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Multi-Cell Massive MIMO Uplink with Underlay Spectrum Sharing

Hayder Al-Hraishawi, Gayan Amarasuriya Aruma Baduge, *Member, IEEE*, Hien Quoc Ngo, *Member, IEEE*, and Erik G. Larsson, *Fellow, IEEE*

Abstract—The achievable rates are investigated for multi-cell multi-user massive multiple-input multiple-output (MIMO) systems with underlay spectrum sharing. A general pilot sharing scheme and two pilot sequence designs (PSDs) are investigated via fully-shared (PSD-1) and partially-shared (PSD-2) uplink pilots. The number of simultaneously served primary users (PUs) and secondary users (SUs) in the same time-frequency resource block by the PSD-1 is higher than that of PSD-2. The transmit power constraints for the SUs are derived to mitigate the secondary co-channel interference (CCI) inflicted at the primary base-station (PBS) subject to a predefined primary interference temperature (PIT). The optimal transmit power control coefficients for the SUs with max-min fairness, and the common achievable rates are derived. The cumulative detrimental effects of channel estimation errors, CCI and intra-cell/inter-cell pilot contamination are investigated. The secondary transmit power constraint and the achievable rates for the perfect channel state information (CSI) case become independent of the PIT when the number of PBS antennas grows unbounded. Therefore, the primary and secondary systems can be operated independent of each other as both intra-cell and inter-cell interference can be asymptotically mitigated at the massive MIMO PBS and secondary base-station (SBS). Nevertheless, the achievable rates and secondary power constraints for the imperfect CSI case with PSD-1 are severely degraded due to the presence of intra-cell and inter-cell pilot contamination. These performance metrics depend on the PIT even in the asymptotic PBS antenna regime. Hence, the primary and secondary systems can no longer be operated independently for imperfect CSI with PSD-1. However, PSD-2 provides achievable rate gains over PSD-1 despite the requirement of lengthier pilot sequences of the former than that of the latter.

Index Terms—Massive MIMO and underlay spectrum-sharing

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) serves many spatially distributed user nodes simultaneously in the same time-frequency resource block by using aggressive spatial multiplexing techniques with linear precoders/detectors [2]. Moreover, massive MIMO can provide not only unprecedented spectral and energy efficiency gains, but also robust withstanding to the detrimental effects of inter-user/inter-cell co-channel interference (CCI) [2], [3]. Hence, massive MIMO has been identified as one of the key enabling technologies for the next-generation wireless standards [4].

Cognitive radio systems can be used to mitigate the radio-frequency (RF) spectrum underutilization and hence to cir-

cumvent the issues of spectrum scarcity in the next-generation wireless systems [5]. Cognitive radios leverage the concept of opportunistic utilization of the spectrum, in which the licensed spectrum of the primary systems can be accessed and utilized by the unlicensed secondary systems without causing detrimental CCI [5]–[9]. The cognitive radios can access the licensed spectrum of the primary system opportunistically by using underlay, overlay and interweave spectrum sharing techniques [10]. Specifically, in underlay spectrum sharing cognitive radios, the secondary users (SUs) concurrently communicate by accessing the same licensed spectrum of the primary users (PUs) under a predefined peak transmit power constraint [10]. This peak transmit power constraint protects the primary systems by maintaining the detrimental CCI inflicted at the primary receivers to an acceptable level, which is less than a predefined primary interference temperature (PIT) [11].

Prior related research: In [6]–[9], conventional MIMO techniques with small antenna arrays at the base-stations (BSs) are exploited to boost the achievable diversity order or/and spatial multiplexing gains of cognitive radios with spectrum sharing techniques subject to the fundamental diversity-rate trade-off. Recently, massive MIMO techniques have been investigated for cognitive radios with underlay spectrum sharing [12]–[22]. In [12], the performance of a path-loss inversion based power control is investigated for a random massive MIMO cognitive secondary system underlaid upon another random primary system. In [13], the secondary interference for a random cognitive massive MIMO system is characterized by investigating pilot contamination, path-loss-inversion power control, receiver association policies, and spatially random nodes. In [12], [13], a Matern cluster process is used for positioning the secondary nodes, whereas a homogeneous Poisson point process is employed for the primary system. In [14], the downlink achievable rate of a secondary massive MIMO system, which is underlaid in a multi-user primary massive MIMO system, is investigated. The BSs of the system set-up in [14] employ the maximum ratio transmission (MRT) and the achievable rate of a single-user, which is selected opportunistically, is derived. In [15], underlay single-user massive MIMO cognitive radio systems are investigated, and thereby, pilot decontamination techniques are proposed to asymptotically mitigate the residual interference. Reference [16] investigates a reciprocity-based cognitive radio beamforming technique by exploiting the cross-link channel estimation through a per-user reciprocity-calibration scheme. In [17], the wireless energy harvesting for cognitive massive MIMO systems is studied by deriving the fundamental energy-rate trade-off. In [18], [19], the achievable secrecy rates of single-cell cognitive massive MIMO systems with passive/active eavesdroppers are investigated. In [20],

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[21], the achievable rates of cognitive MIMO relay systems with underlay spectrum sharing are derived, and thereby, the detrimental effects of inter/intra cell pilot contamination are investigated. In [22], relay selection strategies for maximizing the achievable rates of cognitive massive MIMO two-way relay networks are investigated for perfect channel state information (CSI). Moreover, in [22], the asymptotic signal-to-noise-plus-interference ratio (SINR) is derived, and thereby, asymptotic achievable rates are derived for the best relay selection.

Motivation: The motivation of this work can be summarized as follows: The aforementioned research [12]–[18], [20]–[22] on spectrum sharing massive MIMO systems investigates the downlink transmission of single-cell secondary systems. Nevertheless, the uplink transmissions of multi-cell multi-user massive MIMO secondary systems, which are underlaid in multi-cell multi-user primary massive MIMO systems have not yet been investigated in the open literature. Even for the cognitive massive MIMO downlink, the impact of practical transmission impairments such as the intra/inter-cell pilot contamination, CCI and channel estimation errors have not yet been investigated in the context of multi-cell deployments. Moreover, all prior related references [12]–[17], [20], [22] investigate the asymptotic performance metrics for infinitely many antennas at the primary base-station (PBS) and secondary base-station (SBS). Hence, the achievable rates, which are valid even in finitely many PBS/SBS antennas, have not yet been investigated for the imperfect CSI case. Thus, this paper fills these gaps in cognitive massive MIMO literature by investigating the performance of the multi-cell multi-user cognitive massive MIMO uplink with estimated/imperfect CSI, and thereby, the quantifying the detrimental effects of practical transmission impairments.

Our contribution: The technical contribution of this work can be summarized as follows: The PUs-to-PBS and SUs-to-SBS channels are estimated by using a general pilot sharing scheme and two pilot sequence designs (PSDs) designed based on fully-shared and partially-shared pilots sent by the PUs and SUs. In PSD-1, non-orthogonal pilot sequences are allocated for PUs and SUs in all L co-channel cells. In PSD-2, orthogonal pilot sequences are allocated for PUs and SUs in the same cell, and non-orthogonal pilot sequences used in $L-1$ co-channel cells. The secondary transmit power constraints of SUs are derived for finite and infinite PBS antenna regimes for the purpose of underlay massive MIMO spectrum sharing. Moreover, the achievable rates for the PUs and SUs are derived for PSD-1 and PSD-2 by exploring three antenna configurations at the PBS and SBS. Thereby, the sum rate losses due to detrimental effects of channel estimation errors, CCI and intra/inter-cell pilot contamination are investigated and compared against the genie-aided perfect CSI case. An optimal transmit power control scheme for the SUs is designed based on max-min fairness criterion. Thereby, the optimal transmit power allocation coefficients and the optimal common SU rates are derived in closed-form.

Design insights: Our analysis for the perfect CSI case reveals that the performance metrics become independent of the PIT when the number of PBS antennas grows without bound. Consequently, the underlay spectrum-sharing secondary system

Fig. 1. Cognitive multi-user MIMO system with underlay spectrum sharing. can be operated at its peak average transmit power level as the secondary CCI inflicted at the PBS can be asymptotically mitigated. Moreover, the detrimental effects of CCI at both primary and secondary systems due to the concurrent transmissions in the $L-1$ co-channel cells can be asymptotically mitigated in the limit of infinitely many PBS and SBS antennas. Nevertheless, when the pilot sequences are partially/fully reused in the PUs and SUs in co-channel cells, the aforementioned claims become invalid, and hence, primary and secondary systems can no longer be operated independently. Thus, the corresponding performance gains promised by the perfect CSI case diminish for two cases of more practically viable PSDs. Specifically, if the same pilot sequences are reused among PUs and SUs in all L cells, then the achievable sum rates are severely degraded by the residual interference due to cumulative effects of intra/inter-cell pilot contamination even in the limit of infinitely many PBS/SBS antennas. However, the severity of intra-cell pilot contamination effects can be circumvented by using orthogonal pilot sequences among PUs and SUs within the same cell. On the contrary, the pilot sequence length of PSD-1 is shorter than that of PSD-2. Thus, the number SUs and PUs that can be served simultaneously in the same time-frequency resource block by the PSD-1 is higher than that of PSD-2. Hence, by using our analysis, this fundamental trade-off between the number of served PUs/SUs and their achievable rates is investigated. The optimal transmit power control coefficients depend only on the long-term channel statistics, and hence, the proposed power control can be readily employed at SUs/PUs despite not having access to the instantaneous CSI. Our analysis/numerical results reveal that massive MIMO can be exploited to significantly boost the achievable rates of multi-cell multi-user cognitive systems with underlay spectrum sharing.

Notation: $\mathbf{Z}^*$, $\mathbf{Z}^T$, $\mathbf{Z}^H$, and $[\mathbf{Z}]_{k,l}$ denote the conjugate, transpose, conjugate transpose, and the (k,l) th element of a matrix $\mathbf{Z}$, respectively. $\mathbb{E}[z]$ is the expected value of z , and $\otimes$ denotes the Kronecker product. $E_1(z)$ is the exponential integral for the positive values of the real part of z . The notation $\tilde{\mathbf{Z}} \sim \mathcal{CN}_{m \times n}(\mathbf{0}_{m \times n}, \mathbf{I}_m \otimes \mathbf{I}_n)$ denotes that the elements of $m \times n$ matrix $\tilde{\mathbf{Z}}$ are independent and circularly symmetric complex Gaussian distributed with zero mean and unit variance.

II. SYSTEM, CHANNEL, AND SIGNAL MODEL

A. System and channel model

We consider a multi-user cognitive MIMO network with underlay spectrum sharing having L co-channel cells (see Fig. 1). Each cell consists of a licensed primary multi-user massive MIMO system and a multi-user secondary system, which shares the same licensed frequency spectrum by exploiting the cognitive underlay spectrum sharing concepts [10]. The primary system consists of K spatially-distributed single-antenna PUs and an N_P -antenna PBS. The secondary system consists of M single-antenna SUs and an N_S -antenna SBS. The ratio between the numbers of antennas at the PSB and SBS is defined as $\beta = N_P/N_S$, where $N_P > K$ and $N_S > M$. The primary and secondary uplink transmissions are assumed to be synchronized perfectly to prevent undesired PU-SU interference.

For the sake of brevity of exposition, we consider the channel model of the l th cell, where $l \in \{1, \dots, L\}$. The channel between the PUs in the i th cell and the PBS in the l th cell is denoted by $\mathbf{F}_{li}$, where $i \in \{1, \dots, L\}$. The channel between the SUs in the i th cell and the SBS in the l th cell is denoted by $\mathbf{G}_{li}$. The interference channel between the PUs in the i th cell and the SBS in the l th cell is denoted by $\mathbf{U}_{li}$. Moreover, the interference channel between the SUs in the i th cell and the PBS in the l th cell is denoted by $\mathbf{V}_{li}$. These four channels can be modeled in a unified form as

$$\mathbf{H}_{li} = \tilde{\mathbf{H}}_{li} \mathbf{D}_{H_{li}}^{1/2}, \quad (1)$$

where $\tilde{\mathbf{H}}_{li} \sim \mathcal{CN}_{m \times n}(\mathbf{0}_{m \times n}, \mathbf{I}_m \otimes \mathbf{I}_n)$ captures the independent, quasi-static Rayleigh fading. The $n \times n$ diagonal matrix $\mathbf{D}_{H_{li}} = \text{diag}(\zeta_{H_{li,1}}, \dots, \zeta_{H_{li,n}})$ accounts for the large-scale fading including path-loss and shadowing. In (1), $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, and the tuple $\{m, n\}$ corresponding to the channel matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{U}$, and $\mathbf{V}$ are defined by $\{N_P, K\}$, $\{N_S, M\}$, $\{N_S, K\}$, and $\{N_P, M\}$, respectively. The k th diagonal element of $\mathbf{D}_{H_{li}}$ for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$ is denoted by $\zeta_{H_{li,k}}$ and is used to denote the path-loss and shadowing between the k th PU/SU in the i th cell and any antenna at the PBS/SBS in the l th cell. In modeling $\mathbf{D}_{H_{li}}$, it is assumed that the PUs/SUs are spatially-distributed, and multiple-antennas at the BSs are co-located [2], [3]. Hence, the large-scale fading coefficients between a specific PU/SU and any antenna at the PBS/SBS is the same. The large-scale fading coefficients are assumed to be known a priori as they change very slowly, and this is a common assumption in massive MIMO literature [2], [3]. Thus, they need to be estimated once about every 40 coherence time intervals for a typical urban wireless channel model [23].

B. Channel state information acquisition

In practice, a certain portion of the coherence interval having τ_C symbols are used for uplink channel training. During the uplink training phase, all PUs/SUs send pilots to the BSs [2], [3]. Then, these pilot sequences are used to estimate the channel matrices $\mathbf{F}_{li}$ and $\mathbf{G}_{li}$ at the PBS and SBS, respectively.

First, a general pilot sharing strategy is presented, and thereby, two special PSDs are investigated. To this end, we

assume that a number of pilots defined by $Q \leq \min(K, M)$ is shared among PUs and SUs in each cell, and then, the same pilot assignment is reused in all L co-channel cells. Thus, Φ_P and Φ_S can be modeled as¹

$$\Phi_P = [\Phi; \tilde{\Phi}_p] \quad \text{and} \quad \Phi_S = [\Phi; \tilde{\Phi}_s], \quad (2)$$

where $\Phi \in \mathbb{C}^{Q \times \tau_p}$ denotes the shared pilot sequences by Q PUs and Q SUs having a length of τ_p symbols. Moreover, $\tilde{\Phi}_p \in \mathbb{C}^{(K-Q) \times \tau_p}$ is the set of pilot sequences assigned to the remaining $K - Q$ PUs. Here, $\tau_p \geq \max(K, M) = U_{\max}$ is the pilot sequence length measured in symbol durations. Similarly, $\tilde{\Phi}_s \in \mathbb{C}^{(M-Q) \times \tau_p}$ is assigned to remaining $M - Q$ SUs. The orthogonality property among these pilot sequences is defined as $\Phi \Phi^H = \mathbf{0}$, $\Phi \tilde{\Phi}_p^H = \mathbf{0}$ and $\tilde{\Phi}_s \tilde{\Phi}_p^H = \mathbf{0}$ because $K + M - Q$ number of pilot sequences are used. Thus, $\Phi_P \Phi_S^H$ can be derived as

$$\Phi_S \Phi_P^H = \begin{bmatrix} \mathbf{I}_Q & \mathbf{0}_{Q \times (K-Q)} \\ \mathbf{0}_{(M-Q) \times Q} & \mathbf{0}_{(M-Q) \times (K-Q)} \end{bmatrix} = \tilde{\mathbf{I}}_{M \times K}. \quad (3)$$

The received pilot signal at the PBS in the l th cell can be written as

$$\begin{aligned} \mathbf{Y}_{Pl} = & \underbrace{\sqrt{\tau P_U} \mathbf{F}_{ll} \Phi_P}_{\text{desired pilots}} + \underbrace{\sqrt{\tau P_U} \sum_{i=1, i \neq l}^L \mathbf{F}_{li} \Phi_P}_{\text{inter-cell PU pilot contamination}} + \underbrace{\sqrt{\tau P_U} \mathbf{V}_{ll} \Phi_S}_{\text{intra-cell SU pilot contamination}} \\ & + \underbrace{\sum_{i=1, i \neq l}^L \sqrt{\tau P_U} \mathbf{V}_{li} \Phi_S}_{\text{inter-cell SU pilot contamination}} + \underbrace{\mathbf{N}_{Pl}}_{\text{AWGN}}, \end{aligned} \quad (4)$$

where P_U is the average transmit power of each PU and SU and $\mathbf{N}_{Pl}$ is additive white Gaussian noise (AWGN) matrix at the l th cell PBS having i.i.d. $\mathcal{CN}(0, 1)$ elements. Similarly, the received pilot signal at the SBS in the l th cell is written as

$$\begin{aligned} \mathbf{Y}_{Sl} = & \underbrace{\sqrt{\tau P_U} \mathbf{G}_{ll} \Phi_S}_{\text{desired pilots}} + \underbrace{\sqrt{\tau P_U} \sum_{i=1, i \neq l}^L \mathbf{G}_{li} \Phi_S}_{\text{inter-cell SU pilot contamination}} + \underbrace{\sqrt{\tau P_U} \mathbf{U}_{ll} \Phi_P}_{\text{intra-cell PU pilot contamination}} \\ & + \underbrace{\sum_{i=1}^L \sqrt{\tau P_U} \mathbf{U}_{li} \Phi_P}_{\text{inter-cell PU pilot contamination}} + \underbrace{\mathbf{N}_{Sl}}_{\text{AWGN}}, \end{aligned} \quad (5)$$

where $\mathbf{N}_{Sl}$ is the AWGN matrix at the l th cell SBS having i.i.d. $\mathcal{CN}(0, 1)$ elements.

The PBS and SBS perform a de-spreading operation by correlating the received pilot signal with the same pilot sequences Φ_P and Φ_S , respectively, as follows:

$$\mathbf{Y}_{Pl} = \frac{1}{\sqrt{p_0}} \mathbf{Y}_{Pl}' \Phi_P^H = \mathbf{F}_{ll} + \sum_{i=1, i \neq l}^L \mathbf{F}_{li} + \sum_{i=1}^L \mathbf{V}_{li} \tilde{\mathbf{I}} + \frac{\mathbf{N}_{F_{ll}}}{\sqrt{p_0}}, \quad (6a)$$

$$\mathbf{Y}_{Sl} = \frac{1}{\sqrt{p_0}} \mathbf{Y}_{Sl}' \Phi_S^H = \mathbf{G}_{ll} + \sum_{i=1, i \neq l}^L \mathbf{G}_{li} + \sum_{i=1}^L \mathbf{U}_{li} \tilde{\mathbf{I}}^H + \frac{\mathbf{N}_{G_{ll}}}{\sqrt{p_0}}, \quad (6b)$$

where $p_0 = \tau_p P_U$, $\mathbf{N}_{F_{ll}} = \mathbf{N}_{Pl} \Phi_P^H$ and $\mathbf{N}_{G_{ll}} = \mathbf{N}_{Sl} \Phi_S^H$.

Next, minimum mean square-error (MMSE) channel estimates at the PBS and SBS are derived for two PSDs, which are special cases of the general pilot sharing scheme.

¹Here, the subscript l is omitted for the sake of notational simplicity as the same pilot assignment is reused in all L co-channel cells.

1) *Pilot sequence design-1 (PSD-1)*: The MMSE channel estimates for PSD-1 with $K \neq M$ can be obtained by letting $Q = \min(K, M) = U_{\min}$. On one hand, if $K < M$ then the orthogonal pilots assigned for K PUs are shared among K SUs, while $M - K$ remaining SUs are assigned with pilots, which are orthogonal to those used by K PUs/SUs. On the other hand, if $K > M$, then the orthogonal pilots allocated for M SUs are shared among M PUs, while $K - M$ remaining PUs are allocated with pilots, which are orthogonal to those used by M PUs/SUs. This pilot assignment is reused in all L co-channel cells. Thus, PSD-1 requires only $\tau_1 \geq \max(K, M)$ pilots and represents the worst-case scenario in terms of inter/intra-cell pilot contamination. By using (4), the MMSE estimate of $\mathbf{F}_{ll}$ can be derived as [3], [24]

$$\begin{aligned}\hat{\mathbf{F}}_{ll} &= \frac{1}{\sqrt{p_0}} \mathbf{Y}_{P_l} \Phi_P^H \left(\sum_{i=1}^L (\mathbf{D}_{F_{li}} + \tilde{\mathbf{D}}_{V_{li}}) + \frac{\mathbf{I}_K}{p_0} \right)^{-1} \mathbf{D}_{F_{ll}} \\ &= \left(\sum_{i=1}^L (\mathbf{F}_{li} + \tilde{\mathbf{V}}_{li}) + \frac{\mathbf{N}_{F_{ll}}}{\sqrt{p_0}} \right) \left(\sum_{i=1}^L (\mathbf{D}_{F_{li}} + \tilde{\mathbf{D}}_{V_{li}}) + \frac{\mathbf{I}_K}{p_0} \right)^{-1} \mathbf{D}_{F_{ll}},\end{aligned}\quad (7)$$

where $\tilde{\mathbf{V}}_{li} = \mathbf{V}_{li} \tilde{\mathbf{I}} \in \mathbb{C}^{N_P \times K}$ and $\tilde{\mathbf{D}}_{V_{li}} = \tilde{\mathbf{I}}^H \mathbf{D}_{V_{li}} \tilde{\mathbf{I}} \in \mathbb{C}^{K \times K}$, respectively. In (7), $\mathbf{F}_{ll}$, $\tilde{\mathbf{V}}_{li}$ and $\mathbf{N}_{F_{ll}}$ are statistically independent. By letting $\mathcal{E}_{F_{ll}}$ be the estimation error matrix, the true channel $\mathbf{F}_{ll}$ can be written in terms of its estimate $\hat{\mathbf{F}}_{ll}$ as

$$\mathbf{F}_{ll} = \hat{\mathbf{F}}_{ll} + \mathcal{E}_{F_{ll}}, \quad (8)$$

where $\hat{\mathbf{F}}_{ll}$ and $\mathcal{E}_{F_{ll}}$ are independent due to orthogonality principle of MMSE estimation [24]. The rows of $\hat{\mathbf{F}}_{ll}$ and $\mathcal{E}_{F_{ll}}$ are mutually independent and distributed as $\mathcal{CN}(\mathbf{0}, \hat{\mathbf{D}}_{F_{ll}})$ and $\mathcal{CN}(\mathbf{0}, \mathbf{D}_{F_{ll}} - \hat{\mathbf{D}}_{F_{ll}})$, respectively. The k th diagonal element of the diagonal matrix $\hat{\mathbf{D}}_{F_{ll}}$ can be derived as [3], [24]

$$\sigma_{\hat{F}_{ll,k}}^2 = \frac{\zeta_{F_{ll,k}}^2}{\sum_{i=1}^L (\zeta_{F_{li,k}} + \mathcal{I}_k \zeta_{V_{li,k}}) + 1/p_0}, \quad (9)$$

where $\mathcal{I}_k = 1$ for $1 \leq k \leq Q$, and $\mathcal{I}_k = 0$ otherwise. By following steps similar to those used for (7), the MMSE estimate of $\mathbf{G}_{ll}$ can be derived as

$$\begin{aligned}\hat{\mathbf{G}}_{ll} &= \left(\sum_{i=1}^L (\mathbf{G}_{li} + \tilde{\mathbf{U}}_{li}) + \frac{\mathbf{N}_{G_{ll}}}{\sqrt{p_0}} \right) \\ &\quad \times \left(\sum_{i=1}^L (\mathbf{D}_{G_{li}} + \tilde{\mathbf{D}}_{U_{li}}) + \frac{\mathbf{I}_M}{p_0} \right)^{-1} \mathbf{D}_{G_{ll}},\end{aligned}\quad (10)$$

where $\tilde{\mathbf{U}}_{li} = \mathbf{U}_{li} \tilde{\mathbf{I}}^H \in \mathbb{C}^{N_S \times M}$ and $\tilde{\mathbf{D}}_{U_{li}} = \tilde{\mathbf{I}} \mathbf{D}_{U_{li}} \tilde{\mathbf{I}}^H \in \mathbb{C}^{M \times M}$, respectively. Both $\mathbf{G}_{ll}$ and $\mathbf{N}_{G_{ll}}$ in (10) are statistically independent [24]. The true channel $\mathbf{G}_{ll}$ can be decomposed into its MMSE estimate $\hat{\mathbf{G}}_{ll}$ and estimation error $\mathcal{E}_{G_{ll}}$ as

$$\mathbf{G}_{ll} = \hat{\mathbf{G}}_{ll} + \mathcal{E}_{G_{ll}}, \quad (11)$$

where $\hat{\mathbf{G}}_{ll}$ and $\mathcal{E}_{G_{ll}}$ are independent [24]. The rows of $\hat{\mathbf{G}}_{ll}$ and $\mathcal{E}_{G_{ll}}$ are mutually independent and distributed as $\mathcal{CN}(\mathbf{0}, \hat{\mathbf{D}}_{G_{ll}})$ and $\mathcal{CN}(\mathbf{0}, \mathbf{D}_{G_{ll}} - \hat{\mathbf{D}}_{G_{ll}})$, respectively. The m th diagonal element of the diagonal matrix $\hat{\mathbf{D}}_{G_{ll}}$ is given by

$$\sigma_{\hat{G}_{ll,m}}^2 = \frac{\zeta_{G_{ll,m}}^2}{\sum_{i=1}^L (\zeta_{G_{li,m}} + \mathcal{I}_m \zeta_{U_{li,m}}) + 1/p_0}. \quad (12)$$

2) *Pilot sequence design-2 (PSD-2)*: In PSD-2, the PUs and SUs located within the same l th cell are assigned with mutually orthogonal pilot sequences; $\Phi_{P_l} \Phi_{S_l}^H = \mathbf{0}_{K \times M}$ and $Q = 0$. Moreover, these pilot sequences are reused for PUs and SUs located in all L co-channel cells; $\Phi_{P_l} = \Phi_P$ and $\Phi_{S_l} = \Phi_S$, where $l \in \{1, \dots, L\}$. The pilot sequence length of PSD-2 is denoted by τ_2 which satisfies $\tau_2 \geq M + K$, and hence, this represents a trade-off between the pilot contamination and the number of simultaneously served PUs/SUs. Thus, $\hat{\mathbf{F}}_{ll}$ is not contaminated by the pilots transmitted by the SUs in any of the L cells. Similarly, $\hat{\mathbf{G}}_{ll}$ is not contaminated by the pilots transmitted by the PUs in any of the L cells. Thus, the MMSE estimates of $\mathbf{F}_{ll}$ and $\mathbf{G}_{ll}$ for PSD-2 can be derived as [24]

$$\begin{aligned}\hat{\mathbf{F}}_{ll} &= \left(\sum_{i=1}^L \mathbf{F}_{li} + \frac{\mathbf{N}_{F_{ll}}}{\sqrt{p_0}} \right) \left(\sum_{i=1}^L \mathbf{D}_{F_{li}} + \frac{\mathbf{I}_K}{p_0} \right)^{-1} \mathbf{D}_{F_{ll}}, \\ \hat{\mathbf{G}}_{ll} &= \left(\sum_{i=1}^L \mathbf{G}_{li} + \frac{\mathbf{N}_{G_{ll}}}{\sqrt{p_0}} \right) \left(\sum_{i=1}^L \mathbf{D}_{G_{li}} + \frac{\mathbf{I}_M}{p_0} \right)^{-1} \mathbf{D}_{G_{ll}},\end{aligned}\quad (13)$$

where $p_0 = \tau_2 P_U$, $\hat{\mathbf{F}}_{ll}$ and $\hat{\mathbf{G}}_{ll}$ are mutually independent of their estimation error matrices $\mathcal{E}_{F_{ll}}$ and $\mathcal{E}_{G_{ll}}$ due to the MMSE property and their Gaussianity. Hence, the variances of the k th diagonal element in $\hat{\mathbf{F}}_{ll}$ distribution and the m th diagonal element in $\hat{\mathbf{G}}_{ll}$ distribution are

$$\sigma_{\hat{F}_{ll,k}}^2 = \frac{\zeta_{F_{ll,k}}^2}{\sum_{i=1}^L \zeta_{F_{li,k}} + 1/p_0} \quad \text{and} \quad \sigma_{\hat{G}_{ll,m}}^2 = \frac{\zeta_{G_{ll,m}}^2}{\sum_{i=1}^L \zeta_{G_{li,m}} + 1/p_0}. \quad (14)$$

C. Signal model

In this section, the signal models for the primary and secondary systems are presented.

1) *Signal model for the primary system*: The signal received at the l th cell PBS after applying the ZF² detector can be written as

$$\begin{aligned}y_{P_l} &= \underbrace{\sqrt{P_P} \hat{\mathbf{W}}_{P_{ll}}^T \mathbf{F}_{ll} \mathbf{x}_{P_l}}_{\text{desired signal}} + \underbrace{\sum_{i=1, i \neq l}^L \sqrt{P_P} \hat{\mathbf{W}}_{P_{ll}}^T \mathbf{F}_{li} \mathbf{x}_{P_i}}_{\text{inter-cell PU CCI}} \\ &\quad + \underbrace{\sqrt{P_S} \hat{\mathbf{W}}_{P_{ll}}^T \mathbf{V}_{ll} \mathbf{x}_{S_l}}_{\text{intra-cell SU CCI}} + \underbrace{\sum_{i=1, i \neq l}^L \sqrt{P_S} \hat{\mathbf{W}}_{P_{ll}}^T \mathbf{V}_{li} \mathbf{x}_{S_i}}_{\text{inter-cell SU CCI}} + \underbrace{\hat{\mathbf{W}}_{P_{ll}}^T \mathbf{n}_{P_l}}_{\text{AWGN}},\end{aligned}\quad (15)$$

where P_P and P_S are the user transmit powers of the PUs and SUs in the l th cell, respectively, for $l \in \{1, \dots, L\}$. Moreover, $\mathbf{x}_{P_l}$ and $\mathbf{x}_{S_l}$ are the transmit signal vectors of the PUs and SUs in the l th cell, respectively, satisfying $\mathbb{E}[\mathbf{x}_{P_l} \mathbf{x}_{P_l}^H] = \mathbf{I}_K$ and $\mathbb{E}[\mathbf{x}_{S_l} \mathbf{x}_{S_l}^H] = \mathbf{I}_M$. In (15), the AWGN satisfies $\mathbb{E}[\mathbf{n}_{P_l} \mathbf{n}_{P_l}^H] = \mathbf{I}_{N_P} \sigma_P^2$, and $\hat{\mathbf{W}}_{P_{ll}}^T$ is the ZF detector at the l th cell PBS and is defined as

$$\hat{\mathbf{W}}_{P_{ll}}^T = (\hat{\mathbf{F}}_{ll}^H \hat{\mathbf{F}}_{ll})^{-1} \hat{\mathbf{F}}_{ll}^H. \quad (16)$$

²ZF detector performs better than matched-filter detector in terms of inter-pair interference mitigation in finite BS antenna regime with imperfect [3].

2) *Signal model for secondary system:* The signal received at the l th cell SBS after applying the ZF detector is given by

$$\begin{aligned} \mathbf{y}_{S_l} = & \underbrace{\sqrt{P_S} \hat{\mathbf{W}}_{S_{ll}}^T \mathbf{G}_{ll} \mathbf{x}_{S_l}}_{\text{desired signal}} + \underbrace{\sum_{i=1, i \neq l}^L \sqrt{P_S} \hat{\mathbf{W}}_{S_{ll}}^T \mathbf{G}_{li} \mathbf{x}_{S_i}}_{\text{inter-cell SU CCI}} \\ & + \underbrace{\sqrt{P_P} \hat{\mathbf{W}}_{S_{ll}}^T \mathbf{U}_{ll} \mathbf{x}_{P_l}}_{\text{intra-cell PU CCI}} + \underbrace{\sum_{i=1, i \neq l}^L \sqrt{P_P} \hat{\mathbf{W}}_{S_{ll}}^T \mathbf{U}_{li} \mathbf{x}_{P_i}}_{\text{inter-cell PU CCI}} + \underbrace{\hat{\mathbf{W}}_{S_{ll}}^T \mathbf{n}_{S_l}}_{\text{AWGN}}, \end{aligned} \quad (17)$$

where $\hat{\mathbf{W}}_{S_{ll}}^T$ is the ZF detector at the SBS in the l th cell and is defined as

$$\hat{\mathbf{W}}_{S_{ll}}^T = (\hat{\mathbf{G}}_{ll}^H \hat{\mathbf{G}}_{ll})^{-1} \hat{\mathbf{G}}_{ll}^H. \quad (18)$$

In (17), the AWGN satisfies $\mathbb{E}[\mathbf{n}_{S_l} \mathbf{n}_{S_l}^H] = \mathbf{I}_{N_S} \sigma_S^2$.

Remark 1: The system model presented in Section II-A can be readily extended to investigate MIMO-enabled PUs/SUs for boosting the achievable UL rates [25]. However, the UL pilot sequence length linearly increases with the number of antennas at the PUs/SUs.

III. SECONDARY TRANSMIT POWER CONSTRAINTS

According the cognitive underlay spectrum sharing concept [10], the transmit power of the SUs (P_S) in the l th cell is constrained to maintain the intra-cell CCI inflicted at the PBS due to concurrent secondary transmissions in the same cell as³

$$P_S = \min \left(P_{S_{\max}}, \frac{I_P}{\mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{ll}}^T \mathbf{V}_{ll} \mathbf{V}_{ll}^H \hat{\mathbf{W}}_{P_l}^* \right) \right]} \right), \quad (19)$$

where $P_{S_{\max}}$ is the maximum SU transmit power and I_P is the PIT, which is the maximum interference level that the PBS can endure without hindering its performance. The transmit power constraint in (19) is designed by only considering the same cell secondary CCI. However, under a cooperative secondary system, a more stringent transmit power constraint on the secondary transmissions can be defined by considering the both inter-cell and intra-cell secondary CCI as

$$P_S = \min \left(P_{S_{\max}}, \frac{I_P}{\sum_{i=1}^L \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \mathbf{V}_{li} \mathbf{V}_{li}^H \hat{\mathbf{W}}_{P_l}^* \right) \right]} \right). \quad (20)$$

A. Secondary power constraints for finite PBS antenna regime

The transmit power constraints corresponding to PSD-1 and PSD-2 in the finite PBS antenna regime can be derived by using (20) as (see Appendix A-A for the derivation)

$$P_S = \min (P_{S_{\max}}, I_P/Z), \quad (21)$$

where $Z \in \{Z^{\text{PSD-1}}, Z^{\text{PSD-2}}\}$ depends on PSD-1 and PSD-2 in Section II-B1 and Section II-B2, respectively, and can be defined as follows:

³The SUs do not have access to instantaneous downlink CSI since no downlink pilots are sent for estimating channels at the SUs. Thus, the secondary transmit power constraint at the SUs is designed based on the statistical CSI.

$$\begin{aligned} Z^{\text{PSD-1}} = & \sum_{i=1}^L \text{Tr}(\mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}^2 \mathbf{D}_{F_{li}}^{-1}) \\ & + \sum_{i=1}^L \frac{\text{Tr}(\hat{\mathbf{D}}_{F_{li}}^{-1})}{N_P - K} \text{Tr}(\mathbf{D}_{V_{li}} - \mathbf{D}_{V_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{D}}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}), \end{aligned} \quad (22)$$

$$Z^{\text{PSD-2}} = \frac{\text{Tr}(\hat{\mathbf{D}}_{F_{li}}^{-1})}{N_P - K} \sum_{i=1}^L \text{Tr}(\mathbf{D}_{V_{li}}). \quad (23)$$

Remark 2: The transmit power constraints given in (21)-(23) depend on I_P when the number of PBS antenna is finite and does not grow without bound with respect to the number of PUs/SUs. Thus, the secondary system cannot be operated independent of the primary system in finite PBS antenna regime.

B. Secondary power constraints for infinite PBS antenna regime

In this subsection, the transmit power constraints at the SUs are derived for the case in which the number of PBS antennas grows without bound with respect to the number of PUs/SUs.

1) *Asymptotic transmit power constraint for PSD-1:* The asymptotic transmit power constraint at the SUs for PSD-1 can be derived by letting $N_P \rightarrow \infty$ in (21) as

$$P_S \xrightarrow{N_P \rightarrow \infty} \min \left(P_{S_{\max}}, \frac{I_P}{\sum_{i=1}^L \left[\text{Tr}(\mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}^2 \mathbf{D}_{F_{li}}^{-1}) \right]} \right). \quad (26)$$

Remark 3: For PSD-1, the asymptotic secondary transmit power constraint (26) depends on the PIT. Thus, the secondary system with PSD-1 cannot be operated independent of the primary system even in the limit of infinitely many PBS antennas.

2) *Asymptotic transmit power constraint for PSD-2:* The asymptotic transmit power constraint in (20) can be derived for PSD-2 by letting the numbers of antennas at PBS and SBS grow without bound as follows:

$$\min_{N_P \rightarrow \infty} P_S = P_{S_{\max}}. \quad (27)$$

Remark 3: For PSD-2, the asymptotic secondary transmit power constraint (26) becomes independent of the primary interference temperature and approaches the allowed maximum value. Thus, the secondary system can be operated independent of the primary system even for the imperfect CSI case with PSD-2.

IV. ACHIEVABLE SUM RATE DEFINITIONS

In this section, the achievable sum rate definitions for the primary and secondary system are presented.

A. Achievable sum rate definition for the primary system

In order to capture the joint impact of detection uncertainty, interference and filtered AWGN, the k th PU data substream received at the l th cell PBS is written by using (15) as [3]

$$[\mathbf{y}_{P_l}]_k = \sqrt{P_P} \mathbb{E}[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k}] x_{P_{l,k}} + \tilde{\mathbf{n}}_{P_{l,k}}, \quad (28)$$

where the first term accounts for the desired signal component. The second term represents the effective noise capturing joint effects of interference arises from detection uncertainty with imperfect CSI, intra/inter-cell PU/SU CCI and filtered AWGN. This effective noise term can be defined as [3]

$$\begin{aligned} \tilde{\mathbf{n}}_{P_l,k} = & \underbrace{\sqrt{P_P} \left(\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k} - \mathbb{E}[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k}] \right)}_{\text{detection uncertainty}} x_{P_l,k} \\ & + \sum_{j=1, j \neq k}^K \underbrace{\sqrt{P_P} \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,j} x_{P_l,j}}_{\text{Inter-pair PU CCI}} + \sum_{i=1, i \neq l}^L \underbrace{\sqrt{P_P} \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{F}_{li} \mathbf{x}_{P_i}}_{\text{inter-cell PU CCI}} \\ & + \sum_{i=1}^L \underbrace{\sqrt{P_S} \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{V}_{li} \mathbf{x}_{S_i}}_{\text{intra/inter-cell SU CCI}} + \underbrace{\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{n}_{P_l}}_{\text{filtered AWGN}}. \end{aligned} \quad (29)$$

In (28)-(29), $\hat{\mathbf{w}}_{P_l,k}$ and $\mathbf{f}_{ll,k}$ are the k th columns of $\hat{\mathbf{W}}_{P_l}$ and $\mathbf{F}_{ll}$, respectively. Moreover, $x_{P_l,k}$ is the k th element of $\mathbf{x}_{P_l}$.

In (28), the desired signal and the effective noise are uncorrelated. By treating the worst-case uncorrelated additive noise as independently distributed Gaussian noise having the same variance, the achievable sum rate of the primary system in the l th cell can be defined as [3]

$$\tilde{\mathcal{R}}_{P_l} = \psi_{P_l} \sum_{k=1}^K \log_2 \left(1 + \frac{P_P \left| \mathbb{E}[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k}] \right|^2}{P_P \text{Var}(\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k}) + \sum_{j=1}^4 P_{I_j}} \right), \quad (30)$$

where $\psi_{P_l} = (\tau_{C_P} - \tau_1)/\tau_{C_P}$ for PSD-1, and $\psi_{P_l} = (\tau_{C_P} - \tau_2)/\tau_{C_P}$ for PSD-2. Here, τ_{C_P} is the coherence interval of the PU-to-PBS channel, and P_{I_j} 's for $j \in \{1, \dots, 4\}$ can be defined as

$$P_{I_1} = \sum_{j=1, j \neq k}^K P_P \mathbb{E} \left[\left| \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,j} \right|^2 \right], \quad (31a)$$

$$P_{I_2} = \sum_{i=1, i \neq l}^L P_P \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{F}_{li} \right\|^2 \right], \quad (31b)$$

$$P_{I_3} = \sum_{i=1}^L P_S \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{V}_{li} \right\|^2 \right] \text{ and } P_4 = \sigma_P^2 \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{P_{ll,k}}^T \right\|^2 \right]. \quad (31c)$$

B. Achievable sum rate definition for the secondary system

By using (17), the m th data substream received at the SBS in the l th cell can be written as [3]

$$[\mathbf{y}_{S_l}]_m = \sqrt{P_S} \mathbb{E}[\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m}] x_{S_l,m} + \tilde{\mathbf{n}}_{S_l,m}, \quad (32)$$

where the effective noise, $\tilde{\mathbf{n}}_{S_l,m}$, can be defined as

$$\begin{aligned} \tilde{\mathbf{n}}_{S_l,m} = & \sqrt{P_S} \left(\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m} - \mathbb{E}[\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m}] \right) x_{S_l,m} \\ & + \sum_{j=1, j \neq m}^M \sqrt{P_S} \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,j} x_{S_l,j} + \sum_{i=1, i \neq l}^L \sqrt{P_S} \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{G}_{li} \mathbf{x}_{S_i} \\ & + \sum_{i=1}^L \sqrt{P_P} \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{U}_{li} \mathbf{x}_P + \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{n}_{S_l}, \end{aligned} \quad (33)$$

where $\hat{\mathbf{w}}_{S_{ll,m}}$ and $\mathbf{g}_{ll,m}$ are the m th columns of $\hat{\mathbf{W}}_{S_{ll}}$ and $\mathbf{G}_{ll}$, respectively. Furthermore, $x_{S_l,m}$ is the m th element of $\mathbf{x}_{S_l}$. By invoking the worst-case Gaussian technique, the achievable sum rate at the SBS in the l th cell can be derived as

$$\tilde{\mathcal{R}}_{S_l} = \psi_{S_l} \sum_{m=1}^M \log_2 \left(1 + \frac{P_S \left| \mathbb{E}[\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m}] \right|^2}{P_S \text{Var}(\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m}) + \sum_{i=1}^4 P_{J_i}} \right), \quad (34)$$

where $\psi_{S_l} = (\tau_{C_S} - \tau_1)/\tau_{C_S}$ for PSD-1, and $\psi_{S_l} = (\tau_{C_S} - \tau_2)/\tau_{C_S}$ for PSD-2. Here, τ_{C_S} is the coherence interval of the SU-to-SBS channel. Moreover, P_{J_i} 's for $j \in \{1, \dots, 4\}$ are given by

$$P_{J_1} = \sum_{j=1, j \neq m}^M P_S \mathbb{E} \left[\left| \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,j} \right|^2 \right], \quad (35a)$$

$$P_{J_2} = \sum_{i=1, i \neq l}^L P_S \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{G}_{li} \right\|^2 \right], \quad (35b)$$

$$P_{J_3} = \sum_{i=1}^L P_P \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{U}_{li} \right\|^2 \right] \text{ and } P_{J_4} = \sigma_S^2 \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{S_{ll,m}}^T \right\|^2 \right] \quad (35c)$$

V. ACHIEVABLE SUM RATE ANALYSIS FOR IMPERFECT CSI

In this section, the achievable sum rate analysis for primary/secondary systems is presented for finite and asymptotic PBS/SBS antenna regimes in the presence of imperfect CSI.

A. Achievable sum rates for finite PBS/SBS antenna regime

In this subsection, the achievable sum rates for primary and secondary systems are presented for the case in which the number of PBS/SBS antennas does not grow without bound with respect to the number PUs/SUs.

1) *Achievable sum rate for primary system:* For the case of finitely-many PBS/SBS antennas⁴, the achievable sum rate for the primary system in the l th cell can be derived by using (30) as follows (see Appendix A-B for the derivation):

$$\mathcal{R}_{P_l,k} = \psi_{P_l} \sum_{k=1}^K \log_2 \left(1 + \frac{P_P}{\Delta_P} \right), \quad (36)$$

where $\Delta_P \in \{\Delta_P^{\text{PSD-1}}, \Delta_P^{\text{PSD-2}}\}$ depends on PSD-1 and PSD-2 described in Section II-B1 and Section II-B2, respectively and can be defined as follows:

$$\begin{aligned} \Delta_P^{\text{PSD-1}} = & \frac{\sigma_{P_l}^2}{\sigma_{\hat{\mathbf{F}}_{ll,k}}^2 (N_P - K)} + \frac{P_P}{\sigma_{\hat{\mathbf{F}}_{ll,k}}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_{ll,j}} - \sigma_{\hat{\mathbf{F}}_{ll,j}}^2 \right) \\ & + \sum_{i=1, i \neq l}^L P_P \zeta_{F_{li,k}}^{-2} \zeta_{F_{li,k}}^2 \left(1 + \frac{1}{\sigma_{\hat{\mathbf{F}}_{li,k}}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_{li,j}} - \sigma_{\hat{\mathbf{F}}_{li,j}}^2 \right) \right) \\ & + \sum_{i=1}^L P_S \left[\zeta_{F_{ll,k}}^{-2} \zeta_{V_{ll,k}}^2 + \frac{1}{\sigma_{\hat{\mathbf{F}}_{ll,k}}^2 (N_P - K)} \text{Tr}(\mathcal{A}_i) \right], \end{aligned} \quad (37)$$

$$\begin{aligned} \Delta_P^{\text{PSD-2}} = & \frac{\sigma_{P_l}^2}{\sigma_{\hat{\mathbf{F}}_{ll,k}}^2 (N_P - K)} + \frac{P_P}{\sigma_{\hat{\mathbf{F}}_{ll,k}}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_{ll,j}} - \sigma_{\hat{\mathbf{F}}_{ll,j}}^2 \right) \\ & + \sum_{i=1, i \neq l}^L P_P \zeta_{F_{li,k}}^{-2} \zeta_{F_{li,k}}^2 \left(1 + \frac{1}{\sigma_{\hat{\mathbf{F}}_{li,k}}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_{li,j}} - \sigma_{\hat{\mathbf{F}}_{li,j}}^2 \right) \right) \\ & + \sum_{i=1}^L \frac{P_S \sum_{j=1}^K \zeta_{V_{li,j}}}{\sigma_{\hat{\mathbf{F}}_{li,k}}^2 (N_P - K)}, \end{aligned} \quad (38)$$

where $\mathcal{A}_i = \mathbf{D}_{V_{li}} - \mathbf{D}_{V_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{D}}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}$.

⁴This corresponds to the case in which the number of antennas at PBS/SBS is not too large with respect to the number of PUs/SUs.

$$\bar{\mathcal{R}}_{P_l} = \psi_{P_l} \sum_{k=1}^K \mathbb{E} \left[\log_2 \left(1 + \frac{P_P |\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,k}|^2}{\sum_{j=1, j \neq k}^K P_P |\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,j}|^2 + \sum_{i=1, i \neq l}^L P_P \|\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{F}_{li}\|^2 + \sum_{i=1}^L P_S \|\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{V}_{li}\|^2 + \sigma_{P_l}^2 \|\hat{\mathbf{w}}_{P_{ll,k}}^T\|^2} \right) \right] \quad (42)$$

$$\bar{\mathcal{R}}_{S_l} = \psi_{S_l} \sum_{m=1}^M \mathbb{E} \left[\log_2 \left(1 + \frac{P_S |\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,m}|^2}{\sum_{j=1, j \neq m}^M P_S |\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{g}_{ll,j}|^2 + \sum_{i=1, i \neq l}^L P_S \|\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{G}_{li}\|^2 + \sum_{i=1}^L P_P \|\hat{\mathbf{w}}_{S_{ll,m}}^T \mathbf{U}_{li}\|^2 + \sigma_{S_l}^2 \|\hat{\mathbf{w}}_{S_{ll,m}}^T\|^2} \right) \right] \quad (43)$$

2) *Achievable sum rate for secondary system:* For the case of finitely-many PBS/SBS antennas, the achievable sum rate for the secondary system in the l th cell can be derived by using (34) as⁵

$$\tilde{\mathcal{R}}_{S_l} = \psi_{S_l} \sum_{m=1}^M \log_2 \left(1 + \frac{P_S}{\Delta_S} \right), \quad (39)$$

where $\Delta_S \in \{\Delta_S^{\text{PSD-1}}, \Delta_S^{\text{PSD-2}}\}$ depends on PSD-1 and PSD-2 described in Section II-B1 and Section II-B2, respectively and can be defined as

$$\begin{aligned} \Delta_S^{\text{PSD-1}} &= \frac{\sigma_{S_l}^2}{\sigma_{\hat{G}_{ll,m}}^2 (N_S - M)} + \frac{P_S}{\sigma_{\hat{G}_{ll,m}}^2 (N_S - M)} \sum_{j=1}^M (\zeta_{G_{ll,j}} - \sigma_{\hat{G}_{ll,j}}^2) \\ &+ \sum_{i=1, i \neq l}^L P_S \zeta_{G_{ll,m}}^{-2} \zeta_{G_{li,m}}^2 \left(1 + \frac{1}{\sigma_{\hat{G}_{li,m}}^2 (N_S - M)} \sum_{j=1}^M (\zeta_{G_{li,j}} - \sigma_{\hat{G}_{li,j}}^2) \right) \\ &+ \sum_{i=1}^L P_P \left[\zeta_{G_{ll,k}}^{-2} \zeta_{U_{ll,k}}^2 + \frac{1}{\sigma_{\hat{U}_{ll,k}}^2 (N_S - M)} \text{Tr}(\mathbf{B}_i) \right], \end{aligned} \quad (40)$$

$$\begin{aligned} \Delta_S^{\text{PSD-2}} &= \frac{\sigma_{S_l}^2}{\sigma_{\hat{G}_{ll,m}}^2 (N_S - M)} + \frac{P_S}{\sigma_{\hat{G}_{ll,m}}^2 (N_S - M)} \sum_{j=1}^M (\zeta_{G_{ll,j}} - \sigma_{\hat{G}_{ll,j}}^2) \\ &+ \sum_{i=1, i \neq l}^L P_S \zeta_{G_{ll,m}}^{-2} \zeta_{G_{li,m}}^2 \left(1 + \frac{1}{\sigma_{\hat{G}_{li,m}}^2 (N_S - M)} \sum_{j=1}^M (\zeta_{G_{li,j}} - \sigma_{\hat{G}_{li,j}}^2) \right) \\ &+ \sum_{i=1}^L \frac{P_P \sum_{j=1}^M \zeta_{U_{li,j}}}{\sigma_{\hat{G}_{ll,m}}^2 (N_S - M)}, \end{aligned} \quad (41)$$

where $\mathbf{B}_i = \mathbf{D}_{U_{li}} - \mathbf{D}_{U_{li}} \mathbf{D}_{G_{li}}^{-1} \hat{\mathbf{D}}_{G_{li}} \mathbf{D}_{G_{li}}^{-1} \mathbf{D}_{U_{li}}$.

Remark 4: The insights that can be obtained via the sum rate analysis in Section IV can be summarized as follows: The achievable rates in (36) and (39) capture the detrimental effects of channel estimation errors, CCI and intra/inter-cell pilot contamination. For instance, $\sigma_{\hat{F}_{ll,m}}^2$ and $\sigma_{\hat{G}_{ll,m}}^2$ in (36) and (39) capture the channel estimation errors. Moreover, $\Delta_P^{\text{PSD-}i}$ and $\Delta_S^{\text{PSD-}i}$, where $i \in \{1, 2\}$, reveal that CCI and intra/inter-cell pilot contamination significantly degrades the achievable rates for PUs and SUs. Hence, simple linear ZF detectors cannot be used to mitigate CCI in spectrum-sharing systems with smaller antenna arrays at the PBS and SBS.

In order to assess the accuracy of our analysis via the worst-case Gaussian technique in Section IV, the achievable sum rates in (36) and (39) are compared against the ergodic achievable sum rates of the PBS/SBS computed via a stronger capacity bound [3, Section 2.3.5]. To this end, the ergodic achievable sum rates at the PBS and SBS can be defined as in (42) and (43), respectively, at the top of this page.

⁵This derivation follows steps similar to those in Appendix A-B and hence is omitted for the sake of brevity.

Since the channel codes may span several realizations of the small-scale fading process, the ergodic sum rate is used as a benchmark for comparison purposes. Thus, in Section VIII, the accuracy of our closed-form achievable sum rates in (36) and (39) is investigated by comparing their tightness with the Monte-Carlo simulations of the ergodic sum rates in (42) and (43).

B. Asymptotic achievable rate analysis for imperfect CSI

In this subsection, the asymptotic achievable sum rates are derived when N_P and N_S grow without bound, while keeping a fixed ratio $\beta = N_P/N_S$. These asymptotic performance metrics can readily be derived by letting $N_P \rightarrow \infty$ or $N_S \rightarrow \infty$ in the corresponding metric for the finite PBS/SBS antenna regime, and hence, their derivations are omitted for the sake of brevity. The pilot transmit power is assumed to be fixed [26], and the transmit power at PUs and SUs are scaled as follows: $P_P = E_P/N_P$ and $P_S = E_S/N_S$, where E_P and E_S are constants.

1) *Asymptotic achievable sum rate for PSD-1:* The asymptotic achievable sum rate of the primary system in the l th cell can be derived by scaling the transmit power as $P_P = E_P/N_P$ and $P_S = E_S/N_S$ and by letting $N_P \rightarrow \infty$ and $N_S \rightarrow \infty$ in (36) and (37) as follows:

$$\mathcal{R}_{P_l}^\infty \xrightarrow{N_P, N_S \rightarrow \infty} \psi_{P_l} \sum_{k=1}^K \log_2 \left(1 + \frac{E_P}{\Delta_{P_l}^{\infty, \text{PSD-1}}} \right), \quad (44)$$

where $\Delta_{P_l}^{\infty, \text{PSD-1}}$ is given by

$$\Delta_{P_l}^{\infty, \text{PSD-1}} = \frac{\sigma_{P_l}^2}{\sigma_{\hat{F}_{ll,k}}^2} + \sum_{i=1, i \neq l}^L \frac{E_P \zeta_{F_{li,k}}^2}{\zeta_{F_{ll,k}}^2} + \beta E_S \sum_{i=1}^L \frac{\zeta_{V_{li,k}}^2}{\zeta_{F_{ll,k}}^2}. \quad (45)$$

The asymptotic sum rate of the secondary system in the l th cell can be derived from (39) and (40) as

$$\mathcal{R}_{S_l}^\infty \xrightarrow{N_P, N_S \rightarrow \infty} \psi_{S_l} \sum_{m=1}^M \log_2 \left(1 + \frac{E_S}{\Delta_S^{\infty, \text{PSD-1}}} \right), \quad (46)$$

where $\Delta_{S_l}^{\infty, \text{PSD-1}}$ is defined as

$$\Delta_{S_l}^{\infty, \text{PSD-1}} = \frac{\sigma_{S_l}^2}{\sigma_{\hat{G}_{ll,m}}^2} + \sum_{i=1, i \neq l}^L \frac{E_S \zeta_{G_{li,m}}^2}{\zeta_{G_{ll,m}}^2} + \frac{E_P}{\beta} \sum_{i=1}^L \frac{\zeta_{U_{li,m}}}{\zeta_{G_{ll,m}}^2}. \quad (47)$$

2) *Asymptotic achievable sum rate for PSD-2:* Next, by using (36), the asymptotic achievable sum rate of the PUs in the l th cell can be derived by scaling the transmit powers as $P_P = E_P/N_P$ and $P_S = E_S/N_S$ and by letting $N_P \rightarrow \infty$ and $N_S \rightarrow \infty$ as follows:

$$\mathcal{R}_{P_l}^\infty \xrightarrow{N_P, N_S \rightarrow \infty} \psi_{P_l} \sum_{k=1}^K \log_2 \left(1 + \frac{E_P}{\frac{\sigma_{P_l}^2}{\sigma_{\hat{F}_{ll,k}}^2} + E_P \sum_{i=1, i \neq l}^L \frac{\zeta_{F_{li,k}}^2}{\zeta_{F_{ll,k}}^2}} \right). \quad (48)$$

The asymptotic achievable rate for the SUs in the l th cell can be derived by using (39) as

$$\mathcal{R}_{S_l}^\infty \xrightarrow{N_P, N_S \rightarrow \infty} \psi_{S_l} \sum_{m=1}^M \log_2 \left(1 + \frac{E_S}{\frac{\sigma_{S_l}^2}{\sigma_{G_{ll,m}}^2} + E_S \sum_{i=1, i \neq l}^L \frac{\zeta_{G_{li,m}}^2}{\zeta_{G_{li,m}}^2}} \right). \quad (49)$$

Remark 4: The important insights obtained through the aforementioned asymptotic analysis with imperfect CSI can be summarized as follows: In PSD-1, the same pilot sequences are reused for PUs and SUs in all L cells. Hence, as per (44) and (46), the asymptotic achievable sum rates of SUs and PUs are severely degraded by the both intra-cell and inter-cell pilot contamination effects. Thus, the achievable rates yielded by PSD-1 corresponds to the worst-case scenario in terms of pilot contamination. Nevertheless, in PSD-2, two orthogonal sets of pilot sequences for allocated for PUs and SUs in the same cell, and these two pilot sets are then reused across the $L - 1$ co-channel cells. Hence, according to (48) and (49), the detrimental effects of intra-cell pilot contamination can be completely canceled. Nevertheless, the inter-cell pilot contamination exists for both PSDs and cannot be mitigated even in the infinite PBS/SBS antenna regimes. Moreover, the pilot sequence lengths for PSD-1 and PSD-2 are $\tau_1 \geq \max(K, M)$ and $\tau_2 \geq M + K$, respectively. Hence, the minimum pilot sequence length of PSD-1 is at least twice less than that of PSD-2. Consequently, the total number of PUs/SUs that can be simultaneously served by PSD-1 is higher than that of PSD-2 although the former yields significantly higher intra-cell pilot contamination over the latter. Thus, there is a fundamental trade-off between the achievable sum rates and the number of PUs/SUs that can be served simultaneously in the same time-frequency resource block.

VI. UPLINK TRANSMIT POWER CONTROL

Max-min power control has been shown to be optimal in the sense of guaranteeing user-fairness in the presence of near-far effect in the uplink [3]. Moreover, max-min power allocation problems are polynomial-time solvable [27]. Thus, in this section, an uplink transmit power policy is investigated for the secondary and primary systems aiming to achieve max-min user-fairness. To make power control problem mathematically tractable and hence to draw design insights, our analysis is limited to the single-cell ($L = 1$) case. To this end, the transmit power allocation coefficients and the achievable common rates are derived.

A. Uplink transmit power control for PUs/SUs for finite PBS/SBS antennas

In this subsection, the max-min fairness transmit power control for the PUs and SUs is investigated. The power allocation coefficients for the k th PU and the m th SU are denoted by μ_k and η_m , where $0 \leq \mu_k \leq 1$ and $0 \leq \eta_m \leq 1$, respectively. Hence, the transmit power allocations of the k th PU and the m th SU are given by $\mu_k P_{P_k}$ and $\eta_m P_{S_m}$, respectively. Here, P_{P_k} and P_{S_m} are the available transmit powers at the k th PU and the m th SU, respectively.

By letting $L = 1$ in (36) and (39), the SINRs for the k th PU and the m th SU for the single-cell case are deduced as

$$\tilde{\gamma}_{P_k}^n = \frac{\mu_k P_{P_k}}{\Delta_{P_k}^n} \quad \text{and} \quad \tilde{\gamma}_{S_m}^n = \frac{\eta_m P_{S_m}}{\Delta_{S_m}^n}, \quad (50)$$

where $n \in \{\text{PSD-1}, \text{PSD-2}\}$, respectively. In (50), $\Delta_{P_k}^n$ and $\Delta_{S_m}^n$ are defined as

$$\Delta_{P_k}^{\text{PSD-1}} = \frac{\sigma_P^2}{\sigma_{F_k}^2 (N_P - K)} + \frac{1}{\sigma_{F_k}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_j} - \sigma_{F_j}^2 \right) \mu_j P_{P_j} + \eta_k P_{S_k} \frac{\zeta_{V_k}^2}{\zeta_{F_k}^2} + \frac{1}{\sigma_{F_k}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{V_k} - \frac{\zeta_{V_k}^2 \zeta_{F_k}}{\zeta_{F_k}^2} \right) \eta_j P_{S_j}, \quad (51a)$$

$$\Delta_{P_k}^{\text{PSD-2}} = \frac{\sigma_P^2}{\sigma_{F_k}^2 (N_P - K)} + \frac{1}{\sigma_{F_k}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{F_{ll,j}} - \sigma_{F_{ll,j}}^2 \right) \mu_j P_{P_j} + \frac{\sum_{j=1}^K \zeta_{V_j} \eta_j P_{S_j}}{\sigma_{F_k}^2 (N_P - K)}, \quad (51b)$$

$$\Delta_{S_m}^{\text{PSD-1}} = \frac{\sigma_S^2}{\sigma_{G_m}^2 (N_S - M)} + \frac{1}{\sigma_{G_m}^2 (N_S - M)} \sum_{j=1}^M \left(\zeta_{G_j} - \sigma_{G_j}^2 \right) \eta_j P_{S_j} + \frac{\mu_m P_{P_m} \zeta_{U_m}^2}{\zeta_{G_m}^2} + \frac{1}{\sigma_{G_m}^2 (N_S - M)} \sum_{j=1}^M \left(\zeta_{U_m} - \frac{\zeta_{U_m}^2 \zeta_{G_m}}{\zeta_{G_m}^2} \right) \mu_j P_{P_j}, \quad (51c)$$

$$\Delta_{S_m}^{\text{PSD-2}} = \frac{\sigma_S^2}{\sigma_{G_m}^2 (N_S - M)} + \frac{1}{\sigma_{G_m}^2 (N_S - M)} \sum_{j=1}^M \left(\zeta_{G_j} - \sigma_{G_j}^2 \right) \eta_j P_{S_j} + \frac{\sum_{j=1}^M \zeta_{U_j} \mu_j P_{P_j}}{\sigma_{G_m}^2 (N_S - M)}. \quad (51d)$$

The transmit power allocation problem ($\mathcal{OP}_1$) is formulated such that the PUs achieve predefined SINR thresholds, while max-min fairness is achieved among the SUs as follows:

$$\mathcal{OP}_1 : \text{maximize}_{\eta_m} \min_m \tilde{\gamma}_{S_m}^n, \quad (52a)$$

$$\text{subject to} \quad C_1 : 0 \leq \eta_m \leq 1, \quad \forall m \quad \text{and} \quad 0 \leq \mu_k \leq 1, \quad \forall k,$$

$$\text{subject to} \quad C_2 : \tilde{\gamma}_{P_k}^n \geq \Gamma_{P_k}^n, \quad \forall k, \quad (52b)$$

$$\text{subject to} \quad C_3 : P_{S_m}^n = \min \left(P_{S_m}^{\max}, I_P / Z_m^n \right), \quad \forall m, \quad (52c)$$

where $\Gamma_{P_k}^n$ is the predefined SINR threshold for the k th PU. In (52c), $P_{S_m}^{\max}$ is the maximum available transmit power at the m th SU, and Z_m^n for $n \in \{\text{PSD-1}, \text{PSD-2}\}$ is defined as

$$Z_m^{\text{PSD-1}} = \left(\frac{\zeta_{V_m}^2}{\zeta_{F_m}^2} + \frac{1}{\sigma_{F_m}^2 (N_P - K)} \sum_{j=1}^K \left(\zeta_{V_j} - \frac{\zeta_{V_j}^2 \zeta_{F_j}}{\zeta_{F_j}^2} \right) \right), \quad (53a)$$

$$Z_m^{\text{PSD-2}} = \sum_{j=1}^K \zeta_{V_j} / \left(\sigma_{F_m}^2 (N_P - K) \right). \quad (53b)$$

The third constraint in (52c) is the secondary transmit power constraint, which depends only on the long-term channel statistics. Hence, the maximum SU transmit power $P_{S_m}^n$ can be determined prior to the power allocation such that (52c) is satisfied. Then, the transmit power for the m th SU can be allocated as a fraction of the maximum SU transmit, $\eta_m P_{S_m}^n$, as per $\mathcal{OP}_1$ in (52a).

The underlying concept of max-min fairness power allocation is to set a lower bound for the SINR targets $\bar{\Gamma}_{S^n}$ of all SUs and then search for the largest possible value for the SINR targets in terms of power allocation coefficients. Hence, by using (50) and an epigraph form of $\mathcal{OP}_1$, the transmit power allocation problem can be reformulated ($\mathcal{OP}_2$) as

$$\mathcal{OP}_2 : \text{maximize}_{\eta_m} \bar{\Gamma}_{S^n}, \quad (54a)$$

$$\text{subject to} \quad C_1 : 0 \leq \eta_m \leq 1, \quad \forall m \quad \text{and} \quad 0 \leq \mu_k \leq 1, \quad \forall k,$$

$$\text{subject to} \quad C_2 : \Gamma_{P_k}^n \Delta_{P_k}^n \leq \mu_k P_{P_k}, \quad \forall k, \quad (54b)$$

$$\text{subject to} \quad C_4 : \bar{\Gamma}_{S^n} \Delta_{S_m}^n \leq \eta_m P_{S_m}, \quad \forall m, \quad (54c)$$

where $\Delta_{P_k}^n$ and $\Delta_{S_m}^n$ are defined in (51a)-(51b) and (51c)-(51d), respectively. Here, in (54c), P_{S_m} is selected such that (52c) is satisfied. The objective function in $\mathcal{OP}_2$ is a monomial and the constraints C_1 , C_2 and C_4 are posynomials. Hence, the power allocation problem in $\mathcal{OP}_2$ is a geometric program and can be efficiently solved by using optimization tools such as CVX - a Matlab software for disciplined convex programming [28]. The exact closed-form derivation of optimal power allocation coefficients, μ_k and η_k , in (54a) for finitely many PBS/SBS antennas arrears mathematically intractable [29]. Hence, the asymptotic power allocation coefficients are derived when the number of PBS/SBS antennas grows unbounded.

B. Uplink transmit power control for PUs/SUs for infinitely many PBS/SBS antennas

To begin with, when number of antennas at PBS grows without bound, the asymptotic secondary transmit power constraints for PSD-1 and PSD-2 can be defined as

$$P_{S_m}^n \xrightarrow{N_P \rightarrow \infty} \min \left(P_{S_m}^{\max}, \frac{I_P}{\zeta_{F_m}^{-2} \zeta_{V_m}^2} \right) \text{ and } P_{S_m}^n \xrightarrow{N_P \rightarrow \infty} P_{S_m}^{\max}, \quad (55)$$

where $n \in \{\text{PSD-1}, \text{PSD-2}\}$. Then, by scaling the PU and SU transmit power as $P_{P_k} = E_{P_k}/N_P$ and $P_{S_m} = E_{S_m}/N_S$, where E_{P_k} and E_{S_m} are fixed, the asymptotic SINRs for PSD-1 and PSD-2 when $N_P \rightarrow \infty$ can be derived as

$$\tilde{\gamma}_{P_k, \infty}^n = \frac{\mu_k E_{P_k}}{\Delta_{P_k, \infty}^n} \quad \text{and} \quad \tilde{\gamma}_{S_m, \infty}^n = \frac{\eta_m E_{S_m}}{\Delta_{S_m, \infty}^n}, \quad (56)$$

where $\Delta_{P_k, \infty}^n$ and $\Delta_{S_m, \infty}^n$ for $n \in \{\text{PSD-1}, \text{PSD-2}\}$ can be defined as

$$\Delta_{P_k, \infty}^{\text{PSD-1}} = \frac{\sigma_P^2}{\sigma_{F_k}^2} + \eta_k \beta E_{S_k} \zeta_{F_k}^{-2} \zeta_{V_k}^2 \quad \text{and} \quad \Delta_{P_k, \infty}^{\text{PSD-2}} = \frac{\sigma_P^2}{\sigma_{F_k}^2}, \quad (57a)$$

$$\Delta_{S_m, \infty}^{\text{PSD-1}} = \frac{\sigma_S^2}{\sigma_{G_m}^2} + \frac{m u_m}{\beta} E_{P_m} \zeta_{G_m}^{-2} \zeta_{U_m}^2 \quad \text{and} \quad \Delta_{S_m, \infty}^{\text{PSD-2}} = \frac{\sigma_S^2}{\sigma_{G_m}^2}, \quad (57b)$$

where $\beta = N_P/N_S$. It can be observed from (56) and (57a) that the asymptotic SINR of the k th PU becomes independent of the transmit powers of the other PUs. In underlay spectrum-sharing cognitive systems, the PUs are given higher priority than the SUs. Hence, in order to investigate the worst-case scenario for the SUs, the maximum available transmit power is utilized at each PU by letting $\mu_k = 1, \forall k$. Thereby, a simple max-min fairness power allocation problem can be reformulated for the SUs.

1) *Transmit power allocation for SUs with PSD-1:* In this subsection, the transmit power control coefficients for the SUs are derived in closed-form for PSD-1. To this end, by allocating maximum available transmit powers to the PUs, a max-min power control problem for the SUs can be reformulated as

$\mathcal{OP}_3^\infty$:

$$\text{maximize}_{\eta_m} \min_m \eta_m E_{S_m} / \left(\sigma_S^2 / \sigma_{G_m}^2 + E_{P_m} \zeta_{G_m}^{-2} \zeta_{U_m}^2 / \beta \right), \quad (58a)$$

$$\text{subject to } C_1: 0 \leq \eta_m \leq 1, \quad \forall m, \quad \text{and}, \quad (58b)$$

$$\text{subject to } C_2: E_{P_k} / \left(\sigma_P^2 / \sigma_{F_k}^2 + \eta_k \beta E_{S_k} \zeta_{F_k}^{-2} \zeta_{V_k}^2 \right) \geq \Gamma_{P_k, \infty}^n, \quad \forall k, \quad (58c)$$

where $\Gamma_{P_k, \infty}^n$ is the pre-defined SINR threshold for the k th PU such that $\Gamma_{P_k, \infty}^n \leq E_{P_k} \sigma_{F_k}^2 / \sigma_P^2$. By subjecting η_i in (58c) and by using (58b), it can be shown

that $\eta_i \leq \min(\bar{\eta}, 1)$ for $i \in \{1, \dots, K\}$, where $\bar{\eta} = \min_k \left[\left(\Gamma_{P_k, \infty}^n - \sigma_P^2 E_{P_k}^{-1} / \sigma_{F_k}^2 \right) E_{P_k} / \left(\beta E_{S_k} \zeta_{F_k}^{-2} \zeta_{V_k}^2 \right) \right]$. Then, in order to achieve max-min fairness among SUs, the SINR targets of SUs are set to a common SINR, $\Gamma_{S_\infty^{\text{PSD-1}}}$, as follows:

$$\eta_m E_{S_m} / \theta_m = \Gamma_{S_\infty^{\text{PSD-1}}}. \quad (59a)$$

where $\theta_m = \sigma_S^2 / \sigma_{G_m}^2 + E_{P_m} \zeta_{G_m}^{-2} \zeta_{U_m}^2 / \beta$. By using the fact that $0 \leq \eta_m \leq \min(\bar{\eta}, 1)$, a maximum for $\Gamma_{S_\infty^{\text{PSD-1}}}$ can be derived as

$$\begin{aligned} 0 \leq \eta_m &= E_{S_m}^{-1} \Gamma_{S_\infty^{\text{PSD-1}}} \theta_m \leq \min(\bar{\eta}, 1), \quad \forall m, \\ \Gamma_{S_\infty^{\text{PSD-1}}} &\leq E_{S_m} \min(\bar{\eta}, 1) / \theta_m, \quad \forall m, \\ \max\{\Gamma_{S_\infty^{\text{PSD-1}}}\} &= \min(\bar{\eta}, 1) / \max_m (E_{S_m}^{-1} \theta_m), \end{aligned} \quad (60)$$

where η_m can be calculated as

$$\eta_m = \frac{E_{S_m}^{-1} \left(\sigma_S^2 / \sigma_{G_m}^2 + E_{P_m} \zeta_{G_m}^{-2} \zeta_{U_m}^2 / \beta \right) \min(\bar{\eta}, 1)}{\max_m \left(E_{S_m}^{-1} \sigma_S^2 / \sigma_{G_m}^2 + E_{S_m}^{-1} E_{P_m} \zeta_{G_m}^{-2} \zeta_{U_m}^2 / \beta \right)}, \quad \forall m. \quad (61)$$

The achievable common achievable rate at the SUs can then be written as

$$\mathcal{R}_S^\infty \xrightarrow{N_P, N_S \rightarrow \infty} \psi_S \log \left(1 + \frac{\min(\bar{\eta}, 1)}{\max_m \left(\frac{\sigma_S^2}{E_{S_m} \sigma_{G_m}^2} + \frac{E_{P_m} \zeta_{G_m}^2}{E_{S_m} \zeta_{G_m}^2 \beta} \right)} \right). \quad (62)$$

2) *Transmit power allocation for SUs with PSD-2:* In this subsection, the transmit power coefficients for SUs with PSD-2 are derived. As per Remark 3, for PSD-2, the primary and secondary systems can be operated independent of each other when $N_P \rightarrow \infty$. This is because the secondary transmit power constraints and the achievable rates asymptotically become independent of the PIT when PSD-2 is invoked. Hence, the transmit power allocation for SUs does not affect the asymptotic achievable rates of the PUs. By following steps similar to those in (60), $\max\{\Gamma_{S_\infty^{\text{PSD-2}}}\}$ and η_m for PSD-2 can be derived as

$$\max\{\Gamma_{S_\infty^{\text{PSD-2}}}\} = \frac{1}{\max_m \left(E_{S_m}^{-1} \sigma_S^2 / \sigma_{G_m}^2 \right)}, \quad (63a)$$

$$\eta_m = \frac{E_{S_m}^{-1} \sigma_S^2 / \sigma_{G_m}^2}{\max_m \left(E_{S_m}^{-1} \sigma_S^2 / \sigma_{G_m}^2 \right)}. \quad (63b)$$

The common achievable rate of the SUs for PSD-2 can be derived as

$$\mathcal{R}_S^\infty \xrightarrow{N_P, N_S \rightarrow \infty} = \psi_S \log \left(1 + \left[\max_m \left(E_{S_m}^{-1} \sigma_S^2 / \sigma_{G_m}^2 \right) \right]^{-1} \right). \quad (64)$$

VII. PERFORMANCE ANALYSIS FOR PERFECT CSI

In this section, the performance metrics for the perfect CSI case are investigated. Thereby, the amounts of performance degradation due to practical impairments such as channel estimation errors, CCI and pilot contamination can be quantified. To begin with, the ZF detectors at the PBS and SBS can be constructed with the availability of the genie-aided perfect CSI as follows:

$$\mathbf{W}_{P_{ll}}^T = (\mathbf{F}_{ll}^H \mathbf{F}_{ll})^{-1} \mathbf{F}_{ll}^H \quad \text{and} \quad \mathbf{W}_{S_{ll}}^T = (\mathbf{G}_{ll}^H \mathbf{G}_{ll})^{-1} \mathbf{G}_{ll}^H. \quad (65)$$

By replacing $\mathbf{W}_{P_{ll}}^T$ and $\mathbf{W}_{S_{ll}}^T$ with $\hat{\mathbf{W}}_{P_{ll}}^T$ and $\hat{\mathbf{W}}_{S_{ll}}^T$, respectively, in (20) and (15)-(17), the secondary transmit power constraints and the achievable rates are derived for the perfect CSI case can be derived as shown in Table I (see Appendix B for the derivations).

Remark 5: The design insights, which can be obtained via our performance analysis for the perfect CSI case in Table I, can be summarized as follows: The asymptotic secondary transmit power constraint and the achievable sum rates become independent of the PIT (I_P) when the number of PBS antennas grows unbounded. Hence, the secondary system can be operated at its peak transmit power in the asymptotic PBS antenna regime. This observation reveals that the primary and secondary systems can be operated independent of each other in underlay spectrum-sharing mode when the PBS/SBS is massive MIMO enabled. When the number of antennas at the PBS/SBS is finite, the secondary power constraints and other performance metrics dependent on the PIT. Hence, the primary and secondary systems can no longer be operated independently. Moreover, in this antenna regime, the inter-cell and intra-cell CCI cannot be mitigated with simple ZF detectors even with perfect CSI. Consequently, the achievable sum rates are degraded by inter-cell and intra-cell CCI in the finite PBS/SBS antenna regime. Nevertheless, when the PBS and SBS are equipped with massive MIMO, the detrimental effects of the intra-cell and inter-cell secondary/primary CCI can be asymptotically mitigated. These design insights reveal that massive MIMO techniques can be exploited to operate the secondary systems in underlay spectrum-sharing mode without degrading the asymptotic performance of the primary systems.

VIII. NUMERICAL RESULTS

The channels are modeled as independent, quasi-static/block Rayleigh fading with a coherence interval of 196 symbol durations, and the pilot sequence length is set to $\tau = M = K$ symbol durations for PSD-1 and $\tau = 2M = 2K$ for PSD-2. The distance vectors between PUs-PBS, SUs-SBS, PUs-SBS and SUs-PBS are denoted by $\mathbf{d}_F$, $\mathbf{d}_G$, $\mathbf{d}_U$, and $\mathbf{d}_V$, respectively. All channels are modeled as Rayleigh fading. Unless otherwise stated, the k th diagonal element of $\mathbf{D}_{H_{ll}}$ for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$ captures the large-scale fading and can be modeled as $\zeta_{H_{ll,p}} = \text{PL}_{ll,p} \times 10^{\psi_{ll,p}/10}$, where $\text{PL}_{ll,p}$ accounts for the path-loss, and $10^{\psi_{ll,p}/10}$ captures the shadow fading effects with $\psi_{ll,p} \sim \mathcal{N}(0, 4^2)$ [30]. The path-loss is modeled as $\text{PL}_{ll,p} = (d_k/d_0)^{-n}$, where d_k is the corresponding distance between the transmitter-receiver pair, d_0 is a close-in reference distance, and n is the path-loss exponent, respectively. Moreover, $\mathbf{D}_{H_{li}} = \delta_{H_{li}} \mathbf{D}_{H_{ll}}$ for $i \in \{1, \dots, L\}$ and $i \neq l$. The average transmit SNR is defined as $\bar{\gamma}_T = P_P/\sigma_P^2$ or $\bar{\gamma}_T = P_{S_{\max}}/\sigma_S^2$, where σ_P^2 and σ_S^2 are the AWGN variances at the receivers in PBS and SBS, respectively. The Monte-Carlo simulations are based on the ergodic achievable sum rates in (42) and (43).

In Fig. 2 and Fig. 3, the behavior of the SU transmit power in the underlay spectrum-sharing massive MIMO systems is observed as functions of the PIT (I_P) and the maximum secondary transmit power $P_{S_{\max}}$. In Fig. 2, the secondary transmit power constraint for PSD-1 is plotted against I_P by

Fig. 2. Transmit power constraint of PSD-1 versus I_P for $\beta = 2$, $L = 10$, $K = M = 10$, $P_{S_{\max}} = 20$ dBm, $\sigma_P^2 = \sigma_S^2 = 0$ dBm, and $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K) \text{ m}$, $n = 2.4$ and $d_0 = 100$ m. Hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$, $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

Fig. 3. Transmit power constraint of PSD-2 versus $P_{S_{\max}}$ for $\beta = 2$, $L = 10$, $K = M = 10$, $I_P = -30$ dBm, $\sigma_P^2 = \sigma_S^2 = 0$ dBm, and $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K) \text{ m}$, $n = 2.4$ and $d_0 = 100$ m. Hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$, $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

gradually increasing the number of PBS antennas (N_P). The asymptotic SU power constraint is plotted by using (26), and the curves corresponding to finite PBS antenna regime are plotted by using (21). Fig. 2 clearly reveals that the secondary transmit power constraint linearly increases with I_P , and hence, the secondary system cannot be operated independent of the primary system even in the asymptotic PBS antenna regime. This asymptotic constraint can be achieved when N_P grows without bound. Nevertheless, the dependency of the SU power constraint vanishes as I_P increases beyond -30 dBm. This is because the endurance level at the PBS against the secondary CCI increases with increasing I_P . In Fig. 3, the secondary power constraint for PSD-2 is plotted against the maximum available SU transmit power ($P_{S_{\max}}$) by gradually increasing N_P . The analytical power constraints are plotted by using (27) and (21). Fig. 3 reveals that the asymptotic secondary power constraint increases with $P_{S_{\max}}$ for a fixed $I_P = -30$ dBm. As per (27), in PSD-2, the secondary power

TABLE I
PERFORMANCE METRICS FOR THE PERFECT CSI CASE

Perfect CSI analysis	Performance metric
Secondary power constraint (N_P is finite)	$\bar{P}_S = \min \left(P_{S_{\max}}, \frac{I_P(N_P - K)}{\sum_{i=1}^L \text{Tr}(\mathbf{D}_{V_{li}}) \text{Tr}(\mathbf{D}_{F_{li}}^{-1})} \right)$
Secondary power constraint ($N_P \rightarrow \infty$)	$\min_{N_P \rightarrow \infty} P_S = P_{S_{\max}}$
Sum rate for PUs (N_P and N_S are finite)	$\bar{\mathcal{R}}_{P_l}^{lb} = \sum_{k=1}^K \log_2 \left(1 + \frac{P_P \zeta_{F_{ll,k}}(N_P - K)}{\sigma_P^2 + \sum_{i=1, i \neq l}^L P_P \text{Tr}(\mathbf{D}_{F_{li}}) + \sum_{i=1}^L \bar{P}_S \text{Tr}(\mathbf{D}_{V_{li}})} \right)$
Sum rate for SUs (N_P and N_S are finite)	$\bar{\mathcal{R}}_{S_l}^{lb} = \sum_{m=1}^M \log_2 \left(1 + \frac{\bar{P}_S \zeta_{G_{ll,m}}(N_S - M)}{\sigma_S^2 + \sum_{i=1, i \neq l}^L \bar{P}_S \text{Tr}(\mathbf{D}_{G_{li}}) + \sum_{i=1}^L P_P \text{Tr}(\mathbf{D}_{U_{li}})} \right)$
Sum rate for PUs ($N_P \rightarrow \infty$ and N_S is finite)	$\mathcal{R}_{P_l}^{\infty} = \sum_{k=1}^K \log_2 \left(1 + E_P \zeta_{F_{ll,k}} / \sigma_P^2 \right)$
Sum rate for SUs ($N_P \rightarrow \infty$ and N_S is finite)	$\bar{\mathcal{R}}_{S_l}^{lb} = \sum_{m=1}^M \log_2 \left(1 + \frac{P_{S_{\max}} \zeta_{G_{ll,m}}(N_S - M)}{\sigma_S^2 + \sum_{i=1, i \neq l}^L P_{S_{\max}} \text{Tr}(\mathbf{D}_{G_{li}})} \right)$
Asymptotic sum rate for PUs ($N_P \rightarrow \infty$ and $N_S \rightarrow \infty$)	$\mathcal{R}_{P_l}^{\infty} = \sum_{k=1}^K \log_2 \left(1 + E_P \zeta_{F_{ll,k}} / \sigma_P^2 \right)$
Asymptotic sum rate for SUs ($N_P \rightarrow \infty$ and $N_S \rightarrow \infty$)	$\mathcal{R}_{S_l}^{\infty} = \sum_{m=1}^M \log_2 \left(1 + E_S \zeta_{G_{ll,m}} / \sigma_S^2 \right)$

Fig. 4. Achievable sum rate of PSD-1 with γ_T for $N_S = 20, \beta = 2, K = M = 5, \sigma_P^2 = \sigma_S^2 = 0$ dBm, $I_P = 30$ dBm and $P_{S_{\max}} = P_P = \gamma_T$. Moreover, $d_0 = 100$ m, $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K)$ m, and hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$ and $\mathbf{D}_{H_{li}} = 0.1 \mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

constraint becomes independent of I_P in the asymptotic PBS antenna regime, and hence, the secondary system can be operated at the peak transmit power without hindering the asymptotic PU sum rate limit. However, Fig. 3 shows that the secondary power constraint saturates when $P_{S_{\max}}$ increases beyond 0 dBm in the finite PBS antenna regime, and hence, the SU transmit power can no longer be increased until $P_{S_{\max}}$. Monte-Carlo simulations validate our analysis in (26), (27) and (21).

In Fig. 4, the achievable sum rate of the PUs and SUs for PSD-1 are plotted for the single-cell and five-cell cases in order to investigate the detrimental effects of intra/inter-cell pilot contamination and CCI in the finite PBS/SBS antenna

Fig. 5. Achievable sum rate with number of PUs/SUs for $N_S = 20, \beta = 2, L = 1, I_P = 30$ dBm and $P_{S_{\max}} = P_P = \gamma_T$ with $d_0 = 100$ m, $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K)$ m. Hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$ and $\mathbf{D}_{H_{li}} = 0.4 \mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

regime. The analytical sum rate curves of the PUs and SUs are plotted by using (36) and (39), respectively, and are compared against the Monte-Carlo simulations of the ergodic sum rates. Fig. 4 reveals that the inter/intra-cell pilot contamination and CCI severely degrade the achievable sum rate. For instance, at an average SNR of 10 dB, about 60.9% sum rate loss is observed when the number of co-channel cells increases from $L = 1$ to $L = 5$ for both primary and secondary systems. Since the secondary transmit power is constrained as per (21) in the finite antenna regime, the achievable sum rate of the PUs outperforms that of the SUs by 3.09 bits/s/Hz at an average SNR of 10 dB. Moreover, the achievable sum rate of the secondary system decreases when the average transmit SNR

Fig. 6. Achievable sum rate of PUs/SUs for PSD-1 with $K = M = 5$, $N_S = 20$, $\beta = 2$, $I_P = 20$ dBm, $E_P = P_P/N_P$, $E_{S_{\max}} = P_{S_{\max}}/N_S$ and $E_P = E_{S_{\max}} = 30$ dBm, $\sigma_S^2 = \sigma_P^2 = 5$ dBm. Moreover, $d_0 = 100$ m, $\mathbf{d}_F = [100 \ 150 \ 300 \ 200 \ 120]$ m, $\mathbf{d}_G = [100 \ 150 \ 200 \ 120 \ 170]$ m, $\mathbf{d}_U = \mathbf{d}_V = 200 \text{ones}(1, K)$ m, and $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

grows beyond 15 dB. This is due to enforcement of secondary transmit power constraints at the SUs for underlay spectrum-sharing to ensure that the secondary co-channel interference falls below the PIT. Our analysis is compared against the Monte-Carlo simulations to validate the fact that the achievable rates derived via the worst-case Gaussian technique provides tight lower bounds to the ergodic rates in the useful SNR regime.

In Fig. 5, the achievable sum rates for PSD-1 and PSD-2 are plotted against the number of PUs/SUs, which is same as the pilot sequence length (τ). The analytical curves are plotted by letting $\tau = K$ and $\tau = 2K$ for PSD-1 and PSD-2, respectively, in (36) and (39). Fig. 5 reveals that the number of PUs/SUs that can be served simultaneously in the same time-frequency resource block by PSD-2 is substantially less than that of PSD-1. This is due to the fact that in order to maintain the orthogonal pilots for the PUs and SUs in the same-cell, PSD-2 requires twice the pilot sequence length of PSD-1. Nevertheless, Fig. 5 shows that PSD-2 outperforms PSD-1 in terms of the achievable sum rate when the number of PUs/SUs increases until 36/81. For example, when the $M = K = 25$, PSD-2 achieves 19.46% and 100.2% sum rate gains by the primary and secondary systems, respectively, over that of PSD-1. This is because PSD-2 can mitigate intra-cell pilot contamination, while the achievable sum rate of PSD-1 is degraded by both intra-cell and inter-cell pilot contamination.

In Fig. 6, the achievable sum rates of the PUs and SUs for PSD-1 are plotted against the number of SBS antennas ($N_P = \beta N_S$) for investigating the detrimental effects of intra/inter-cell pilot contamination and CCI in the asymptotic PBS/SBS antenna regime. The asymptotic sum rate curves are plotted by using (44) and (46), respectively, and are compared against the Monte-Carlo simulations. Fig. 6 shows that the intra/inter-cell pilot contamination results in asymptotic sum rate losses of 13.07 bits/s/Hz and 7.21 bits/s/Hz for primary and secondary systems, respectively, when the number of co-channel cells increases from $L = 1$ to $L = 10$. Moreover, the PUs outperforms SUs by 30.42% and 17.7% in terms of the asymptotic sum rate for $L = 1$ and $L = 10$, respectively. Fig. 6 reveals that the theoretical/asymptotic sum rate limits in

Fig. 7. Comparison of PU sum rate for PSD-1 and PSD-2 with $K = M = 5$, $N_S = 100$, $\beta = 2$, $I_P = 20$ dBm, $P_{S_{\max}} = P_P = 30$ dBm. Here, $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ones}(1, K)$ m and $\mathbf{D}_{H_{li}} = 0.2\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

Fig. 8. Achievable SU rates for PSD-1 with power control. Here, $K = M = 4$, $N_S = 10$, $\beta = 2$, $L = 1$, $I_P = 30$ dBm, $P_{P_k} = P_{S_{\max}} = \gamma T$, $\sigma_P^2 = \sigma_S^2 = 0$ dBm, $d_0 = 100$ m, $\mathbf{d}_F = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ones}(1, K)$ m, $\mathbf{d}_G = [110 \ 120 \ 170 \ 190]$ m and $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

(44) and (46) can be achieved when the number of PBS/SBS antennas grows unbounded.

In Fig. 7, the percentage sum rate gain of PSD-2 over PSD-1 is plotted for the primary system as a function of the number of co-channel cells (L). This percentage sum rate gain is defined as $[(\mathcal{R}_{P_i}^{\text{PSD-2}} - \mathcal{R}_{P_i}^{\text{PSD-1}})/\mathcal{R}_{P_i}^{\text{PSD-1}} \times 100\%]$. The asymptotic curve is plotted by using (44) and (48), while the curves corresponding to finite PBS antenna regime is plotted by using (36). Fig. 7 reveals that the PSD-2 provides the largest sum rate gain over PSD-1 for the single-cell ($L = 1$) case. This is due to the fact that the intra-cell pilot contamination can be mitigated by using PSD-2, while the achievable rate of PSD-1 is severely affected by the intra-cell pilot contamination. Moreover, the sum rates of both PSD-1 and PSD-2 are not affected by the inter-cell pilot contamination for $L = 1$ case. Nevertheless, the percentage sum rate gain of PSD-2 gradually decreases when L increases. This is because the achievable sum rate of PSD-2 gets affected by the inter-cell pilot contamination for $L > 1$ cases. Moreover, as $L > 6$, PSD-2 provides diminishing gains over PSD-1 as the inter-cell pilot contamination becomes more dominant sum rate deteriorating factor.

In Fig. 8, the achievable user rates for PSD-1 is plotted

Fig. 9. Achievable PU/SU rates with power control for PSD-1 and $K = M = 4$, $N_S = 10$, $\beta = 2$, $L = 1$, $I_P = 30$ dBm, $P_{P_k} = P_{S_{\max}} = \bar{\gamma}_T$, $\sigma_P^2 = \sigma_S^2 = 0$ dBm, $d_0 = 100$ m, $\mathbf{d}_F = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K) \text{ m}$, $\mathbf{d}_G = [110, 120, 170, 190] \text{ m}$ and $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

Fig. 10. The effect of CCI on the achievable sum rate for the perfect CSI case with $E_P = 34$ dBm, $E_S = 32$ dBm, $\sigma_P^2 = \sigma_S^2 = 5$ dBm, $I_P = 20$ dBm, $\beta = 2$, $M = 6$, $K = 10$, $n = 2.6$, $d_0 = 1$ m, $\mathbf{d}_F = \mathbf{d}_G = 10$ m and $\mathbf{d}_U = \mathbf{d}_V = 100$ m. Hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$ and $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

against the $\bar{\gamma}_T$ for (i) the equal power allocation and (ii) optimal power allocation with max-min fairness among SUs, while the predefined SINR thresholds are achieved by PUs. The SUs are spatially distributed as $\mathbf{d}_G = [110, 120, 170, 190] \text{ m}$. The power allocation coefficients for the optimal power allocation are computed by solving the geometric program in (54a)-(54c) by using Matlab CVX. To this end, in Fig. 8, the achievable SU rates are compared for the equal and optimal power control policies. The achievable SU rates with the equal power control depends heavily on the SU spatial location. However, our max-min fairness power control provides a common achievable rate for every SU regardless of their spatial locations. In Fig. 9, the achievable PU/SU rates are plotted for PSD-1 with optimal power control. The SINR thresholds for the PUs are set as $\Gamma_{P_k}^{\text{PSD-1}} = [\bar{\gamma}_T, 0.8\bar{\gamma}_T, 0.6\bar{\gamma}_T, 0.4\bar{\gamma}_T]$. Fig. 9 reveals that the our power control policy achieves the predefined SINR thresholds for PUs, while ensuring the max-min fairness among SUs.

In Fig. 10, the achievable sum rate of the PUs and SUs are plotted against the number of PBS antennas by varying the number of co-channel interferers (L) for the perfect CSI case. The asymptotic sum rate curves are plotted by using $\mathcal{R}_{P_i}^\infty$ and

Fig. 11. The achievable user rate comparison of PSD-1 and PSD-2 with the perfect CSI for $\beta = 2$, $I_P = 20$ dBm, $K = M = 5$, $N_S = 40$, $P_{P_k} = P_{S_{\max}} = \bar{\gamma}_T$, $d_0 = 100$ m, and $\mathbf{d}_F = \mathbf{d}_G = \mathbf{d}_U = \mathbf{d}_V = 100 \text{ ones}(1, K) \text{ m}$. Hence, $\mathbf{D}_{H_{ll}} = \mathbf{I}$ and $\mathbf{D}_{H_{li}} = 0.1\mathbf{D}_{H_{ll}}$, where $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}, \mathbf{U}, \mathbf{V}\}$, $i \in \{1, \dots, L\}$ and $i \neq l$.

$\mathcal{R}_{S_i}^\infty$ in Table I, whereas the ergodic sum rate curves are plotted by using Monte-Carlo simulations. The sum rate curve corresponding to $L = 1$ only accounts for intra-cell CCI, whereas the curves pertinent to $L = \{3, 6, 12\}$ capture effects of both intra-cell and inter-cell CCI. Fig. 10 reveals that the cumulative effects of intra-cell and inter-cell interference indeed degrade the achievable sum rate when the PBS/SBS is equipped with arbitrarily small amount of antennas. Nevertheless, the sum rate curves gradually approach the corresponding asymptotic limits as the number of PBS/SBS antennas grows unbounded irrespective of L in the presence of perfect CSI. Hence, both intra-cell and inter-cell interference can asymptotically be mitigated by exploiting very large antenna arrays at the PBS/SBS. This observation validates the insights summarized in Remark 5. Therefore, massive MIMO can be exploited to operate spectrum sharing secondary systems without degrading the asymptotic performance of the primary system.

In Fig. 11, the percentage sum rate loss of estimated/imperfect CSI (PSD-1 and PSD-2) over perfect CSI case is plotted against the average SNR for the primary system. Fig. 11 shows that this sum rate loss is more dominant in low SNR regime. Moreover, this sum rate degradation becomes more severe when the number of co-channel cells increases. The reason for the kink/dip in sum rate of loss at the average transmit SNR of 3 dB is due to the fact that the rate loss decreases gradually with the SNR until 3 dB and then the rate of decrease of sum rate loss reduces beyond the SNR of 3 dB due to secondary transmit power constraints.

IX. CONCLUSION

The performance of multi-cell multi-user cognitive massive MIMO systems with underlay spectrum sharing has been investigated. The secondary transmit power constraints and the achievable sum rates have been derived for imperfect/perfect CSI cases in finite/infinite PBS/SBS antenna regimes. The cumulative performance losses due to imperfect CSI, CCI and inter/intra-cell pilot contamination have been investigated for PSDs. The uplink transmit power control based on max-min user fairness has been investigated. Thereby, the optimal power allocation coefficients and the optimal common achievable rate for SUs have been derived. Our analysis for the perfect

CSI case reveals that the spectrum-sharing secondary system can be operated independent of the primary system as the asymptotic performance metrics no longer depend on the PIT when the number of PBS antennas grows unbounded. Nevertheless, the aforementioned observation vanishes when the PU pilot sequences are shared among the SUs and PUs in the co-channel cells according to the PSD-1. Consequently, in PSD-1, the achievable rates of PUs/SUs degrade severely by both intra-cell and inter-cell pilot contamination. However, the detrimental effects of intra-cell pilot contamination can be mitigated by using PSD-2. Specifically, in PSD-2, the secondary power constraint asymptotically becomes independent of the PIT. Consequently, the SUs can be operated at the peak transmit power when the number of PBS antenna grows unbounded. Hence, a certain fraction of performance gains promised by the perfect CSI case can also be obtained via PSD-2. Moreover, the SU power allocation coefficients depend only on the long-term channel statistics, and hence, the proposed max-min fairness power control can be implemented at the SUs, which rely on the channel hardening only. Our analysis and simulation results reveal that massive MIMO can be exploited to significantly boost the achievable rates of the PUs and SUs in cognitive systems with underlay spectrum sharing.

APPENDIX A

DERIVATIONS FOR THE PERFORMANCE METRICS FOR IMPERFECT CSI CASE

A. Derivation of the transmit power constraint for the secondary system:

To begin with, the derivations of the transmit power constraints at the SUs for PSD-1 and PSD-2 are outlined. By using (7), $\hat{\mathbf{F}}_{li}$ can be re-written as

$$\hat{\mathbf{F}}_{li} = \hat{\mathbf{F}}_{li} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{F_{li}}. \quad (66)$$

Then, one can show that

$$\begin{aligned} \hat{\mathbf{W}}_{P_{li}}^T &= (\hat{\mathbf{F}}_{li}^H \hat{\mathbf{F}}_{li})^{-1} \hat{\mathbf{F}}_{li}^H \\ &= (\mathbf{D}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{F}}_{li}^H \hat{\mathbf{F}}_{li} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{F_{li}})^{-1} \mathbf{D}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{F}}_{li}^H \\ &= \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{F_{li}} (\hat{\mathbf{F}}_{li}^H \hat{\mathbf{F}}_{li})^{-1} \hat{\mathbf{F}}_{li}^H = \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{F_{li}} \tilde{\mathbf{W}}_{P_{li}}^T. \end{aligned} \quad (67)$$

For the sake of mathematical manipulations, in the case of PSD-1, MMSE channel estimation for $\mathbf{V}_{li}$ can be derived from (4) as follows:

$$\begin{aligned} \hat{\mathbf{V}}_{li} &= \frac{1}{\sqrt{p_0}} \mathbf{Y}_{P_{li}} \Phi^H \left(\sum_{i=1}^L (\mathbf{D}_{F_{li}} + \mathbf{D}_{V_{li}}) + \frac{\mathbf{I}_K}{p_0} \right)^{-1} \mathbf{D}_{V_{li}} \\ &= \left(\sum_{i=1}^L (\mathbf{F}_{li} + \mathbf{V}_{li}) + \frac{\mathbf{N}_{F_{li}}}{\sqrt{p_0}} \right) \left(\sum_{i=1}^L (\mathbf{D}_{F_{li}} + \mathbf{D}_{V_{li}}) + \frac{\mathbf{I}_K}{p_0} \right)^{-1} \mathbf{D}_{V_{li}}, \end{aligned} \quad (68)$$

where Φ is the shared pilot sequence among primary and secondary users. By using (7), (68) and (66), with several mathematical manipulations, it can be shown that

$$\hat{\mathbf{F}}_{li} \mathbf{D}_{F_{li}}^{-1} = \hat{\mathbf{V}}_{li} \mathbf{D}_{V_{li}}^{-1}. \quad (69)$$

Next, Z appears in the transmit power constraint (20) can be re-written by using (68) as

$$\begin{aligned} Z &= \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \mathbf{V}_{li} \mathbf{V}_{li}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \\ &= \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \left(\hat{\mathbf{V}}_{li} + \mathcal{E}_{V_{li}}^H \right) \left(\hat{\mathbf{V}}_{li}^H + \mathcal{E}_{V_{li}}^H \right) \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \\ &= \underbrace{\mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \hat{\mathbf{V}}_{li} \hat{\mathbf{V}}_{li}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right]}_{\mathcal{I}_1} + \underbrace{\mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \mathcal{E}_{V_{li}}^H \mathcal{E}_{V_{li}}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right]}_{\mathcal{I}_2}, \end{aligned} \quad (70)$$

where $\mathcal{E}_{V_{li}}$ is the estimation error of $\mathbf{V}_{li}$. By using (67) and (69), $\mathcal{I}_1$ and $\mathcal{I}_2$ can be calculated as

$$\begin{aligned} \mathcal{I}_1 &= \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \hat{\mathbf{V}}_{li} \hat{\mathbf{V}}_{li}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \\ &\stackrel{(a)}{=} \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \hat{\mathbf{F}}_{li} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}^2 \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{F}}_{li}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \\ &\stackrel{(b)}{=} \mathbb{E} \left[\text{Tr} \left(\mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{F_{li}} \tilde{\mathbf{W}}_{P_{li}}^T \hat{\mathbf{F}}_{li} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}^2 \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{F}}_{li}^H \tilde{\mathbf{W}}_{P_{li}}^* \mathbf{D}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \right) \right] \\ &= \text{Tr} \left(\mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}^2 \mathbf{D}_{F_{li}}^{-1} \right), \end{aligned} \quad (71a)$$

$$\begin{aligned} \mathcal{I}_2 &= \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \mathcal{E}_{V_{li}}^H \mathcal{E}_{V_{li}}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \\ &= \mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \hat{\mathbf{W}}_{P_{li}}^* \right) \right] \mathbb{E} \left[\text{Tr} \left(\mathcal{E}_{V_{li}}^H \mathcal{E}_{V_{li}}^H \right) \right] / N_P \\ &= \frac{\text{Tr} \left(\hat{\mathbf{D}}_{F_{li}}^{-1} \right)}{N_P - K} \text{Tr} \left(\mathbf{D}_{V_{li}} - \hat{\mathbf{D}}_{V_{li}} \right) \\ &= \frac{\text{Tr} \left(\hat{\mathbf{D}}_{F_{li}}^{-1} \right)}{N_P - K} \text{Tr} \left(\mathbf{D}_{V_{li}} - \mathbf{D}_{V_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{D}}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}} \right). \end{aligned} \quad (71b)$$

In (71a), step (a) and (b) are achieved by invoking (69) and (67), respectively. By substituting (71a) and (71b) into (70), the transmit power constraint for PSD-1 can be derived as shown in (21) and (23). Whereas, the transmit power constraint for PSD-2 can be derived as shown in (21) and (23) by using the following identity [31]

$$\mathbb{E} \left[\text{Tr} \left(\hat{\mathbf{W}}_{P_{li}}^T \mathbf{V}_{li} \mathbf{V}_{li}^H \hat{\mathbf{W}}_{P_{li}}^* \right) \right] = \frac{\text{Tr}(\mathbf{D}_{V_{li}}) \text{Tr}(\hat{\mathbf{D}}_{F_{li}}^{-1})}{N_P - K}. \quad (72)$$

B. Derivation of the achievable rate at the primary system for PSD-1:

To begin with, the term $(\hat{\mathbf{W}}_{P_{li}}^T \mathbf{F}_{li})$ in (30) can be expanded by invoking (16) and (8) as follows:

$$\hat{\mathbf{W}}_{P_{li}}^T \mathbf{F}_{li} = \hat{\mathbf{W}}_{P_{li}}^T (\hat{\mathbf{F}}_{li} + \mathcal{E}_{F_{li}}) = \mathbf{I}_K + \hat{\mathbf{W}}_{P_{li}}^T \mathcal{E}_{F_{li}}. \quad (73)$$

Hence, the k th row can be written as

$$\hat{\mathbf{w}}_{P_{li},k}^T \mathbf{f}_{li,k} = 1 + \hat{\mathbf{w}}_{P_{li},k}^T \mathcal{E}_{f_{li,k}}, \quad (74)$$

where $\mathcal{E}_{f_{li,k}}$ is the k th column of $\mathcal{E}_{F_{li}}$. Since $\hat{\mathbf{w}}_{P_{li},k}^T$ and $\mathcal{E}_{f_{li,k}}$ are uncorrelated and $\mathcal{E}_{f_{li,k}}$ is a zero-mean random variable, $\mathbb{E}[\hat{\mathbf{w}}_{P_{li},k}^T \mathcal{E}_{f_{li,k}}] = 0$. Thus, one can show that

$$\mathbb{E} \left[\hat{\mathbf{w}}_{P_{li},k}^T \mathbf{f}_{li,k} \right] = 1. \quad (75)$$

The first term of the effective noise in (30), $\text{Var}(\hat{\mathbf{w}}_{P_{li},k}^T \mathbf{f}_{li,k})$, can be derived via (74)-(75) as

$$\begin{aligned} \text{Var}(\hat{\mathbf{w}}_{P_{li},k}^T \mathbf{f}_{li,k}) &= \mathbb{E} \left[\left| \hat{\mathbf{w}}_{P_{li},k}^T \mathcal{E}_{f_{li,k}} \right|^2 \right] \\ &= (\zeta_{F_{li},k} - \sigma_{\hat{F}_{li},k}^2) \mathbb{E} \left[\left\| \hat{\mathbf{w}}_{P_{li},k} \right\|^2 \right] = (\zeta_{F_{li},k} - \sigma_{\hat{F}_{li},k}^2) \mathbb{E} \left[\left(\hat{\mathbf{F}}_{li}^H \hat{\mathbf{F}}_{li} \right)^{-1} \right]_{k,k} \\ &= \frac{\zeta_{F_{li},k} - \sigma_{\hat{F}_{li},k}^2}{\sigma_{\hat{F}_{li},k}^2 K} \mathbb{E}[\text{Tr}(\mathbf{X}^{-1})] = \frac{\zeta_{F_{li},k} - \sigma_{\hat{F}_{li},k}^2}{\sigma_{\hat{F}_{li},k}^2 (N_P - K)}, \end{aligned} \quad (76)$$

where $\mathbf{X}$ is a $K \times K$ central Wishart matrix with N_P degrees of freedom and covariance matrix $\mathbf{I}_K$, where $\mathbb{E}[\text{Tr}(\mathbf{X}^{-1})] = K/(N_P - K)$ [31]. The computations of the remaining terms of the effective noise power in (31) can be outlined as follows:

- Computation of P_{I_1} : From (73), $\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,j} = \hat{\mathbf{w}}_{P_{ll,k}}^T \mathcal{E}_{f_{ll,j}}$ for $j \neq k$. Since $\hat{\mathbf{w}}_{P_{ll,k}}^T$ and $\mathcal{E}_{f_{ll,j}}$ are uncorrelated, then it can be shown that

$$\begin{aligned} \mathbb{E} \left[\left| \hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{f}_{ll,j} \right|^2 \right] &= (\zeta_{F_{ll,j}} - \sigma_{\hat{F}_{ll,j}}^2) \mathbb{E} \left[\left(\hat{\mathbf{F}}_{ll}^H \hat{\mathbf{F}}_{ll} \right)^{-1} \right]_{k,k} \\ &= \frac{\zeta_{F_{ll,j}} - \sigma_{\hat{F}_{ll,j}}^2}{\sigma_{\hat{F}_{ll,k}}^2 (N_P - K)}. \end{aligned} \quad (77)$$

- Computation of P_{I_2} : By using (67), P_{I_2} in (31b) can be re-written as

$$\begin{aligned} P_{I_2} &= \sum_{i=1, i \neq l}^L P_P \mathbb{E} \left[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{F}_{li} \mathbf{F}_{li}^H \hat{\mathbf{w}}_{P_{ll,k}}^* \right] \\ &= \sum_{i=1, i \neq l}^L P_P \mathbb{E} \left[\frac{\zeta_{F_{li,k}}}{\zeta_{F_{ll,k}}} \hat{\mathbf{w}}_{P_{li,k}}^T \left(\hat{\mathbf{F}}_{li} + \mathcal{E}_{F_{li}} \right) \left(\hat{\mathbf{F}}_{li}^H + \mathcal{E}_{F_{li}}^H \right) \hat{\mathbf{w}}_{P_{li,k}}^* \frac{\zeta_{F_{li,k}}}{\zeta_{F_{ll,k}}} \right] \\ &= \sum_{i=1, i \neq l}^L P_P \mathbb{E} \left[\frac{\zeta_{F_{li,k}}^2}{\zeta_{F_{ll,k}}^2} \left(\tilde{\mathbf{w}}_{P_{li,k}}^T \hat{\mathbf{F}}_{li} \hat{\mathbf{F}}_{li}^H \tilde{\mathbf{w}}_{P_{li,k}}^* + \tilde{\mathbf{w}}_{P_{li,k}}^T \mathcal{E}_{F_{li}} \mathcal{E}_{F_{li}}^H \tilde{\mathbf{w}}_{P_{li,k}}^* \right) \right] \\ &= \sum_{i=1, i \neq l}^L P_P \zeta_{F_{ll,k}}^{-2} \zeta_{F_{li,k}}^2 \left(1 + \frac{\sum_{j=1}^K (\zeta_{F_{li,j}} - \sigma_{\hat{F}_{li,j}}^2)}{\sigma_{\hat{F}_{li,k}}^2 (N_P - K)} \right). \end{aligned} \quad (78)$$

- Computation of P_{I_3} : By using (67) and (69), and following steps similar to those used in (71a) and (71b), P_{I_3} can be derived as

$$\begin{aligned} P_{I_3} &= \sum_{i=1}^L P_S \mathbb{E} \left[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{V}_{li} \mathbf{V}_{li}^H \hat{\mathbf{w}}_{P_{ll,k}}^* \right] \\ &= \sum_{i=1}^L P_S \left(\zeta_{F_{ll,k}}^{-2} \zeta_{V_{li,k}}^2 + \frac{\hat{\zeta}_{F_{ll,k}}^{-1}}{N_P - K} \text{Tr}(\mathcal{A}_i) \right). \end{aligned} \quad (79)$$

where $\mathcal{A}_i = \mathbf{D}_{V_{li}} - \mathbf{D}_{V_{li}} \mathbf{D}_{F_{li}}^{-1} \hat{\mathbf{D}}_{F_{li}} \mathbf{D}_{F_{li}}^{-1} \mathbf{D}_{V_{li}}$.

- Computation of P_4 : In (31c), similarly P_4 obtains as

$$P_4 = \sigma_P^2 \mathbb{E} \left[\hat{\mathbf{w}}_{P_{ll,k}}^T \hat{\mathbf{w}}_{P_{ll,k}}^* \right] = \frac{\sigma_P^2}{\sigma_{\hat{F}_{ll,k}}^2 (N_P - K)}. \quad (80)$$

By substituting (75), (76), (77), (78), (79), and (80) into (30), the achievable sum rate at the l th PBS can be derived as (36).

The achievable sum rate for PSD-2 can be derived by following steps similar to those in (73)-(80). The only difference is with the computation of P_{I_3} , in which the properties of uncorrelated random variables, $\hat{\mathbf{W}}_{P_{ll}}^T$ and $\mathbf{V}_{li}$, can be exploited to simplify the derivation as follows:

$$P_{I_3} = \sum_{i=1}^L P_S \mathbb{E} \left[\hat{\mathbf{w}}_{P_{ll,k}}^T \mathbf{V}_{li} \mathbf{V}_{li}^H \hat{\mathbf{w}}_{P_{ll,k}}^* \right] = \sum_{i=1}^L \frac{P_S \sum_{j=1}^K \zeta_{V_{li,j}}}{\sigma_{\hat{F}_{ll,k}}^2 (N_P - K)}. \quad (81)$$

APPENDIX B

DERIVATIONS OF THE PERFORMANCE METRICS FOR PERFECT CSI

A. Derivation for transmit power constraint at the secondary system:

To begin with, by assuming perfect CSI and by substituting (65) into (20), the transmit power constraint is re-written as

$$P_S = \min \left(P_{S_{\max}}, \frac{I_P}{\sum_{i=1}^L \mathbb{E}[Z]} \right), \quad (82)$$

where Z for the case of perfect CSI can be written as

$$\begin{aligned} \mathbb{E}[Z] &= \text{Tr} \left(\mathbb{E} \left[(\mathbf{F}_{ll}^H \mathbf{F}_{ll})^{-1} (\mathbf{F}_{ll}^H \mathbf{V}_{li}) (\mathbf{V}_{li}^H \mathbf{F}_{ll}) (\mathbf{F}_{ll}^H \mathbf{F}_{ll})^{-1} \right] \right) \\ &= \text{Tr} \left(\mathbb{E} \left[(\mathbf{F}_{ll}^H \mathbf{F}_{ll})^{-1} \right] \right) \text{Tr} \left(\mathbb{E} [\mathbf{V}_{li} \mathbf{V}_{li}^H] \right) / N_P \\ &= \text{Tr}(\mathbf{D}_{F_{ll}}^{-1}) \text{Tr}(\mathbf{D}_{V_{li}}) / (N_P - K). \end{aligned} \quad (83)$$

B. Derivation of $\bar{\mathcal{R}}_{S_l}^{lb}$ when both N_P and N_S are finite:

To begin with, by substituting (65) into (17), one can readily show that

$$\mathbf{w}_{S_{ll,m}}^T \mathbf{g}_{ll,m} = 1 \quad \text{and} \quad \sum_{j=1, j \neq m}^M \mathbf{w}_{S_{ll,m}}^T \mathbf{g}_{ll,j} = 0. \quad (84)$$

By using (15) and (84), the SINR of the k th data substream at the SBS in the l th cell can be written as

$$\gamma_{S_{l,k}} = \frac{P_S}{\sum_{i=1, i \neq l}^L P_P \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{G}_{li} \right\|^2 + \sum_{i=1}^L P_P \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{U}_{li} \right\|^2 + \sigma_{S_l}^2 \left\| \mathbf{w}_{S_{ll,m}}^T \right\|^2}. \quad (85)$$

Then, the achievable sum rate can be tightly lower bound as

$$\bar{\mathcal{R}}_{S_l} = \sum_{m=1}^M \mathbb{E} [\log(1 + \gamma_{S_{l,k}})] \geq \bar{\mathcal{R}}_{S_l}^{lb} = \sum_{m=1}^M \log \left(1 + \frac{1}{\mathbb{E}[\gamma_{S_{l,k}}^{-1}]} \right), \quad (86)$$

where $\mathbb{E}[\gamma_{S_{l,k}}^{-1}]$ can be written from (85) as follows:

$$\begin{aligned} \mathbb{E}[\gamma_{S_{l,k}}^{-1}] &= P_S^{-1} \left(\sum_{i=1, i \neq l}^L \mathbb{E} \left[P_S \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{G}_{li} \right\|^2 \right] \right. \\ &\quad \left. + \sum_{i=1}^L \mathbb{E} \left[P_P \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{U}_{li} \right\|^2 \right] + \sigma_{S_l}^2 \mathbb{E} \left[\left\| \mathbf{w}_{S_{ll,m}}^T \right\|^2 \right] \right). \end{aligned} \quad (87)$$

Next, by using [31], the following identities can be written:

$$\mathbb{E} \left[\left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{G}_{li} \right\|^2 \right] = \frac{\text{Tr}(\mathbf{D}_{G_{li}}) \zeta_{G_{li,m}}^{-1}}{N_S - M}, \quad (88a)$$

$$\mathbb{E} \left[\left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{U}_{li} \right\|^2 \right] = \frac{\text{Tr}(\mathbf{D}_{U_{li}}) \zeta_{U_{li,m}}^{-1}}{N_S - M}, \quad (88b)$$

$$\mathbb{E} \left[\left\| \mathbf{w}_{S_{ll,m}}^T \right\|^2 \right] = \frac{\zeta_{G_{ll,m}}^{-1}}{N_S - M}. \quad (88c)$$

By substituting (88b) into (87), (87) can be derived in closed-form as follows:

$$\mathbb{E}[\gamma_{S_{l,k}}^{-1}] = \frac{\left(\sigma_S^2 + \sum_{i=1, i \neq l}^L P_S \text{Tr}(\mathbf{D}_{G_{li}}) + \sum_{i=1}^L P_P \text{Tr}(\mathbf{D}_{U_{li}}) \right)}{P_S \zeta_{G_{ll,m}} (N_S - M)}. \quad (89)$$

By substituting (89) into (86), $\bar{\mathcal{R}}_{S_l}^{lb}$ can be derived as given in Table I.

C. Derivation for the transmit power constraint at the secondary system when $N_P \rightarrow \infty$:

To begin with, we recall the following results from the random matrix theory [32]

$$\lim_{N_P \rightarrow \infty} \frac{\mathbf{F}_{ll}^H \mathbf{F}_{ll}}{N_P} = \mathbf{D}_{F_{ll}} \quad \text{and} \quad \lim_{N_P \rightarrow \infty} \frac{\mathbf{F}_{li} \mathbf{V}_{li}}{N_P} = \mathbf{0}. \quad (90)$$

In (82), Z can be alternatively written as

$$Z = \text{Tr} \left[\left(\frac{\mathbf{F}_{ll}^H \mathbf{F}_{ll}}{N_P} \right)^{-1} \left(\frac{\mathbf{F}_{ll}^H \mathbf{V}_{li}}{N_P} \right) \left(\frac{\mathbf{V}_{li}^H \mathbf{F}_{ll}}{N_P} \right) \left(\frac{\mathbf{F}_{ll}^H \mathbf{F}_{ll}}{N_P} \right)^{-1} \right]. \quad (91)$$

By letting $N_P \rightarrow \infty$ in (91) and then by invoking (90), it can be shown that $\lim_{N_P \rightarrow \infty} Z = 0$. The transmit power constraint in (82) approaches $P_{S_{\max}}$ asymptotically as shown in Table I.

D. Derivation for $\mathcal{R}_{S_l}^\infty$ for the asymptotic SBS antenna regime:

By following steps similar to those in (91) and invoking (90), we have

$$\lim_{N_S \rightarrow \infty} \sum_{i=1, i \neq l}^L \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{G}_{li} \right\|^2 = 0, \quad \lim_{N_S \rightarrow \infty} \sum_{i=1}^L \left\| \mathbf{w}_{S_{ll,m}}^T \mathbf{U}_{li} \right\|^2 = 0, \quad (92a)$$

$$\lim_{N_S \rightarrow \infty} \left\| \mathbf{w}_{S_{ll,m}}^T \right\|^2 = \lim_{N_S \rightarrow \infty} N_S \zeta_{G_{ll,m}}^{-1}, \quad (92b)$$

where in deriving (92a), the following asymptotic results are invoked: $\lim_{N_S \rightarrow \infty} \mathbf{G}_{ll}^H \mathbf{G}_{li} / N_S = \mathbf{0}$, $\lim_{N_S \rightarrow \infty} \mathbf{G}_{ll}^H \mathbf{U}_{li} / N_S = \mathbf{0}$. By substituting (92a) into (85), by scaling the transmit powers as $P_P = E_P / N_P$ and $P_{S_{\max}} = E_{S_{\max}} / N_S$ and by letting N_P and N_S grow without bound, while having a fixed ratio, the desired asymptotic SINR can be derived as

$$\lim_{N_P \rightarrow \infty} \gamma_{S_{l,k}} = E_{S_{\max}} \zeta_{G_{ll,m}} / \sigma_{S_l}^2. \quad (93)$$

Thereby, the desired asymptotic sum rate expression can be derived as $\mathcal{R}_{S_l}^\infty$ in Table I.

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