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Cognitive Wireless Powered Communication Networks with Secondary User Selection and QoS in Primary Networks

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and Sang Quang Nguyen

Abstract—In this research, we investigate the outage probability of the secondary network in a cognitive wireless powered communication network (WPCN). Energy-constrained secondary users harvest energy from a hybrid access-point and a primary transmitter in the first phase. In the second phase, we select a secondary user based on two different schemes, namely the best uplink channel selection (UCS) and the minimal interference channel selection (MICS), to transfer information to the hybrid access-point. In this setup, the secondary network can share the spectrum with the primary network ensuring that a desired outage probability constraint in the primary network is always met. This constraint represents the quality-of-service (QoS) of the primary network. The analytical expressions and asymptotic expressions of the outage probability of the secondary network are provided and verified. We demonstrate that increasing the number of secondary users can considerably improve system performance. We show that the transmit power of the selected secondary user, energy harvesting time and relaxing the QoS constraint of the primary network have a significant impact on the outage probability of the secondary network. The results show that UCS outperforms MICS.

I. INTRODUCTION

Energy harvesting can scavenge energy from the surrounding environment. In particular, radio-frequency (RF) energy harvesting has drawn considerable attention from academia and industry [1]–[3]. Compared to solar or wind energy harvesting, RF energy harvesting is more flexible, self-sustainable and stable since more and more ambient transmitters will be deployed as new sources to harvest energy. It is worth noting that some systems are already commercially available, for example Powercast can harvest energy operating at 915MHz, the RF energy harvesting is 3.5mW at a distance of 0.6 meters, and 1uW at a distance of 11 meters [4]. In addition, there is growing interest in studying wireless powered communication networks (WPCN) [5]–[8], where the battery of wireless communication devices can be remotely replenished by RF signals.

Thanks to the latest development in wireless networks, different scenarios of WPCN have been recently investigated in the literature ([9]–[12]). In [9], a “harvest-then-transmit” protocol is first studied in multi-user WPCN, where sum-throughput maximization solves the doubly near-far problem with time-division-multiple-access (TDMA) based wireless information transmission (WIT). Afterwards, the system model in [9] was extended to a full-duplex hybrid access-point (H-AP) that enable simultaneous wireless energy transfer (WET)

in the downlink (DL) and WIT in the uplink (UL) in [10]. In addition, [11] extends the study in [9] to a multi-antenna WPCN, where beamforming obtained more efficient DL WET and better throughput performance in the UL WIT. Besides, [12] investigated a WPCN massive multiple-input-multiple-output (MIMO) system where H-AP is equipped with a large number of antennas to improve WET/WIT efficiency.

Spectrum is currently highly limited due to a boom in the growth of wireless devices and services while most of licenced spectrum bands are occupied [13]. It is urgent to deploy new technologies to optimise the current spectrum usage. Fortunately, cognitive radio techniques [14] can efficiently enable unlicensed secondary users to transfer messages over the licenced primary users spectrum in an opportunistic manner. The combination of cognitive radio and energy harvesting technologies can bring great advantages to WPCN. In [15], the impact of the primary network on the secondary network in cognitive WPCN was investigated. The wireless-powered cognitive radio network was studied in [16], where secondary users harvest energy and reuse spectrum from primary users based on stochastic-geometry models. Furthermore, a cognitive WPCN shares the same spectrum for its WET and WIT by jointly optimizing the time and power allocations in the secondary network in [17].

However, key issues such as secondary user selection schemes and the impact of guaranteeing QoS in primary networks for cognitive WPCN in an energy harvesting context have not been addressed by previous models. Our research addresses these key issues by proposing a new model and studying the impact on the secondary network outage probability. Motivated by this, we take our attention to analyse the outage performance of cognitive WPCN over Rayleigh fading.

The contribution of this paper is summarised as follows:

- We take into account the QoS of the primary network and study how relaxing the QoS constraint affects the secondary network. The outage probability constraint of the primary network is always satisfied. This constraint represents the QoS of the primary network, which dictates the transmit power of secondary users. We develop the analytical expressions and asymptotic expressions of the outage probability of the secondary network.
- Our model considers two secondary user selection schemes and also assesses the impact of varying the number of secondary users. Two selection schemes are

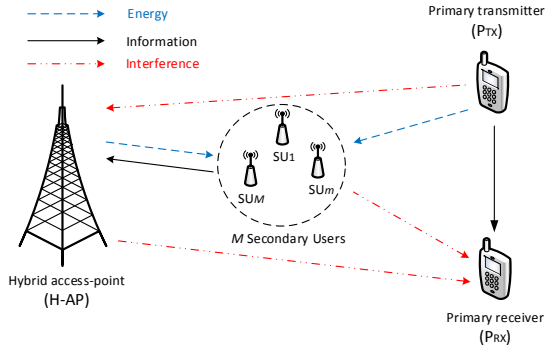


Fig. 1: Cognitive WPCN considering QoS in the primary network.

proposed, namely, UCS which prioritizes the uplink channel and MICS which prioritizes minimizing interference to the primary user. We assess the impact of these two schemes upon outage probability.

- We also study the impact of a varying harvesting energy time upon outage probability in order to find out an optimal value to minimize outage probability. This value varies with the transmit power.

The rest of this paper is organized as follows. Section II describes the system and channel models. Outage probability expressions and asymptotic outage probability analysis are derived in Section III. The numerical results based on Monte-Carlo simulations are shown in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM AND CHANNEL MODELS

We consider a cognitive WPCN consisting of a single antenna hybrid access-point H-AP, one primary transmitter P_{TX} , one primary receiver P_{RX} , M single antenna secondary users SU_m for $m = \{1, \dots, M\}$ as shown in Fig.1. In this network, one secondary user SU_s will be selected from the SU_m based on two selection schemes. In this system, we assume all nodes are located sufficiently far from each other so that $H-AP \rightarrow SU_s$, $H-AP \rightarrow P_{RX}$, $SU_s \rightarrow H-AP$, $SU_s \rightarrow P_{RX}$, $P_{TX} \rightarrow P_{RX}$, $P_{TX} \rightarrow SU_s$ and $P_{TX} \rightarrow H-AP$ experience independent and identically distributed Rayleigh fading, in which the channel power gains are exponential distributed with parameters λ_X for $X = \{HS, HR, SH, SR, TR, PS, TH\}$, respectively. The noise at P_{RX} and H-AP is modeled as the additive white Gaussian noise (AWGN) with zero mean and variances N_0 and N_p respectively.

A. Secondary Network Transmit Power Constraints

The QoS of the primary network is characterized by its desired outage probability \mathbb{P}_{out} . The primary network \mathbb{P}_{out} should be below a desired \mathbb{P}_{out} constraint \mathcal{K} , which limits the transmit powers of H-AP and SU_s . The desired \mathbb{P}_{out} is given

as

$$\mathbb{P}\{\Psi_{P_{RX},n} < \beta_{P_{RX}}\} \leq \mathcal{K} \quad (1)$$

where $R_{P_{RX}}$ is the target rate of the primary network, $\beta_{P_{RX}} = 2^{R_{P_{RX}}} - 1$ and $0 < \mathcal{K} < 1$ is the QoS parameter or constraint that represents the desired \mathbb{P}_{out} of the primary network. In the primary network, P_{TX} sends information to P_{RX} through channel h_{TR} with transmit power \mathcal{P}_T . In the first phase, the signal-to-interference-plus-noise-ratio (SINR) at P_{RX} is given as

$$\Psi_{P_{RX},1} = \frac{\mathcal{P}_T |h_{TR}|^2}{\mathcal{P}_H |h_{HR}|^2 + N_0} \quad (2)$$

where \mathcal{P}_H is the transmit power of H-AP which satisfies the primary network \mathbb{P}_{out} constraint \mathcal{K} , h_{TR} is the channel coefficient of $P_{TX} \rightarrow P_{RX}$ link, and h_{HR} is the channel coefficient of $H-AP \rightarrow P_{RX}$ link. In the second phase, the SINR at P_{RX} is given as

$$\Psi_{P_{RX},2} = \frac{\mathcal{P}_T |h_{TR}|^2}{\mathcal{P}_S^{Thr} |h_{SR}|^2 + N_0} \quad (3)$$

where \mathcal{P}_S^{Thr} is the maximum threshold transmit power of SU_s allowed by \mathbb{P}_{out} constraint \mathcal{K} to protect the primary network from interference, h_{SR} is the channel coefficient of $SU_s \rightarrow P_{RX}$ link. From (1), \mathcal{P}_H and \mathcal{P}_S^{Thr} can be derived as follows:

$$\mathbb{P}\left\{\frac{\mathcal{P}_T |h_{TR}|^2}{\mathcal{P}_H |h_{HR}|^2 + N_0} < \beta_{P_{RX}}\right\} \leq \mathcal{K} \quad (4)$$

$$\mathbb{P}\left\{\frac{\mathcal{P}_T |h_{TR}|^2}{\mathcal{P}_S^{Thr} |h_{SR}|^2 + N_0} < \beta_{P_{RX}}\right\} \leq \mathcal{K} \quad (5)$$

From (4), \mathcal{P}_H can be derived as

$$\mathcal{P}_H = \begin{cases} \mathcal{P}_T \lambda_{HR} \xi, & \text{if } \xi > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

where $\gamma_T = \frac{\mathcal{P}_T}{N_0}$ and

$$\xi = \frac{1}{\beta_{P_{RX}} \lambda_{TR}} \left[\frac{1}{1 - \mathcal{K}} \exp\left(\frac{-\lambda_{TR} \beta_{P_{RX}}}{\gamma_T}\right) - 1 \right]. \quad (7)$$

Similarly, from (6), \mathcal{P}_S^{Thr} can be derived as

$$\mathcal{P}_S^{Thr} = \begin{cases} \mathcal{P}_T \lambda_{SR} \xi, & \text{if } \xi > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

B. Selected Schemes at Secondary Users

Motivated by wireless sensor networks and clustering, the same cluster sensor nodes can co-operate. Therefore, to improve performance, a given SU_s can be selected from SU_m to transmit information. Two selection schemes are deployed, namely UCS and MICS.

1) *Uplink Channel Selection (UCS)*: In the UCS scheme, selection based on the CSI of $SU_s \rightarrow H-AP$ link to choose best uplink. The secondary user SU_s is chosen as follows:

$$|h_{S_s H}|^2 = \max_{m=1 \dots M} [|h_{S_m H}|^2], \quad (9)$$

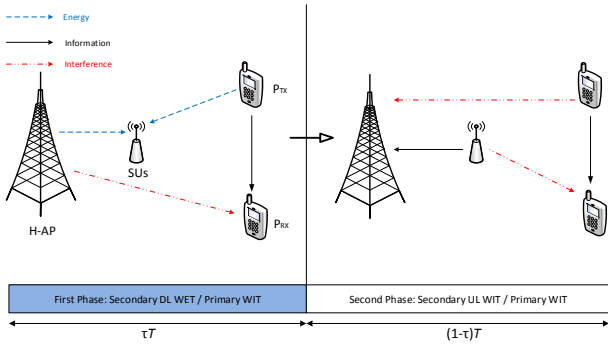


Fig. 2: Time-switching-based EH protocol

2) *Minimal Interference Channel Selection (MICS)*: The MICS scheme is based on the CSI of $SU_s \rightarrow P_{RX}$ link to guarantee that minimal interference affects the primary receiver. The secondary user SU_s is chosen as follows:

$$|h_{S_s R}|^2 = \min_{m=1 \dots M} [|h_{S_m R}|^2], \quad (10)$$

C. Achievable Rate

SU_s harvests energy from H-AP and P_{TX} in the first phase by implementing time-switching-based architecture as shown in Fig. 2 while other SU_m enter the idle mode, then SU_s uses the harvested energy to transmit information to H-AP in the second phase. The harvested power in SU_s is given as

$$\mathcal{P}_S^{Har} = \alpha(\mathcal{P}_H |h_{HS}|^2 + \mathcal{P}_T |h_{TS}|^2) \quad (11)$$

where $\alpha = \frac{\eta\tau}{1-\tau}$, $0 < \eta < 1$ is the conversion efficiency coefficient, h_{HS} is the channel power gain of H-AP \rightarrow SU_s link and h_{TS} is the channel power gain of $P_{TX} \rightarrow$ SU_s link.

In the second phase, to protect the primary network, the transmit power of SU_s must satisfy the QoS constraint with the threshold transmit power, given as

$$\mathcal{P}_S^* = \min[\mathcal{P}_S^{Har}, \mathcal{P}_S^{Thr}]. \quad (12)$$

The SINR at H-AP can be given as

$$\Psi_{SH} = \frac{\mathcal{P}_S^* |h_{S_s H}|^2}{\mathcal{P}_T |h_{TH}|^2 + N_p}. \quad (13)$$

The achievable rate of $SU_s \rightarrow$ H-AP link is given as

$$C_{SH} = (1 - \tau) \log_2(1 + \Psi_{SH}), \quad (14)$$

III. OUTAGE PROBABILITY

The \mathbb{P}_{out} of the secondary network is the probability that communication rate of $SU_s \rightarrow$ H-AP link is smaller than a threshold rate. The \mathbb{P}_{out} can be formulated as

$$\begin{aligned} \mathbb{P}_{out} &= \mathbb{P}\{C_{SH} < R_{th}\} \\ &= \mathbb{P}\{\Psi_{SH} < \beta\} \\ &= F_{\Psi_{SH}}(\beta), \end{aligned} \quad (15)$$

where R_{th} is the target rate of the secondary network, $\beta = 2^{\frac{R_{th}}{1-\tau}} - 1$, and $F_{\Psi_{SH}}(x)$ is the cumulative distribution function (CDF) of Ψ_{SH} .

The OP of the secondary network can be written as

$$\begin{aligned} \mathbb{P}_{out} &= \mathbb{P}\left\{\frac{\mathcal{P}_S^* |h_{S_s H}|^2}{\mathcal{P}_T |h_{TH}|^2 + N_p} < \beta\right\} \\ &= \mathbb{P}\left\{\frac{\mathcal{P}_S^{Thr}}{N_p} < \frac{\beta(\gamma_T |h_{TH}|^2 + 1)}{|h_{S_s H}|^2}\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} > \mathcal{P}_S^{Thr}\right\} \\ &+ \mathbb{P}\left\{\frac{\mathcal{P}_S^{Har}}{N_p} < \frac{\beta(\gamma_T |h_{TH}|^2 + 1)}{|h_{S_s H}|^2}\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} < \mathcal{P}_S^{Thr}\right\} \end{aligned} \quad (16)$$

where $\gamma_T = \frac{\mathcal{P}_T}{N_p}$.

A. Uplink Channel Selection

To facilitate finding the \mathbb{P}_{out} with UCS scheme, we denote

$$\mathcal{Z}_U = \gamma_T \lambda_{HR} \alpha \xi |h_{HS}|^2 + \gamma_T \alpha |h_{TS}|^2, \quad (17)$$

$$\mathcal{Y}_U = \frac{\beta(\gamma_T |h_{TH}|^2 + 1)}{\max_{m=1 \dots M} \{|h_{S_m H}|^2\}}. \quad (18)$$

From (16), the \mathbb{P}_{out} of UCS scheme can be rewritten as

$$\begin{aligned} \mathbb{P}_{out} &= \mathbb{P}\left\{\frac{\mathcal{P}_S^{Thr}}{N_p} < \mathcal{Y}_U\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} > \mathcal{P}_S^{Thr}\right\} \\ &+ \mathbb{P}\left\{\mathcal{Z}_U < \mathcal{Y}_U\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} < \mathcal{P}_S^{Thr}\right\} \end{aligned} \quad (19)$$

The CDF of \mathcal{Z}_U and \mathcal{Y}_U are given as follows

$$\begin{aligned} F_{\mathcal{Z}_U}(z) &= 1 - \frac{\lambda_{TS} \lambda_{HR} \xi \exp\left[-\frac{\lambda_{HS}}{\alpha \gamma_T \lambda_{HR} \xi} z\right]}{\lambda_{TS} \lambda_{HR} \xi - \lambda_{HS}} \\ &- \frac{\lambda_{HS} \exp\left[-\frac{\lambda_{TS}}{\gamma_T \alpha} z\right]}{\lambda_{HS} - \lambda_{TS} \lambda_{HR} \xi}, \end{aligned} \quad (20)$$

$$\begin{aligned} F_{\mathcal{Y}_U}(y) &= \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \frac{y}{m \beta \gamma_T \frac{\lambda_{SH}}{\lambda_{TH}} + y} \\ &\times \exp\left[-\frac{m \beta \lambda_{SH}}{y}\right], \end{aligned} \quad (21)$$

The PDF of \mathcal{Y}_U is given as

$$\begin{aligned} f_{\mathcal{Y}_U}(y) &= \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \exp\left[-\frac{m \beta \lambda_{SH}}{y}\right] \\ &\times \frac{(\beta \lambda_{SH} + \beta \gamma_T \frac{\lambda_{SH}}{\lambda_{TH}}) m y + \frac{\gamma_T (m \beta \lambda_{SH})^2}{\lambda_{TH}}}{y(y + m \beta \gamma_T \frac{\lambda_{SH}}{\lambda_{TH}})^2}, \end{aligned} \quad (22)$$

The OP of the secondary network with UCS scheme is given as follows:

$$\begin{aligned} \mathbb{P}_{out} &= 1 - \frac{\alpha \gamma_T \lambda_{HR} \xi}{\lambda_{TS} \lambda_{HR} \xi - \lambda_{HS}} \left(\frac{\lambda_{TS}}{\gamma_T \alpha} \Theta_1 [m \varrho_1 + m \varrho_2, \right. \\ &\quad \left. \varrho_1 \varrho_2, \varrho_2, \frac{\lambda_{HS}}{\gamma_T \lambda_{HR} \xi \alpha}, \varrho_1] - \frac{\lambda_{HS}}{\gamma_T \lambda_{HR} \xi \alpha} \right. \\ &\quad \left. \Theta_1 [m \varrho_1 + m \varrho_2, \varrho_1 \varrho_2, \varrho_2, \frac{\lambda_{TS}}{\gamma_T \alpha}, \varrho_1] \right) \end{aligned} \quad (23)$$

where $\varrho_1 = \beta\lambda_{SH}$, $\varrho_2 = \beta\gamma_T \frac{\lambda_{SH}}{\lambda_{TH}}$ and Θ_1 is given as

$$\Theta_1(a, b, c, d, e) = \int_0^{\mathcal{P}_S^{Thr}} \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \frac{ay + b}{y(y+c)^2} \times \exp\left[-dy - \frac{e}{y}\right] dy$$

with $(a > 0, b > 0, c > 0, d > 0, e > 0)$,

Proof: The proof is given in Appendix A. ■

We now analyse the asymptotic \mathbb{P}_{out} .

Corollary 1: When $\gamma_T \rightarrow \infty$, the asymptotic \mathbb{P}_{out} of the system with UCS scheme can be approximated as (see equation (24))

where $\varsigma_1 = \frac{\beta\lambda_{SH}\lambda_{HS}\lambda_{TS}}{\lambda_{TS}\lambda_{TH}\lambda_{HR}\alpha\xi' - \alpha\lambda_{TH}\lambda_{HS}}$, $\varsigma_2 = \frac{\lambda_{HS}\beta\lambda_{sH}}{\lambda_{HR}\lambda_{TH}\xi'\alpha}$, $\varsigma_3 = \frac{\lambda_{TS}\beta\lambda_{sH}}{\alpha\lambda_{TH}}$, $\xi' = \frac{m}{\beta_{PRX}\lambda_{TR}} \left[\frac{1}{1-\mathcal{K}} - 1 \right]$, defined in [18, (3.352.1)], $Ei(\cdot)$ is the exponential integral function.

Proof: The proof is given in Appendix:B. ■

B. Minimal Interference Channel Selection

ξ and \mathcal{P}_S^{Thr} in MICS scheme is given as:

$$\xi_M = \frac{m}{\beta_{PRX}\lambda_{TR}} \left[\frac{1}{1-\mathcal{K}} \exp\left(\frac{-\lambda_{TR}\beta_{PRX}}{\gamma_T}\right) - 1 \right]. \quad (25)$$

$$\mathcal{P}_M^{Thr} = \begin{cases} \mathcal{P}_T\lambda_{SR}\xi_M, & \text{if } \xi_M > 0 \\ 0, & \text{otherwise.} \end{cases} \quad (26)$$

To facilitate finding the \mathbb{P}_{out} with MICS scheme, we denote

$$\mathcal{Z}_M = \gamma_T\lambda_{HR}\alpha\xi_M|h_{HS}|^2 + \gamma_T\alpha|h_{TS}|^2, \quad (27)$$

$$\mathcal{Y}_M = \frac{\beta(\gamma_T|h_{TH}|^2 + 1)}{|h_{SH}|^2}. \quad (28)$$

From (16), the OP with MICS scheme can be rewritten as

$$\mathbb{P}_{out} = \mathbb{P}\left\{ \frac{\mathcal{P}_S^{Thr}}{N_p} < \mathcal{Y}_M \right\} \cdot \mathbb{P}\left\{ \mathcal{P}_S^{Har} > \mathcal{P}_M^{Thr} \right\} + \mathbb{P}\left\{ \mathcal{Z}_M < \mathcal{Y}_M \right\} \cdot \mathbb{P}\left\{ \mathcal{P}_S^{Har} < \mathcal{P}_M^{Thr} \right\} \quad (29)$$

The CDF of \mathcal{Z}_M and \mathcal{Y}_M are given, respectively, as follows

$$F_{\mathcal{Z}_M}(z) = 1 - \frac{\lambda_{TS}\lambda_{HR}\xi_M \exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi_M} z\right]}{\lambda_{TS}\lambda_{HR}\xi_M - \lambda_{HS}} \quad (30)$$

$$-\frac{\lambda_{HS} \exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha} z\right]}{\lambda_{HS} - \lambda_{TS}\lambda_{HR}\xi_M}, \quad (31)$$

$$F_{\mathcal{Y}_M}(y) = \frac{y}{\beta\gamma_T \frac{\lambda_{SH}}{\lambda_{TH}} + y} \exp\left[-\frac{\beta\lambda_{SH}}{y}\right] \quad (32)$$

The PDF of \mathcal{Y}_M is given as

$$f_{\mathcal{Y}_M}(y) = \frac{(\beta\lambda_{SH} + \beta\gamma_T \frac{\lambda_{SH}}{\lambda_{TH}})y + \gamma_T \frac{(\beta\lambda_{SH})^2}{\lambda_{TH}}}{y(y + \beta\gamma_T \frac{\lambda_{SH}}{\lambda_{TH}})^2} \exp\left[-\frac{\beta\lambda_{SH}}{y}\right] \quad (33)$$

The OP of the secondary network with MICS scheme is given as follows:

$$\mathbb{P}_{out} = 1 - \frac{\alpha\gamma_T\lambda_{HR}\xi_M}{\lambda_{TS}\lambda_{HR}\xi_M - \lambda_{HS}} \left(\frac{\lambda_{TS}}{\gamma_T\alpha} \Theta_2\left[\varrho_1 + \varrho_2, \varrho_1\varrho_2, \varrho_2, \frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi_M}, \varrho_1\right] - \frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi_M} \Theta_2\left[\varrho_1 + \varrho_2, \varrho_1\varrho_2, \varrho_2, \frac{\lambda_{TS}}{\gamma_T\alpha}, \varrho_1\right] \right) \quad (34)$$

where $\varrho_1 = \beta\lambda_{SH}$, $\varrho_2 = \beta\gamma_T \frac{\lambda_{SH}}{\lambda_{TH}}$ and Θ_2 is given as

$$\Theta_2(a, b, c, d, e) = \int_0^{\mathcal{P}_{MICS}^{Thr}} \frac{ay + b}{y(y+c)^2} \exp\left[-dy - \frac{e}{y}\right] dy$$

with $(a > 0, b > 0, c > 0, d > 0, e > 0)$,

Proof: Similar analysis as Appendix A. ■

Corollary 2: When $\gamma_T \rightarrow \infty$, the asymptotic \mathbb{P}_{out} of the system with MICS scheme can be approximated as (see equation (35))

where $\xi'_M = \frac{1}{\beta_{PRX}\lambda_{TR}} \left[\frac{1}{1-\mathcal{K}} - 1 \right]$.

Proof: Similar analysis as in Appendix B. ■

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, Monte Carlo simulations are provided to validate the theoretical analyses. Without loss of generality, the following parameters are set: $\eta = 0.5$, $R_{PRX} = 0.6$ bits/s/Hz and $R_{th} = 0.5$ bits/s/Hz, respectively.

Fig. 3 plots the \mathbb{P}_{out} versus γ_T for different number of secondary users from $M=1$ to 3 with $\mathcal{K} = 0.05$, $\tau = 0.6$. The asymptotic \mathbb{P}_{out} varies for different number of secondary users. From Fig. 3, we observed that increasing γ_T will lead \mathbb{P}_{out} to decrease. In addition, as γ_T increases beyond a certain value, \mathbb{P}_{out} converges to its floor. We can also observe that increasing the number of SU_m results in a reduction in \mathbb{P}_{out} , and the gap between curves will be smaller with higher number of secondary users. The UCS scheme shows lower outage probability than the MICS scheme. As γ_T increases, the transmit power of SU_s is allowed to increase too, which result in a reduction in \mathbb{P}_{out} . Eventually, SU_s transmit power is limited in order to satisfy the primary network \mathbb{P}_{out} constraint \mathcal{K} , and \mathbb{P}_{out} reaches the floor when converging to the asymptotic value. By increasing the number of SU_m , the selected SU_s has a higher probability to get a better uplink channel in the UCS scheme and smaller interference to the primary user in the MICS scheme. The UCS scheme guarantees the best uplink channel to H-AP, while the MICS only guarantees the minimal interference to primary user but does not guarantees a good uplink to H-AP. This results in the UCS scheme having lower outage probability than MICS scheme. Increasing γ_T and the number of SU_m can reduce \mathbb{P}_{out} , and \mathbb{P}_{out} eventually reaches the floor.

Fig. 4 plots the \mathbb{P}_{out} versus γ_T for different values of \mathcal{K} . In this figure, we set $M=3$, $\tau = 0.6$. We can observe from Fig. 4 that increasing the value of \mathcal{K} can reduce \mathbb{P}_{out} . When relaxing the QoS requirement of the primary network, the SU_s can transmit information with higher transmit power to have

$$\begin{aligned}
\mathbb{P}_{\text{out}} \approx & \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} m \varsigma_1 \cdot \exp(m \varsigma_2) \left[\text{Ei} \left(-\frac{\lambda_{HS} \lambda_{SR}}{\lambda_{HR} \alpha} - m \varsigma_2 \right) - \text{Ei} \left(-m \varsigma_2 \right) \right] \\
& + \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} m \varsigma_1 \exp(m \varsigma_3) \left[\text{Ei} \left(-\frac{\lambda_{TS} \lambda_{SR} \xi'}{\alpha} - m \varsigma_3 \right) - \text{Ei} \left(-m \varsigma_3 \right) \right] \\
& + \left[1 - \sum_{m=1}^M \binom{M}{m} (-1)^{m+1} \frac{\lambda_{SR} \xi'}{m \beta \frac{\lambda_{SH}}{\lambda_{TH}} + \lambda_{SR} \xi'} \right] \left(\frac{\lambda_{TS} \lambda_{HR} \xi' \exp \left[-\frac{\lambda_{HS} \lambda_{SR}}{\lambda_{HR} \alpha} \right]}{\lambda_{TS} \lambda_{HR} \xi' - \lambda_{HS}} + \frac{\lambda_{HS} \exp \left[-\frac{\lambda_{TS} \lambda_{SR} \xi'}{\alpha} \right]}{\lambda_{HS} - \lambda_{TS} \lambda_{HR} \xi'} \right) \quad (24)
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}_{\text{out}} \approx & \varsigma_1 \cdot \exp(\varsigma_2) \left[\text{Ei} \left(-\frac{\lambda_{HS} \lambda_{SR}}{\lambda_{HR} \alpha} - \varsigma_2 \right) - \text{Ei} \left(-\varsigma_2 \right) \right] + \varsigma_1 \cdot \exp(\varsigma_3) \left[\text{Ei} \left(-\frac{\lambda_{TS} \lambda_{SR} \xi'_M}{\alpha} - \varsigma_3 \right) - \text{Ei} \left(-\varsigma_3 \right) \right] \\
& + \left[1 - \frac{\lambda_{SR} \xi'}{\beta \frac{\lambda_{SH}}{\lambda_{TH}} + \lambda_{SR} \xi'_M} \right] \left(\frac{\lambda_{TS} \lambda_{HR} \xi'_M \exp \left[-\frac{\lambda_{HS} \lambda_{SR}}{\lambda_{HR} \alpha} \right]}{\lambda_{TS} \lambda_{HR} \xi'_M - \lambda_{HS}} + \frac{\lambda_{HS} \exp \left[-\frac{\lambda_{TS} \lambda_{SR} \xi'_M}{\alpha} \right]}{\lambda_{HS} - \lambda_{TS} \lambda_{HR} \xi'_M} \right) \quad (35)
\end{aligned}$$

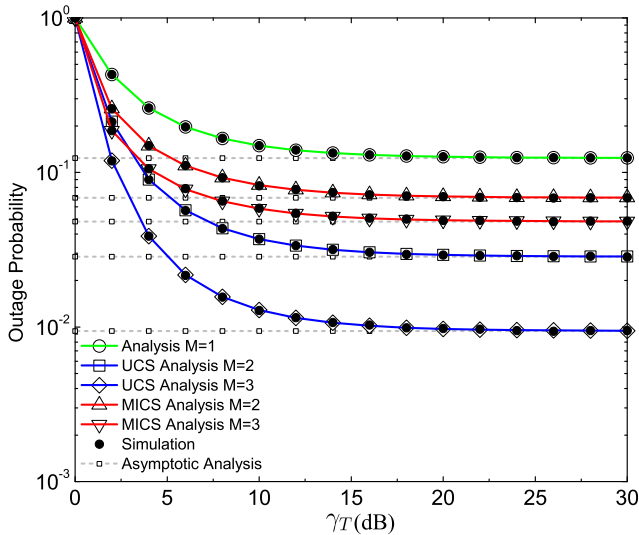


Fig. 3: \mathbb{P}_{out} versus γ_T for different numbers of secondary users. ($\mathcal{K} = 0.05$, $\tau = 0.6$)

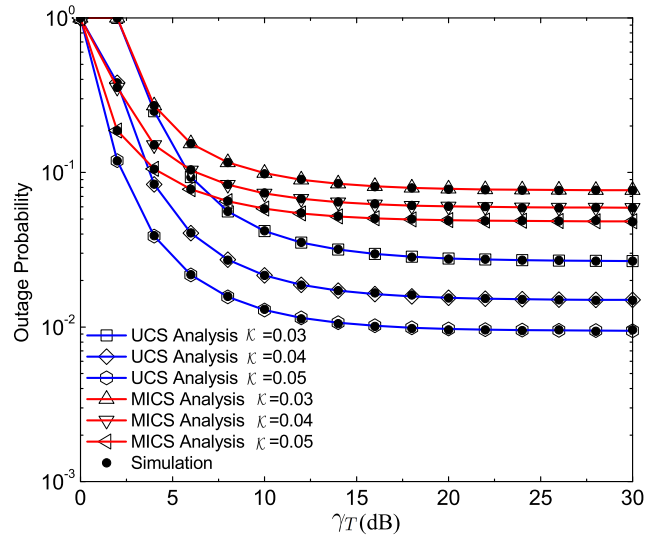


Fig. 4: \mathbb{P}_{out} versus γ_T for different value of desired \mathbb{P}_{out} constraint. ($M=3$, $\tau = 0.6$)

a lower \mathbb{P}_{out} . Consequently, increasing the value of \mathcal{K} (i.e. relaxing the constraint) will result in a decrease in \mathbb{P}_{out} .

Fig. 5 examines \mathbb{P}_{out} versus τ for different values of γ_T . In this figure, we set $M=3$, $\mathcal{K} = 0.05$. We can see that \mathbb{P}_{out} is a convex function varying with τ , and the function reaches a minimum. We can observe from the figure that, \mathbb{P}_{out} decreases as γ_T increases, and \mathbb{P}_{out} reaches a lower minimum with higher γ_T . When τ is small, increasing τ results in \mathbb{P}_{out} decreasing because the SU_s has more time to harvest energy to transmit information with higher power. However, if τ is larger than certain value, \mathbb{P}_{out} will increase because the transmission time is too small. Therefore, there is an optimal value τ which can be observed from Fig. 5. Furthermore, SU_s need less time to harvest enough energy as γ_T increases. Therefore, \mathbb{P}_{out} has a lower minimum value when γ_T increases.

V. CONCLUSION

In this paper, we investigate the outage probability of cognitive wireless powered communication networks considering QoS in the primary networks. The secondary user is powered by the energy harvested from an H-AP and a primary transmitter. Secondary users use the harvested energy to transmit information to the H-AP in the uplink. The transmitting secondary user is selected from the user which has the best uplink to H-AP or the minimal interference to primary user. Two proposed selection schemes enhance the system's outage probability. The analytical and asymptotic expressions of the outage probability system are derived. The results have shown that increasing the transmit power and the number of secondary users leads to a decrease of outage probability. As the transmit power of the primary transmitter increases

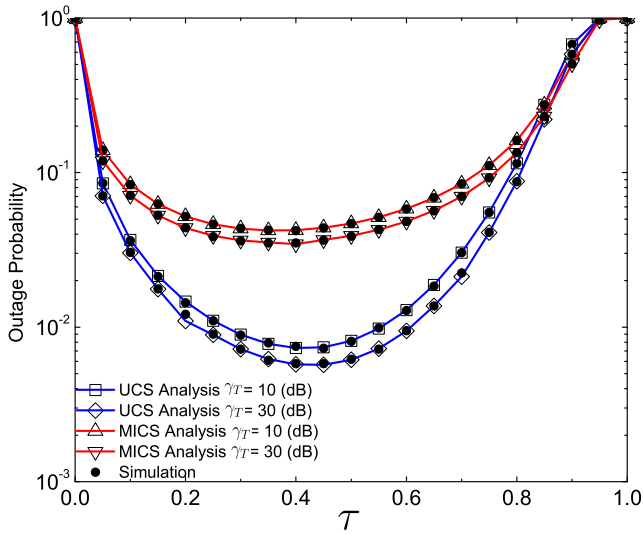


Fig. 5: \mathbb{P}_{out} versus τ for different value of γ_T . ($M=3$, $\mathcal{K} = 0.05$)

beyond a certain value, it converges to the outage probability floor. In addition, relaxing the QoS requirement of the primary network improves the performance of the secondary network because information can be transmitted with higher power by the secondary user. Besides, there is an optimal value of energy harvested time. This optimal value will vary and will be lower with higher transmit power of primary transmitter. Finally, the numerical results are provided to validate our correctness.

APPENDIX A PROOF OF LEMMA 1

From, the \mathbb{P}_{out} of the secondary network can be written as

$$\mathbb{P}_{\text{out}} = \underbrace{\mathbb{P}\left\{\frac{\mathcal{P}_S^{Thr}}{N_p} < \mathcal{Y}\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} > \mathcal{P}_S^{Thr}\right\}}_{Q_1} + \underbrace{\mathbb{P}\left\{\mathcal{Z} < \mathcal{Y}\right\} \cdot \mathbb{P}\left\{\mathcal{P}_S^{Har} < \mathcal{P}_S^{Thr}\right\}}_{Q_2} \quad (36)$$

Q_1 can be calculated as follows:

$$Q_1 = \left[1 - \frac{\mathcal{P}_S^{Thr}}{\varrho_2 + \mathcal{P}_S^{Thr}} \exp\left(-\frac{\varrho_1}{\mathcal{P}_S^{Thr}}\right)\right] \left(\frac{\lambda_{TS}\lambda_{HR}\xi \exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi}\mathcal{P}_S^{Thr}\right]}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} + \frac{\lambda_{HS}\exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha}\mathcal{P}_S^{Thr}\right]}{\lambda_{HS} - \lambda_{TS}\lambda_{HR}\xi}\right) \quad (37)$$

where $\varrho_1 = \beta\lambda_{SH}$ and $\varrho_2 = \beta\gamma_T\frac{\lambda_{SH}}{\lambda_{TH}}$. Q_2 can be calculated as follows (see equation (38) at the upper of next page)

From (37) and (38), \mathbb{P}_{out} is given as follow

$$\mathbb{P}_{\text{out}} = 1 - \int_0^{\mathcal{P}_S^{Thr}} \frac{\frac{\lambda_{TS}}{\gamma_T\alpha}(\varrho_1 y + \varrho_2 y + \varrho_1 \varrho_2)}{y(y + \varrho_2)^2} \times \frac{\alpha\gamma_T\lambda_{HR}\xi}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} \exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi_{\text{MICS}}}y - \frac{\varrho_1}{y}\right] dy - \int_0^{\mathcal{P}_S^{Thr}} \frac{\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi}(\varrho_1 y + \varrho_2 y + \varrho_1 \varrho_2)}{y(y + \varrho_2)^2} \times \frac{\alpha\gamma_T\lambda_{HR}\xi}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} \exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha}y - \frac{\varrho_1}{y}\right] dy \quad (39)$$

APPENDIX B PROOF OF LEMMA 2

Based on the preceding results, an asymptotic \mathbb{P}_{out} will be now carried out in order to evaluate the behaviour of \mathbb{P}_{out} in the high-SNR regime which we assume $\gamma_T = \infty$. Therefore, we rewrite the equation (13)

$$\Psi_{SH} = \frac{\min(\gamma_T\lambda_{HR}\xi\alpha|h_{HS}|^2 + \gamma_T\alpha|h_{TS}|^2, \gamma_T\lambda_{SR}\xi) |h_{SH}|^2}{1 + \gamma_T|h_{TH}|^2} \approx \frac{\min(\lambda_{HR}\xi'\alpha|h_{HS}|^2 + \alpha|h_{TS}|^2, \lambda_{SR}\xi') |h_{SH}|^2}{|h_{TH}|^2} \quad (40)$$

cause $\gamma_T = \infty$, the $1 + \gamma_T|h_{TH}|^2$ can be simplify to $\gamma_T|h_{TH}|^2$, from (7) and (40), ξ can be rewrite as

$$\xi' = \frac{m}{\beta_{\text{PRX}}\lambda_{TR}} \left[\frac{1}{1-\mathcal{K}} - 1\right]$$

To facilitate finding the asymptotic \mathbb{P}_{out} , we denote

$$\mathcal{A} = |h_{TH}|^2, \quad (41)$$

$$\mathcal{B} = |h_{SH}|^2, \quad (42)$$

$$\mathcal{X} = \lambda_{HR}\xi'\alpha|h_{HS}|^2 + \alpha|h_{TS}|^2, \quad (43)$$

$$\mathcal{U} = \min(\mathcal{X}, \lambda_{SR}\xi') \quad (44)$$

The PDF of \mathcal{X} is given as

$$f_{\mathcal{X}}(x) = \frac{\lambda_{TS}\lambda_{HS}\exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi'}x\right]}{\alpha\gamma_T\lambda_{TS}\lambda_{HR}\xi' - \alpha\gamma_T\lambda_{HS}} + \frac{\lambda_{TS}\lambda_{HS}\exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha}x\right]}{\alpha\gamma_T\lambda_{HS} - \alpha\gamma_T\lambda_{TS}\lambda_{HR}\xi'} \quad (45)$$

From (40), the \mathbb{P}_{out} can be rewrite as

$$\mathbb{P}_{\text{out}} \approx \left\{\frac{\mathcal{U} \cdot \mathcal{B}}{\mathcal{A}} < \beta\right\} \approx \left\{\mathcal{B} < \frac{\beta\mathcal{A}}{\mathcal{U}}\right\} \quad (46)$$

Calculate the \mathbb{P}_{out} conditioned on \mathcal{U}

$$\mathbb{P}_{\text{out}|\mathcal{U}} \approx \int_0^{\infty} F_{\mathcal{B}|\mathcal{U}}\left(\frac{\beta\mathcal{A}}{\mathcal{U}}\right) f_{\mathcal{A}}(x) dx \approx \int_0^{\infty} [1 - \exp(-\lambda_{SH} \cdot \frac{\beta x}{\mathcal{U}})] \lambda_{TH} \exp(-\lambda_{TH}x) dx \approx 1 - \frac{\mathcal{U}}{\beta\frac{\lambda_{SH}}{\lambda_{TH}} + \mathcal{U}} \quad (47)$$

$$\begin{aligned}
Q_2 &= \mathbb{P}\{\mathcal{P}_S^{Har} < \mathcal{P}_S^{Thr}\} \cdot \mathbb{P}\{\mathcal{Y}_S < \mathcal{P}_S^{Thr}\} + \mathbb{P}\{\mathcal{P}_S^{Har} < \mathcal{P}_S^{Thr}\} \cdot \mathbb{P}\{\mathcal{Y} > \mathcal{P}_S^{Thr}\} \\
&= \left[1 - \frac{\mathcal{P}_S^{Thr}}{\varrho_2 + \mathcal{P}_S^{Thr}} \exp\left(-\frac{\varrho_1}{\mathcal{P}_S^{Thr}}\right)\right] \left(1 - \frac{\lambda_{TS}\lambda_{HR}\xi \exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi} \mathcal{P}_S^{Thr}\right]}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} - \frac{\lambda_{HS}\exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha} \mathcal{P}_S^{Thr}\right]}{\lambda_{HS} - \lambda_{TS}\lambda_{HR}\xi}\right) \\
&\quad - \int_0^{\mathcal{P}_S^{Thr}} \frac{\alpha\gamma_T\lambda_{HR}\xi}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} \cdot \frac{\frac{\lambda_{TS}}{\gamma_T\alpha}(\varrho_1 y + \varrho_2 y + \varrho_1 \varrho_2)}{y(y + \varrho_2)^2} \exp\left[-\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi} y - \frac{\varrho_1}{y}\right] dy \\
&\quad - \int_0^{\mathcal{P}_S^{Thr}} \frac{\alpha\gamma_T\lambda_{HR}\xi}{\lambda_{TS}\lambda_{HR}\xi - \lambda_{HS}} \cdot \frac{\frac{\lambda_{HS}}{\alpha\gamma_T\lambda_{HR}\xi_{MICS}}(\varrho_1 y + \varrho_2 y + \varrho_1 \varrho_2)}{y(y + \varrho_2)^2} \exp\left[-\frac{\lambda_{TS}}{\gamma_T\alpha} y - \frac{\varrho_1}{y}\right] dy + \frac{\mathcal{P}_S^{Thr}}{\varrho_2 + \mathcal{P}_S^{Thr}} \exp\left[-\frac{\varrho_1}{\mathcal{P}_S^{Thr}}\right] \quad (38)
\end{aligned}$$

\mathcal{U} can be rewritten as

$$\mathcal{U} = \begin{cases} \mathcal{X}, & \text{if } \mathcal{X} < \lambda_{SR}\xi' \\ \lambda_{SR}\xi', & \text{if } \mathcal{X} > \lambda_{SR}\xi'. \end{cases} \quad (48)$$

calculating the integral conditioned on \mathcal{X} , the \mathbb{P}_{out} is given as

$$\begin{aligned}
\mathbb{P}_{\text{out}} &\approx \int_0^{\lambda_{SR}\xi'} \left[1 - \frac{x}{\beta\frac{\lambda_{SH}}{\lambda_{TH}} + x}\right] f_{\mathcal{X}}(x) dx \\
&\quad + \int_{\lambda_{SR}\xi'}^{\infty} \left[1 - \frac{\lambda_{SR}\xi'}{\beta\frac{\lambda_{SH}}{\lambda_{TH}} + \lambda_{SR}\xi'}\right] f_{\mathcal{X}}(x) dx \\
&\approx \varsigma_1 \cdot \exp(\varsigma_2) \left[\text{Ei}\left(-\frac{\lambda_{HS}\lambda_{SR}}{\lambda_{HR}\alpha} - \varsigma_2\right) - \text{Ei}\left(-\varsigma_2\right)\right] \\
&\quad + \varsigma_1 \cdot \exp(\varsigma_3) \left[\text{Ei}\left(-\frac{\lambda_{TS}\lambda_{SR}\xi'}{\alpha} - \varsigma_3\right) - \text{Ei}\left(-\varsigma_3\right)\right] \\
&\quad + \left[1 - \frac{\lambda_{SR}\xi'}{\beta\frac{\lambda_{SH}}{\lambda_{TH}} + \lambda_{SR}\xi'}\right] \left(\frac{\lambda_{TS}\lambda_{HR}\xi' \exp\left[-\frac{\lambda_{HS}\lambda_{SR}}{\lambda_{HR}\alpha}\right]}{\lambda_{TS}\lambda_{HR}\xi' - \lambda_{HS}}\right. \\
&\quad \left. + \frac{\lambda_{HS}\exp\left[-\frac{\lambda_{TS}\lambda_{SR}\xi'}{\alpha}\right]}{\lambda_{HS} - \lambda_{TS}\lambda_{HR}\xi'}\right) \quad (49)
\end{aligned}$$

where $\varsigma_1 = \frac{\beta\lambda_{SH}\lambda_{HS}\lambda_{TS}}{\lambda_{TS}\lambda_{TH}\lambda_{HR}\alpha\xi' - \alpha\lambda_{TH}\lambda_{HS}}$, $\varsigma_2 = \frac{\lambda_{HS}\beta\lambda_{SH}}{\lambda_{HR}\lambda_{TH}\xi'\alpha}$, $\varsigma_3 = \frac{\lambda_{TS}\beta\lambda_{SH}}{\alpha\lambda_{TH}}$ and $\xi' = \frac{M}{\beta_{\text{PRX}}\lambda_{TR}} \left[\frac{1}{1-\mathcal{K}} - 1\right]$

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