as

$$\mathbf{G}_1 = \frac{1}{\sqrt{2}} \left[\begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array} \right], \ \mathbf{G}_k = \mathbf{G}_{k-1} \otimes \mathbf{G}_1, \qquad (4)$$

where $\otimes$ denotes the Kronecker product. Notice that all its elements are real-valued, which makes the WH suitable for low-complexity implementation. It is also shown from (4) that the size of the WH matrix is required to be $N = 2^k$.

By contrast, the ZC precoding matrix is complex-valued and provides a more flexible solution with any values for its size N. In particular, the ZC matrix is constructed based on the root ZC sequence given by

$$c_n = \begin{cases} e^{-\frac{j2\pi m}{N}\left(\frac{n^2}{2}+qn\right)} & \text{for even } N \\ e^{-\frac{j2\pi m}{N}\left[\frac{n(n+1)}{2}+qn\right]} & \text{for odd } N \end{cases}, \qquad (5)$$

where $n = 1, 2, ..., N$, m is any integer prime to N and q is any integer. The first column of $\mathbf{G}$ is selected as $[c_1, ..., c_N]^T$ and the other columns are defined as the cyclically shifted versions of the first column vector. For example with $N = 4$, we choose $m = 1$ and $q = 0$ in (5) to obtain the root sequence of $(a, -1, a, 1)$, where $a = \left(\sqrt{2} + j\sqrt{2}\right)/2$, and thus the resulting ZC matrix is

$$\mathbf{G} = \frac{1}{2} \left[\begin{array}{cccc} a & 1 & a & -1 \\ -1 & a & 1 & a \\ a & -1 & a & 1 \\ 1 & a & -1 & a \end{array} \right]. \qquad (6)$$

Here, the factor of $1/\sqrt{N}$ is used to normalize the precoding matrix for spreading, i.e., $\|\mathbf{G}\|^2 = 1$.

Both the ZC and WH precoding matrices have two important features as follows: (i) All elements in the precoding matrices have the same magnitude such that each non-zero symbol in $\mathbf{x}$ equally spread to all N available sub-carriers. This is essential to harvest the maximum diversity gain, especially in the M-ary symbol detection under severely fading sub-carriers. As a result, the proposed S-OFDM-IM is expected to achieve the higher diversity order than the existing OFDM-IM schemes which have the diversity order limited by two only; (ii) The two precoding matrices are orthogonal, i.e., $\mathbf{G}^{-1} = \mathbf{G}^H$. Thus, the free-interference transmission can be attained and the Euclidean distances between potential vectors $\mathbf{x}$ remain unchanged after spreading and compressing into $\mathbf{z} = \mathbf{G}\mathbf{x}$. The orthogonality of $\mathbf{G}$ also enables low-complexity detectors, as will be shown in the next section.

III. Low-Complexity MMSE/ZF-LLR Detectors

We propose two low-complexity detectors based on the zero-forcing (ZF) equalizer and the minimum mean square error (MMSE) equalizer, and the log-likelihood ratio (LLR) method. Fig. 2 illustrates the proposed detectors which are termed as ZF-LLR and MMSE-LLR.

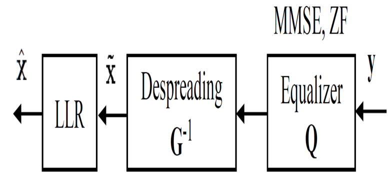

Fig. 2. The proposed low-complexity ZF-LLR and MMSE-LLR detectors.

Particularly, the received vector $\mathbf{y}$ is first multiplied with the equalization matrix which is denoted as $\mathbf{Q} = \mathrm{diag}\{q_1, q_2, ..., q_N\}$, where

$$q_n = \begin{cases} h_n^{-1} & \text{for ZF equalizer} \\ \frac{h_n^*}{|h_n|^2 + \bar{\gamma}^{-1}} & \text{for MMSE equalizer} \end{cases} \qquad (7)$$

is the n-th sub-carrier coefficient. The output of the channel equalizer is then multiplied with the despreading matrix $\mathbf{G}^{-1} (= \mathbf{G}^H)$ to extract the received data symbol vector as

$$\tilde{\mathbf{x}} = \mathbf{G}^H \mathbf{Q} \mathbf{y}. \qquad (8)$$

Due to the orthogonality of the ZF and WH spreading matrices, none of inverse matrix operators are required in (8), which substantially reduces the computational complexity.

After despreading, the LLR method is employed to recover the data symbol vector $\hat{\mathbf{x}}$ through computing the following LLR for each sub-carrier (please refer to [14] for more details)

$$\lambda_n = |\tilde{x}_n|^2 - |\tilde{x}_n - \hat{x}_n|^2, \qquad (9)$$

where $\tilde{x}_n$ is the n-th element of $\tilde{\mathbf{x}}$, and $\hat{x}_n = \arg\min_{x_n \in \mathcal{S}} |\tilde{x}_n - x_n|^2$ for $n = 1, ..., N$. Here, $\mathcal{S}$ is the M-ary modulation constellation. Finally, $\hat{\mathbf{x}}$, or equivalently K active indices and K complex M-ary symbols are directly determined corresponding to K largest LLRs λ_n from (9).

As seen from (8) and (9), the complexity of either ZF-LLR or MMSE-LLR per cluster is $\sim \mathcal{O}(NM)$, which is a linear function of M. The performance of these proposed detectors will be verified in the simulation results.

IV. Performance Analysis

In this section, we analyze the bit error probability (BEP) of S-OFDM-IM with the ML detection. For this, we evaluate the pairwise error probability (PEP) and then provide the upper bound on the BEP.

For given $\mathbf{H}$, the conditional PEP that the transmitted vector $\mathbf{x}$ is incorrectly detected as $\hat{\mathbf{x}} \neq \mathbf{x}$, is given by

$$P(\mathbf{x} \to \hat{\mathbf{x}} | \mathbf{H}) = Q\left(\sqrt{\frac{\|\mathbf{HG}(\mathbf{x} - \hat{\mathbf{x}})\|^2}{2N_0}}\right), \qquad (10)$$

where $Q(.)$ denotes the Gaussian tail probability [16]. Following the approach in [10], the unconditional PEP of (10) can be obtained as

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{i=1}^{N} \left(\frac{\sin^2\phi}{\sin^2\phi + \frac{\eta_i}{4N_0}} \right) d\phi, \qquad (11)$$

where η_i is the i-th diagonal element of $\mathbf{A} = \mathbf{B}^H\mathbf{B}$ with $\mathbf{B} = \text{diag}\left\{\mathbf{G}\left(\mathbf{x} - \hat{\mathbf{x}}\right)\right\}$, i.e., $\eta_i = \left|\mathbf{g}_i\left(\mathbf{x} - \hat{\mathbf{x}}\right)\right|^2$, where $\mathbf{g}_i$ is the i-th row of $\mathbf{G}$. Note that the closed-form solution for the integral (11) can be found in Appendix 5A in [16].

Denote by $u\left(\mathbf{x}\right)$ the number of non-zero elements of the vector $\mathbf{x}$. It can be seen that the diversity order of the PEP in (11) is $u\left(\mathbf{G}\left(\mathbf{x} - \hat{\mathbf{x}}\right)\right)$, which is strongly influenced by the spreading matrix $\mathbf{G}$. Thus, the diversity order achieved by S-OFDM-IM is given by

$$d = \min_{\mathbf{x} \neq \hat{\mathbf{x}}} u\left(\mathbf{G}\left(\mathbf{x} - \hat{\mathbf{x}}\right)\right). \tag{12}$$

Remark 1: As shown in (12), properly designing the spreading matrix $\mathbf{G}$ for given system parameters N, K and M can maximize the diversity order of S-OFDM-IM. However, solving such the optimization problem is out of the scope of this paper. For simplicity, adopting the WH and ZC matrices, we expect to enhance the performance of S-OFDM-IM due to their features presented in Subsection II.A.

We provide the following example to show the effectiveness of using $\mathbf{G}$ to improve diversity order of the PEP (11). Consider a system with $N = 4$, $K = 1$, and a PEP between $\mathbf{x} = \left[x_1, 0, 0, 0\right]^T$ and $\hat{\mathbf{x}} = \left[\hat{x}_1, 0, 0, 0\right]^T$, where $x_1 \neq \hat{x}_1$. For the classical OFDM-IM, this is the worst PEP case when the diversity order is limited to $u\left(\mathbf{x} - \hat{\mathbf{x}}\right) = 1$, while the proposed scheme with both the WH and ZC yields the diversity order of $u\left(\mathbf{G}\left(\mathbf{x} - \hat{\mathbf{x}}\right)\right) = 4$. As a result, the spread OFDM-IM technique significantly improves the diversity gain of OFDM-IM. This will be validated by simulations.

Remark 2: The upper bound on the BEP of S-OFDM-IM can be obtained based on the PEP (11) and the union bound theory as

$$P_b \leq \frac{1}{pT} \sum_{\mathbf{x}} \sum_{\hat{\mathbf{x}}} P\left(\mathbf{x} \to \hat{\mathbf{x}}\right) w\left(\mathbf{x}, \hat{\mathbf{x}}\right), \tag{13}$$

where $T = CM^K$ is for all possible realizations of $\mathbf{x}$ and $w\left(\mathbf{x}, \hat{\mathbf{x}}\right)$ denotes the number of bit errors in the event $\left(\mathbf{x} \to \hat{\mathbf{x}}\right)$.

V. Simulation Results

The simulation results are presented to verify the BEP performance of S-OFDM-IM system, over the Rayleigh fading channel. For comparison, we consider three benchmark schemes including classical OFDM-IM [2], CI-OFDM-IM [10] and OFDM-IM-TD [11]. The upper bound on the BEP of the proposed scheme is also examined.

Fig. 3 compares the BEP of the proposed scheme with three reference schemes at the data rate of 1 bps/Hz, with the ML detection. We consider two configurations of S-OFDM-IM including $(N, K, M) = (4, 1, 4)$ and $(8, 2, 4)$. Both the WH and ZC spreading matrices are employed. It can be seen from Fig 3 that our proposed scheme significantly outperforms reference schemes in whole SNR regions as expected in Remark 1. Especially, S-OFDM-IM with larger N, i.e., $N = 8$ provides the higher diversity order than CI-OFDM-IM and OFDM-IM-TD, which are always limited by two. For example, at BEP $= 10^{-4}$, the proposed scheme

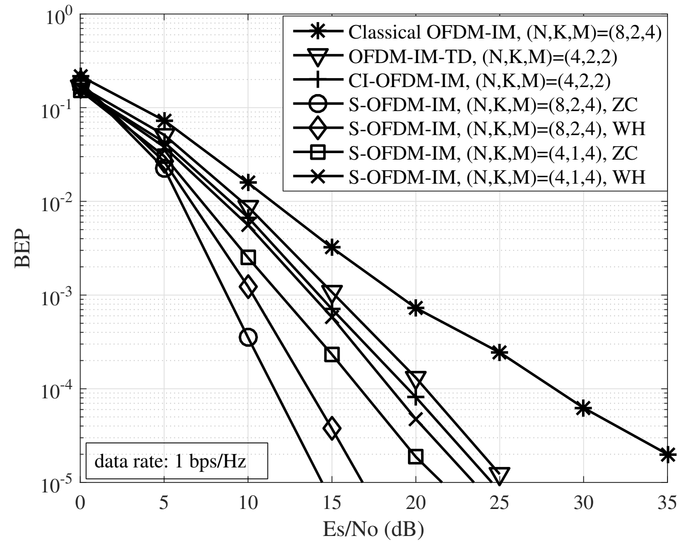

Fig. 3. BEP comparison among two S-OFDM-IM schemes with both WH and ZC spreading, classical OFDM-IM, CI-OFDM-IM and OFDM-IM-TD at the data rate of 1 bps/Hz. All schemes use ML detection.

with $(N, K, M) = (8, 2, 4)$ and ZC spreading achieves SNR gains of 8 dB, 9 dB and 16 dB over CI-OFDM-IM, OFDM-IM-TD and classical OFDM-IM, respectively. This obviously confirms the effectiveness of using the spreading code to improve the BEP performance of S-OFDM-IM. In addition, Fig. 3 shows that the ZC provides the better BEP than the WH. This is because when using the ZC spreading matrix which is complex-valued, the data symbols are spread over both the imaginary and real dimensions, leading to more diversity gain than the real-valued WH matrix.

In Fig. 4, we compare the BEP of S-OFDM-IM using the ZC spreading matrix with its benchmarks at the data rate of 1.5 bps/Hz. All schemes employ the same parameters $(N, K, M) = (4, 2, 4)$ and the ML detection. The upper bound on the BEP of the proposed scheme is also presented. As observed from Fig. 4, S-OFDM-IM is still superior to the reference schemes with SNR gains of 3 dB, 4 dB and 9 dB over CI-OFDM-IM, OFDM-IM-TD and classical OFDM-IM, at BEP $= 10^{-3}$, respectively. In addition, it is shown via Fig. 4 that the upper bound is tight in high SNR regions.

Fig. 5 depicts the BEPs of S-OFDM-IM with two proposed low-complexity detectors: ZF-LLR and MMSE-LLR. The ML performance is also presented for comparison. We consider two configurations of S-OFDM-IM which are the same as Fig. 3. As seen from Fig. 5, the ZF-LLR performs the worst, while the MMSE-LLR achieves a relatively good performance compared to the ML. For example, at BEP $= 10^{-4}$, with S-OFDM-IM of $(N, K, M) = (4, 1, 4)$, the MMSE-LLR is worse than the ML about 2 dB SNR gain, however this gap becomes larger with the scheme of $(N, K, M) = (8, 2, 4)$, which is 5 dB. Moreover, compared with the reference schemes with ML detectors in Fig. 3, the proposed scheme with the MMSE-LLR still provides the better performance. Thus, the MMSE-LLR can be an attractive solution for the low-complexity

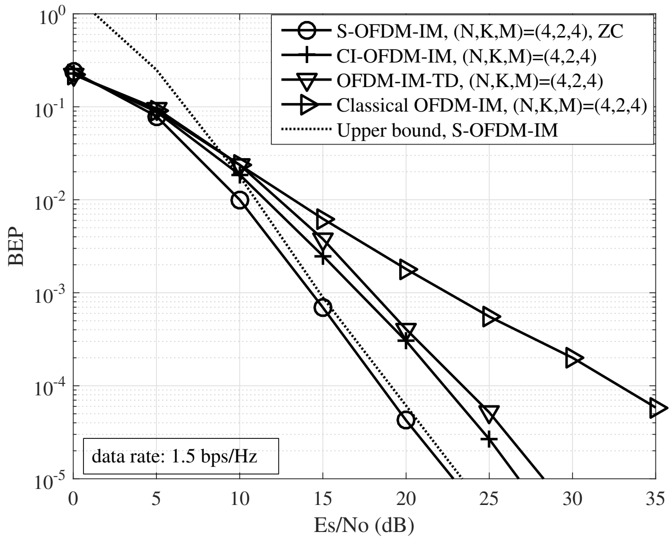

Fig. 4. BEP comparison among S-OFDM-IM with ZC spreading, classical OFDM-IM, CI-OFDM-IM and OFDM-IM-TD at the data rate of 1.5 bps/Hz. ML detector is used for all schemes.

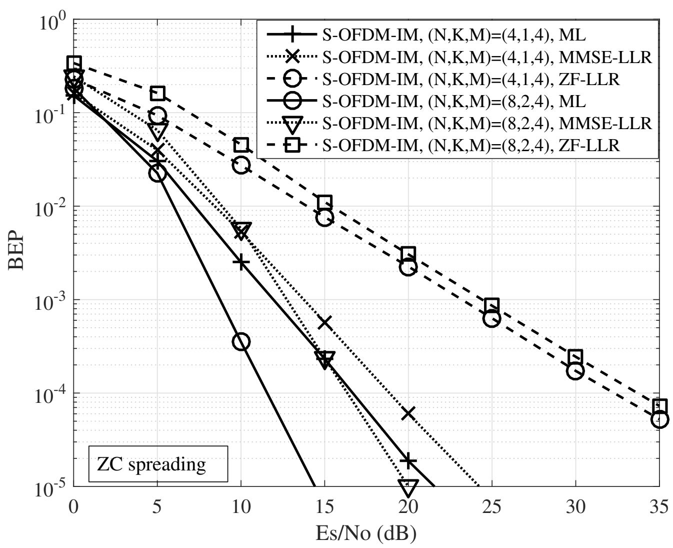

Fig. 5. BEP performance of ML, MMSE-LLR and ZF-LLR detectors of S-OFDM-IM.

implementation of S-OFDM-IM.

VI. Conclusion

We have proposed the precoded OFDM-IM in order to enhance the reliability of OFDM-IM systems, employing various spreading matrices in the OFDM-IM framework. Particularly, we adopted two spreading matrices such as the WH and ZC matrices to be combined with the OFDM-IM transmission and detection designs. Employing the spreading concept to the OFDM-IM, the proposed S-OFDM-IM has aimed to spread non-zero data symbols of active sub-carriers and their active indices to all the sub-carriers. This manner enabled both non-zero data and index symbols to be compressed into each sub-carriers. Devising the ML optimum receiver, we have also devised the two low-complexity detections, taking into account the MMSE-based LLR and the ZF-based LLR, at the cost of performance loss. For the performance analysis, the bit error probability has been analysed. Through the analysis and simulations, it was clearly shown that the spread OFDM-IM scheme is promising to benefit from both multipath and index diversity gains, outperforming the benchmarks. This will be suitable to machine type communications, where high reliability is important at low complexity. Future work will include a comprehensive theoretical analysis to provide more insight into the performance behaviours, and effective receiver with minimum performance loss will be designed

Acknowledgment

This work was supported by GIST Research Institute (GRI) grant funded by the GIST in 2018.

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