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Cognitive Wireless Powered Communication Networks with Secondary User Selection and QoS in Primary Networks

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Abstract—In this research, we investigate the outage probability of the secondary network in a cognitive wireless powered communication network (WPCN). Energy-constrained secondary users harvest energy from a hybrid access-point and a primary transmitter in the first phase. In the second phase, we select a secondary user based on two different schemes, namely the best uplink channel selection (UCS) and the minimal interference channel selection (MICS), to transfer energy to the primary access-point. In this setup, the secondary network can share the spectrum with the primary network ensuring that a desired outage probability constraint in the primary network is always met. This constraint represents the quality-of-service (QoS) of the primary network. The analytical expressions and asymptotic expressions of outage probability of the secondary network are provided and verified. We demonstrate that increasing the number of secondary users can considerably improve system performance. We show that the transmit power of the selected secondary user, energy harvesting time and relaxing the QoS constraint of the primary network have a significant impact on the outage probability of the secondary network. The results show that UCS outperforms MICS.

I. INTRODUCTION

Energy harvesting can scavenge energy from the surrounding environment. In particular, radio-frequency (RF) energy harvesting has drawn considerable attention from academia and industry [1]–[3]. Compared to solar or wind energy harvesting, RF energy harvesting is more flexible, self-sustainable and stable since more and more ambient transmitters will be deployed as new sources to harvest energy. It is worth noting that some systems are already commercially available, for example Powercast can harvest energy operating at 915MHz, the RF energy harvesting is 3.5mW at a distance of 0.6 meters, and 1uW at a distance of 11 meters [4]. In addition, there is growing interest in studying wireless powered communication networks (WPCN) [5]–[8], where the battery of wireless communication devices can be remotely replenished by RF signals.

Thanks to the latest development in wireless networks, different scenarios of WPCN have been recently investigated in the literature ([9]–[12]). In [9], a "harvest-then-transmit" protocol is first studied in multi-user WPCN, where sum-throughput maximization solves the doubly near-far problem with time-division-multiple-access (TDMA) based wireless information transmission (WIT). Afterwards, the system model in [9] was extended to a full-duplex hybrid access-point (H-AP) that enable simultaneous wireless energy transfer (WET) in the downlink (DL) and WIT in the uplink (UL) in [10]. In addition, [11] extends the study in [9] to a multi-antenna WPCN, where beamforming obtained more efficient DL WET and better throughput performance in the UL WIT. Besides, [12] investigated a WPCN massive multiple-input-multiple-output (MIMO) system where H-AP is equipped with a large number of antennas to improve WET/WIT efficiency.

Spectrum is currently highly limited due to a boom in the growth of wireless devices and services while most of licenced spectrum bands are occupied [13]. It is urgent to deploy new technologies to optimise the current spectrum usage. Fortunately, cognitive radio techniques [14] can efficiently enable unlicenced secondary users to transfer messages over the licenced primary users spectrum in an opportunistic manner. The combination of cognitive radio and energy harvesting technologies can bring great advantages to WPCN. In [15], the impact of the primary network on the secondary network in cognitive WPCN was investigated. The wireless-powered cognitive radio network was studied in [16], where secondary users harvest energy and reuse spectrum from primary users based on stochastic-geometry models. Furthermore, a cognitive WPCN shares the same spectrum for its WET and WIT by jointly optimizing the time and power allocations in the secondary network in [17].

However, key issues such as secondary user selection schemes and the impact of guaranteeing QoS in primary networks for cognitive WPCN in an energy harvesting context have not been addressed by previous models. Our research addresses these key issues by proposing a new model and studying the impact on the secondary network outage probability. Motivated by this, we take our attention to analyse the outage performance of cognitive WPCN over Rayleigh fading.

The contribution of this paper is summarised as follows:

• We take into account the QoS of the primary network and study how relaxing the QoS constraint affects the secondary network. The outage probability constraint of the primary network is always satisfied. This constraint represents the QoS of the primary network, which dictates the transmit power of secondary users. We develop the analytical expressions and asymptotic expressions of the outage probability of the secondary network.

• Our model considers two secondary user selection schemes and also assesses the impact of varying the number of secondary users. Two selection schemes are
Hybrid access expressions and asymptotic outage probability analysis are described the system and channel models. Outage probability of SU $m$ $P_{\text{out}}$ is given as

$$P_{\text{out}} = \max_{m=1,...,M} [h_{S_mH}]$$

where $R_{\text{px}}$, $\beta_{\text{px}}$, $\lambda_H$, $\lambda_T$, and $\kappa$ are the target rate of the primary network, the QoS parameter or constraint that represents the desired $P_{\text{out}}$ of the primary network. In the primary network, $P_{\text{tx}}$ sends information to $P_{\text{rx}}$ through channel $h_{\text{TX}}$ with transmit power $P_T$. In the first phase, the signal-to-interference-plus-noise-ratio (SINR) at $P_{\text{rx}}$ is given as

$$\Psi_{\text{px},1} = \frac{P_T |h_{\text{TX}}|^2}{P_H |h_{\text{HR}}|^2 + N_0}$$

where $P_H$ is the transmit power of H-AP which satisfies the primary network $P_{\text{out}}$ constraint $\kappa$, $h_{\text{TX}}$ is the channel coefficient of $P_{\text{tx}} \rightarrow P_{\text{RX}}$ link, and $h_{\text{HR}}$ is the channel coefficient of H-AP $\rightarrow P_{\text{RX}}$ link. In the second phase, the SINR at $P_{\text{rx}}$ is given as

$$\Psi_{\text{px},2} = \frac{P_T |h_{\text{TX}}|^2}{P_{\text{S}} |h_{\text{SR}}|^2 + N_0}$$

where $P_{\text{S}}$ is the maximum threshold transmit power of $SU_s$ allowed by $P_{\text{out}}$ constraint $\kappa$, $h_{\text{SR}}$ is the channel coefficient of $SU_s \rightarrow P_{\text{RX}}$ link. From (1), $P_H$ and $P_{\text{S}}$ can be derived as follows:

$$P_{\text{out}} = \max \left\{ \frac{P_T |h_{\text{TX}}|^2}{P_H |h_{\text{HR}}|^2 + N_0} < \beta_{\text{px}}, \frac{P_T |h_{\text{TX}}|^2}{P_{\text{S}} |h_{\text{SR}}|^2 + N_0} < \beta_{\text{px}} \right\}$$

From (4), $P_H$ can be derived as

$$P_H = \left\{ \begin{array}{l} P_T \lambda_{\text{HR}} \xi, \quad \text{if } \xi > 0, \\ 0, \quad \text{otherwise.} \end{array} \right.$$

where $\gamma_T = \frac{P_T \lambda_{\text{HR}}}{N_0}$ and

$$\xi = \frac{1}{\beta_{\text{px}} \lambda_T} \left[ \frac{1}{1 - \kappa} \exp \left( -\frac{\lambda_T \beta_{\text{px}}}{\gamma_T} \right) - 1 \right].$$

Similarly, from (6), $P_{\text{S}}$ can be derived as

$$P_{\text{S}} = \left\{ \begin{array}{l} P_T \lambda_{\text{SR}} \xi, \quad \text{if } \xi > 0, \\ 0, \quad \text{otherwise.} \end{array} \right.$$

B. Selected Schemes at Secondary Users

Motivated by wireless sensor networks and clustering, the same cluster sensor nodes can co-operate. Therefore, to improve performance, a given $SU_s$ can be selected from $SU_m$ to transmit information. Two selection schemes are deployed, namely UCS and MICS.

1) Uplink Channel Selection (UCS): In the UCS scheme, selection based on the CSI of $SU_s \rightarrow$H-AP link to choose best uplink. The secondary user $SU_s$ is chosen as follows:

$$|h_{S_sH}|^2 = \max_{m=1,...,M} [h_{S_mH}]^2$$

Fig. 1: Cognitive WPCN considering QoS in the primary network.
2) Minimal Interference Channel Selection (MICS): The MICS scheme is based on the CSI of SU to SU. The harvested power in SU by implementing time-switching-based architecture as shown in Fig. 2. The threshold transmit power, given as

\[ P_{\text{Thr}} = \alpha \left( P_{\text{IN}} |h_{\text{SU}}|^2 + P_{\text{TX}} |h_{\text{SU}}|^2 \right) \]

where \( \alpha = \frac{\eta}{\gamma} \), \( 0 < \eta < 1 \) is the conversion efficiency coefficient, \( h_{\text{SU}} \) is the channel power gain of SU link and \( h_{\text{TX}} \) is the channel power gain of TX link.

In the second phase, to protect the primary network, the transmit power of SU must satisfy the QoS constraint with the threshold transmit power, given as

\[ P_{\text{Thr}} = \alpha \left( P_{\text{IN}} |h_{\text{SU}}|^2 + P_{\text{TX}} |h_{\text{SU}}|^2 \right) \]

The SINR at H-AP can be given as

\[ \Psi_{\text{SU}} = \frac{P_{\text{IN}} |h_{\text{SU}}|^2}{P_{\text{IN}} |h_{\text{TX}}|^2 + N_p} \]

The achievable rate of SU-H-AP link is given as

\[ C_{\text{SU}} = (1 - \tau) \log_2(1 + \Psi_{\text{SU}}) \]

III. OUTAGE PROBABILITY

The \( P_{\text{out}} \) of the secondary network is the probability that communication rate of SU-H-AP link is smaller than a threshold rate. The \( P_{\text{out}} \) can be formulated as

\[ P_{\text{out}} = P \left\{ C_{\text{SU}} < R_{th} \right\} = P \left\{ \Psi_{\text{SU}} < \beta \right\} = F_{\Psi_{\text{SU}}} (\beta) \]

where \( R_{th} \) is the target rate of the secondary network, \( \beta = \frac{\eta}{\gamma} - 1 \), and \( F_{\Psi_{\text{SU}}} (x) \) is the cumulative distribution function (CDF) of \( \Psi_{\text{SU}} \).

The OP of the secondary network can be written as

\[ P_{\text{out}} = P \left\{ \frac{P_{\text{IN}} |h_{\text{SU}}|^2}{P_{\text{TX}} |h_{\text{TX}}|^2 + N_p} < \beta \right\} = P \left\{ \frac{\beta(\gamma_{\text{SU}} |h_{\text{SU}}|^2 + 1)}{\gamma_{\text{SU}} |h_{\text{TX}}|^2} < \frac{\beta(\gamma_{\text{SU}} |h_{\text{SU}}|^2 + 1)}{\gamma_{\text{SU}} |h_{\text{TX}}|^2} \right\} \cdot P \left\{ \Psi_{\text{SU}} > \beta \right\} \]

where \( \gamma_{\text{SU}} = \frac{P_{\text{IN}}}{P_{\text{TX}}} \).

A. Uplink Channel Selection

To facilitate finding the \( P_{\text{out}} \) with UCS scheme, we denote

\[ Z_U = \gamma_{\text{SR}} \lambda_{\text{HR}} \xi \exp \left( -\frac{\lambda_{\text{SR}} \lambda_{\text{HR}} \xi \gamma_{\text{SU}}}{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi} \right) \]

\[ Y_U = \max_{m=1}^{M} \left( \frac{|h_{\text{SU}}|^2}{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi} \right) \]

The CDF of \( Z_U \) and \( Y_U \) are given as follows

\[ F_{Z_U} (z) = 1 - \frac{\lambda_{\text{TS}} \lambda_{\text{HR}} \xi \exp \left( -\frac{\lambda_{\text{SR}} \lambda_{\text{HR}} \xi \gamma_{\text{SU}}}{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi} \right)}{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi} \]

\[ F_{Y_U} (y) = \left( \frac{1}{m} \right) \left( \frac{y}{m \beta_{\gamma_{\text{TM}}} \lambda_{\text{SR}} \lambda_{\text{HR}} \xi} \right) \]

The PDF of \( Y_U \) is given as

\[ f_{Y_U} (y) = \left( \frac{1}{m} \right) \left( \frac{y}{m \beta_{\gamma_{\text{TM}}} \lambda_{\text{SR}} \lambda_{\text{HR}} \xi} \right) \]

The OP of the secondary network with UCS scheme is given as follows:

\[ P_{\text{out}} = 1 - \frac{\alpha_{\gamma_{\text{SU}}} \lambda_{\text{HR}} \xi \lambda_{\text{HR}} \xi}{\lambda_{\text{HR}} \gamma_{\text{SU}} \lambda_{\text{HR}} \xi} \left( \frac{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi}{\lambda_{\text{HR}} \gamma_{\text{SU}} \xi} \right)^{-1} \left( m q_1 + m q_2, \right. \]

\[ \left. q_1 q_2, \frac{1}{\gamma_{\text{SU}}} \lambda_{\text{HR}} \lambda_{\text{HR}} \xi, \frac{1}{\gamma_{\text{SU}}} \lambda_{\text{HR}} \lambda_{\text{HR}} \xi \right) = \frac{\lambda_{\text{HR}} \xi}{\gamma_{\text{SU}}} \]

\[ \Theta \left( m q_1 + m q_2, q_1 q_2, \frac{1}{\gamma_{\text{SU}}} \lambda_{\text{HR}} \lambda_{\text{HR}} \xi, \frac{1}{\gamma_{\text{SU}}} \lambda_{\text{HR}} \lambda_{\text{HR}} \xi \right) \]
where $q_1 = \beta \lambda SH$, $q_2 = \beta \gamma_T \lambda_{SH}/\lambda_{TH}$ and $\Theta_1$ is given as

$$
\Theta_1(a, b, c, d, e) = \int_0^{p_{ Thr}^{ MICS}} \sum_{m=1}^{M} \left( \frac{1}{m} \right) (-1)^{m+1} ay + b y(y+c)^2 
\times \exp \left[ - \frac{dy - e}{y} \right] dy
\quad \text{with } (a > 0, b > 0, c > 0, d > 0, e > 0),
$$

**Proof:** The proof is given in Appendix A.

We now analyse the asymptotic $P_{out}$.

**Corollary 1:** When $\gamma_T \to \infty$, the asymptotic $P_{out}$ of the system with UCS scheme can be approximated as (see equation (24))

$$
P_{out} = 1 - \frac{\alpha \gamma_T \lambda_r H_{SM}}{\lambda_{TS} \lambda_r H_{SM} - \lambda_{HS}} \left( \frac{\lambda_{TS}}{\alpha \gamma_T \lambda_r H_{SM}} \right)(q_1 + q_2),
$$

where $q_1 = \beta \lambda_{SH}$, $q_2 = \beta \gamma_T \lambda_{SH}/\lambda_{TH}$, and $\Theta_2$ is given as

$$
\Theta_2(a, b, c, d, e) = \int_0^{p_{ Thr}^{ MICS}} \frac{ay + b}{y(y+c)^2} \exp \left[ - \frac{dy - e}{y} \right] dy
\quad \text{with } (a > 0, b > 0, c > 0, d > 0, e > 0),
$$

**Proof:** Similar analysis as Appendix A.

**Corollary 2:** When $\gamma_T \to \infty$, the asymptotic $P_{out}$ of the system with MICS scheme can be approximated as (see equation (35))

$$
P_{out} = 1 - \frac{\alpha \gamma_T \lambda_r H_{SM}}{\lambda_{TS} \lambda_r H_{SM} - \lambda_{HS}} \left( \frac{\lambda_{TS}}{\alpha \gamma_T \lambda_r H_{SM}} \right)(q_1 + q_2),
$$

where $q_1 = \beta \lambda_{SH}$, $q_2 = \beta \gamma_T \lambda_{SH}/\lambda_{TH}$ and $\Theta_2$ is given as

$$
\Theta_2(a, b, c, d, e) = \int_0^{p_{ Thr}^{ MICS}} \frac{ay + b}{y(y+c)^2} \exp \left[ - \frac{dy - e}{y} \right] dy
\quad \text{with } (a > 0, b > 0, c > 0, d > 0, e > 0),
$$

**Proof:** Similar analysis as in Appendix B.

**IV. Numerical Results and Discussions**

In this section, Monte Carlo simulations are provided to validate the theoretical analyses. Without loss of generality, the following parameters are set: $\eta = 0.5$, $R_{tx} = 0.6$ bits/s/Hz and $R_{th} = 0.5$ bits/s/Hz, respectively.

Fig. 3 plots the $P_{out}$ versus $\gamma_T$ for different number of secondary users from $M=1$ to 3 with $K = 0.05$, $\tau = 0.6$. The asymptotic $P_{out}$ varies for different number of secondary users. From Fig. 3, we observed that increasing $\gamma_T$ will lead $P_{out}$ to decrease. In addition, as $\gamma_T$ increases beyond a certain value, $P_{out}$ converges to its floor. We can also observe that increasing the number of SU$_m$ results in a reduction in $P_{out}$, and the gap between curves will be smaller with higher number of secondary users. The UCS scheme shows lower outage probability than the MICS scheme. As $\gamma_T$ increases, the transmit power of SU$_m$ is allowed to increase too, which result in a reduction in $P_{out}$. Eventually, SU$_m$ transmit power is limited in order to satisfy the primary network $P_{out}$ constraint $K$, and $P_{out}$ reaches the floor when converging to the asymptotic value. By increasing the number of SU$_m$, the selected SU$_m$ has a higher probability to get a better uplink channel in the UCS scheme and smaller interference to the primary user in the MICS scheme. The UCS scheme guarantees the best uplink channel to H-AP, while the MICS only guarantees the minimal interference to primary user but does not guarantees a good uplink to H-AP. This results in the UCS scheme having lower outage probability than MICS scheme. Increasing $\gamma_T$ and the number of SU$_m$ can reduce $P_{out}$, and $P_{out}$ eventually reaches the floor.

Fig. 4 plots the $P_{out}$ versus $\gamma_T$ for different values of $K$. In this figure, we set $M=3$, $\tau = 0.6$. We can observe from Fig. 4 that increasing the value of $K$ can reduce $P_{out}$. When relaxing the QoS requirement of the primary network, the SU$_m$ can transmit information with higher transmit power to have
relaxing the constraint) will result in a decrease in a lower minimum value when K this figure, we set M=3, harvest enough energy as γ because the SU s is too small. Therefore, there is an optimal value P c certain value, τ is small, increasing T P out increases, and T P out reaches a lower minimum with higher γ. When τ is small, increasing τ results in P out decreasing because the SU s has more time to harvest energy to transmit information with higher power. However, if τ is larger than certain value, P out will increase because the transmission time is too small. Therefore, there is an optimal value τ which can be observed from Fig. 5. Furthermore, SU s need less time to harvest enough energy as γ T increases. Therefore, P out has a lower minimum value when γ T increases.

\[ P_{out} \approx \sum_{m=1}^{M} \left( \frac{M}{m} \right) (-1)^{m+1} m c_1 \cdot \exp(m c_2) \left[ \text{Ei} \left( -\frac{\lambda H S \lambda S R}{\lambda H R o} - m c_2 \right) - \text{Ei} \left( -m c_2 \right) \right] + \sum_{m=1}^{M} \left( \frac{M}{m} \right) (-1)^{m+1} m c_1 \exp(m c_3) \left[ \text{Ei} \left( -\frac{\lambda T S \lambda S R \epsilon^t}{\alpha} - m c_3 \right) - \text{Ei} \left( -m c_3 \right) \right] + \left[ 1 - \sum_{m=1}^{M} \left( \frac{M}{m} \right) (-1)^{m+1} \frac{\lambda S R \epsilon^t}{m} \lambda^H S \rho T + \lambda S R \lambda^H S \right] \left( \text{Ei} \left( -\frac{\lambda H S \lambda S R}{\lambda H R o} - \frac{\lambda H S \lambda S R}{\lambda H S} \right) + \lambda H S \exp \left( -\frac{\lambda T S \lambda S R \epsilon^t}{\alpha} \right) \right) \] (24)

\[ P_{out} \approx c_1 \cdot \exp \left( c_2 \right) \left[ \text{Ei} \left( -\frac{\lambda H S \lambda S R}{\lambda H R o} - c_2 \right) - \text{Ei} \left( -c_2 \right) \right] + c_1 \cdot \exp \left( c_3 \right) \left[ \text{Ei} \left( -\frac{\lambda T S \lambda S R \epsilon^t}{\alpha} - c_3 \right) - \text{Ei} \left( -c_3 \right) \right] + \left[ 1 - \frac{\lambda S R \epsilon^t}{\rho T} \right] \left( \text{Ei} \left( -\frac{\lambda H S \lambda S R \epsilon^t}{\alpha} - \frac{\lambda H S \lambda S R}{\lambda H S} \right) + \lambda H S \exp \left( -\frac{\lambda T S \lambda S R \epsilon^t}{\alpha} \right) \right) \] (25)

Fig. 3: \( P_{out} \) versus \( \gamma_T \) for different numbers of secondary users. (\( K = 0.05, \tau = 0.6 \))

Fig. 4: \( P_{out} \) versus \( \gamma_T \) for different value of desired \( P_{out} \) constraint. (\( M=3, \tau = 0.6 \))

V. Conclusion

In this paper, we investigate the outage probability of cognitive wireless powered communication networks considering QoS in the primary networks. The secondary user is powered by the energy harvested from an H-AP and a primary transmitter. Secondary users use the harvested energy to transmit information to the H-AP in the uplink. The transmitting secondary user is selected from the user which has the best uplink to H-AP or the minimal interference to primary user. Two proposed selection schemes enhance the system’s outage probability. The analytical and asymptotic expressions of the outage probability system are derived. The results have shown that increasing the transmit power and the number of secondary users leads to a decrease of outage probability. As the transmit power of the primary transmitter increases
Fig. 5: $P_{\text{out}}$ versus $\tau$ for different value of $\gamma_T$. (M=3, $K=0.05$)

beyond a certain value, it converges to the outage probability floor. In addition, relaxing the QoS requirement of the primary network improves the performance of the secondary network because information can be transmitted with higher power by the secondary user. Besides, there is an optimal value of energy harvested time. This optimal value will vary and will be lower with higher transmit power of primary transmitter. Finally, the numerical results are provided to validate our correctness.

APPENDIX A
PROOF OF LEMMA 1

From the $P_{\text{out}}$ of the secondary network can be written as

$$P_{\text{out}} = \mathbb{P}\{P_{S}^{\text{Thr}} < Y\} \cdot \mathbb{P}\{P_{S}^{\text{Thr}} < P_{S}^{\text{Thr}}\}$$

$$+ \mathbb{P}\{Z < Y\} \cdot \mathbb{P}\{P_{S}^{\text{Thr}} < P_{S}^{\text{Thr}}\}$$

(36)

$Q_1$ can be calculated as follows:

$$Q_1 = \left[1 - \frac{P_{S}^{\text{Thr}}}{q_2 + P_{S}^{\text{Thr}}} \exp\left(-\frac{q_1}{P_{S}^{\text{Thr}}}\right)\right]$$

$$\left(\frac{H_{S}^{\text{HR}} \exp\left[-\frac{H_{S}^{\text{HR}}}{\beta H_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]}{\beta H_{S}^{\text{HR}} \exp\left[-\frac{1}{\beta H_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]}\right)$$

(37)

where $q_1 = \beta H_{S}^{\text{HR}}$ and $q_2 = \beta H_{S}^{\text{HR}}$

$Q_2$ can be calculated as follows (see equation (38) at the upper of next page)

From (37) and (38), $P_{\text{out}}$ is given as follows

$$P_{\text{out}} = 1 - \int_{0}^{\frac{P_{S}^{\text{Thr}}}{q_2}} \frac{\beta H_{S}^{\text{HR}} (1 +q_2 y + q_1 y)}{y(y + q_2)^2} \frac{\beta H_{S}^{\text{HR}}}{\beta H_{S}^{\text{HR}} - \lambda_{S}^{\text{HR}} y - \frac{q_1}{y}} dy$$

$$\times \frac{\lambda_{S}^{\text{HR}} \gamma_{S}^{\text{HR}} \exp\left[-\frac{\gamma_{S}^{\text{HR}}}{\gamma_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]}{\lambda_{S}^{\text{HR}} \gamma_{S}^{\text{HR}} - \lambda_{S}^{\text{HR}} y - \frac{q_1}{y}} dy$$

(39)

APPENDIX B
PROOF OF LEMMA 2

Based on the preceding results, an asymptotic $P_{\text{out}}$ will be now carried out in order to evaluate the behaviour of $P_{\text{out}}$ in the high-SNR regime which we assume $\gamma_T = \infty$. Therefore, we rewrite the equation (13)

$$\Psi_{SH} = \min_{\xi} \left(\gamma_T \lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{SH}|^2 + \gamma_T \lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{TH}|^2\right)$$

$$\approx \min_{\xi} \left(\lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{SH}|^2 + \lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{TH}|^2\right)$$

(40)

cause $\gamma_T = \infty$, the $1 + \gamma_T |h_{TH}|^2$ can be simplify to $\gamma_T |h_{TH}|^2$, from (7) and (40), $\xi$ can be rewrite as

$$\xi = \frac{m}{\beta H_{S}^{\text{HR}}} \left[\frac{1}{2} - 1\right]$$

To facilitate finding the asymptotic $P_{\text{out}}$, we denote

$$A = |h_{TH}|^2,$$

$$B = |h_{SH}|^2,$$

$$X = \lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{SH}|^2 + \lambda_{S}^{\text{HR}} \xi \gamma_{H}|h_{TH}|^2,$$

(41)

(42)

(43)

(44)

The PDF of $X$ is given as

$$f_X(x) = \frac{\lambda_{S}^{\text{HR}} \lambda_{S}^{\text{TH}} \exp\left[-\frac{\lambda_{S}^{\text{HR}}}{\beta H_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]}{\alpha H_{S}^{\text{HR}} - \alpha H_{S}^{\text{HR}} P_{S}^{\text{Thr}}} \exp\left[-\frac{\alpha H_{S}^{\text{HR}}}{\beta H_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]$$

$$\alpha H_{S}^{\text{HR}} \exp\left[-\frac{\alpha H_{S}^{\text{HR}}}{\beta H_{S}^{\text{HR}} P_{S}^{\text{Thr}}}\right]$$

(45)

From (40), the $P_{\text{out}}$ can be rewrite as

$$P_{\text{out}} \approx \left\{\frac{U \cdot B}{A} < \beta\right\}$$

$$\approx \left\{B < \beta A\right\}$$

From the upper of next page)

(46)

Calculate the $P_{\text{out}}$ conditioned on $U$

$$P_{\text{out}}|U \approx \int_{0}^{\infty} \int_{0}^{U \cdot B} \frac{B A}{U} f_A(x) dx$$

$$\approx \int_{0}^{\infty} \left[1 - \exp\left(-\lambda_{S}^{\text{HR}} \beta H_{S}^{\text{HR}} x\right)\right] \lambda_{S}^{\text{HR}} \exp\left(-\lambda_{S}^{\text{HR}} x\right)$$

$$\approx 1 - \frac{U}{\beta H_{S}^{\text{HR}} + U}$$

(47)
\[ Q_2 = P_{\text{Thr} S} \cdot P_{\text{Thr} T} \cdot P_{\text{Thr} Y} \cdot P_{\text{Thr} Y} \cdot P_{\text{Thr} Y} \cdot P_{\text{Thr} Y} \]

\[ \mathcal{U} \text{ can be rewritten as} \]

\[ \mathcal{U} = \begin{cases} x, & \text{if } x < \lambda_S R \xi' \\ \lambda_S R \xi', & \text{if } x > \lambda_S R \xi' \end{cases} \quad (48) \]

\[ \text{calculating the integral conditioned on } \mathcal{X}, \text{ the } P_{\text{out}} \text{ is given as} \]

\[ P_{\text{out}} \approx \int_0^\infty \left[ 1 - \frac{x}{\lambda_S R \xi'} - \frac{x}{\lambda_S R \xi'} - x \right] f_X(x) dx \]

\[ \approx \gamma_1 \cdot \exp \left( \gamma_2 \right) \left[ \text{Ei} \left( -\frac{\lambda_{HS} \lambda_{SR}}{\lambda_{HR} \alpha} \cdot \gamma_2 \right) - \text{Ei} \left( -\gamma_2 \right) \right] \]

\[ + \gamma_1 \cdot \exp \left( \gamma_3 \right) \left[ \text{Ei} \left( -\frac{\lambda_{TS} \lambda_{SR} \xi'}{\lambda_{HR} \alpha} \cdot \gamma_3 \right) - \text{Ei} \left( -\gamma_3 \right) \right] \]

\[ + \left[ 1 - \frac{\lambda_{HS} \xi'}{\lambda_{HR} \alpha} \right] \left( \frac{\lambda_{TS} \lambda_{HR} \xi' \cdot \exp \left( -\frac{\lambda_{HS} \lambda_{SR} \xi'}{\lambda_{HR} \alpha} \right)}{\lambda_{TS} \lambda_{HR} \xi' - \lambda_{HS}} \right) \]

\[ \text{where } \gamma_1 = \frac{\beta_{S} \lambda_{S} \lambda_{T H} \lambda_{H} \alpha}{\alpha \lambda_{T H}} \text{ and } \xi' = \frac{M}{\beta_{SR} \lambda_{T H}} \left[ 1 - K - 1 \right] \]

\[ \gamma_2 = \frac{\lambda_{HS} \lambda_{SR} \lambda_{T H} \lambda_{H} \alpha}{\lambda_{HR} \lambda_{T H} \lambda_{H} \alpha} \text{ and } \gamma_3 = \frac{\lambda_{HS} \lambda_{SR} \lambda_{T H} \lambda_{H} \alpha}{\lambda_{HR} \lambda_{T H} \lambda_{H} \alpha} \]

\[ \text{REFERENCES} \]


