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1 Enhanced sparse component analysis for operational modal identification of real-life bridge structures

2  
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7  
8 Abstract:

9 Blind source separation receives increasing attention as an alternative tool for operational modal  
10 analysis in civil applications. However, the implementations on real-life structures in literature are rare,  
11 especially in the case of using limited sensors. In this study, an enhanced version of sparse component  
12 analysis is proposed for output-only modal identification with less user involvement compared with the  
13 existing work. The method is validated on ambient and non-stationary vibration signals collected from  
14 two bridge structures with the working performance evaluated by the classic operational modal analysis  
15 methods, stochastic subspace identification and natural excitation technique combined with the  
16 eigensystem realisation algorithm (NExT/ERA). Analysis results indicate that the method is capable of  
17 providing comparative results about modal parameters as the NExT/ERA for ambient vibration data.  
18 The method is also effective in analysing non-stationary signals due to heavy truck loads or human  
19 excitations and capturing small changes in mode shapes and modal frequencies of bridges. Additionally,  
20 closely-spaced and low-energy modes can be easily identified. The proposed method indicates the  
21 potential for automatic modal identification on field test data.

22  
23 Keywords: Blind source separation; sparse component analysis; operational modal identification; non-  
24 stationary signals.

## 25 1 INTRODUCTION

26 Operational modal analysis (OMA) is targeted at identifying modal characteristics from only response  
27 measurements of structures under ambient or natural excitation [1] and has many applications such as  
28 for structural identification, vibration-based health monitoring and damage detection, etc.

29 Several algorithms have been developed for OMA, including natural excitation technique combined  
30 with the eigensystem realisation algorithm (NExT/ERA) [2,3], stochastic subspace identification (SSI)  
31 approaches [4] and a general auto-regression moving average (ARMA) model [5] in time domain and  
32 frequency domain decomposition [6] in frequency domain. Most of them are parametric identification  
33 methods based on a mathematical model representing the physical phenomenon of structural dynamics.  
34 Their applications are limited to certain situations (e.g. ambient and free vibration signals) due to the  
35 model assumption regarding the nature of excitation forces (e.g. a broadband uncorrelated random

36 process). In addition, the working performance is sensitive to some model parameters (e.g. model order)  
37 and the parameter selection is dependent on users' subjective judgement.

38 Hilbert Huang transform (HHT) [7–9] is a parameter-free time-frequency analysis tool for modal  
39 identification which is capable of dealing with nonlinear and nonstationary signals. One critical step in  
40 the HHT is empirical mode decomposition (EMD), i.e. decomposing one multi-component signal into  
41 a series of mono-component signals. The decomposition process exploits no joint information between  
42 multiple measurement channels and might derive modal responses involving mode mixing [10].  
43 Improved work has been performed by proposing the ensemble EMD [11] and multivariate EMD [12,13]  
44 to overcome limitations.

45 Blind source separation (BSS) offers an alternative for OMA, belonging to non-parametric  
46 identification methods. BSS originates from the audio signal processing field for de-mixing audio  
47 sources from recordings via a mixing matrix. Its physical interpretation for OMA is that with the modal  
48 responses regarded as virtual sources, the mixing matrix is mapped directly to structural vibration  
49 modes [14]. BSS is classified into two types, overdetermined and undetermined cases depending on the  
50 provided measurement channels compared to the number of active modes. Underdetermined BSS is  
51 suitable for civil applications with limited sensors available and has been addressed by different  
52 methods such as sparse component analysis (SCA) and tensor decomposition.

53 The SCA makes use of sparseness in the transformed domain i.e. the time-frequency (TF) domain for  
54 decomposition. The sources are assumed to be sparsely represented after the TF transform e.g. short-  
55 time Fourier transform (STFT) [15] [16] [17], wavelet packet transform [18] and quadratic TF transform  
56 [19]. A mixing matrix (or mode shapes) is estimated using clustering algorithms (e.g. hierarchical  
57 clustering [15], K-hyperline clustering [16], K-means clustering [17] [19] and Fuzzy C-means  
58 clustering [20]) on scatter plots of measurement signals in the transformed domain. Given the mixing  
59 matrix, source signals can be reconstructed based on the source sparsity using  $l^1$  norm minimisation [16]  
60 [20], smoothed zero-norm algorithm [15] or subspace-based algorithm (by identifying active sources at  
61 TF points and estimating the energy each of these sources contributes) [19]. With the source signals (or  
62 modal responses) available, modal frequencies and damping ratios can be estimated using either single-  
63 mode curve fitting in frequency domain or logarithmic decrement method in time domain. The SCA  
64 has been implemented for OMA on a cantilever beam structure [16], a laboratory tower structure under  
65 narrow-band excitations [18] and a column structure in temperature-varying environment [17] with the  
66 working performance evaluated against identification using SSI [16].

67 Tensor decomposition method is an alternative for the underdetermined BSS based on the assumption  
68 that source signals are uncorrelated among different channels but correlated individually in time [21].  
69 The main idea is to decompose the third-order tensor representation (i.e. spatial covariance matrices of  
70 observation signals for different time lags) into a linear combination of a minimal number of rank-1  
71 terms by means of an alternating least squares algorithm. The derived mixing matrix and auto-

72 covariance of modal responses can be used for modal parameter estimation. The method has been  
73 validated at being effective when analysing ambient vibration signals [22], earthquake responses [23]  
74 as well as human-induced vibrations [24] [25].

75 Although there are already a few studies implementing the underdetermined BSS for OMA, most of  
76 these are numerical and laboratory studies, while field tests are rare except on two footbridges [25,26],  
77 one tower structure [27] and two buildings [28,29]. There is no further study to investigate whether the  
78 underdetermined BSS method is capable of offering an effective alternative to classic OMA methods  
79 on field testing data, especially non-stationary vibration signals.

80 In this study, an enhanced method based on the SCA is proposed for OMA suitable for field applications.  
81 In this method, a novel procedure of the two-step clustering is involved to ensure an automatic and  
82 robust estimation of mode shapes that is the basis for the accurate estimation of modal parameters.

83 The proposed method is validated on two full-scale in-operation bridges in both ambient and non-  
84 stationary vibrations (i.e. due to heavy truck loads or passing pedestrians). Wired and wireless  
85 accelerometer sensors with different accuracy levels were used for data acquisition to test the sensitivity  
86 of the proposed method to noise level. Closely-spaced and low-energy modes that are common for  
87 footbridges are considered based on the recorded data. The working performance of the proposed  
88 method is evaluated by comparing with the classic OMA methods i.e. NExT/ERA and SSI.

89 To that end, section 2 introduces the main methodologies of the proposed OMA method based on the  
90 SCA and improvements, mainly in the clustering step, to ensure a robust estimation of mode shapes.  
91 Section 3 describes a validation study on ambient vibrations of a short-span road bridge and evaluates  
92 the performance through comparing the results with those using the NExT/ERA algorithm. Section 4  
93 describes a load test on the same bridge and investigates the feasibility of the proposed method on non-  
94 stationary signals. Section 5 analyses non-stationary vibration data from a footbridge under pedestrian  
95 excitation and validates the effectiveness of the proposed method for vibration signals under narrow-  
96 band excitation and for extracting closely-spaced modes.

## 97 2 ENHANCED SPARSE COMPONENT ANALYSIS FOR OMA

98 The BSS is a powerful tool for separating mixed signals when the sources and the mixing methodology  
99 are unknown. The simple form of BSS in the noiseless case is to determine a mixing matrix  $\mathbf{A}$  (using  
100 statistical and data structure information [30]) and to recover the  $M$ -component source data  $\mathbf{s}$  from  
101 their linear mixture in the  $N$ -component observational data  $\mathbf{X}$ , expressed as

$$102 \quad \mathbf{X}(t) = \mathbf{A}\mathbf{s}(t) . \quad (1)$$

103 Consistent with the expression in BSS, the vibration measurement  $\mathbf{X}$  could be decomposed via the  
104 mode shape matrix  $\Phi$  into single-mode response signals  $q(t)$ , similar to the BSS expression in  
105 Equation (1).

$$106 \quad \mathbf{X}(t) = \Phi\mathbf{q}(t) . \quad (2)$$

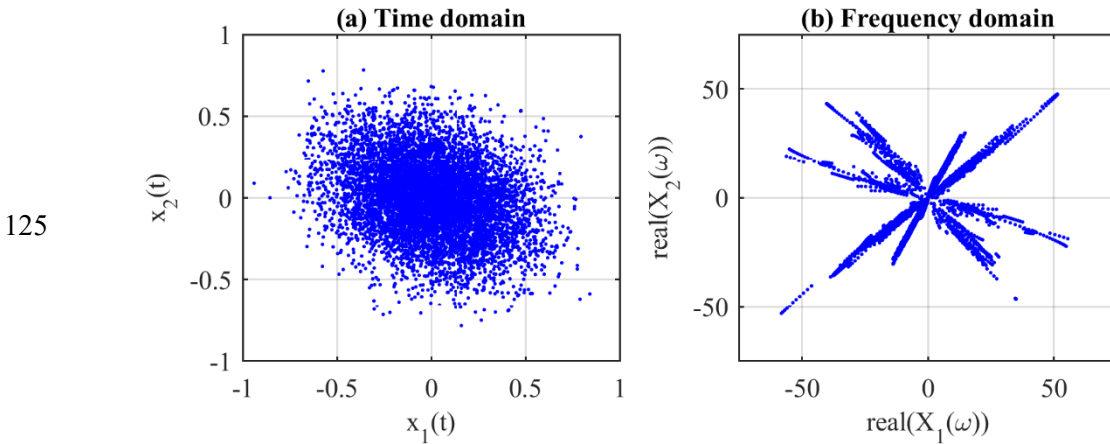
107 Thus, BSS methods have been successfully utilised for OMA [30], i.e. estimating mode shapes and  
 108 identifying modal parameters from the recovered single-mode response signals  $q(t)$ .

109 The case of underdetermined BSS, where the number of active modes is larger than the number of  
 110 measurement channels ( $M > N$ ), is common for civil applications. To solve the underdetermined BSS  
 111 problem, the SCA provides a simple framework based on source sparseness [31]. The main algorithms  
 112 and procedures of the SCA are presented in section 2.1; and an enhanced SCA targeted for OMA in  
 113 civil applications is described in section 2.2.

### 114 2.1 Sparse component analysis for OMA

115 SCA is a relatively simple tool for separating a number of sources from observed mixtures, primarily  
 116 for underdetermined cases. The underlying assumption is data sparsity, e.g. at each point  $t$ , a single  
 117 source is significantly more active than others [31]. In a scatter plot of observational data mixtures, the  
 118 collection of points dominated by the same source signal forms into one straight line passing the origin  
 119 and could be separated as one cluster with the line direction representing the mixing vector.

120 The original form of data mixture generally does not fit the assumption about sparsity. Figure 1(a)  
 121 demonstrates the temporal scatter plot of two mixture signals ( $x_1(t)$  and  $x_2(t)$ ) from five sources (data  
 122 from the numerical example in section 2.3). The figure indicate no apparent line alignment and the  
 123 sources could not be regarded as disjoint support in time domain. Therefore, a pre-processing step i.e.  
 124 sparse signal representation is essential before any clustering.



126 Figure 1 Scatter plots of two-channel signals (involving five sources) in time domain (a) and frequency domain  
 127 (b).

128 A sparse representation of observed mixtures could facilitate the mixing matrix estimation. Linear time-  
 129 frequency (TF) transforms like short-time Fourier transform (STFT) and wavelet transform are  
 130 commonly applied to measured signals of each channel ( $c_x^\Psi := \mathbf{X}\Psi$ ) for sparsity. Through the linearity  
 131 of the transform, the source separation problem has an exact analogue in the transformed domain as

$$132 \quad C_x^\Psi(k) = A C_s^\Psi(k) \quad (3)$$

133 and the sources  $C_s^\Psi(k)$  in the transformed domain are expected to be reasonably disjoint. Figure 1 (b)  
 134 presents the TF scatter plot of the two-channel signals, indicating approximately directions of five  
 135 aligned straight lines.

136 The second step of SCA consists of estimating the mixing matrix by means of clustering from a scatter  
 137 plot of the TF coefficients  $\{C_x^\Psi(k)\}$ . The performance of mixing matrix estimation using a clustering  
 138 algorithm degrades when the sources are non-disjoint in the transformed domain. This problem could  
 139 be resolved by refining the TF coefficients  $\{C_x^\Psi(k)\}$  for clustering through detecting only single source  
 140 points (SSPs) i.e. where a single source dominates. The common criteria for SSP detection include the  
 141 complex ratio of the mixtures over a small window in the transformed domain [17,32] and directional  
 142 alignment of the real and imaginary parts of TF coefficients [33][15]. Scatter plots of sparse coefficients  
 143  $\{C_x^\Psi(k_i)\}$  yield clear lines of orientation corresponding to the vectors constituting the mixing matrix.

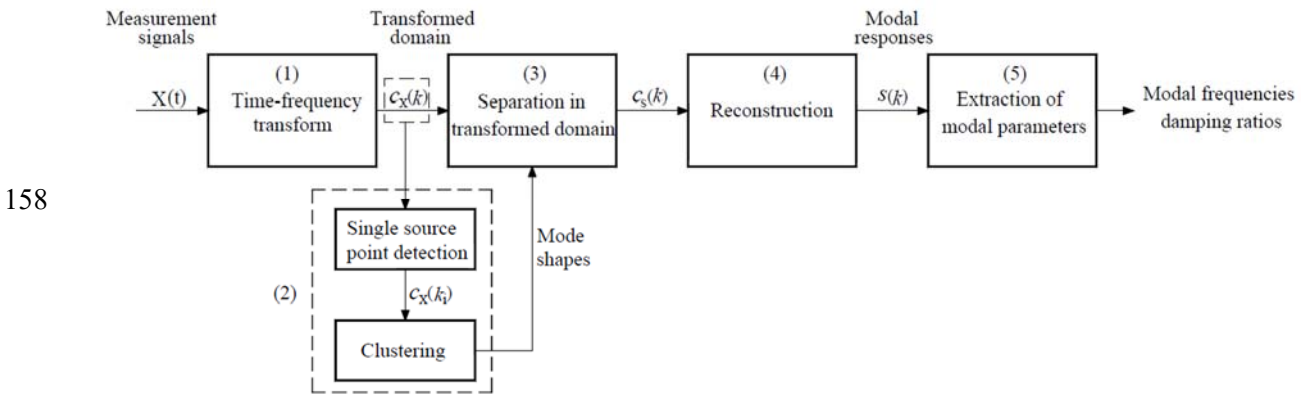
144 For convenience, unit vectors of the normalised TF coefficients  $\{\bar{C}_x^\Psi(k_i)\}$  are imported for  
 145 classification with cluster centroids denoting the mixing matrix or mode shapes directly. Clustering  
 146 algorithms used for OMA applications include hierarchical clustering algorithm [33][15], K-means  
 147 algorithm [17][19], K-hyperline clustering [16] and Fuzzy C-Means clustering [20].

148 Given the estimated mixing matrix  $\hat{A}$ , the source TF representation  $C_s^\Psi(k)$  in Equation (3) is estimated  
 149 based on the source sparsity by finding the solution that minimises the  $l^q$  norm [34],

$$150 \quad \hat{C}_s^\Psi(k) := \arg \min \|C_s^\Psi\|_q \text{ subject to } \hat{A} C_s^\Psi(k) = C_x^\Psi(k), \quad q \leq 1. \quad (4)$$

151 For example,  $l^1$  norm minimisation could be interpreted as a maximum likelihood estimate of source  
 152 TF coefficients assuming the coefficients have a Laplacian distribution. The sparsity criteria used in  
 153 literature include  $l^1$  norm [16,20] and an improved  $l^0$  norm named smoothed zero norm algorithm [15].

154 In the fourth step, source signals are reconstructed to the time domain by inverse TF transform. Finally,  
 155 the modal parameters can be extracted from source signals (modal responses) by using either single-  
 156 mode curve fitting in frequency domain or logarithmic decrement method in time domain. The SCA  
 157 flowchart is summarised in Figure 2; further details can be found in [17][31].



159 Figure 2 Flowchart of the SCA for OMA (modified from Figure 10.2 in [31]).

## 160 2.2 Enhanced sparse component analysis

161 Accurate estimates of mode shapes in the second step are critical for the robustness of source separation  
162 that has direct influence on the accuracy of identified modal parameters. The existing problem in the  
163 SCA method for OMA is that some mode shapes of a structural system estimated based on limited  
164 sensors are of high similarity and might be incorrectly assigned to one cluster, contributing together for  
165 the estimation of a single mode shape. For example, torsion modes could not be distinguished from  
166 bending modes by the SCA method when sensors for data collection are located on one longitudinal  
167 side of a bridge structure.

168 Compared with existing work [15–20] implementing the SCA method for OMA, the proposed method  
169 in this study made improvement to the second step, mode shape estimation. To overcome the ambiguity  
170 of mode shape representation using limited sensors, a novel two-step clustering procedure is shown in  
171 Figure 3, i.e. first clustering frequency values  $\{f(k)\}$  and then clustering TF coefficients  $\{C_x^\Psi(k)\}$ .

172 After TF transform of measurement signals in the first step, both the TF coefficients  $\{C_x^\Psi(k)\}$  and the  
173 corresponding frequency values  $\{f(k)\}$  are stored for analysis. The SSPs are detected using a threshold  
174 angle  $\Delta\theta$  based on directional alignment of the real parts ( $R\{C_x^\Psi(k)\}$ ) and the imaginary parts  
175 ( $I\{C_x^\Psi(k)\}$ ) in TF coefficients [33] expressed as

$$176 \frac{\left| R\{C_x^\Psi(k)\}^T I\{C_x^\Psi(k)\} \right|}{\left\| R\{C_x^\Psi(k)\} \right\| \cdot \left\| I\{C_x^\Psi(k)\} \right\|} > \cos(\Delta\theta). \quad (5)$$

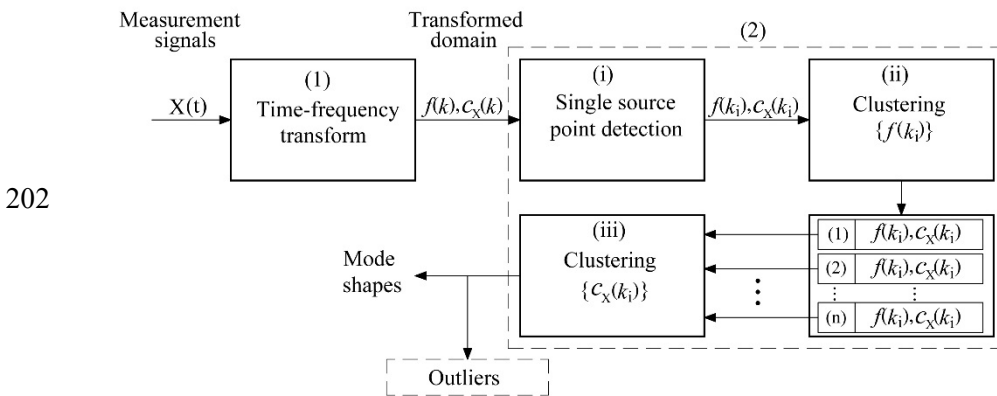
177 Instead of clustering TF coefficients directly for mode shape estimation, a frequency-clustering step is  
178 added to avoid any ambiguity of mode shape representation. The stored frequency values of these  
179 identified SSPs are analysed first using hierarchical clustering, leading to a few groups of SSPs with  
180 different frequency ranges. Since similar modes based on limited sensors usually have apparent  
181 deviations in modal frequency values, the purpose of this step is to separate them into different groups  
182 before clustering TF coefficients. Note that this step is not aimed at modal frequency estimation and  
183 also it is acceptable to include several closely-spaced modes into one group.

184 For each group of SSPs, the normalised TF coefficients are clustered to identify mode shape candidates  
185 which are real-valued. Implementation of some clustering methods used in literature like K-means and  
186 Fuzzy C-Means requires prior specification of the number of clusters. This is problematic as the existing  
187 mode number in measurement signals is unknown before the analysis. Another problem in these  
188 methods is the sensitivity to the initialisation and that poor choices of initialised cluster centroids can  
189 lead to sub-optimal configuration of cluster assignment.

190 In this study, a probabilistic method using Dirichlet process mixture models [35] is used for cluster  
 191 analysis of the TF coefficients. The main idea of this method is to fit the data to a Dirichlet process  
 192 mixture model (i.e. an infinite mixture model) that maximises the overall posterior probability of cluster  
 193 assignment. As a random variable in the model, the number of clusters is estimated as an intrinsic part  
 194 of the algorithm. This clustering method has been validated to be robust to the presence of outliers (by  
 195 assigning them into separate clusters) [35] that is common for data collected from field tests. Detailed  
 196 description of this method is in the reference [35].

197 Among the extracted mode shape candidates, outliers are automatically removed according to a  
 198 minimum sample number and statistical information about sample distribution in each cluster using two  
 199 parameters, standard deviations of modal frequency values and standard deviations of point distance to  
 200 the cluster centroid.

201 Procedures (steps 3-5) after mode shape estimation follow the SCA flowchart in Figure 2.



203 Figure 3 Procedures of mode shape estimation using two-step clustering proposed in this study

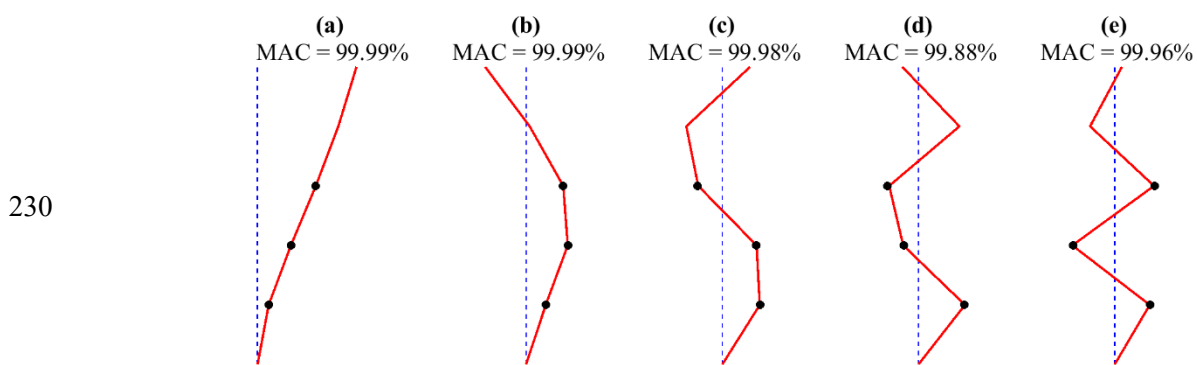
### 204 2.3 Numerical illustration

205 A five degree-of-freedom (DOF) building model [25] is set up to validate the proposed method. The  
 206 natural frequencies are 0.91 Hz, 3.37 Hz, 7.11 Hz, 10.66 Hz and 12.73 Hz while damping ratios are  
 207 assumed as 2% in all modes. The system is excited by white noise (zero mean unit variance Gaussian  
 208 process) at all the five floor level and integration scheme based on state space representation is  
 209 implemented to obtain the time-history responses of the system at the sample rate of 128 Hz. White  
 210 noise is added to the simulated acceleration data and the noise level is taken as 5% root mean square  
 211 (RMS) noise-to-signal ratio.

212 Acceleration data with the duration of 60 s for the bottom three floors are used in modal analysis. The  
 213 data are firstly transformed to TF coefficients using the STFT with the sliding (Hamming) window  
 214 length of 1024 and window shift size of 2. SSPs are then detected based on the specified threshold (4  
 215 degrees) related to directional alignment of TF coefficients in Equation(5). For those SSPs, the real and  
 216 imaginary parts of TF coefficients are collected together for the two-stepping clustering. Hierarchical  
 217 cluster analysis is applied to SSP frequency values, classifying the SSPs into five groups with the  
 218 frequency centroids at 0.95 Hz, 3.32 Hz, 7.09 Hz, 10.60 Hz and 12.66 Hz, respectively. The clustering

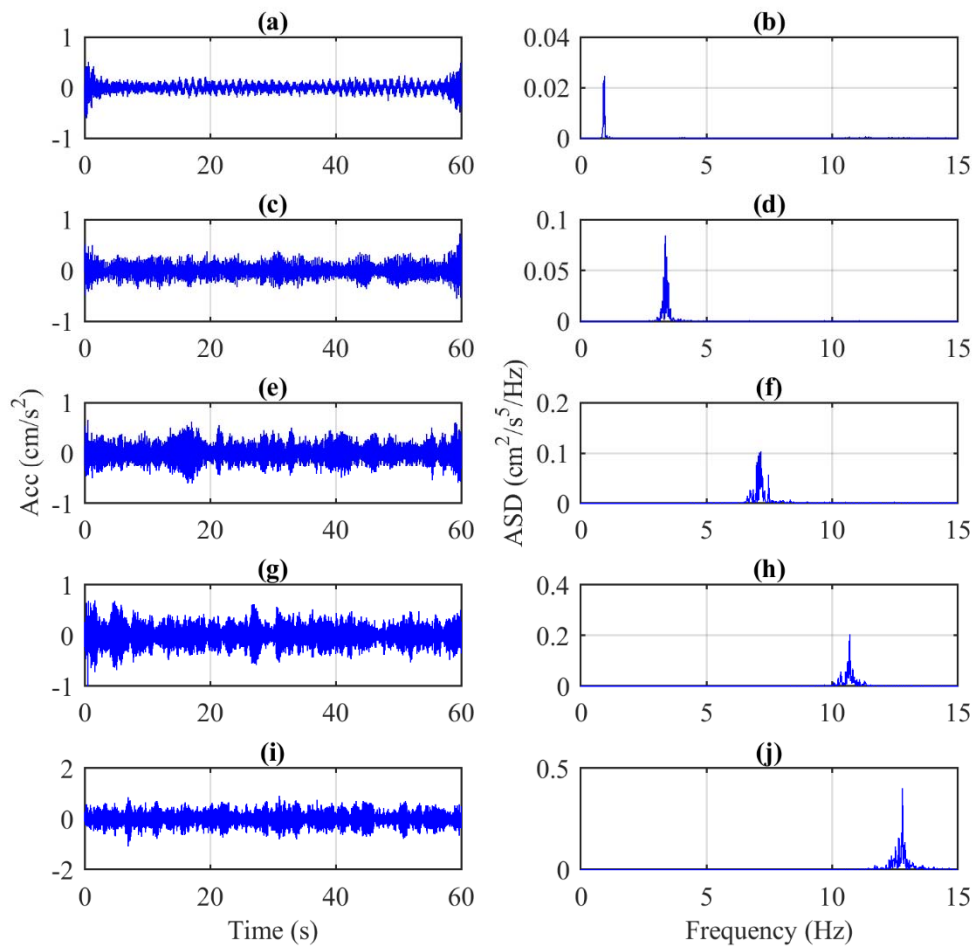


219 algorithm in [35] is then implemented for each frequency group to classify TF coefficients. Outliers are  
 220 automatically removed based on the criterion for cluster distribution, i.e. minimum sample number  
 221 (e.g. >100), standard deviation of modal frequency values (<0.05) and standard deviation of point  
 222 distance to cluster centroid (<0.05). Five clusters (i.e. 0.92 Hz, 3.39 Hz, 7.07 Hz, 10.70 Hz and 12.64  
 223 Hz) are derived with the corresponding mode shapes indicated in Figure 4. Dot markers represent  
 224 estimated mode shape ordinates using the enhanced SCA method while the solid curves are the  
 225 theoretical ones taken as the reference. Compared with the reference, the estimation results have the  
 226 MAC values over 99.8%. For comparison, two other window functions (i.e. rectangular and Hann) are  
 227 implemented in the STFT for TF transform. The achieved MAC values using either window function  
 228 are high (over 99.6%) for all the five modes. Thus, the method is not sensitive to the window function  
 229 chosen for STFT.



231 Figure 4 Mode shapes of a 5-DOF building system: solid curves represent the reference mode shapes in the  
 232 simulation; and dot markers denote the modal shape ordinate estimated by the enhanced SCA method using three-  
 233 channel acceleration data. Modal assurance criteria (MAC) compared with the reference mode shapes are given  
 234 in subplot titles.

235 Given the mode shape matrix, the source TF representation is separated based on  $l^1$  norm minimisation  
 236 using an open source package SPGL1 [36,37] and then recovered to the time domain using inverse  
 237 discrete Fourier transform. Figure 5 shows the estimated sources and the corresponding auto-spectral  
 238 densities (ASD), indicating that the five sources are clearly identified using three-channel measurement.



240 Figure 5 Five source signals recovered from three-channel measurement using the enhanced SCA method (the left  
 241 column) and the corresponding auto-spectral densities (the right column).

242 TF transform representation is the necessary step in the SCA method for data sparsity. The simplest TF  
 243 transform (i.e. STFT) is demonstrated to be effective in this numerical example and will be used with  
 244 field data as described in sections 3 to 5, although there are other feasible alternatives e.g. wavelet  
 245 packet transform [18] and quadratic TF transform [19].

246 Application to field test data collected from two bridges is described next. In section 3, the enhanced  
 247 SCA method is firstly validated for OMA of a short-span road bridge using data collected by wired and  
 248 wireless accelerometer sensors. The feasibility of the proposed method for analysing non-stationary  
 249 vibration data is investigated in section 4 using the data from the same bridge under a heavy truck  
 250 passage, while section 5 employing human-induced vibration data from a footbridge.

### 251 3 FIELD TEST ON A ROAD BRIDGE DURING NORMAL OPERATION

252 This section reports the validation study of the enhanced SCA method for OMA on a short-span road  
 253 bridge. The extracted modal parameters using the new method are evaluated through comparison with

254 the results by the NEXt/ERA procedure [38]. NEXt/ERA operational modal analysis procedure is  
 255 chosen as the reference due to its long experience of use and availability in a custom software [39].  
 256 Essentially, section 3.1 introduces the test configuration on the bridge and demonstrates the  
 257 measurement results. Modal analysis results using enhanced SCA are described in section 3.2 for mode  
 258 shape estimation and in section 3.3 for modal parameter extraction.

259 *3.1 Test configuration and measurement results*

260 Station Road Bridge in Figure 6(a) is a steel girder bridge with 36 m span near Exeter St David’s railway  
 261 station. A modal test was performed on the bridge (also reported in [40]) using two types of sensors,  
 262 wired Honeywell QA-750 accelerometers and APDM Opal™ wireless inertial measurement units  
 263 (IMUs).

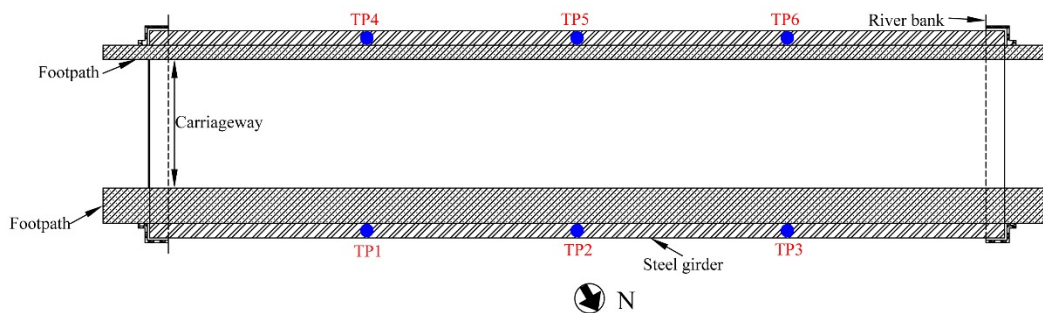
264 The QA-750 accelerometers are DC-response devices with a resolution better than 1  $\mu\text{g}$  and sensor  
 265 noise floor better than 7  $\mu\text{g} / \sqrt{\text{Hz}}$  in 0-10 Hz band from manufacture data. The IMU Opal sensor  
 266 includes a tri-axial accelerometer with the resolution of 240  $\mu\text{g}$  and 730  $\mu\text{g}$  for the sensing ranges of  $\pm 2$   
 267 g and  $\pm 6$  g, respectively and noise floor one or two orders of magnitude inferior to the QAs.

268 Sensors were arranged in six test points on the bridge,  $\frac{1}{4}$  points (TP1 and TP4), mid-span (TP2 and TP5)  
 269 and  $\frac{3}{4}$  points (TP3 and TP6) of north and south sides, as indicated in Figure 6(b). With four QA  
 270 accelerometers available, two runs of recordings were performed to cover all the six test points: two  
 271 QAs were kept at the same locations (TP3 and TP5) while the other two were moved from TP1 and TP2  
 272 in the first run to TP4 and TP6 in the second run. Six Opal sensors were arranged in the six test points  
 273 with one run of data recorded directly to the memory of each IMU. The sample rates for two sensing  
 274 systems were both set as 128 Hz.

275 (a)

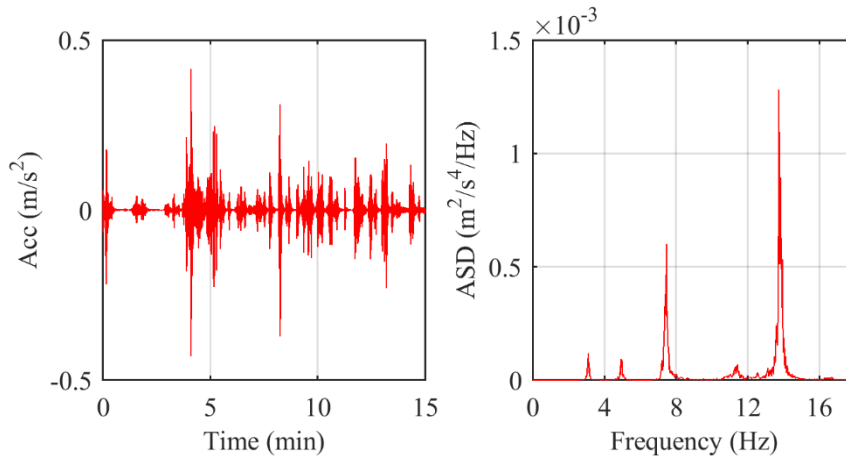


277 (b)



279 Figure 6 Bridge information and test point locations: (a) bridge elevation taken from the north side of the bridge;  
 280 and (b) configuration of test points for accelerometer sensors.

281 The vertical acceleration signals from a 15-minute recording are truncated for the modal analysis.  
 282 Vibration data collected by a QA accelerometer at TP3 in Run 1 are demonstrated in Figure 7 as an  
 283 example. The experienced maximum acceleration reaches  $0.43 \text{ m/s}^2$  and the auto-spectral density  
 284 (ASD) indicates five modes lower than 18 Hz at approximately 3.1 Hz, 5.0 Hz, 7.5 Hz, 11.4 Hz and  
 285 13.7 Hz.



286  
 287 Figure 7 Time histories of vertical acceleration and the corresponding auto-spectral density (ASD) at test point  
 288 TP3 in Run 1 by QA accelerometer.

### 289 3.2 Estimation of mode shapes by enhanced SCA method

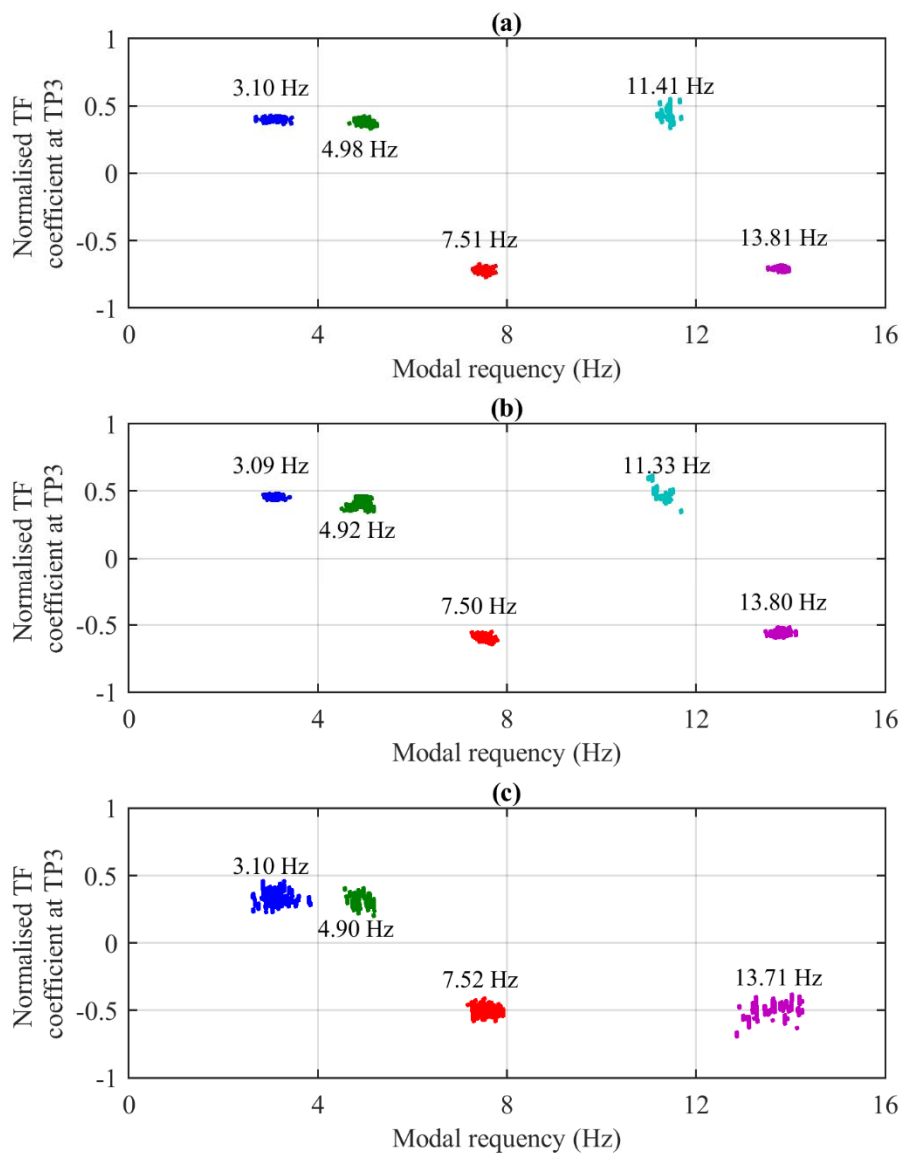
290 The vibration data collected by QA and Opal sensors were analysed following the procedures of the  
 291 enhanced SCA method in section 2.2. Firstly, the acceleration data were transformed to the TF domain  
 292 by STFT using Hamming windows with the window length 14400 and the hop size 20 (i.e. the number  
 293 of samples between the begin-steps of adjacent windows).

294 The second step is to estimate mode shapes using TF coefficients of the SSPs. The SSPs were detected  
 295 based on directional alignment of the real and imaginary parts in TF coefficients using a threshold angle  
 296  $\Delta\theta$ . A smaller threshold angle  $\Delta\theta$  corresponds to imposing a tougher requirement on the qualified  
 297 SSPs, leading to a smaller number of SSPs. The specified angle  $\Delta\theta$  for QA data is 2 degrees providing  
 298 45088 SSPs while the same value applied to Opal data leads to only 52 qualified SSPs and then failure  
 299 of cluster analysis. A large value for  $\Delta\theta$  (5 degrees) was taken to analyse the Opal data, providing  
 300 5544 qualified SSPs.

301 The two-step clustering results for the detected SSPs are shown in Figure 8: X and Y axes correspond  
 302 to the frequency values (lower than 20 Hz) and the normalised TF coefficients at the test point TP3.

- 303 • The QA data in Run 1 are assigned to five groups shown in (a) with frequency centroids at 3.10 Hz,  
 304 4.98 Hz, 7.51 Hz, 11.41 Hz and 13.81 Hz, similar to observations from ASD plot in Figure 7 (b).
- 305 • Analysis results for QA data in Run 2 shown in (b) are similar to (a) but with slight difference in  
 306 frequency centroid values.
- 307 • In (c), Opal data are assigned to four groups at 3.10 Hz, 4.90 Hz, 7.52 Hz and 13.71 Hz. For the  
 308 SSPs in every cluster, the normalised TF coefficients (along the y axis) have larger variation ranges

309 compared with QA results possibly due to a more flexible criterion (higher threshold angle) on SSP  
 310 detection. One cluster near 11.41 Hz that is visible from QA data is missed by Opal data.

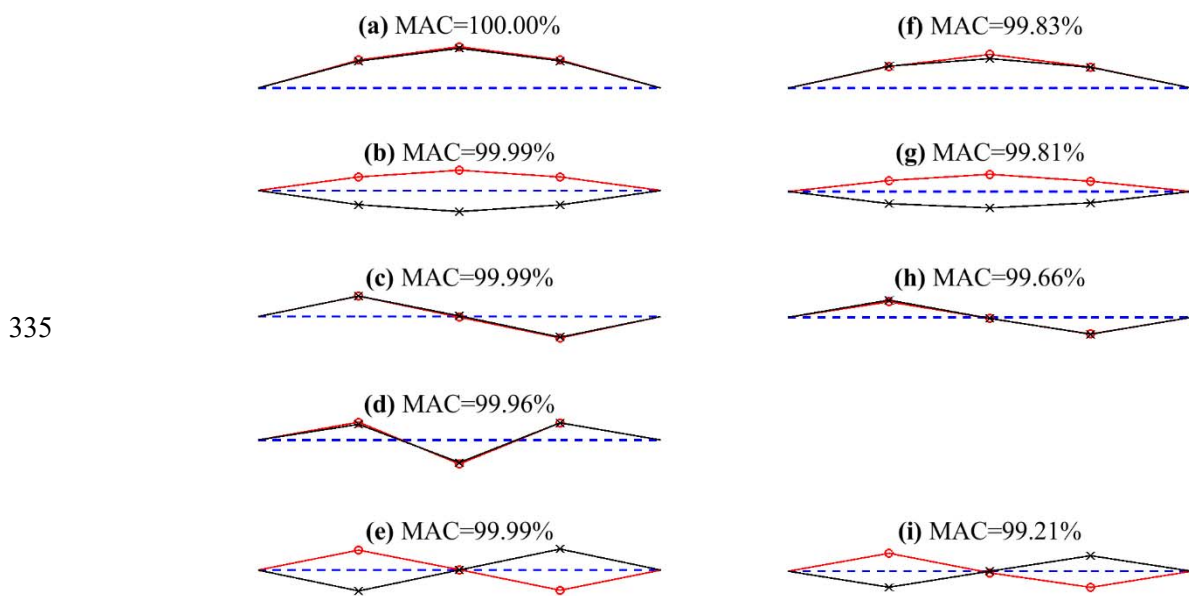


312 Figure 8 Clustering results about normalised TF coefficients from acceleration measurement: (a) clustering results  
 313 for acceleration measurement in Run 1 by QA accelerometers; (b) clustering results for acceleration measurement  
 314 in Run 2 by QA accelerometers; and (c) clustering results for acceleration measurement by Opal IMUs.

315 For each cluster, the centroid of normalised TF coefficients represents directly one mode shape vector  
 316 at test points. Mode shape vectors in two runs of QA measurement were merged based on the two  
 317 reference test points with the results shown in the left column of Figure 9. In the figure, the red lines  
 318 with circular markers and the black lines with 'x' shaped markers represent the mode shape ordinate of  
 319 the two longitudinal sides of the bridge in the north and south, respectively. Subplots (b) and (e) indicate  
 320 the first two torsion modes of the bridge while the other three are bending modes.

321 The mode shapes estimated from QA data using NExT/ERA procedures in [40] are taken as the  
 322 reference to evaluate the estimation accuracy of the enhanced SCA method. For QA data, all the  
 323 extracted five modes indicate high similarity with the reference and the modal assurance criteria (MAC)  
 324 reach over 99.96%. In Run 1, four QA sensors were located at the three test points (i.e.  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{3}{4}$   
 325 span) in the north side, and the  $\frac{1}{2}$  span point in south. Based on four channel measurement, mode shape  
 326 vectors of the third bending and first torsion modes (Figure 9(c) and (e)) are of high similarity and might  
 327 be judged as one mode using traditional SCA method. The enhanced SCA method employs a two-step  
 328 clustering procedure for automatic classification without any signal pre-processing and captures all the  
 329 modes of interest accurately. It indicates that the proposed method is effective for the underdetermined  
 330 case using limited sensors.

331 The mode shapes extracted from Opal IMU data are indicated in the right column of Figure 9. Compared  
 332 with the reference, the estimation results have the MAC values over 99.2%. The third bending mode at  
 333 approx. 11.41 Hz is missed due to a small number ( $< 100$ ) of SSPs available in the adjacent frequency  
 334 range.



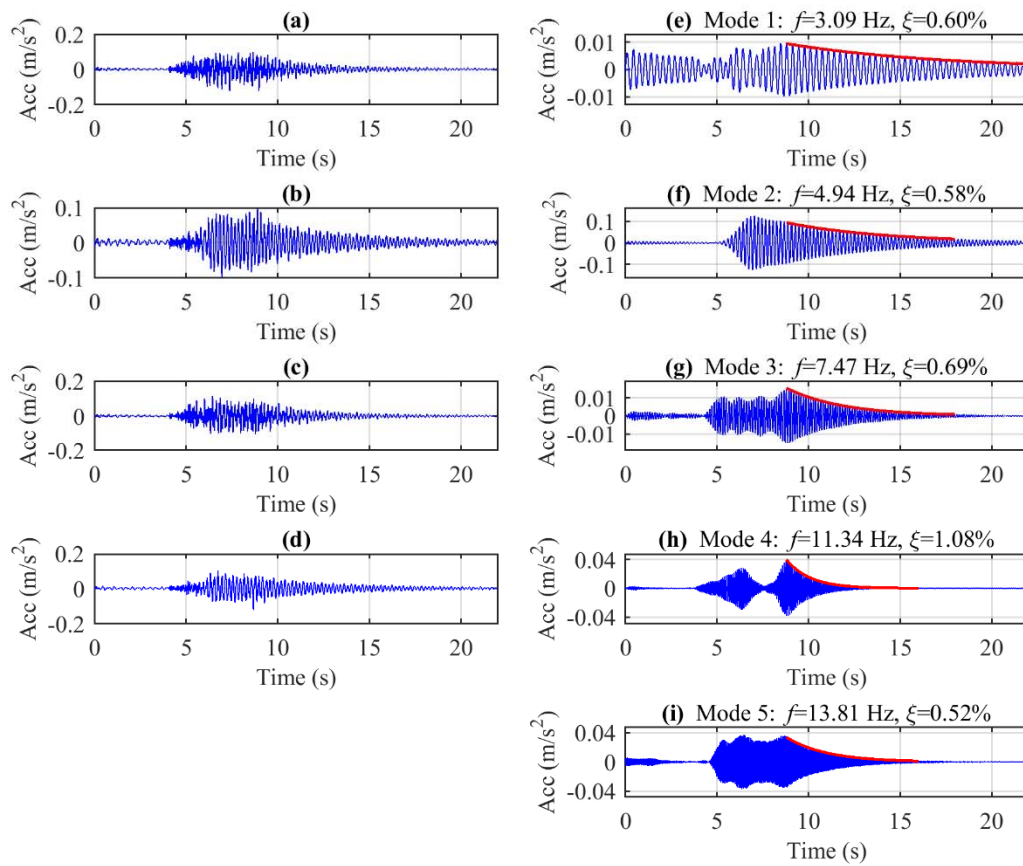
336 Figure 9 Estimated mode shapes by the enhanced SCA method: the left column corresponds to the first five modes  
 337 of the bridge estimated from QA data (a-e); and the right column corresponds to the four modes of the bridge  
 338 estimated from Opal IMU data (f-i). Dashed lines denote initial location of the bridge, solid lines with circular  
 339 markers denote the mode shape ordinate of the north side of the bridge (TP1~3) and solid lines with 'x' shaped  
 340 markers denote the mode shape ordinate of the south side of the bridge (TP4~6). The modal assurance criteria  
 341 (MAC) compared with the mode shapes estimated by NExT/ERA method using QA measurement data [40] are  
 342 given in the subplot titles.

343 3.3 *Extraction of modal parameters*

344 Given the estimated mode shapes, the TF representations of modal responses were separated based on  
345  $l^1$  norm minimisation using a MATLAB toolbox for the SPGL1 solver [36,37] and then reconstructed  
346 to the time domain by inverse STFT.

347 QA acceleration data covering a free decay period were truncated for modal parameter estimation with  
348 the duration of 22 seconds, presenting in the left column of Figure 10. The modal responses were  
349 separated from the measurement signals based on the estimated mode shapes. Due to the existence of  
350 very similar mode shapes that are indistinguishable using four test points, the output of modal responses  
351 might carry two or more frequency components and are not necessarily single degree-of-freedom  
352 signals. Hence a band-pass filter with the bandwidth of 2 Hz around the frequency value of a cluster  
353 centroid (in section 3.2) was applied to the modal response with the results shown in the right column  
354 of Figure 10.

355 The modal frequencies were estimated by the peak-picking method from the auto-spectral densities of  
356 the filtered modal responses. The damping ratios were derived from the free decay parts using the  
357 logarithmic decrement method and the fitted envelopes are indicated as red lines in modal response  
358 plots. The estimation results of modal parameters are given in Table 1 compared with those by the  
359 NExT/ERA method in [40]. The frequency estimates match very well with difference within 0.3%.  
360 Although the estimated damping ratios are much smaller than the values by the NExT/ERA method,  
361 the estimates from free-decay signals in this study could be reliable since the fitted envelopes of damped  
362 vibration curves in Figure 10(e) to (i) match very well with the actual ones.



364 Figure 10 Truncated 22 s signals of QA acceleration measurement in Run 1 and the corresponding modal response  
 365 signals separated by  $l^1$  norm minimisation: the left column (a-d) corresponds to acceleration measurement by four  
 366 QA accelerometers at test locations TP1, TP2, TP3 and TP5; and the right column (e-i) corresponds to the modal  
 367 response signals separated by the SCA method after implementing a band-pass filter with the bandwidth of 2 Hz  
 368 around the modal frequencies. Logarithmic decrement method is used for damping ratio estimation with the fitted  
 369 envelopes (red lines) indicated in modal response plots (in the right column).

370 Table 1 Modal frequencies ( $f$ ) and damping ratios ( $\xi$ ) of the first five modes estimated by the enhanced SCA  
 371 method and the NExT/ERA procedure [40] using QA data

Mode number	Enhanced SCA		NExT/ERA	
	$f$ (Hz)	$\xi$	$f$ (Hz)	$\xi$
1	3.09	0.60%	3.10	1.75%
2	4.94	0.58%	4.94	0.99%
3	7.47	0.69%	7.47	1.07%
4	11.34	1.08%	11.35	1.87%
5	13.81	0.52%	13.78	0.84%



372 After validating the enhanced SCA method for the OMA of ambient vibration of a road bridge, the  
373 proposed method is applied to modal identification of truck-induced non-stationary vibration data for  
374 the same bridge in section 4 and then for pedestrian-induced vibration data of a footbridge in section 5.

#### 375 4 FIELD TEST ON A ROAD BRIDGE DURING HEAVY TRUCK PASSAGE

376 Some of the classic OMA methods like the NExT and SSI impose the assumption of stationary  
377 excitation process and thus are challenging for analysing non-stationary signals such as truck-induced  
378 and human-induced vibrations. The SCA-based method is feasible in this case because the underlying  
379 assumption for the SCA is essentially geometrical about the sparsity of sources [31].

380 In this section, the enhanced SCA method is implemented for the modal identification of non-stationary  
381 signals recorded on a road bridge (the same bridge as in section 3) under heavy truck passages. Section  
382 4.1 introduces the test configuration on the bridge and demonstrates the measurement results while  
383 section 4.2 presents the estimated results of mode shapes using the enhanced SCA method. The step of  
384 modal parameter estimation is not presented in this section as it is very similar to the content in section  
385 3.3.

##### 386 4.1 Test configuration and measurement results

387 The truck used in the test had a total weight of 32 t with four axles shown in Figure 11(a). Sensors used  
388 for recording consisted of four QA accelerometers located at the  $\frac{1}{4}$  point (TP1), mid-span points (TP2  
389 and TP5), and  $\frac{3}{4}$  point (TP3) in Figure 11(b). The sample rate was set as 256 Hz. The truck passed the  
390 bridge, without stopping, from the west to the east using the north lane in Run 1 and from the east to  
391 the west using the south lane in Run 2.

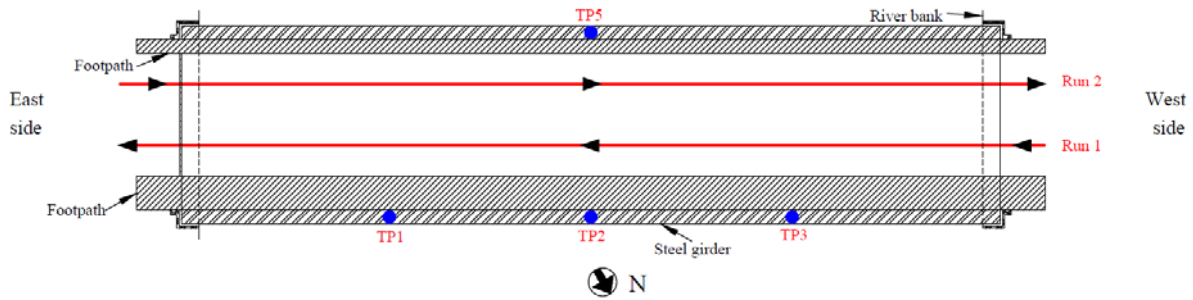
392 (a)



393

394 (b)

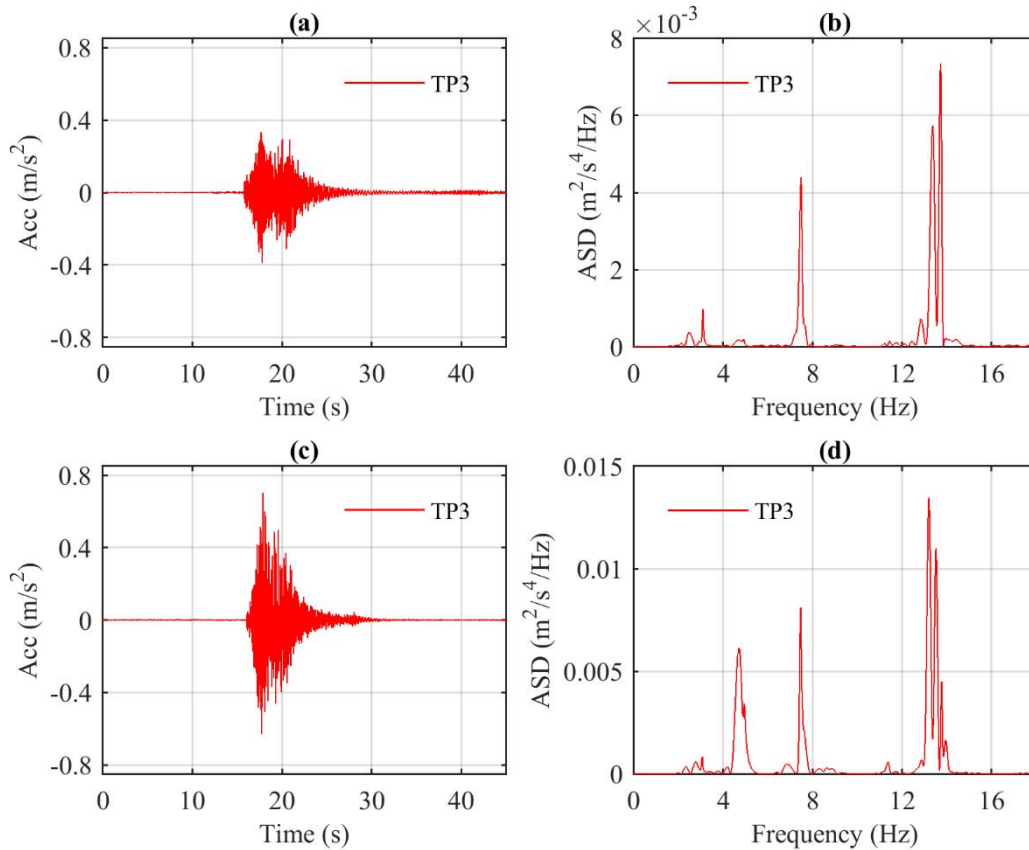
395



396 Figure 11 Truck information and test point locations: (a) the truck used in the test; and (b) locations of four QA  
 397 accelerometers and truck passage routes in two runs.

398 Vibration data during the truck passages were truncated for the analysis. The time series data at TP3 in  
 399 two runs are shown in Figure 12 (a) and (c) and the maximum acceleration experienced was  $0.33 \text{ m/s}^2$   
 400 and  $0.70 \text{ m/s}^2$ , respectively. Auto-spectral densities of the signals shown in (a) and (c) are estimated  
 401 using the Welch's method and the results are shown in Figure 12(b) and (d) respectively. Modes that  
 402 received more energy in Run 1 are the second bending mode at 7.5 Hz and the second torsion mode at  
 403 13.8 Hz while the first torsion mode at 4.95 Hz becomes more apparent in Run 2. This is likely related  
 404 to the fact that during Run 2 the truck was closer to the edge of the deck than in is in Run 1 due to the  
 405 narrower footpath on the south side of the bridge. Two or three peaks with high energy are observed  
 406 near 13.8 Hz that indicate the non-stationary and time-varying feature of the vibration signals.

407



408 Figure 12 Vertical acceleration measurement at TP3 during the truck passages in two runs and the corresponding  
 409 auto-spectral densities: (a) acceleration measurement recorded when the truck passed the bridge from the west to  
 410 the east in Run 1; (b) auto-spectral densities of acceleration data in (a); (c) acceleration measurement recorded  
 411 when the truck passed the bridge from the east to the west in Run 2; and (d) auto-spectral densities of acceleration  
 412 data in (c).

#### 413 4.2 Estimation of mode shapes by enhanced SCA method

414 Mode shapes of the bridge were estimated following the procedures of the enhanced SCA method in  
 415 section 2.2. The TF transform applied to vibration data is STFT using Hamming windows with the  
 416 window length 5760 and the hop size 2. The threshold angle  $\Delta\theta$  for SSP detection was taken as 2  
 417 degrees, same as in section 3.2.

418 Table 2 provides the estimation results of modal frequencies and also the MAC values compared with  
 419 the references that are mode shapes estimated in section 3 using QA data.

- 420 • For the first bending mode initially at 3.09 Hz, three mode shapes in Run 1 at 2.11 Hz, 2.57 Hz and  
 421 3.09 Hz are observed reaching high MAC values ( $>99.5\%$ ) compared with the reference. In Run 2,  
 422 four modes at 2.29 Hz, 2.59 Hz, 3.08 Hz and 3.27 Hz are identified with similar mode shapes as  
 423 the reference. Initially the appearance of multiple frequencies that have the same apparent mode  
 424 shape is surprising. However, it is to do with the fact that when the truck is on the bridge, the  
 425 frequencies of this coupled system consisting of the vehicle and the bridge can vary with truck  
 426 position, resulting in a non-stationary vibration signal. This phenomena is not the focus of this paper  
 427 so is not discussed further here, but has been reported in detail in [41].
- 428 • For the first torsion mode initially at 4.94 Hz, the mode shape estimates at 4.53 Hz from the data in  
 429 Run 1 has the MAC of 95.87%. In Run 2, two modes at 4.16 Hz and 4.81 Hz are identified with the  
 430 MAC values of 99.19% and 99.97%, respectively. These modes are demonstrated in Figure 13. In  
 431 Run 1 when the truck passed from the west to the east using the north lane of the carriageway, the  
 432 modal displacement in the north side (TP1~3) apparently decreased while that for TP5 in the south  
 433 side increased slightly. Mode shape changes in Run 2 are less obvious.
- 434 • The information of the other three modes lower than 15 Hz is given in Table 2. The 3<sup>rd</sup> bending  
 435 mode is missed when analysing the vibration data in Run 2. The MAC values between the identified  
 436 mode shapes and the reference are higher than 98.8%.

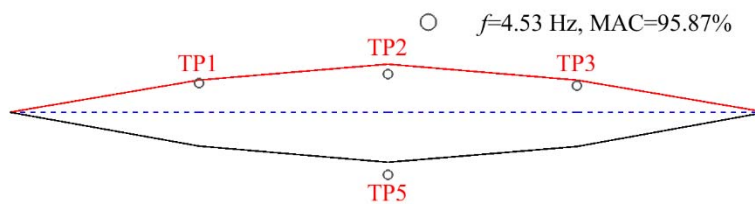
437 Table 2 Modal frequency estimates during truck passages and the MAC values compared with the mode shapes  
 438 estimated in section 3 using ambient vibration data recorded by QA accelerometers.

Mode No.	Test runs	Modal Frequency (Hz)	MAC
<b>1<sup>st</sup> bending</b>	Reference	3.09	--
	Run 1	2.11	99.81%
		2.57	99.70%
		3.09	99.99%

		2.29	98.98%
	Run 2	2.59	99.90%
		3.08	99.99%
		3.27	99.56%
<b>1<sup>st</sup> torsion</b>	Reference	4.94	--
	Run 1	4.53	<b>95.87%</b>
	Run 2	4.16	99.19%
		4.84	99.97%
<b>2<sup>nd</sup> bending</b>	Reference	7.47	--
	Run 1	7.52	99.96%
	Run 2	7.45	99.26%
<b>3<sup>rd</sup> bending</b>	Reference	11.34	--
	Run 1	11.36	99.68%
	Run 2	Not available	Not available
<b>2<sup>nd</sup> torsion</b>	Reference	13.81	--
	Run 1	13.75	99.35%
	Run 2	13.23	98.86%
		13.56	99.67%

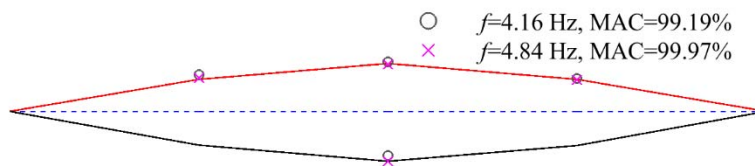
439

(a) Run 1



440

(b) Run 2



441 Figure 13 Estimation results of the first torsion modes during truck passages in Run 1 and Run 2. Dashed lines  
 442 denote initial location of the bridge; two solid lines denote the reference mode shape ordinate of the north and  
 443 south sides of the bridge estimated using ambient vibration data of QA accelerometers in Section 3. Circular and  
 444 'x' shaped markers denote the mode shape ordinate estimated using vibration data during truck passages; and the  
 445 corresponding modal frequencies and MAC values compared with the reference mode shapes are given in the  
 446 legends.

447 Analysis results indicate that the enhanced SCA method is capable of analysing non-stationary vibration  
 448 signals, i.e. identifying accurately bridge mode shapes and capturing additional modes due to changes

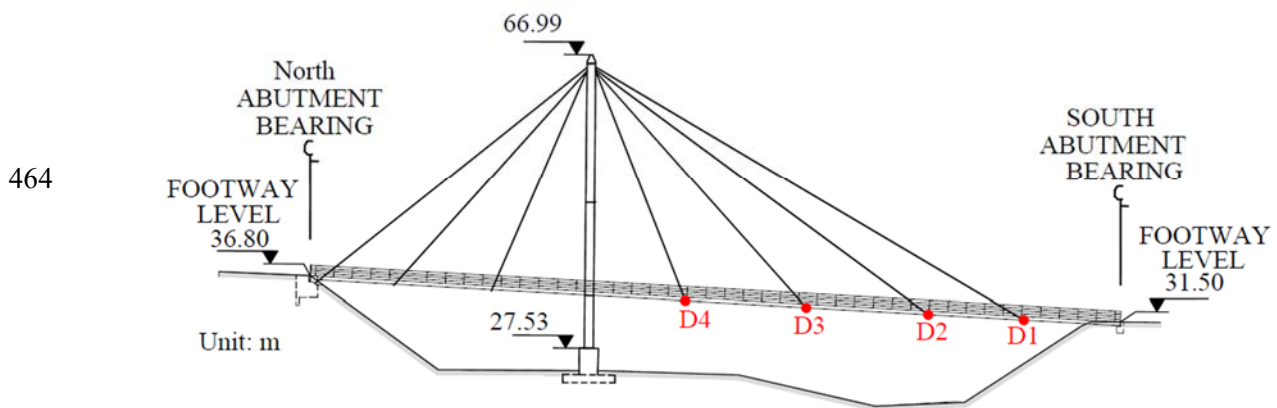
449 of system properties. The procedures are intended to identify such changes while their interpretation  
450 requires structural engineering expertise supported by numerical modelling and further investigations.

## 451 5 FIELD TEST ON A FOOTBRIDGE

452 In this section, the enhanced SCA method is implemented for modal identification of vibration signals  
453 recorded on a cable-stayed footbridge. There are some challenges of implementing classical OMA  
454 methods e.g. NExT/ERA and SSI to capture modal information completely for this bridge as it has  
455 several closely-spaced modes and experiences high energy only in frequency components close to  
456 pedestrian pacing rates. Section 5.1 introduces the bridge and the test configuration and then  
457 demonstrates the measurement results, while section 5.2 and 5.3 presents the estimated results of mode  
458 shapes and modal frequencies using the enhanced SCA method.

### 459 5.1 Test configuration and measurement results

460 Baker Bridge, shown in Figure 14, is a cable-stayed footbridge with the span length 109 m in Exeter,  
461 UK. The bridge links Digby & Sowton railway station in the north to the Sandy Park Stadium in the  
462 south that is the home ground of Exeter Chiefs Rugby Club. The bridge has six vertical modes lower  
463 than 3.5 Hz [42] and thus experiences considerable dynamic response to pedestrian traffic.



465 (a)

466



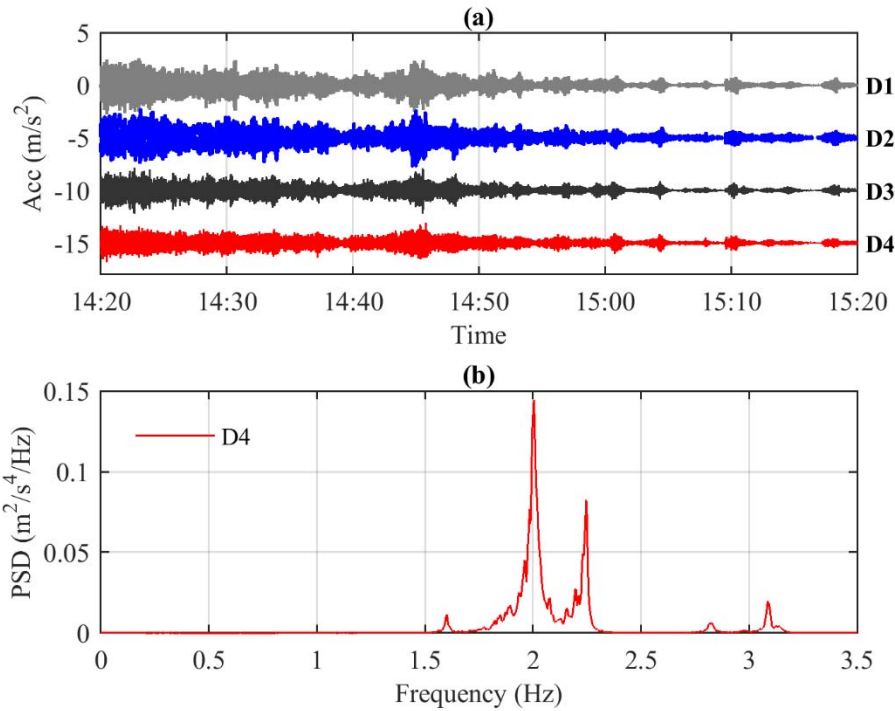
467

468 (b)

469 Figure 14 Bridge information and sensor locations: (a) bridge elevation and locations of four Opal IMUs at D1 to  
470 D4 in the southwest side of the bridge; and (b) west elevation of the south span of the bridge at 14:28:30 PM from  
471 a recorded video file on the test day.

472 Four APDM Opal™ IMU sensors were installed on the south span adjacent to the west parapet at D1  
473 to D4, as shown in Figure 14(a) on a match day. The match kick-off time was 15:00 PM. The sample  
474 rate was set as 128 Hz.

475 Vibration data from 14:20 PM to 15:20 PM were truncated for modal identification. The vertical  
476 acceleration measurement shown in Figure 15(a) indicates that the bridge became very quiet after the  
477 match kick-off at 15:00 PM. Auto-spectral densities estimated using the Welch's method are shown in  
478 Figure 15(b). The two modes with the frequencies close to the normal walking pace (2 Hz) are the  
479 strongest and most obvious.



480

481 Figure 15 Acceleration measurement by four Opal IMUs in the vertical direction and the corresponding auto-  
 482 spectral densities: (a) acceleration measurement from 14:20 PM to 15:20 PM; and (b) the auto-spectral densities  
 483 of acceleration measurement at D4 in (a).

484 *5.2 Estimation of mode shapes by enhanced SCA method*

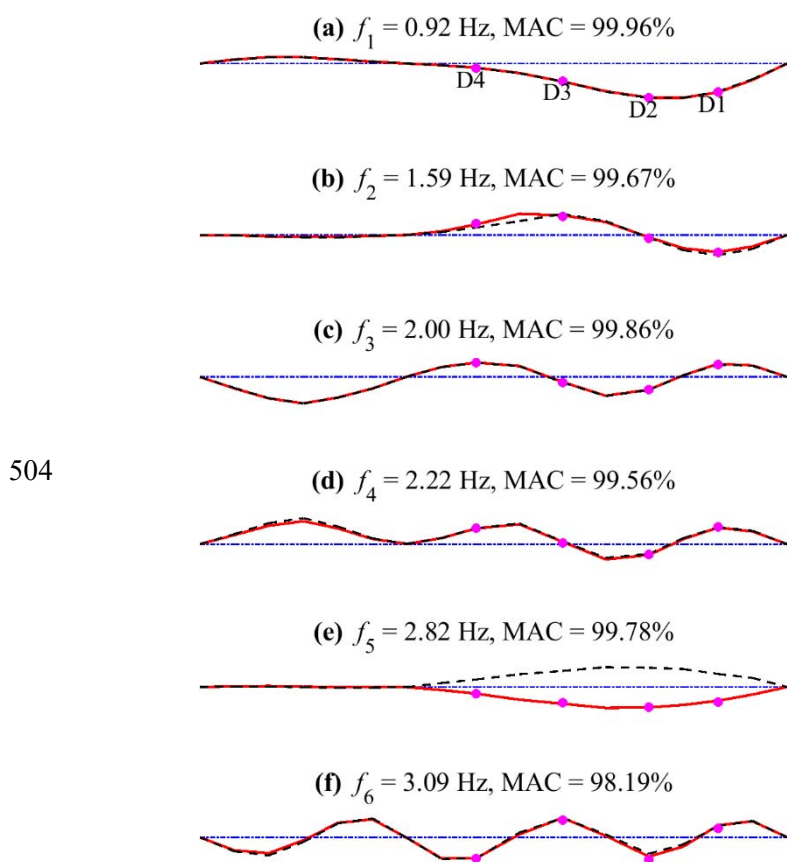
485 Following the two-step clustering described in section 2.2, mode shapes and frequencies were extracted  
 486 from each cluster centroid of normalised TF coefficients. The threshold angle  $\Delta\theta$  for SSP detection  
 487 was taken as 5 degrees, same as for Opal measurement in section 3.2.

488 Figure 16 demonstrates the first six mode shapes estimated by the enhanced SCA method together with  
 489 the reference that is derived from the previous ambient modal test [42] using NExT/ERA procedures.  
 490 In this ambient modal test, six wireless accelerometer sensors were used to record bridge vibrations.  
 491 Two sensors were kept at the same points as the reference while the other four were ‘roved’ over the  
 492 remaining 30 test points (covering bridge two sides) in several recordings. Mode shape and modal  
 493 frequency information for the first six modes have been demonstrated in [42] and now are re-interpreted  
 494 here as the reference: Solid and dashed curves correspond to modal shape ordinate of the west and east  
 495 sides of the bridge.

496 The dot markers denote the modal shape ordinate at D1~D4 on the west side of the bridge estimated by  
 497 the enhanced SCA method using Opal IMU data. The first five mode shape estimates using the enhanced  
 498 SCA method match well with the reference with the MAC values over 99.5% while the MAC value for  
 499 the six mode is slightly lower (98.19%).

500 In this example, mode shape vectors for the third and fourth bending modes (Figure 16(c) and (d)) are  
 501 of high similarity based on the four channel measurement (D1-4) and should be challenging to be

502 distinguished via traditional SCA method. The demonstration example validated again the feasibility of  
 503 the enhanced SCA method for the underdetermined case using limited sensors.



504

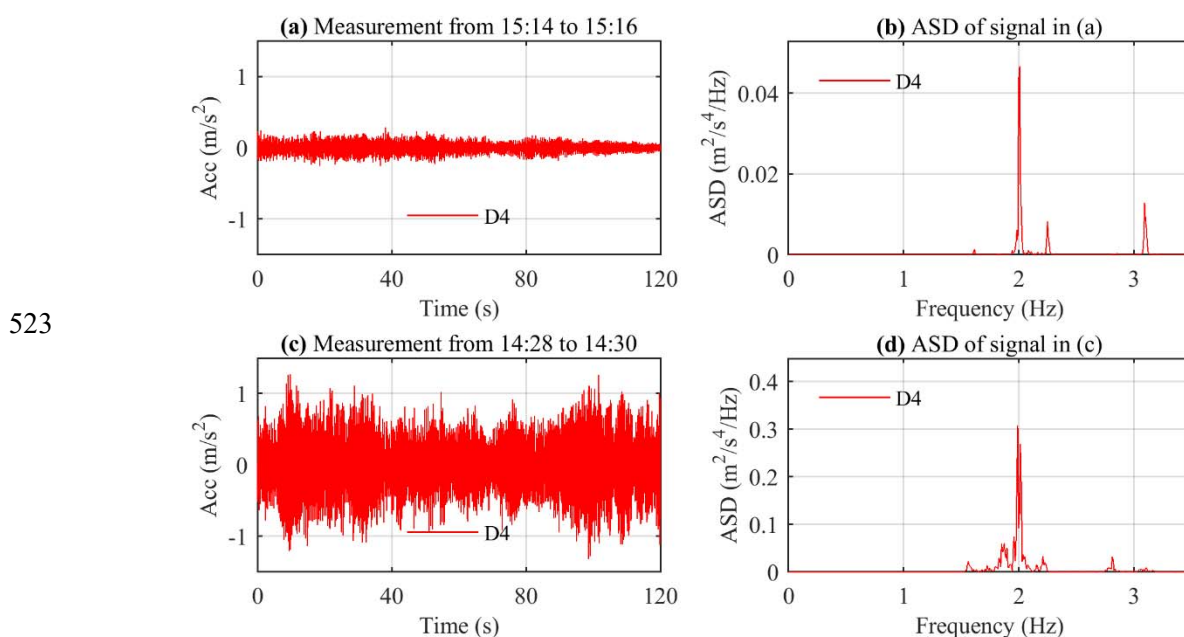
505 Figure 16 Estimated mode shapes by the enhanced SCA method together with reference mode shapes of the bridge:  
 506 solid and dashed curves denote the reference mode shape ordinate from a previous modal test using the  
 507 NExT/ERA method [42] in the west and east sides of the bridge, respectively; and dot markers denote the modal  
 508 shape ordinate estimated by the enhanced SCA method using acceleration data (in Figure 15(a)) from four Opal  
 509 IMUs located at the southwest side of the bridge. Estimates by the enhanced SCA method including modal  
 510 frequencies and modal assurance criteria (MAC) compared with the reference mode shapes are given in subplot  
 511 titles.

### 512 5.3 Extraction of modal parameters

513 Based on vibration signals involving some periods with a crowd of pedestrians shown in Figure 14(b),  
 514 the mode shape estimates in Figure 16 still have good match with the results in a previous ambient  
 515 modal test. This indicates that the non-stationary feature of human-induced vibrations in this study is  
 516 not apparently reflected in mode shape changes.



517 To investigate the time-varying characteristics, two time intervals of the separated modal responses  
 518 with the duration of two minutes were truncated for modal parameter estimation when the bridge was  
 519 occupied by a few pedestrians and a crowd, respectively. The raw acceleration measurement at D4  
 520 during these two selected periods is shown in Figure 17(a) and (c). The experienced maximum  
 521 acceleration reaches  $0.29 \text{ m/s}^2$  and  $1.31 \text{ m/s}^2$ , respectively and the strongest mode is both at 2 Hz,  
 522 where with the auto-spectral density for the crowd is almost an order of magnitude stronger.



524 Figure 17 Acceleration measurement at D4 during two time intervals (when the bridge was occupied by a few  
 525 pedestrians and a crowd, respectively) and the corresponding auto-spectral densities: (a) vertical acceleration  
 526 measurement at D4 from 15:14 to 15:16; (b) auto-spectral densities of the acceleration signal in (a); (c) vertical  
 527 acceleration measurement at D4 from 14:28 to 14:30; and (d) auto-spectral densities of the acceleration signal in  
 528 (c).

529 Figure 18 and Figure 19 provide the separated modal responses and the corresponding auto-spectral  
 530 densities recovered from the two time intervals.

531 For the estimated results in the first time interval (Figure 18), the modal responses are close to single-  
 532 degree-of-freedom signals except in (a) for the first bending mode (at approximately 0.94 Hz) where  
 533 some frequency components near 2.1 Hz (slightly deviated from the third bending mode frequency of  
 534 2.0 Hz) also contain considerable energy.

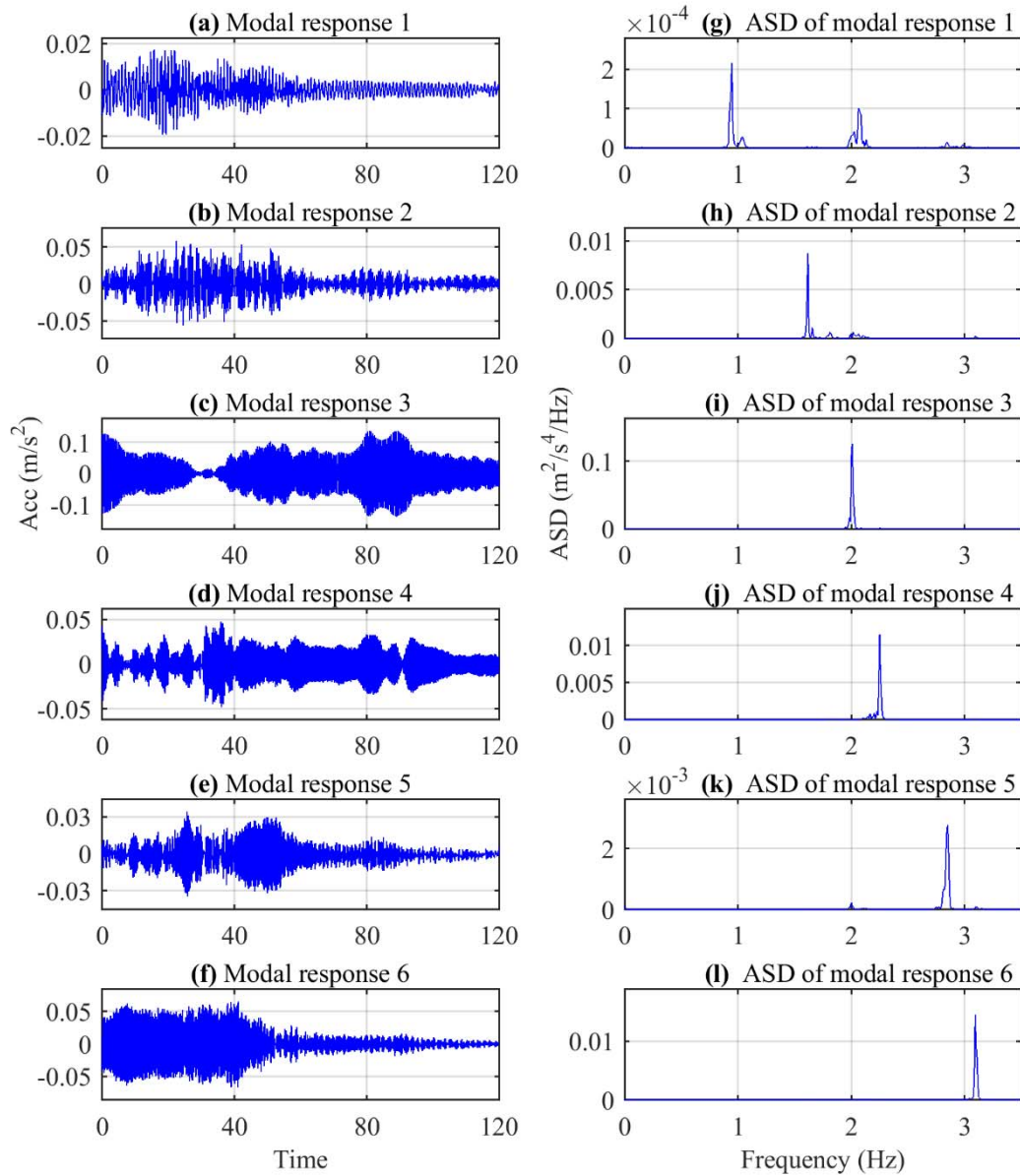
535 For the estimated results in the second time interval (Figure 19), clear peaks near dominant modal  
 536 frequencies are indicated in the auto-spectral density plots of the third, fifth and sixth modal responses.  
 537 The first, second and fourth modal responses involve considerable energy in the frequency components  
 538 between 1.8 Hz and 2.1 Hz that are probably due to the excitations of walking pedestrians.

539 The modal frequencies were extracted from the auto-spectral density plots by the peak-picking method.  
 540 As a comparison, the acceleration signals were also analysed directly by the covariance-driven SSI

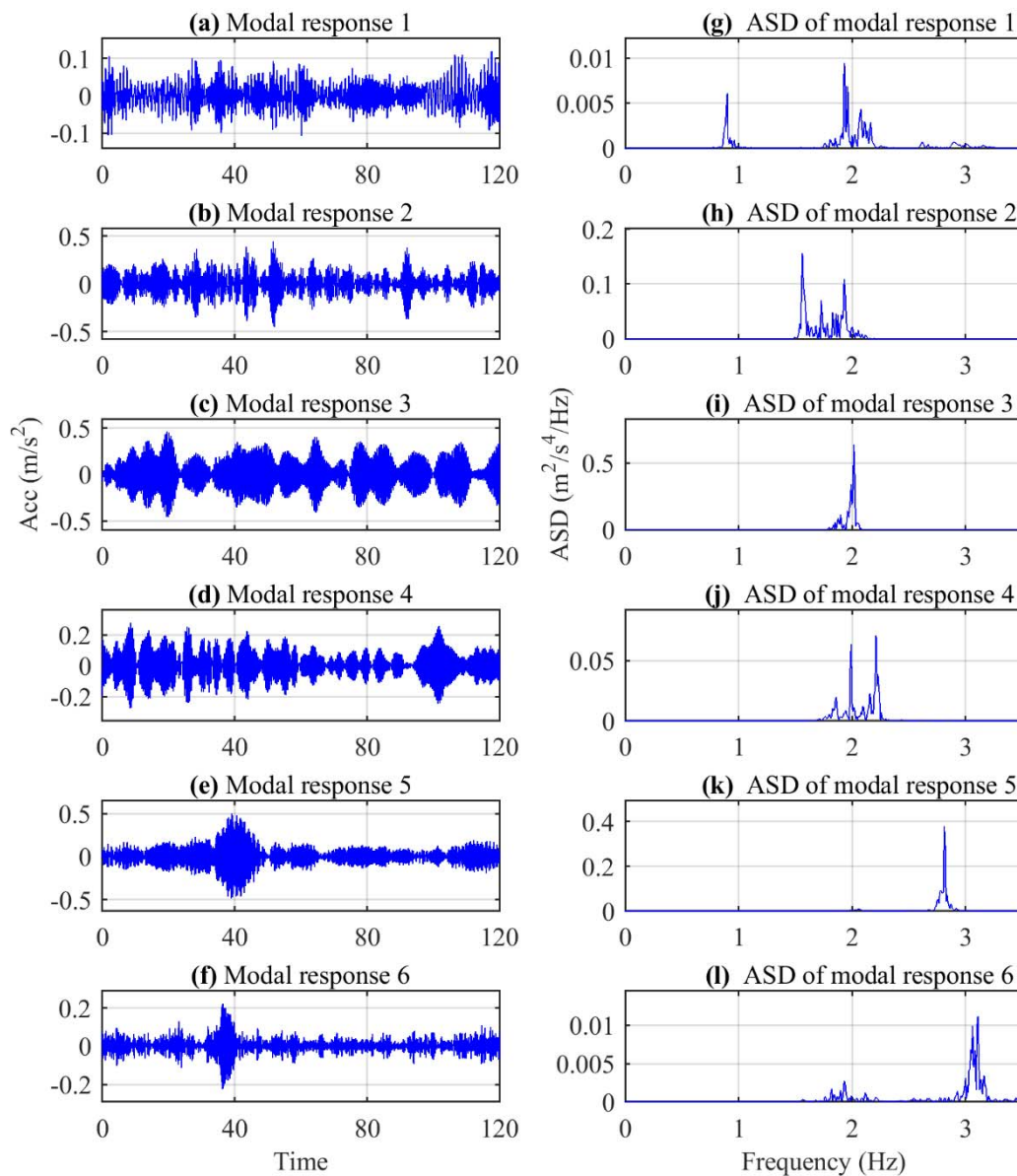
541 method for modal frequency estimation. The variables were set as 180 points in the covariance function  
542 and maximum order of 80 poles. Table 3 provides modal frequency estimates by the two methods as  
543 well as a reference from the previous ambient modal test [42] using NExT/ERA method. Observations  
544 in Table 3 show that,

- 545 • The enhanced SCA method identifies all the first six modes lower than 3.5 Hz from vibration data  
546 in either quiet or busy periods. The SSI method fails to capture some modes (e.g. 0.94 Hz and 3.09  
547 Hz) even when weighting algorithms (e.g. Canonical Variate Analysis, Principal Components or  
548 Unweighted Principal Components) are considered for performance improvement. It is possibly  
549 due to the low-energy in the adjacent frequency ranges as shown in Figure 17(b) and (d). The  
550 enhanced SCA method is feasible for the low-energy modes because the information used for  
551 cluster analysis is the set of unit vectors of normalised TF coefficients, and their scales are neglected.
- 552 • Compared with the previous ambient modal test results, the modal frequency estimates in the first  
553 time interval match very well, while in the second time interval the first two modal frequencies are  
554 apparently reduced due to heavy pedestrian occupation.

555



556 Figure 18 Modal response signals and the corresponding auto-spectral densities in the first time interval when the  
557 bridge was occupied by a few pedestrians: the left column (a-f) corresponds to the modal response signals  
558 separated by the SCA method; and the right column (g-l) corresponds to the auto-spectral densities of the signals  
559 in the left column.



561 Figure 19 Modal response signals and the corresponding auto-spectral densities in the second time interval when  
 562 the bridge was occupied by crowds of pedestrians: the left column (a-f) corresponds to the modal response signals  
 563 separated by the SCA method; and the right column (g-l) corresponds to the auto-spectral densities of the signals  
 564 in the left column.

565 Table 3 Estimated modal frequencies of Baker Bridge: the 2<sup>nd</sup> column denotes modal frequencies estimated in a  
 566 previous ambient modal test [42] by NExT/ERA method; the 3<sup>rd</sup> to 4<sup>th</sup> columns represent the modal frequencies  
 567 estimated from acceleration data in the first time interval by the enhanced SCA and SSI methods; and the 5<sup>th</sup> to  
 568 6<sup>th</sup> columns represent the modal frequencies estimated from acceleration data in the second time interval by the  
 569 enhanced SCA and SSI methods.

Modal frequency (Hz)	Previous modal test	Time interval 1		Time interval 2	
		By enhanced SCA	By SSI	By enhanced SCA	By SSI
Mode 1	0.94	0.95	--	0.90	--
Mode 2	1.62	1.62	1.60	1.56	1.56
Mode 3	2.00	2.01	2.01	2.02	1.98
Mode 4	2.24	2.25	2.25	2.21	--
Mode 5	2.84	2.85	2.83	2.81	2.81
Mode 6	3.08	3.09	3.10	3.11	--

570

571 Results indicate the enhanced SCA method provides accurate estimates of mode shapes and frequencies  
572 for human-induced vibrations and is capable to capture low-energy modes that is infeasible by SSI  
573 method. The non-stationary characteristics are reflected in the reconstructed modal responses with time-  
574 varying modal frequencies and possibly including the components of pedestrian excitations.

## 575 6 CONCLUSIONS

576 This study proposes an enhanced SCA method for structural modal identification. Through direct  
577 application to field test data, the method is validated to be capable of providing comparative results  
578 about modal parameters from ambient vibration data as the classic OMA method NExT/ERA.  
579 Compared with traditional SCA method, the proposed method has the advantage of accurately  
580 identifying highly similar modes that is beneficial for structural modal testing using limited sensors.

581 The enhanced SCA method has no assumption regarding the nature of excitation forces and is validated  
582 to be effective for analysing non-stationary signals including vehicle-induced and human-induced  
583 vibrations. For vehicle-induced vibrations, small changes in mode shapes and modal frequencies due to  
584 the time-varying feature can be captured. For human-induced vibrations, the mode shape changes are  
585 negligible in this study while the recovered modal response signals are non-stationary, reflecting small  
586 changes of modal frequencies as well as the components of pedestrian excitations. The proposed method  
587 could identify easily the low-energy and closely-spaced modes, indicating better performance than the  
588 SSI method.

589 Compared with other OMA methods, the enhanced SCA method in this study has less dependence on  
590 parameter selection and potentially fits the requirements of automatic modal identification on field test  
591 data.

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599 the two anonymous reviewers for their constructive comments.  
600

## 601 8 DECLARATION OF CONFLICTING INTERESTS

602 The authors declare that there is no conflict of interest.

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