**QUEEN'S
UNIVERSITY
BELFAST**

Prediction of in situ strengths in composites: Some considerations

Catalanotti, G. (2019). Prediction of in situ strengths in composites: Some considerations. *Composite Structures*, 207, 889-893. <https://doi.org/10.1016/j.compstruct.2018.09.075>

Published in:
Composite Structures

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
[Link to publication record in Queen's University Belfast Research Portal](#)

Publisher rights

Copyright 2018 Elsevier.

This manuscript is distributed under a Creative Commons Attribution-NonCommercial-NoDerivs License (<https://creativecommons.org/licenses/by-nc-nd/4.0/>), which permits distribution and reproduction for non-commercial purposes, provided the author and source are cited.

General rights

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.

Prediction of *in situ* strengths in composites: some considerations

G. Catalanotti^{a,*}

^a*Advanced Composites Research Group (ACRG), School of Mechanical and
Aerospace Engineering, Queen's University Belfast, Belfast BT9 5AH, UK*

Abstract

The classic formulation that relates the fracture toughness with the *in situ* strength of the ply implicitly assumes that a fracture process zone fully develops within the ply. This assumption, reasonable for conventional composite laminates, may not be appropriate when the ply thickness is very small or the fracture process zone very large. In the following it is shown how considering the R-curve of the material, the *in situ* strength for the cases when the fracture process zone cannot develop completely can be correctly computed. Closed form solutions are found for the *in situ* strengths, and for their maximum values that are obtained when the ply thickness approaches zero.

Key words:

In situ strength, Intralaminar R-curve, Analytical model, *Thin-ply* laminates, Thermoplastics

---

---

* Corresponding author

Email address: g.catalanotti@qub.ac.uk (G. Catalanotti).

1 Introduction

Strength prediction methods of composite structures rely on the ability of failure criteria to predict the ply damage onset associated with a given failure mechanism. When the ply is embedded in a laminate, phenomenological failure criteria [1–7] are usually written in terms of the *in situ* strengths, i.e. the strengths exhibited by the ply when embedded in a laminate. It is well known that when a ply is part of a multidirectional laminate it exhibits a transverse strength that is usually higher than the strength of the same ply when part of a unidirectional laminate [8]; this effect is usually referred as *in situ* effect. The *in situ* effect depends, in a first approximation, on the thickness of the ply [9], and in a second approximation, on other parameters as the lay-up of the laminate and the manufacturing process.

An analytical model to predict the *in situ* strengths was proposed by Camanho et al. [9] for thick and thin plies under transverse tensile, Y_T^{is} , and in-plane shear loading, S_L^{is} . Later, it was also shown that the transverse strength in compression, Y_C^{is} , and the transverse shear strength, S_T^{is} , are *in situ* parameters that can be estimated if the *in situ* strength in shear, S_L^{is} , is known [6]. All these models have been intensively used, and the ability to correctly estimate the strength of the material have been proved for conventional laminates (e.g. laminates manufactured using a thermosetting resin with a thickness in the range of 0.1 to 0.2 mm).

New material systems have however challenged the ability of the classic formulation to predict the *in situ* strength. For thermoplastic composites [10], and for (ultra)-*thin-ply* laminates [11,12], the use of the classic formulation may provide a substantial overestimation of the *in situ* strengths.

Considering a constant value of the fracture toughness is more than reasonable for conventional composites, but leads to well known pathological issues: first of all,

the *in situ* strengths asymptotically tend to infinity when the thickness approaches zero. The following shows how taking into account the R-curve of the material could mitigate and/or eliminate all these issues, and could lead to a correct estimation of the *in situ* strengths.

2 Analytical Model

2.1 In situ tensile strength of a thin embedded ply

The *in situ* tensile strength of a thin embedded ply, Y_T^{is} , can be obtained solving the following Equation [9,13]:

$$\mathcal{G}_{Ic} = \frac{\pi t}{8} \Lambda_{22}^o Y_T^{is2} \quad (1)$$

where t is the thickness of the ply, $\mathcal{G}_{Ic}$ is the composite transverse fracture toughness in mode I, and Λ_{22}^o is a parameter defined as [13,14]:

$$\Lambda_{22}^o = 2 \left(\frac{1}{E_2} - \frac{\nu_{12}^2}{E_1} \right) = \frac{2}{E_2} (1 - \nu_{12}\nu_{21}) \quad (2)$$

being E_1 and E_2 the ply Young's moduli in the longitudinal and transverse directions, and ν_{12} and ν_{21} the major and minor Poisson's ratios, respectively.

For a very thin ply, or for a thermoplastic composite for which the resin exhibits an R-curve characterized by a large fracture of process zone, it may happen that $l_{fpz} \geq t/2$. In this case, Equation (1) is not appropriate because it assumes that the slit crack of Figure 1 propagates along the transverse direction, developing a full fracture process zone.

Fig. 1. Slit crack (after [13]).

In this case, the R-curve of the material should be considered; it can be expressed in analytical form as [15]:

$$\mathcal{R} = \begin{cases} \mathcal{R}_{ss} (1 - (1 - \Delta a / l_{fpz})^\kappa) & \text{if } \Delta a \leq l_{fpz} \\ \mathcal{R}_{ss} & \text{if } \Delta a > l_{fpz} \end{cases} \quad (3)$$

where Δa is the crack increment, $\mathcal{R}_{ss}$ is the steady state value of the R-curve, and κ is a dimensionless parameter. Equation (3) can be expressed in a more compact form as:

$$\mathcal{R}_{\Delta a} = \mathcal{R}_{ss} \left(\left(1 - \left(1 - \frac{\Delta a}{l_{fpz}} \right)^\kappa \right) H[l_{fpz} - \Delta a] + H[\Delta a - l_{fpz}] \right) \quad (4)$$

where $H[x]$ is the Heaviside step function defined as:

$$H[x] = \begin{cases} 1 & \text{if } x > 0 \\ 1/2 & \text{if } x = 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (5)$$

Replacing the fracture toughness in Equation (1) with the R-curve of Equation (4) yields:

$$\bar{\mathcal{R}}_{t/2} = \bar{\mathcal{R}}_{ss} \left(\left(1 - \left(1 - \frac{t/2}{\bar{l}_{fpz}} \right)^{\bar{\kappa}} \right) H \left[\bar{l}_{fpz} - \frac{t}{2} \right] + H \left[\frac{t}{2} - \bar{l}_{fpz} \right] \right) = \frac{\pi t}{8} \Lambda_{22}^o Y_T^{is2} \quad (6)$$

where the bar accent denotes any parameter that defines the mode I intralaminar R-curve. Equation (6) can be solved for the *in situ* tensile strength, Y_T^{is} :

$$Y_T^{is} = \sqrt{\frac{8 \bar{\mathcal{R}}_{t/2}}{\pi t \Lambda_{22}^o}} \quad (7)$$

The maximum value of the *in situ* strength, $\hat{Y}_T^{is}$, can be calculated as the limit of (7), when t approaches 0, as:

$$\hat{Y}_T^{is} = \lim_{t \rightarrow 0} \sqrt{\frac{8 \bar{\mathcal{R}}_{t/2}}{\pi t \Lambda_{22}^o}} = \lim_{t \rightarrow 0} \sqrt{\frac{8 \bar{\mathcal{R}}_{ss} \left(1 - \left(1 - \frac{t}{2 \bar{l}_{fpz}} \right)^{\bar{\kappa}} \right)}{\pi t \Lambda_{22}^o}} \quad (8)$$

which is an indeterminate form. However, noting that the Maclaurin expansion of $\left(1 - \frac{t}{2 \bar{l}_{fpz}} \right)^{\bar{\kappa}}$ reads:

$$\left(1 - \frac{t}{2 \bar{l}_{fpz}} \right)^{\bar{\kappa}} \Big|_{t=0} = 1 - \frac{\bar{\kappa}}{2 \bar{l}_{fpz}} t + o[t] \quad (9)$$

where $o[t]$ denotes a *little-o* of t , the limit in (8) can easily be calculated as:

$$\hat{Y}_T^{is} = \sqrt{\frac{4 \bar{\kappa} \bar{\mathcal{R}}_{ss}}{\pi \bar{l}_{fpz} \Lambda_{22}^o}} \quad (10)$$

2.2 In situ shear strength of a thin embedded ply

The *in situ* shear strength for a thin embedded ply can be calculated as the real root of [9,13]:

$$\frac{S_L^{is^2}}{8 G_{12}} + \frac{3}{16} \beta S_L^{is^4} = \frac{\mathcal{G}_{IIc}}{\pi t} \quad (11)$$

where G_{12} and $\mathcal{G}_{IIc}$ are the ply shear in-plane modulus and the composite transverse fracture toughness in mode II, respectively; β is the parameter used by Hahn and Tsai [16] to approximate the non-linear behaviour of the ply in shear:

$$\gamma_{12} = \frac{1}{G_{12}} \tau_{12} + \beta \tau_{12}^3 \quad (12)$$

where τ_{12} and γ_{12} are the shear stress and strain, respectively. If in Equation (11) the R-curve is included it follows:

$$\frac{S_L^{is^2}}{8 G_{12}} + \frac{3}{16} \beta S_L^{is^4} = \frac{\tilde{\mathcal{R}}_{t/2}}{\pi t} \quad (13)$$

where the tilde accent indicates any material parameter that defines the mode II R-curve (see Equation (6)). The real root of Equation (13) reads [9]:

$$S_L^{is} = \sqrt{\frac{(1 + \beta \phi G_{12}^2)^{\frac{1}{2}} - 1}{3 \beta G_{12}}} \quad (14)$$

where:

$$\phi = \frac{48 \tilde{\mathcal{R}}_{t/2}}{\pi t} \quad (15)$$

The maximum value of the *in situ* shear strength, $\hat{S}_L^{is}$, is calculated as:

$$\hat{S}_L^{is} = \lim_{t \rightarrow 0} \sqrt{\frac{(1 + \beta \phi G_{12}^2)^{\frac{1}{2}} - 1}{3 \beta G_{12}}} \quad (16)$$

In an analogous way, as previously done in Equation (8), the Maclaurin expansion of the function $\left(1 - \frac{t}{2\tilde{l}_{fpz}}\right)^{\tilde{\kappa}}$ can be calculated, and the limit of ϕ as t approaches zero reads:

$$\hat{\phi} = \lim_{t \rightarrow 0} \phi = \lim_{t \rightarrow 0} \frac{48 \tilde{\mathcal{R}}_{t/2}}{\pi t} = \frac{24 \tilde{\mathcal{R}}_{ss} \tilde{\kappa}}{\pi \tilde{l}_{fpz}} \quad (17)$$

Replacing Equation (17) into Equation (16) yields:

$$\hat{S}_L^{is} = \sqrt{\frac{(1 + \beta \hat{\phi} G_{12}^2)^{\frac{1}{2}} - 1}{3 \beta G_{12}}} \quad (18)$$

2.3 In situ strength of a thin outer ply

The formula to be used for the transverse and shear *in situ* strengths for a thin outer ply are the same as reported in [9,13] and will not be reported here for the sake of conciseness. It should be observed that since the surface crack propagates until $a = t$ the fracture toughness should be replaced by the value of the R-curve for $\Delta a = t$, i.e. $\mathcal{R}_t$.

2.4 In situ transverse compressive strength and transverse shear strength

The *in situ* transverse strength in compression, Y_C^{is} , and transverse shear strength, S_T^{is} , can be estimated as [6]:

$$Y_C^{is} = \frac{(1 - 2 \cos^2 \alpha_0)}{\eta_L \cos^2 \alpha_0} S_L^{is} \quad (19)$$

$$S_T^{is} = \frac{(2 \sin^2 \alpha_0 - 1)}{\eta_L \sin 2\alpha_0} S_L^{is} \quad (20)$$

where α_0 is the fracture angle under pure compression, and η_L is the friction coefficient in the longitudinal direction that corresponds to the slope in the σ_{22} - τ_{12} diagram when $\sigma_{22} = 0$, and is defined as:

$$\eta_L = - \left. \frac{\partial \tau_{12}}{\partial \sigma_{22}} \right|_{\sigma_{22}=0} \quad (21)$$

The *in situ* in-plane shear strength from Equation (14) is replaced into Equations (19-20). Once again the maximum values of the *in situ* transverse strength in compression, $\hat{Y}_C^{is}$, and transverse shear strength, $\hat{S}_T^{is}$, are calculate replacing ϕ by $\hat{\phi}$ in Equations (19-20). The value of $\hat{\phi}$ to be used is obtained from Equation (17) for an embedded ply.

3 Discussion

Let us consider the material system reported in Table 1. The fracture toughness for a slit crack propagating in the transverse direction of the lamina is not usually available because of the lack of test methods, and it is usually approximated with the

interlaminar fracture toughness measured using the double cantilever beam (DCB) and the end notched flexural (ENF) tests. This has become common practice because the transverse intralaminar and interlaminar fracture are matrix-dominated mechanisms with the crack propagating parallel to the fibres direction. However, while interlaminar fracture propagation is usually characterised by bridging phenomena that influences the shape of the R-curve (i.e. the length of the fracture process zone) and, ultimately, its steady-state value, the fracture process zone of a propagating intralaminar crack is fundamentally due to matrix plasticity and fibre-matrix decohesion effects. Therefore, the parameters l_{fpz} and κ are not generally available for mode I and II transverse cracks, and appropriate test methods should be proposed in the future to obtain them. Since the main purpose of the paper is to show how the R-curve affects the *in situ* strengths, at this stage, it is assumed that the length of fracture process zone, l_{fpz} , is equal to 0.02 mm for both mode I and II.

The second parameter that affects the R-curve, κ , is chosen to be equal to $\kappa = 1, 1.5, 3, \infty$. Figure 2 shows how κ affects the R-curve. When $\kappa = \infty$ the R-curve is constant, and when $\kappa = 1$, the R-curve is a ramp. The curves with $\kappa = 1.5$ and $\kappa = 3$ represents two possible in-between cases. It should be noted that the solutions for the *in situ* strength proposed by Camanho et al. [9] for the thin ply can be obtained as a particular case of the proposed model when $\kappa = \infty$.

Figure 3 shows the *in situ* transverse tensile strength as a function of the ply thickness for different values of κ . The solution for the thick embedded ply ($Y_T^{is} = 1.12\sqrt{2}Y_T^{ud}$ where Y_T^{ud} is the transverse strength for the unidirectional laminate) is also reported as calculated in [9]. It is noted that when $\kappa = \infty$, $t = 0$ is a vertical asymptote for the curve (red dotted curve in Figure 3), as in Camanho et al. [9]. If the R-curve is properly considered, the *in situ* transverse tensile strength approaches a maximum finite value for $t = 0$. To obtain the maximum value Equation (10) can

Table 1

Material parameters.

E_1 [Mpa]	171420
E_2 [Mpa]	9080
ν_{12} [-]	0.32
G_{12} [MPa]	5290
β [MPa ⁻³]	$3.6 \cdot 10^{-8}$
$\bar{\mathcal{R}}_{ss}$ [N/mm]	0.2
$\tilde{\mathcal{R}}_{ss}$ [N/mm]	0.7
$l_{fpz} = \bar{l}_{fpz} = \tilde{l}_{fpz}$ [mm]	0.02
Y_T^{ud} [MPa]	62.3
S_L^{ud} [MPa]	92.3

be used. It should be noted that all four curves are coincident for $t \geq 2l_{fpz} = 0.04$ mm. For $\kappa = 1$, the *in situ* strength is constant for $t \leq 2l_{fpz}$, and therefore it could be concluded that for a material that exhibits an R-curve with $\kappa \approx 1$ decreasing the thickness does not provide a substantial improvement in terms of *in situ* strengths.

Same considerations as before can be done for Figures 4 which show the in-plane *in situ* shear strength as a function of the thickness t for the embedded ply. The solution for the thick embedded ply is also reported as calculated in [9]. In this case the maximum value of the *in situ* strength is calculated using Equation (18), where $\hat{\phi}$ is reported in Equation (17).

It is worth mentioning the work performed by Arteiro et al. [17], who using micro-

Fig. 2. R-curve as a function of κ .

mechanical analysis showed that when the ply is sufficiently thick several cracks propagate through the thickness; however, when the thickness is sufficiently small through-thickness crack propagation is followed by diffused damage due to the plasticity of the resin. It is clear that when this occurs the failure mechanism changes and the proposed model is possibly not rigorous; it is conceivable, however, to expect that the proposed method yields a conservative value of the *in situ* strength. In fact, all the dissipative phenomena due to plasticity cause the blunting of the intralaminar cracks that may be present, thus delaying the fracture of the ply.

Furthermore, if the *in situ* strength predicted from the model is sufficiently high, its actual value for $t = 0$ is of no practical relevance. In fact the stresses of the outer plies may exceed their strength. Therefore, the first ply failure of the laminate will depend on the outer ply.

Fig. 3. Y_T^{is} as a function of the ply thickness t for different shapes of the R-curve.

Fig. 4. S_L^{is} for the embedded ply as a function of t for different shapes of the R-curve.

It should also be observed that the analyses performed in [17] consider only the variation of the thickness, while the diameter of the fibre, d , remains constant. Rigorously, the modification of the cracking mechanism, and the presence of overall plasticity on very thin plies, depend more on the ratio between the thickness of the ply and the diameter of the fibre, t/d , than on the thickness alone. It can be speculated that if the diameter of the fibre will be scaled with the thickness of the ply, the cracking mechanism will not change because the material will remain homogeneous at ply level. Therefore, very thin plies could exhibit the same cracking mechanism of conventional plies if very thin fibres are used: this could be achievable, for example, using ultrafine continuous nanofibres composites.

Ultrafine continuous nanofibres show tremendous improvements on the mechanical properties when the diameter of the fibre is reduced. For example, for polyacrylonitrile nanofibers, it was found that a reduction of the diameter from $2.8\text{ }\mu\text{m}$ to 100 nm resulted in an increase of the Young's modulus from 0.36 GPa to 48 GPa , and of the strength from 15 MPa to 1750 MPa [18]. The reduction of the diameter may modify the matrix damage mechanism (due to geometrical effects), while the improvement of the fibre strength may lead to reconsidering the importance of correctly calculating the *in situ* strengths of the ply. In fact, if the strength of the fibre is very high, intralaminar damage may occur before fibre fracture. On the other hand, a further decrease of the thickness of the ply achievable using nanofibres could result in a further increase of the *in situ* strength of the ply. However this would depend on the R-curve of the material. For the case of $\kappa \approx 1$ this improvement could be negligible.

4 Conclusions

The proposed analytical model provides the solutions of the *in situ* strength of fibre reinforced composites when the length of the fracture process zone associated with transverse intralaminar crack propagation is larger or comparable to the thickness of the ply.

The proposed formulation, including the R-curve of the material, enables to obtain closed-form solutions for the *in situ* strengths. It is shown that when the ply thickness is small, not only the steady-state value, but also the shape of the R-curve (in mode I and II) should be taken into account; therefore, the length of fracture process zone, l_{fpz} , the steady-state value of the fracture toughness, $\mathcal{R}_{ss}$, and the dimensionless parameter κ are required. Although the transverse intralaminar fracture toughness is usually approximated using the DCB and ENF test methods, reliable measurements of the steady-state value of the R-curve, l_{fpz} and κ cannot be obtained, and alternative measurement procedures must be proposed.

Acknowledgements

The author would like to acknowledge Dr Albertino Arteiro for his helpful inputs and valuable comments.

References

- [1] Puck A, Schürmann H. Failure analysis of FRP laminates by means of physically based phenomenological models. *Compos Sci Technol*, 58:1045–1067, 1998.

- [2] Puck A, Schürmann H. Failure analysis of FRP laminates by means of physically based phenomenological models. *Compos Sci Technol*, 62:1633–1662, 2002.
- [3] Dávila CG, Camanho PP. Failure criteria for FRP laminates in plane stress. Technical Report NASA/TM-2003-212663, NASA Langley Research Center, Hampton, Virginia, November 2003.
- [4] Davila CG, Camanho PP, Rose CA. Failure criteria for FRP laminates. *J Compos Mater*, 39(4):323–345, 2005.
- [5] Pinho ST, Iannucci L, Robinson P. Physically-based failure models and criteria for laminated fibre-reinforced composites with emphasis on fibre kinking: Part I: Development. *Compos Part A-Appl S*, 37:63–73, 2006.
- [6] Catalanotti G, Camanho PP, Marques AT. Three-dimensional failure criteria for fiber-reinforced laminates. *Compos Struct*, 95:63–79, 2013.
- [7] P.P. Camanho, A. Arteiro, A.R. Melro, G. Catalanotti, and M. Vogler. Three-dimensional invariant-based failure criteria for fibre-reinforced composites. *International Journal of Solids and Structures*, 55:92 – 107, 2015. Special Issue Computational and Experimental Mechanics of Advanced Materials A workshop held at King Abdullah University of Science and Technology Jeddah, Kingdom of Saudi Arabia July 1-3, 2013.
- [8] Parvizi A, Garrett K, Bailey J. Constrained cracking in glass fibre reinforced epoxy cross-ply laminates. *J Mater Sci*, 13:195–201, 1978.
- [9] Camanho PP, Dávila CG, Pinho ST, Iannucci L, Robinson P. Prediction of in situ strengths and matrix cracking in composites under transverse tension and in-plane shear. *Compos Part A-Appl S*, 37:165–176, 2006.
- [10] M Versteegh. *Investigation on transverse in- situ tensile strength of thermo- plastic Carbon/PEKK FC material*. PhD thesis, University of Twente, 2016.
- [11] A. Arteiro, G. Catalanotti, A.R. Melro, P. Linde, and P.P. Camanho. Micro-mechanical analysis of the in situ effect in polymer composite laminates. *Composite*

Structures, 116:827 – 840, 2014.

- [12] J. Cugnoni, R. Amacher, S. Kohler, J. Brunner, E. Kramer, C. Dransfeld, W. Smith, K. Scobbie, L. Sorensen, and J. Botsis. Towards aerospace grade thin-ply composites: Effect of ply thickness, fibre, matrix and interlayer toughening on strength and damage tolerance. *Composites Science and Technology*, 2018.
- [13] Dvorak GJ, Laws N. Analysis of progressive matrix cracking in composite laminates II. First ply failure. *J Compos Mater*, 21:30929, 1987.
- [14] P Maimí, EV González, and PP Camanho. Comment to the paper analysis of progressive matrix cracking in composite laminates ii. first ply failure by george j dvorak and norman laws. *Journal of Composite Materials*, 48(9):1139–1141, 2014.
- [15] Z.P. Bazant and J. Planas. *Fracture and Size Effect in Concrete and Other Quasibrittle Materials*. New Directions in Civil Engineering. Taylor & Francis, 1997.
- [16] Hahn HT, Tsai SW. Nonlinear elastic behaviour of unidirectional composite laminate. *J Compos Mater*, 7:102–10, 1973.
- [17] Albertino Arteiro, Giuseppe Catalanotti, José Reinoso, Peter Linde, and Pedro P. Camanho. Simulation of the mechanical response of thin-ply composites: From computational micro-mechanics to structural analysis. *Archives of Computational Methods in Engineering*, Sep 2018.
- [18] D. Papkov, Y. Zou, M.N., Andalib, A. Goponenko, S.Z. Cheng, and Y.A. Dzenis. Simultaneously strong and tough ultrafine continuous nanofibers. *ACS Nano*, 7(4):3324–31, 2013.