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Spectral Efficiency of Multipair Massive MIMO Two-Way Relaying with Imperfect CSI

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Abstract—We consider a two-way half-duplex relaying system where multiple pairs of single-antenna users exchange information assisted by a multiple-antenna relay. Taking into account the practical constraint of imperfect channel knowledge, we study the achievable sum spectral efficiency (SE) of the amplify-and-forward protocol, assuming that the relay employs maximum ratio processing. We derive a closed-form expression for the sum SE for arbitrary system parameters and a large-scale approximation for the sum SE when the number of relay antennas, M , becomes sufficiently large. In addition, we study how the transmit power reduces with M to maintain a desired SE. Our results show that by using a large number of relay antennas, the transmit powers of the user, relay, and pilot symbol can be scaled down proportionally to $1/M^\alpha$, $1/M^\beta$, and $1/M^\gamma$ for certain combinations of α , β , and γ , respectively. This elegant power scaling law reveals a fundamental tradeoff between the transmit powers of the user/relay and pilot symbol. Finally, capitalizing on the new expressions for the sum SE, novel power allocation schemes are designed to further improve the sum SE.

Index Terms—Amplify-and-forward, geometric programming, massive MIMO, power scaling law, two-way relaying.

I. INTRODUCTION

Relaying is a low-complexity and cost-effective means to extend the network coverage and provide spatial diversity, which has attracted a great deal of research attention from both academia and industry [1]–[5]. Thus far, most practical

relaying systems are assumed to operate in the half-duplex mode where the relay does not transmit and receive signals simultaneously. Yet, such half-duplex mechanism incurs a 50% spectral efficiency (SE) loss. To reduce this loss in SE, two-way relaying was proposed in [6]–[12], where the two communicating nodes perform bidirectional simultaneous data transmission.

A. Related Work

Multipair two-way relaying is a sophisticated generalization of single-pair two-way relaying, where multiple pairs of users simultaneously establish a communication link with the aid of a single shared relay [13]–[17], hence substantially boosting the system SE. The major challenge is to properly handle the inter-pair interference from co-existing communication pairs. Thus far, a number of advanced techniques have been introduced to mitigate inter-pair interference, such as dirty-paper coding [18] and interference alignment [19]. Unfortunately, the practical implementation of these techniques is in general very complex. On the other hand, the massive multiple-input multiple-output (MIMO) paradigm has demonstrated superior interference suppression capabilities, with very simple and low-complexity linear processing [20]. Besides, the use of massive MIMO has the potential to achieve orders of magnitude spectral efficiency improvement with affordable signal processing complexity. Therefore, deploying large-scale antenna arrays in two-way relaying systems appears to be a very promising solution for inter-pair interference mitigation.

Some initial works have studied the fundamental performance of such systems [13], [14], [21]–[25]. In particular, for the half-duplex amplify-and-forward (AF) relaying, [13] investigated the achievable SE and power scaling laws of the maximum ratio (MR) and zero-forcing (ZF) processing schemes. Moreover, [14] derived a closed-form approximation of the SE of the MR scheme, and addressed the optimal user pair selection problem. Then, given minimum rate constraints, [21] proposed two algorithms to maximize the energy efficiency. For the full-duplex protocol, [22] studied the energy efficiency based on a practical power consumption model, while [23] considered antenna correlation and discovered the power scaling laws under various cases. However, one major limitation of the above works is that perfect channel state information (CSI) is assumed. Since obtaining perfect CSI is a formidable challenge in massive MIMO systems, it is important to look into the realistic scenario of imperfect CSI. An early attempt was made in [24], where the authors studied

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the sum SE performance of training-based systems utilizing the ZF scheme. Later, [25] investigated the asymptotic SE with MR and ZF, and designed an optimal power allocation strategy to maximize the sum SE under certain practical constraints. Although the derived results in [24], [25] are useful for understanding the impact of imperfect CSI on the system performance, a number of important questions remain to be addressed. For instance, the fundamental tradeoff between the transmit powers of pilot, user, and the relay remains an open problem. Our previous work of [26] has revealed this tradeoff for the decode-and-forward relaying system. Since the AF protocol does not need to decode the signals, it is preferred for applications which are sensitive to complexity and time latency, hence, is particularly suitable for the URLLC ecosystem. Therefore, providing a comprehensive analysis of multipair two-way relaying systems with the AF protocol is also of great interest. Motivated by this, we extend the analysis of [26] to the AF relaying systems, and present the achievable rate, the power tradeoff, and the power allocation design for the AF protocol. Our contributions are summarized as follows.

B. Contributions

- We investigate a multipair two-way AF relaying system that employs MR processing with imperfect CSI, and derive the SE expression in closed-form which is applicable for arbitrary system configurations and a large-scale approximation of the SE when $M \rightarrow \infty$, where M is the number of relay antennas.
- We consider three different transmit powers, i.e., the relay's transmit power, the user's transmit power, and the transmit power of each pilot symbol, and characterize the interplay between them, which permits great flexibility in the design of practical systems. In contrast, most of the previous works only consider the transmit powers of the user and the relay, and ignore the power-scaling analysis in the channel training stage.
- We study the power allocation problem that maximizes the sum SE, subject to a total power constraint. Using the same method as in [25], a local optimum solution is obtained by solving a sequence of geometric programming (GP) problems. In addition, a closed-form power allocation strategy is obtained for the special case where users are independent and identically distributed, which suggests that half of the total power should be allocated to the relay while the rest is equally allocated to the users.

C. Paper Outline and Notations

The remainder of the paper is organized as follows: Section II introduces the considered multipair two-way half-duplex relaying system model. Section III presents the SE in closed-form for arbitrary M and a large-scale approximation when $M \rightarrow \infty$, with imperfect CSI, while Section IV studies the power scaling laws of different system configurations. The power allocation problem is discussed in Section V. The numerical results are verified in Section VI. Finally, Section VII provides some concluding remarks.

Notation: We use bold upper case letters to denote matrices, bold lower case letters to denote vectors and lower case letters to denote scalars. Moreover, $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^{-1}$ represent the conjugate transpose operator, the conjugate operator, the transpose operator, and the matrix inverse, respectively. Also, $\|\cdot\|$ is the Euclidian norm, $\|\cdot\|_F$ denotes the Frobenius norm, and $|\cdot|$ is the absolute value. In addition, $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ denotes a circularly symmetric complex Gaussian random vector \mathbf{x} with covariance matrix $\mathbf{\Sigma}$, \mathbf{I}_k is the identity matrix of size k , while $\mathbf{1}$ denotes the vector whose elements are all 1. Finally, the statistical expectation operator is represented by $\mathbb{E}\{\cdot\}$, the variance operator is $\text{Var}(\cdot)$, and the trace is denoted by $\text{tr}(\cdot)$.

II. SYSTEM MODEL

We consider a multipair two-way relaying communication network consisting of a relay T_R with M antennas, and N pairs of single-antenna users $T_{A,i}$ and $T_{B,i}$, $i = 1, \dots, N$. We assume that the direct links between $T_{A,i}$ and $T_{B,i}$ do not exist due to large obstacles or severe shadowing. Thus, they exchange information with each other via T_R , as shown in Fig. 1. The relay operates in the half-duplex mode, i.e., it cannot transmit and receive simultaneously.

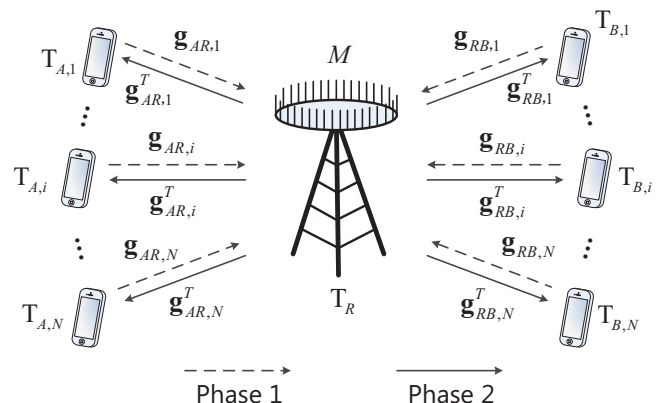


Fig. 1: Illustration of the multipair two-way relaying system.

We assumed that the system operates under the time-division duplex protocol and channel reciprocity holds. The channels from $T_{A,i}$ to T_R , and $T_{B,i}$ to T_R can be respectively denoted as $\mathbf{G}_{AR} \triangleq [\mathbf{g}_{AR,1}, \dots, \mathbf{g}_{AR,N}] \in \mathbb{C}^{M \times N}$ and $\mathbf{G}_{RB} \triangleq [\mathbf{g}_{RB,1}, \dots, \mathbf{g}_{RB,N}] \in \mathbb{C}^{M \times N}$. More precisely, by accounting for both small-scale and large-scale fading, the channel vectors can be expressed as $\mathbf{g}_{AR,i} \sim \mathcal{CN}(\mathbf{0}, \beta_{AR,i} \mathbf{I}_M)$ and $\mathbf{g}_{RB,i} \sim \mathcal{CN}(\mathbf{0}, \beta_{RB,i} \mathbf{I}_M)$, where $\beta_{AR,i}$ and $\beta_{RB,i}$ model the large-scale path-loss and shadowing effects that are assumed to be constant over many coherence intervals and known a priori.

A. Channel Training Stage

In the channel training stage, the channels \mathbf{G}_{AR} and \mathbf{G}_{RB} are estimated by utilizing uplink pilots [20]. In each coherence interval of length τ_c , $T_{A,i}$ and $T_{B,i}$ simultaneously transmit their mutually orthogonal pilot sequences of length τ_p to T_R ,

for $i = 1, \dots, N$. Thus, the received pilot matrix at the relay is given by

$$\mathbf{Y}_p = \sqrt{\tau_p p_p} \mathbf{G}_{AR} \Phi_A^T + \sqrt{\tau_p p_p} \mathbf{G}_{RB} \Phi_B^T + \mathbf{N}_p, \quad (1)$$

where p_p is the transmit power of each pilot symbol, \mathbf{N}_p is AWGN matrix including i.i.d. $\mathcal{CN}(0, 1)$ elements.¹ The matrices $\Phi_A \in \mathbb{C}^{\tau_p \times N}$ and $\Phi_B \in \mathbb{C}^{\tau_p \times N}$ are respectively the pilot sequences transmitted from $T_{A,i}$ and $T_{B,i}$, and satisfy $\tau_p \geq 2N$, $\Phi_A^T \Phi_A = \mathbf{I}_N$, $\Phi_B^T \Phi_B = \mathbf{I}_N$, and $\Phi_A^T \Phi_B = \mathbf{0}_N$ to preserve orthogonality of the pilots.

As in [1], [24], [27], we assume that T_R uses the minimum mean-square-error (MMSE) estimator to estimate the channels \mathbf{G}_{AR} and \mathbf{G}_{RB} . From the property of MMSE channel estimation, the channel vectors can be decomposed as

$$\mathbf{g}_{AR,i} = \hat{\mathbf{g}}_{AR,i} + \mathbf{e}_{AR,i}, \quad (2)$$

$$\mathbf{g}_{RB,i} = \hat{\mathbf{g}}_{RB,i} + \mathbf{e}_{RB,i}, \quad (3)$$

where $\hat{\mathbf{g}}_{AR,i}$, $\hat{\mathbf{g}}_{RB,i}$, $\mathbf{e}_{AR,i}$, and $\mathbf{e}_{RB,i}$ are the i -th columns of the estimated matrices $\hat{\mathbf{G}}_{AR}$, $\hat{\mathbf{G}}_{RB}$, and the estimation error matrices \mathbf{E}_{AR} and \mathbf{E}_{RB} , respectively, which are mutually independent. Furthermore, the elements of $\hat{\mathbf{g}}_{AR,i}$, $\mathbf{e}_{AR,i}$ are Gaussian random variables with zero mean, variance $\sigma_{AR,i}^2$ and $\tilde{\sigma}_{AR,i}^2$, respectively, where $\sigma_{AR,i}^2 \triangleq \frac{\tau_p p_p \beta_{AR,i}^2}{1 + \tau_p p_p \beta_{AR,i}}$ and $\tilde{\sigma}_{AR,i}^2 \triangleq \frac{\beta_{AR,i}}{1 + \tau_p p_p \beta_{AR,i}}$. Similarly, the elements of $\hat{\mathbf{g}}_{RB,i}$, and $\mathbf{e}_{RB,i}$ are complex Gaussian random variables with zero mean, variance $\sigma_{RB,i}^2$ and $\tilde{\sigma}_{RB,i}^2$, respectively, where $\sigma_{RB,i}^2 \triangleq \frac{\tau_p p_p \beta_{RB,i}^2}{1 + \tau_p p_p \beta_{RB,i}}$ and $\tilde{\sigma}_{RB,i}^2 \triangleq \frac{\beta_{RB,i}}{1 + \tau_p p_p \beta_{RB,i}}$.

B. Data Transmission Stage

The data transmission stage consists of two separate phases. In the first phase, the N user pairs simultaneously transmit their respective signals to T_R . Thus, the received signal at T_R is given by

$$\mathbf{y}_r = \sum_{i=1}^N (\sqrt{p_{A,i}} \mathbf{g}_{AR,i} x_{A,i} + \sqrt{p_{B,i}} \mathbf{g}_{RB,i} x_{B,i}) + \mathbf{n}_R, \quad (4)$$

where $x_{A,i}$ and $x_{B,i}$ are complex Gaussian signals with zero mean and unit power transmitted by the i -th user pair, $p_{A,i}$ and $p_{B,i}$ are the average transmit power of $T_{A,i}$ and $T_{B,i}$, respectively, and \mathbf{n}_R is a vector of additive white Gaussian noise (AWGN) at T_R , whose elements are identically and independently distributed (i.i.d.) $\mathcal{CN}(0, 1)$.

In the second phase, the relay first applies MR processing on the received signal², and then broadcasts it to the users. Thus, the transmit signal from T_R can be written as

$$\mathbf{y}_t = \rho \mathbf{F} \mathbf{y}_r, \quad (5)$$

where the processing matrix $\mathbf{F} \in \mathbb{C}^{M \times M}$ is given by [14]

$$\mathbf{F} = \mathbf{B}^* \mathbf{A}^H, \quad (6)$$

¹Note that for notational convenience, we set the noise variance to be 1 throughout the paper. With this convention, the transmit power in the paper can be interpreted as the normalized transmit signal to noise (expressed in dB).

²Note that MR is a very attractive linear processing technique in the context of massive MIMO systems due to its low complexity. Most importantly, it can be implemented in a distributed manner [20].

with $\mathbf{A} \triangleq [\hat{\mathbf{G}}_{AR}, \hat{\mathbf{G}}_{RB}]$, and $\mathbf{B} \triangleq [\hat{\mathbf{G}}_{RB}, \hat{\mathbf{G}}_{AR}]$, and ρ is chosen to satisfy the long-term total transmit power constraint at the relay, namely, $\mathbb{E}\{\|\mathbf{y}_t\|^2\} = p_r$, where p_r is the average transmit power of the relay. Thus, we have ρ , which is given by (7), shown on the top of the next page.

As a result, the received signals at $T_{A,i}$ and $T_{B,i}$ are respectively given by

$$z_{A,i} = \mathbf{g}_{AR,i}^T \mathbf{y}_t + n_{A,i}, \quad (8)$$

$$z_{B,i} = \mathbf{g}_{RB,i}^T \mathbf{y}_t + n_{B,i}, \quad (9)$$

where $n_{A,i} \sim \mathcal{CN}(0, 1)$ and $n_{B,i} \sim \mathcal{CN}(0, 1)$ represent the AWGN at $T_{A,i}$ and $T_{B,i}$.

III. SPECTRAL EFFICIENCY

In this section, we derive the SE expression in closed-form for MR processing, which is applicable for arbitrary number of relay antennas. Furthermore, a large-scale approximation of the SE is deduced when antenna arrays at the relay are very large.

Without loss of generality, we focus on the characterization of the achievable SE of user $T_{A,i}$. When $T_{A,i}$ receives the superimposed signal from T_R , it first attempts to subtract its own transmitted message according to its available CSI (known as self-interference cancellation). Here, we consider the realistic case where the users only have the statistical CSI that is obtained by the feedback from the relay. Therefore, after subtracting the partial self-interference term $\rho \sqrt{p_{A,i}} \mathbb{E}\{\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,i}\} x_{A,i}$, the received signal at $T_{A,i}$ is re-expressed as

$$\begin{aligned} \hat{z}_{A,i} &= z_{A,i} - \rho \sqrt{p_{A,i}} \mathbb{E}\{\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,i}\} x_{A,i} \\ &= \underbrace{\rho \sqrt{p_{B,i}} \mathbb{E}\{\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i}\} x_{B,i}}_{\text{desired signal}} \\ &\quad + \underbrace{\rho \sqrt{p_{B,i}} (\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} - \mathbb{E}\{\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i}\}) x_{B,i}}_{\text{gain uncertainty}} \\ &\quad + \underbrace{\rho \sqrt{p_{A,i}} (\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,i} - \mathbb{E}\{\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,i}\}) x_{A,i}}_{\text{residual self-interference}} \\ &\quad + \underbrace{\rho \sum_{j \neq i} \mathbf{g}_{AR,i}^T \mathbf{F} (\sqrt{p_{A,i}} \mathbf{g}_{AR,j} x_{A,j} + \sqrt{p_{B,i}} \mathbf{g}_{RB,j} x_{B,j})}_{\text{inter-user interference}} \\ &\quad + \underbrace{\rho \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{n}_R + n_{A,i}}_{\text{compound noise}}. \end{aligned} \quad (10)$$

Using a standard approach as in [1], [29], an ergodic achievable SE of $T_{A,i}$ is

$$R_{A,i} = \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{A_i}{B_i + C_i + D_i + E_i} \right), \quad (11)$$

where the pre-log factor $1/2$ is introduced for the half-duplex

$$\rho = \sqrt{\frac{p_r}{\sum_{i=1}^N (p_{A,i} \mathbf{E} \{ \|\mathbf{F} \mathbf{g}_{AR,i}\|^2 \} + p_{B,i} \mathbf{E} \{ \|\mathbf{F} \mathbf{g}_{RB,i}\|^2 \}) + \mathbf{E} \{ \|\mathbf{F}\|_F^2 \}}}. \quad (7)$$

mode, and

$$A_i = p_{B,i} |\mathbf{E} \{ \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} \}|^2, \quad (12)$$

$$B_i = p_{B,i} \text{Var} (\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i}), \quad (13)$$

$$C_i = p_{A,i} \text{Var} (\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,i}), \quad (14)$$

$$D_i = \sum_{j \neq i} \mathbf{E} \{ p_{A,i} |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \} \quad (15)$$

$$+ \sum_{j \neq i} \mathbf{E} \{ p_{B,i} |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,j}|^2 \},$$

$$E_i = \mathbf{E} \{ \|\mathbf{g}_{AR,i}^T \mathbf{F}\|^2 \} + \frac{1}{\rho^2}.$$

Thus, the ergodic sum SE of the multipair two-way AF relaying system is given by

$$R = \sum_{i=1}^N (R_{A,i} + R_{B,i}), \quad (16)$$

where $R_{B,i}$ is the SE of $T_{B,i}$, which can be derived in a similar fashion.

Theorem 1: With the AF protocol, the ergodic SE for an arbitrary number of relay antennas is given by (11) with

$$A_i = p_{B,i} M^2 (M+1)^2 \sigma_{AR,i}^4 \sigma_{RB,i}^4, \quad (17)$$

$$B_i = p_{B,i} 2M(M+1) \beta_{AR,i} \beta_{RB,i} \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \quad (18)$$

$$\begin{aligned} &+ p_{B,i} M(M+1) \sigma_{AR,i}^2 \sigma_{RB,i}^2 (\beta_{AR,i} \sigma_{RB,i}^2 + \beta_{RB,i} \sigma_{AR,i}^2) \\ &+ p_{B,i} 2M(M+1) \sigma_{AR,i}^2 \sigma_{RB,i}^2 \tilde{\sigma}_{AR,i}^2 \tilde{\sigma}_{RB,i}^2 \\ &+ p_{B,i} 2M(M+1)^2 \sigma_{AR,i}^4 \sigma_{RB,i}^4 \\ &+ p_{B,i} M(M+1)(M+2) \sigma_{AR,i}^2 \tilde{\sigma}_{AR,i}^2 \sigma_{RB,i}^2 (\sigma_{AR,i}^2 + \sigma_{RB,i}^2), \end{aligned} \quad (19)$$

$$C_i = 4p_{A,i} M(M+1) \beta_{AR,i}^2 \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \quad (20)$$

$$\begin{aligned} &+ 4p_{A,i} \sigma_{AR,i}^2 \sigma_{RB,i}^2 M(M+1)(M+2) \sigma_{AR,i}^4 \\ &+ 4p_{A,i} \sigma_{AR,i}^2 \sigma_{RB,i}^2 M(M+1)(M+5) \sigma_{AR,i}^2 \tilde{\sigma}_{AR,i}^2 \\ &+ 4p_{A,i} \sigma_{AR,i}^2 \sigma_{RB,i}^2 M(M+1) \tilde{\sigma}_{AR,i}^4, \end{aligned}$$

$$D_i = \sum_{j \neq i} 2M(M+1) \beta_{AR,i} t_j \sum_{n \neq i,j} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \quad (21)$$

$$+ \sum_{j \neq i} M \sigma_{AR,i}^2 \sigma_{RB,i}^2 t_j m \sigma_{AR,i}^2$$

$$+ \sum_{j \neq i} M \sigma_{AR,i}^2 \sigma_{RB,i}^2 t_j 2(M+1) \tilde{\sigma}_{AR,i}^2$$

$$+ \sum_{j \neq i} M \beta_{AR,i} \sigma_{AR,j}^2 \sigma_{RB,j}^2 m (p_{A,i} \sigma_{AR,j}^2 + p_{B,i} \sigma_{RB,j}^2)$$

$$+ \sum_{j \neq i} 2(M+1) (p_{A,i} \tilde{\sigma}_{AR,j}^2 + p_{B,i} \tilde{\sigma}_{RB,j}^2),$$

$$E_i = 2M(M+1) \beta_{AR,i} \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \quad (22)$$

$$+ M \sigma_{AR,i}^2 \sigma_{RB,i}^2 (m \sigma_{AR,i}^2 + 2(M+1) \tilde{\sigma}_{AR,i}^2)$$

$$+ \frac{1}{p_r} \sum_{i=1}^N M \sigma_{AR,i}^2 \sigma_{RB,i}^2 m (\sigma_{AR,i}^2 p_{A,i} + \sigma_{RB,i}^2 p_{B,i})$$

$$+ \frac{1}{p_r} \sum_{i=1}^N M \sigma_{AR,i}^2 \sigma_{RB,i}^2 2(M+1) (\tilde{\sigma}_{AR,i}^2 p_{A,i} + \tilde{\sigma}_{RB,i}^2 p_{B,i})$$

$$+ \frac{1}{p_r} \sum_{i=1}^N 2M(M+1) t_i \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2$$

$$+ \frac{1}{p_r} 2M(M+1) \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2,$$

where $t_j = p_{A,i} \beta_{AR,j} + p_{B,i} \beta_{RB,j}$ and $m = (M+1)(M+3)$.

Proof: See Appendix A. \blacksquare

Theorem 1 presents the closed-form SE, which is applicable to arbitrary number of relay antennas. While Theorem 1 enables efficient evaluation of the SE, it is in general difficult to extract insights due to the involved expression. As such, we now look into the asymptotic regime where the relay has a large number of antennas, and derive a simple and accurate approximation for the SE.

Theorem 2: With the AF protocol, as the number of relay antennas grows to infinity, then we have $R_{A,i} - \tilde{R}_{A,i} \xrightarrow{M \rightarrow \infty} 0$, where $\tilde{R}_{A,i}$ is given by

$$\tilde{R}_{A,i} \triangleq \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{p_{B,i} M}{\tilde{B}_i + \tilde{C}_i + \tilde{D}_i + \tilde{E}_i} \right), \quad (23)$$

where

$$\tilde{B}_i \triangleq p_{B,i} \left(\frac{\beta_{RB,i}}{\sigma_{RB,i}^2} + \frac{\beta_{AR,i}}{\sigma_{AR,i}^2} \right), \quad (24)$$

$$\tilde{C}_i \triangleq \frac{4p_{A,i} \beta_{AR,i}}{\sigma_{RB,i}^2}, \quad (25)$$

$$\tilde{D}_i \triangleq \sum_{j \neq i} p_{A,j} \left(\frac{\beta_{AR,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^4 \sigma_{RB,j}^2 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right) \quad (26)$$

$$+ \sum_{j \neq i} p_{B,j} \left(\frac{\beta_{RB,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^4 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right),$$

$$\tilde{E}_i \triangleq \frac{1}{\sigma_{RB,i}^2} \quad (27)$$

$$+ \frac{1}{p_r \sigma_{AR,i}^4 \sigma_{RB,i}^4} \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2 (p_{A,n} \sigma_{AR,n}^2 + p_{B,n} \sigma_{RB,n}^2).$$

Theorem 2 presents a large-scale approximation of the i -th user's SE which is asymptotically tight. Despite being obtained under the massive array assumption, the approximation turns

out to be very accurate even for practical number of relay antennas. Most importantly, it is easy to observe the impact of various factors on the asymptotic SE. It can be seen that the individual user SE $\tilde{R}_{A,i}$ decreases with the number of user pairs N ; this is anticipated since a higher number of users increases the amount of inter-user interference. Now, we focus on studying the impact of the transmit power of i -th user pair $p_{A,i}$ and $p_{B,i}$, the transmit power of the relay p_r , and the transmit power of each pilot symbol p_p on the system performance. As can be seen, when $p_{A,i} \rightarrow \infty$ and $p_{B,i} \rightarrow \infty$, the SE is limited by p_r and p_p ; in contrast, it is limited by $p_{A,i}$, $p_{B,i}$, and p_p when $p_r \rightarrow \infty$.

IV. POWER SCALING LAWS

In this section, we present a detailed analysis of the power scaling laws. In other words, we characterize that how the powers can be reduced proportionally to M while maintaining a desired non-zero SE. Note that power actually refers to the signal-to-noise ratio and thus the analysis shows how to achieve the SE either using less power or extending the coverage. Since our main focus is on the extent of power reduction rather than the behavior of a particular user, we assume that all the users have the same transmit power, i.e., $p_{A,i} = p_{B,i} = p_u$. Then, based on three different scenarios, we characterize the interplay between the relay's transmit power p_r , the transmit power of each user p_u , and the transmit power of each pilot symbol p_p , as the number of relay antennas M grows to infinity.

A. Scenario A

In scenario A, p_u and p_r are fixed, while $p_p = \frac{E_p}{M^\gamma}$ with $\gamma > 0$, and E_p being a constant. Such a scenario represents the potential of power saving in the training stage.

Now define the Condition I: $E_p \ll \tau_p M^\gamma \times \min\{\beta_{AR,i}, \beta_{RB,i}\}$, then we have the following important result:

Theorem 3: For fixed and finite p_u , p_r and E_p , when $p_p = \frac{E_p}{M^\gamma}$ with $\gamma > 0$, as $M \rightarrow \infty$ and Condition I holds, we have

$$R_{A,i} - \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{\tau_p E_p M^{1-\gamma}}{\hat{B}_i + \hat{C}_i + \hat{D}_i + \hat{E}_i} \right) \xrightarrow{M \rightarrow \infty} 0, \quad (28)$$

where

$$\hat{B}_i \triangleq \frac{1}{\beta_{RB,i}} + \frac{1}{\beta_{AR,i}}, \quad (29)$$

$$\hat{C}_i \triangleq \frac{4\beta_{AR,i}}{\beta_{RB,i}^2}, \quad (30)$$

$$\hat{D}_i \triangleq \sum_{j \neq i} \left(\frac{\beta_{AR,j} + \beta_{RB,j}}{\beta_{RB,i}^2} + \frac{\beta_{AR,j}^4 \beta_{RB,j}^2 + \beta_{AR,j}^2 \beta_{RB,j}^4}{\beta_{AR,i}^3 \beta_{RB,i}^4} \right), \quad (31)$$

$$\hat{E}_i \triangleq \frac{1}{p_u \beta_{RB,i}^2} \quad (32)$$

$$+ \frac{1}{p_r \beta_{AR,i}^4 \beta_{RB,i}^4} \sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2).$$

Theorem 3 implies that the large-scale approximation of the SE $R_{A,i}$ depends on the choice of γ . When $\gamma > 1$, $R_{A,i}$ reduces to zero due to the poor channel estimation accuracy caused by over-reducing the pilot transmit power. In contrast, when $0 < \gamma < 1$, $R_{A,i}$ grows without bound, which indicates that the transmit power of each pilot symbol can be scaled down arbitrarily within this regime. Finally, when $\gamma = 1$, $R_{A,i}$ converges to a non-zero limit, which suggests that with large antenna arrays, the transmit power of each pilot symbol can be scaled down at most by $1/M$ to maintain a given SE.

B. Scenario B

In scenario B, p_p is fixed, while $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, with $\alpha \geq 0$ and $\beta \geq 0$, and E_u , E_r are constants. Hence, the channel estimation accuracy remains unchanged, and the objective is to study the potential power savings in the data transmission stage, as well as, the interplay between the user and relay transmit powers.

Now define the following conditions, namely, Condition II:

$$\frac{M^\alpha}{E_u \sigma_{RB,i}^2} + \frac{M^\beta}{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^4} \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2) \gg \frac{\beta_{RB,i}}{\sigma_{RB,i}^2} + \frac{\beta_{AR,i}}{\sigma_{AR,i}^2} + \frac{4\beta_{AR,i}}{\sigma_{RB,i}^2} + \sum_{j \neq i} \left(\frac{\beta_{AR,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^4 \sigma_{RB,j}^2 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right) + \sum_{j \neq i} \left(\frac{\beta_{RB,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^4 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right), \quad \text{Condition III:}$$

$$\frac{1}{E_u} \gg \frac{M^{\beta-1}}{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^2} \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2), \quad \text{and}$$

Condition IV:

$$\frac{M^{\alpha-1}}{E_u} \ll \frac{1}{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^2} \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2),$$

then we have:

Theorem 4: For fixed and finite p_p , E_u , and E_r , when $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, with $\alpha \geq 0$, $\beta \geq 0$, as $M \rightarrow \infty$ and Condition II holds, we have (33), which is shown on the top of the next page.

Theorem 4 reveals that in Scenario B, the estimation error, the residual self-interference, and the inter-user interference vanish completely, and only the compound noise remains, as $M \rightarrow \infty$. The reason is that the power scaling pushes the system into a noise limited regime. Moreover, it is observed that the compound noise consists of two parts, namely Part I and Part II as shown in (33), which represent the noise at the relay and the noise at the user $T_{A,i}$, respectively. This observation can be interpreted as: when both the transmit powers of each user and the relay are scaled down inversely proportional to M , the effect of noise becomes increasingly significant. In addition, we can also see that when the channel estimation accuracy is fixed, the large-scale approximation of the SE $R_{A,i}$ depends on the value of α and β . When we cut down the transmit powers of the relay and/or of each user too much, namely, $\alpha > 1$ and/or $\beta > 1$, $R_{A,i}$ converges to zero. On the other hand, $R_{A,i}$ grows unboundedly for $0 \leq \alpha < 1$ and $0 \leq \beta < 1$. Only if $\alpha = 1$ and/or $\beta = 1$, $R_{A,i}$ converges to a finite limit as detailed in the following corollaries.

Corollary 1: For fixed and finite p_p , E_u , and E_r , when $\alpha = \beta = 1$, namely, $p_u = \frac{E_u}{M}$, $p_r = \frac{E_r}{M}$, as $M \rightarrow \infty$ and Condition II holds, the SE has the limit (34), which is shown on the top of the next page.

$$R_{A,i} - \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{1}{\underbrace{\frac{M^{\alpha-1}}{E_u \sigma_{RB,i}^2}}_{\text{Part I}} + \underbrace{\frac{M^{\beta-1}}{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^4} \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2)}_{\text{Part II}}} \right) \xrightarrow{M \rightarrow \infty} 0. \quad (33)$$

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{1}{\frac{1}{E_u \sigma_{RB,i}^2} + \frac{1}{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^4} \sum_{n=1}^N (\sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2))} \right). \quad (34)$$

From Corollary 1, we observe that when both the transmit powers of the relay and of each user are scaled down with the same speed, i.e., $1/M$, $R_{A,i}$ converges to a non-zero limit. Moreover, this non-zero limit increases with E_u and E_r , as expected. Now consider the special case of all the links having the same large-scale fading, e.g., $\beta_{AR,i} = \beta_{RB,i} = 1$, for $i = 1, \dots, N$, then the sum SE of the system reduces to

$$R \rightarrow \frac{\tau_c - \tau_p}{\tau_c} N \log_2 \left(1 + \frac{\sigma_1^2 E_u E_r}{E_r + 2NE_u} \right), \quad (35)$$

where $\sigma_1^2 = \frac{\tau_p p_p}{\tau_p p_p + 1}$. Therefore, the sum SE in (35) is equal to the one of N parallel single-input single-output channels with transmit power $\frac{\sigma_1^2 E_u E_r}{E_r + 2NE_u}$, without interference and small-scale fading. Note that we only need $\frac{2N(E_u + \tau_p E_p) + E_r}{M}$ amount of power (the transmit power of each user is $\frac{E_u}{M}$, the transmit power of each pilot sequence is $\frac{\tau_p E_p}{M}$, and the transmit power of the relay is $\frac{E_r}{M}$) in multipair two-way AF relaying systems when using very large number of relay antennas. This represents a remarkable power reduction, thereby showcasing the huge benefits from the perspective of radiated energy efficiency by deploying large antenna arrays.

Corollary 2: For fixed and finite p_p , E_u and E_r , when $\alpha = 1$ and $0 \leq \beta < 1$, namely, $p_u = \frac{E_u}{M}$, $p_r = \frac{E_r}{M^\beta}$, as $M \rightarrow \infty$ and Conditions II and III hold, the SE has the limit

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \log_2 (1 + E_u \sigma_{RB,i}^2). \quad (36)$$

Corollary 2 presents an interesting phenomenon, that if the transmit power of each user is overly cut down compared to the reduction of the relay transmit power, the value of Part I is dominant compared to Part II in (33), and thus $R_{A,i}$ converges to a non-zero limit that is determined by the noise at the relay. This observation is intuitive, since when both $T_{A,i}$ and $T_{B,i}$ transmit with extremely low power, the effect of noise at the relay becomes the performance limiting factor. Similarly, when $0 \leq \alpha < \beta = 1$, we have the following corollary.

Corollary 3: For fixed and finite p_p , E_u , and E_r , when $0 \leq \alpha < 1$ and $\beta = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M}$, as

$M \rightarrow \infty$ and Conditions II and IV hold, the SE has the limit

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(1 + \frac{E_r \sigma_{AR,i}^4 \sigma_{RB,i}^4}{\sum_{n=1}^N (\sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,n}^2))} \right). \quad (37)$$

Similar to the analysis in Corollary 2, if the down-scaling of the relay transmit power in the second phase is faster than that of the users' transmit power in the first phase, the limit of $R_{A,i}$ will only depend on the noise at the users.

C. Scenario C

In scenario C, all the transmit powers scale with the number of relay antennas, i.e., $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\alpha \geq 0$, $\beta \geq 0$, and $\gamma > 0$, E_u , E_r , and E_p are constants. This is the most general scenario where we are able to flexibly adjust the transmit powers of the considered system to maintain the desired performance.

Now define the following conditions, namely, Condition V:

$$\frac{M^\alpha}{E_u \beta_{RB,i}^2} + \frac{M^\beta}{E_r \beta_{AR,i}^4 \beta_{RB,i}^4} \sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2) \gg \frac{1}{\beta_{RB,i}} + \frac{1}{\beta_{AR,i}} + \frac{4\beta_{AR,i}}{\beta_{RB,i}^2} + \sum_{j \neq i} \left(\frac{\beta_{AR,j}}{\beta_{RB,i}^2} + \frac{\beta_{AR,j}^4 \beta_{RB,j}^2}{\beta_{AR,i}^3 \beta_{RB,i}^4} \right) + \sum_{j \neq i} \left(\frac{\beta_{RB,j}}{\beta_{RB,i}^2} + \frac{\beta_{AR,j}^4 \beta_{RB,j}^4}{\beta_{AR,i}^3 \beta_{RB,i}^4} \right),$$

Condition VI:

$$\frac{1}{E_u} \gg \frac{M^{\beta+\gamma-1}}{E_r \beta_{AR,i}^4 \beta_{RB,i}^2} \sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2),$$

and Condition VII:

$$\frac{M^{\alpha+\gamma-1}}{E_u} \ll \frac{1}{E_r \beta_{AR,i}^4 \beta_{RB,i}^2} \sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2),$$

then we have:

Theorem 5: For fixed and finite E_u , E_r , and E_p , when $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\alpha \geq 0$, $\beta \geq 0$, and $\gamma > 0$, as $M \rightarrow \infty$ and Conditions I and V hold, we have (38), shown on the top of the next page.

Theorem 5 reveals the coupled relationship between the training power and user (or relay) transmit power. When $\alpha + \gamma > 1$ and/or $\beta + \gamma > 1$, $R_{A,i}$ converges to zero, due

$$R_{A,i} - \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{1}{\frac{M^{\alpha+\gamma-1}}{\tau_p E_p E_u \beta_{RB,i}^2} + \frac{M^{\beta+\gamma-1}}{\tau_p E_p E_r \beta_{AR,i}^4 \beta_{RB,i}^4} \sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2)} \right) \xrightarrow{M \rightarrow \infty} 0. \quad (38)$$

to either poor estimation accuracy or low user/relay transmit power. On the other hand, when $0 < \alpha + \gamma < 1$ and $0 < \beta + \gamma < 1$, $R_{A,i}$ grows without bound. Only if $\alpha + \gamma = 1$ and/or $\beta + \gamma = 1$, $R_{A,i}$ converges to a non-zero limit. In the following, we take a closer look at these particular cases of interest.

Corollary 4: For fixed and finite E_u , E_r , and E_p , when $\alpha = \beta > 0$ and $\alpha + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as $M \rightarrow \infty$ and Conditions I and V hold, the SE has the limit (39), shown on the top of the next page.

Corollary 4 suggests that no matter how α , β , and γ change, as long as the overall power reduction SE at the user/relay and pilot symbol remains the same, i.e., $\alpha + \gamma = 1$, the same asymptotic SE can be attained. In other words, it is possible to balance between the pilot symbol power to the user/relay transmit power.

Corollary 5: For fixed and finite E_u , E_r , and E_p , when $\alpha > \beta \geq 0$ and $\alpha + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as $M \rightarrow \infty$ and Conditions I, V, and VI hold, the SE has the limit

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \log_2 (1 + \tau_p E_p E_u \beta_{RB,i}^2). \quad (40)$$

Corollary 5 shows the same trade-off between α and γ as in Corollary 4. However, similar to Corollary 2, the SE is only related to the noise at the relay.

Corollary 6: For fixed and finite E_u , E_r , and E_p , when $0 \leq \alpha < \beta$ and $\beta + \gamma = 1$, namely, $p_u = \frac{E_u}{M^\alpha}$, $p_r = \frac{E_r}{M^\beta}$, and $p_p = \frac{E_p}{M^\gamma}$, with $\gamma > 0$, as $M \rightarrow \infty$ and Conditions I, V, and VII hold, the SE has the limit

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \times \log_2 \left(1 + \frac{\tau_p E_p E_r \beta_{AR,i}^4 \beta_{RB,i}^4}{\sum_{n=1}^N \beta_{AR,n}^2 \beta_{RB,n}^2 (\beta_{AR,n}^2 + \beta_{RB,n}^2)} \right). \quad (41)$$

Corollary 6 indicates that when we cut down the transmit power of the relay p_r more compared with the transmit power of each user p_u , i.e., $0 \leq \alpha < \beta$, to obtain a constant limit SE, the trade-off between p_r and the transmit power of the relay each pilot symbol p_p should be satisfied, namely, $\beta + \gamma = 1$. This trade-off provides valuable insights, since we can adjust p_r and p_p flexibly based on different demands, to meet the same limit. In addition, Corollary 6 also shows that $R_{A,i}$ is an increasing function with respect to E_p and E_r , while a decreasing function of N . In other words, when the number of user pairs N increases, the relay and/or each pilot symbol should increase their power in order to maintain the same performance. This is due to the fact that a larger transmit

power of the relay and/or more accurate channel estimation can compensate the individual SE loss caused by stronger inter-user interference.

V. POWER ALLOCATION

In this section, we formulate a power allocation problem maximizing the sum SE subject to a total power constraint, i.e., $\sum_{i=1}^N (p_{A,i} + p_{B,i}) + p_r \leq P$. For notational simplicity, we define $\mathcal{N} \triangleq \{1, \dots, N\}$, $\mathbf{p}_A \triangleq [p_{A,1}, \dots, p_{A,N}]^T$, and $\mathbf{p}_B \triangleq [p_{B,1}, \dots, p_{B,N}]^T$.

For mathematical tractability, instead of using the sum SE expression in Theorem 1, we work with the large-scale approximation from Theorem 2 which turns out to be tight for even moderate M in the simulation results of Section VI-A. Thus, the power allocation optimization problem is formulated as [25]

$$\begin{aligned} & \underset{\mathbf{p}_A, \mathbf{p}_B, p_r}{\text{maximize}} && \sum_{i=1}^N (\tilde{R}_{A,i} + \tilde{R}_{B,i}) \\ & \text{subject to} && \sum_{i=1}^N (p_{A,i} + p_{B,i}) + p_r \leq P, \\ & && \mathbf{0} \leq \mathbf{p}_A \leq p_0 \mathbf{1}, \mathbf{0} \leq \mathbf{p}_B \leq p_0 \mathbf{1}, 0 \leq p_r \leq p_1, \end{aligned} \quad (42)$$

where $\tilde{R}_{B,i}$ is the large-scale approximation for the SE of $T_{B,i}$, which can be derived in a similar fashion, p_0 and p_1 are the peak power constraints of $p_{A,i}$ ($p_{B,i}$) and p_r , respectively.

Since $\log(\cdot)$ is an increasing function, (42) can be equivalently reformulated as:

$$\begin{aligned} \mathcal{P}_1 : & \underset{\substack{\mathbf{p}_A, \mathbf{p}_B, p_r \\ \gamma_A, \gamma_B}}{\text{minimize}} && \prod_{i=1}^N (1 + \gamma_{A,i})^{-1} (1 + \gamma_{B,i})^{-1} \\ & \text{subject to} && \gamma_{A,i} \leq \frac{p_{B,i}}{\xi_i}, \gamma_{B,i} \leq \frac{p_{A,i}}{\tilde{\xi}_i}, i \in \mathcal{N} \\ & && \sum_{i=1}^N (p_{A,i} + p_{B,i}) + p_r \leq P, \\ & && \mathbf{0} \leq \mathbf{p}_A \leq p_0 \mathbf{1}, \mathbf{0} \leq \mathbf{p}_B \leq p_0 \mathbf{1}, 0 \leq p_r \leq p_1, \end{aligned}$$

where $\gamma_A \triangleq [\gamma_{A,1}, \dots, \gamma_{A,N}]^T$, $\gamma_B \triangleq [\gamma_{B,1}, \dots, \gamma_{B,N}]^T$, $\gamma_{AR,i}$ and $\gamma_{RB,i}$ are considered as the signal-to-interference-plus-noise ratio (SINR) of $\tilde{R}_{A,i}$ and $\tilde{R}_{B,i}$, respectively. Also, ξ_i and $\tilde{\xi}_i$ are respectively given by

$$\begin{aligned} \xi_i &= \sum_{j=1}^N (a_{i,j} p_{A,j} + b_{i,j} p_{B,j}) \\ &+ p_r^{-1} \sum_{j=1}^N (c_{i,j} p_{A,j} + d_{i,j} p_{B,j}) + e_i, \end{aligned} \quad (43)$$

$$R_{A,i} \rightarrow \frac{\tau_c - \tau_p}{2\tau_c} \log_2 \left(1 + \frac{1}{\frac{1}{\tau_p E_p E_u \beta_{RB,i}^2} + \frac{1}{\tau_p E_p E_r \beta_{AR,i}^4 \beta_{RB,i}^4} \sum_{n=1}^N \left(\beta_{AR,n}^2 \beta_{RB,n}^2 \left(\beta_{AR,n}^2 + \beta_{RB,n}^2 \right) \right)} \right). \quad (39)$$

and

$$\begin{aligned} \tilde{\xi}_i = & \sum_{j=1}^N \left(\tilde{a}_{i,j} p_{A,j} + \tilde{b}_{i,j} p_{B,j} \right) \\ & + p_r^{-1} \sum_{j=1}^N \left(\tilde{c}_{i,j} p_{A,j} + \tilde{d}_{i,j} p_{B,j} \right) + \tilde{e}_i, \end{aligned} \quad (44)$$

where

$$a_{i,j} = \frac{1}{M} \begin{cases} \frac{4\beta_{AR,i}}{\sigma_{RB,i}^2}, & j = i, \\ \frac{\beta_{AR,i}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^2 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4}, & j \neq i, \end{cases} \quad (45)$$

$$b_{i,j} = \frac{1}{M} \begin{cases} \frac{\beta_{RB,i}}{\sigma_{RB,i}^2} + \frac{\beta_{AR,i}}{\sigma_{AR,i}^2}, & j = i, \\ \frac{\beta_{RB,i}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^2 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4}, & j \neq i, \end{cases} \quad (46)$$

$$c_{i,j} = \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^2}{M \sigma_{AR,i}^4 \sigma_{RB,i}^4}, \quad (47)$$

$$d_{i,j} = \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^2}{M \sigma_{AR,i}^4 \sigma_{RB,i}^4}, \quad (48)$$

$$e_i = \frac{1}{M \sigma_{RB,i}^2}, \quad (49)$$

and $\tilde{a}_{i,j}$, $\tilde{b}_{i,j}$, $\tilde{c}_{i,j}$, $\tilde{d}_{i,j}$, \tilde{e}_i are obtained by replacing the subscripts ‘‘AR’’, ‘‘RB’’ with ‘‘RB’’, ‘‘AR’’ in $b_{i,j}$, $a_{i,j}$, $d_{i,j}$, $c_{i,j}$, e_i , respectively. Note that we have replaced the equality ‘‘=’’ with ‘‘ \leq ’’ in the first two constraints of problem \mathcal{P}_1 ; however, this does not change or relax the original problem (42), since the objective function is decreasing with $\gamma_{AR,i}$ and $\gamma_{RB,i}$. Therefore, we can guarantee that these two constraints must be active at any optimal solution of \mathcal{P}_1 .

The above problem \mathcal{P}_1 is identified as a complementary geometric programming problem, which is nonconvex [30], hence, the optimal solution is intractable. Responding to this, we propose an efficient suboptimal solution for problem \mathcal{P}_1 , which significantly outperforms the uniform power allocation scheme. Specifically, noticing that if the objective function is a monomial function, the problem \mathcal{P}_1 becomes a standard GP problem, and can be solved efficiently with standard optimization tools such as CVX [31] or ggpplab [32]. The key idea is to use a monomial function $\omega_{X,i} (\gamma_{X,i})^{\mu_{X,i}}$ to approximate $1 + \gamma_{X,i}$ near an arbitrary point $\hat{\gamma}_{X,i} > 0$, where $X \in \{A, B\}$, $\mu_{X,i} = \frac{\hat{\gamma}_{X,i}}{1 + \hat{\gamma}_{X,i}}$ and $\omega_{X,i} = (\hat{\gamma}_{X,i})^{-\mu_{X,i}} (1 + \hat{\gamma}_{X,i})$. At each iteration, the GP is obtained by replacing the posynomial objective function with its best local monomial approximation near the solution obtained at the previous iteration. Then, a local optimum of the original problem \mathcal{P}_1 can be found by solving a sequence of GPs, capitalizing on the technique

proposed in [33, Lemma 1].³ Now, we outline the successive approximation algorithm to solve the original problem \mathcal{P}_1 in the following.

Algorithm 1 Successive approximation algorithm for \mathcal{P}_1

1) *Initialization.* Define a tolerance ϵ and parameter θ . Set $k = 1$, the initial values of $\hat{\gamma}_{A,i}$ and $\hat{\gamma}_{B,i}$ are chosen according to the SINR in Theorem 2 when letting $p_{A,i} = p_{B,i} = \frac{P}{4N}$, and $p_r = \frac{P}{2}$.

2) *Iteration k.* Compute $\mu_{A,i} = \frac{\hat{\gamma}_{A,i}}{1 + \hat{\gamma}_{A,i}}$ and $\mu_{B,i} = \frac{\hat{\gamma}_{B,i}}{1 + \hat{\gamma}_{B,i}}$. Then, solve the following GP problem:

$$\begin{aligned} \mathcal{P}_2 : & \underset{\substack{p_A, p_B, p_r \\ \gamma_A, \gamma_B}}{\text{minimize}} & \prod_{i=1}^N (\gamma_{A,i})^{-\mu_{A,i}} (\gamma_{B,i})^{-\mu_{B,i}} \\ & \text{subject to} & \theta^{-1} \hat{\gamma}_{A,i} \leq \gamma_{A,i} \leq \theta \hat{\gamma}_{A,i}, i \in \mathcal{N} \\ & & \theta^{-1} \hat{\gamma}_{B,i} \leq \gamma_{B,i} \leq \theta \hat{\gamma}_{B,i}, i \in \mathcal{N} \\ & & \gamma_{A,i} p_{B,i}^{-1} \xi_i \leq 1, i \in \mathcal{N} \\ & & \gamma_{B,i} p_{A,i}^{-1} \tilde{\xi}_i \leq 1, i \in \mathcal{N} \\ & & \sum_{i=1}^N (p_{A,i} + p_{B,i}) + p_r \leq P, \\ & & \mathbf{0} \leq \mathbf{p}_A \leq p_0 \mathbf{1}, \mathbf{0} \leq \mathbf{p}_B \leq p_0 \mathbf{1}, 0 \leq p_r \leq p_1. \end{aligned}$$

Denote the optimal solutions by $\gamma_{A,i}^{(k)}$ and $\gamma_{B,i}^{(k)}$, $i \in \mathcal{N}$.

3) *Stopping criterion.* If $\max_i |\gamma_{A,i}^{(k)} - \hat{\gamma}_{A,i}| < \epsilon$ and/or $\max_i |\gamma_{B,i}^{(k)} - \hat{\gamma}_{B,i}| < \epsilon$, stop; otherwise, go to step 4).

4) *Update initial values.* Set $\hat{\gamma}_{A,i} = \gamma_{A,i}^{(k)}$ and $\hat{\gamma}_{B,i} = \gamma_{B,i}^{(k)}$, and $k = k + 1$. Go to step 2).

Note that we have neglected $\omega_{A,i}$ and $\omega_{B,i}$ in the objective function of \mathcal{P}_2 , since they are constants and do not affect the problem solution at each iteration. Also, some trust region constraints, i.e., the first two constraints, are added, which limit how much the variables are allowed to differ from the current guess $\hat{\gamma}_{A,i}$ and $\hat{\gamma}_{B,i}$. The limit of any convergent sequence generated by Algorithm 1 is a Karush-Kuhn-Tucker point, and the detailed proof can be found in [35]. The parameter $\theta > 1$ controls the desired accuracy. More precisely, when θ is close to 1 it provides good accuracy for the monomial approximation but with slower convergence speed, and vice versa if θ is large. As discussed in [33], [36], [37], $\theta = 1.1$ offers a good tradeoff between the accuracy and convergence speed.

Regarding the complexity of algorithm 1, we notice that algorithm 1 is executed by solving a sequence of GP problems. According to [34], GP can be solved by the interior point

³During the submission of the current work, an independent work has appeared in [25], which also uses the technique proposed in [33] to tackle the power allocation issue in the context of multipair massive MIMO two-way relaying systems.

method with provably polynomial time complexity. Also, it can be efficiently implemented with high-quality software such as the MOSEK package.

To gain further insights, we now consider the special case when all the users transmit with the same power, i.e., $p_{A,i} = p_{B,i} = p_u$. Also, we remove the peak power constraints by assuming that p_0 and p_1 are very high. Then, the optimization problem (42) reduces to

$$\begin{aligned} \mathcal{P}_3 : \text{maximize}_{p_u, p_r} \quad & \sum_{i=1}^N \left(\tilde{R}_{A,i} + \tilde{R}_{B,i} \right) \\ \text{subject to} \quad & 2Np_u + p_r \leq P, \\ & p_u \geq 0, p_r \geq 0. \end{aligned}$$

Theorem 6: \mathcal{P}_3 is a convex optimization problem.

Proof: See Appendix B. \blacksquare

Since the optimization problem \mathcal{P}_3 is convex, the optimal solutions $p_u^{\text{opt}} \in (0, \frac{P}{2N}]$ and $p_r^{\text{opt}} \in (0, P]$ maximizing the sum SE can be obtained efficiently by adopting some standard techniques, such as the bisection method with respect to P . However, we cannot directly obtain closed-form expressions of p_u^{opt} and p_r^{opt} , since the objective function relies on the statistical characteristics of all the channel vectors. In order to simplify the analysis and provide some further insights, we assume that all the users have the same large-scale fading, e.g., $\beta_{AR,i} = \beta_{RB,i} = 1$, thereby resulting in $\sigma_{A,i}^2 = \sigma_{B,i}^2 = \sigma^2$, $\tilde{\sigma}_{A,i}^2 = \tilde{\sigma}_{B,i}^2 = \tilde{\sigma}^2$, $\tilde{R}_{A,i} = \tilde{R}_{B,i}$, and then the optimization problem \mathcal{P}_3 can be analytically solved in the following theorem:

Theorem 7: The optimization problem \mathcal{P}_3 for the scenario where all the users have the same large-scale fading, e.g., $\beta_{AR,i} = \beta_{RB,i} = 1$ is solved by

$$\begin{cases} p_u^{\text{opt}} = \frac{P}{4N}, \\ p_r^{\text{opt}} = \frac{P}{2}. \end{cases} \quad (50)$$

Theorem 7 suggests that, for a given power budget $2Np_u + p_r \leq P$, half of the total power should be allocated to the relay regardless of the number of users, and the remaining half should be equally allocated to the $2N$ users. Such a symmetric power allocation strategy is rather intuitive due to the symmetric system setup. In addition, it can be directly inferred that the optimal power p_u^{opt} decreases monotonically by increasing the number of user pairs N , which serves as a useful guideline for practical system design.

VI. NUMERICAL RESULTS

In this section, we present numerical results to validate the previous analytical results. For all illustrative examples, the following set of parameters are used in simulation. Unless otherwise specified, the length of the coherence interval is $\tau_c = 196$ symbols, chosen by the LTE standard. The length of the pilot sequences is $\tau_p = 2N$ which is the minimum requirement. For simplicity, we set the large-scale fading coefficient $\beta_{AR} = \beta_{RB} = 1$, and assume that each user has the same transmit power, i.e., $p_{A,i} = p_{B,i} = p_u$.

A. Validation of analytical expressions

Fig. 2 shows the sum SE versus the transmit power of each user p_u for different number of relay antennas with $p_p = p_u$ and $p_r = 2Np_u$. Note that the ‘‘Approximations’’ curves are obtained by using (23), and the ‘‘Numerical results’’ curves are generated according to (16) by averaging over 10^4 independent channel realizations, respectively. As can be readily observed, the large-scale approximations are very accurate, especially for large antenna arrays. Also, as expected, increasing the number of relay antennas significantly yields higher SE.

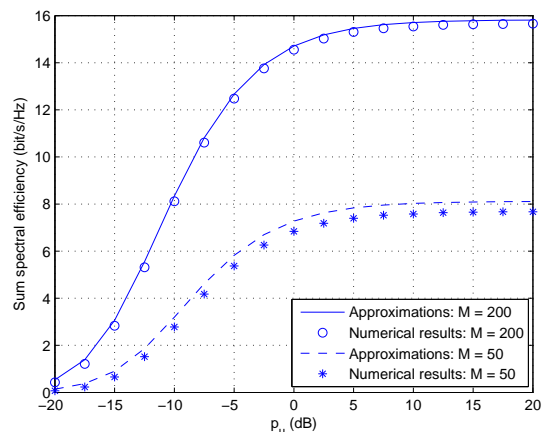


Fig. 2: Sum SE versus p_u for $N = 5$, $p_p = p_u$ and $p_r = 2Np_u$.

B. Power scaling

In this subsection, we present numerical simulation results to verify the previous power scaling law analysis for three different scenarios, and demonstrate the power efficiency of using large number of antennas at the relay.

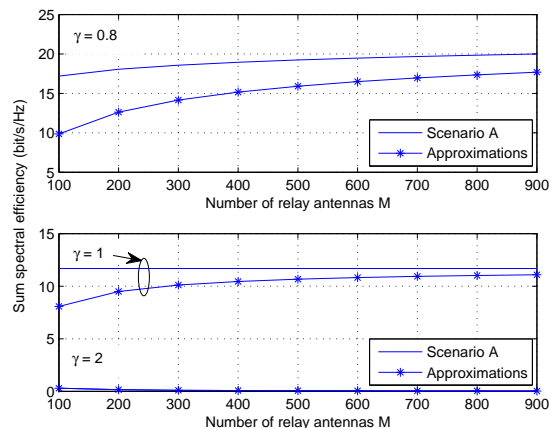


Fig. 3: Sum SE versus the number of relay antennas M for $N = 5$, $p_u = 10$ dB, $p_r = 20$ dB, and $p_p = E_p/M^\gamma$ with $E_p = 10$ dB.

1) *Scenario A*: Fig. 3 verifies the analytical results for Scenario A. Note that the curves labelled as ‘‘Scenario A’’ are plotted according to Theorems 3. We can see that when M is large, the two curves are almost overlapped, which means that the previous asymptotic analysis is very accurate. In addition, when $\gamma > 1$, e.g., $\gamma = 2$, the SE gradually approaches zero. In contrast, when $0 < \gamma < 1$, e.g., $\gamma = 0.8$, the SE grows unbounded. Finally, when $\gamma = 1$, the SE converges to a non-zero limit.

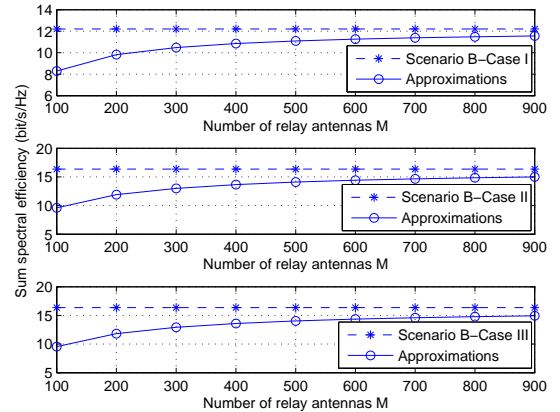
2) *Scenario B*: Fig. 4 investigates how the transmit power of each user $p_u = \frac{E_u}{M^\alpha}$ and the transmit power of the relay $p_r = \frac{E_r}{M^\beta}$ scale with M . To fully evaluate the SE behavior, we consider three different cases based on the values of α and β , namely, 1) Case I: $\alpha = \beta = 1$; 2) Case II: $\alpha = 1, \beta = 0.2$; 3) Case III: $\alpha = 0.4, \beta = 1$. Note that the curves labelled as ‘‘Scenario B’’ are generated by using Theorem 4, while the curves labelled as ‘‘Scenario B-Case X’’ with $X \in \{I, II, III\}$ are plotted according to Corollaries 1–3, respectively. Fig. 4(a) shows that the sum SE saturates in the asymptotical large M regime for all the three cases, which agrees with Corollaries 1–3. Furthermore, Case I has the lowest SE due to simultaneously cutting the transmit powers of each user and of the relay, while Case II and Case III achieve the same performance due to the setting of $E_r = 2NE_u$.

Fig. 4(b) illustrates the system performance when the transmit power down-scaling is either too aggressive or too moderate. As expected, as the number of relay antennas increases, the sum SE gradually reduces to zero for $\alpha > 1, \beta \geq 0$, $\alpha \geq 0, \beta > 1$, or $\alpha > 1, \beta > 1$. However, the speed of reduction varies significantly depending on the scaling parameters. The larger the scaling parameters, the faster the decay of the SE. In contrast, if we cut down the transmit powers of each user and of the relay moderately, the sum SE grows unboundedly.

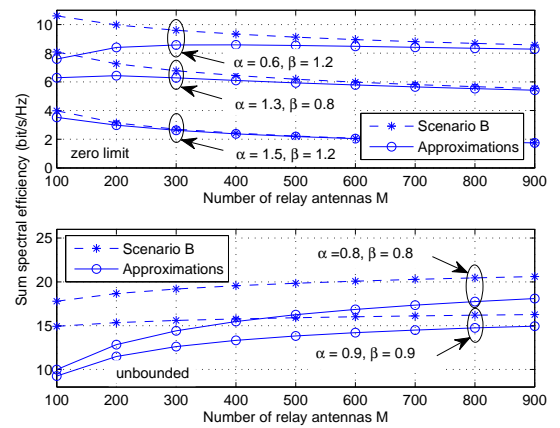
3) *Scenario C*: Fig. 5 presents the tradeoff between the user/relay power and the pilot symbol power. We set two examples, i.e., $\alpha = 1.3, \beta = 1.1, \gamma = 0.5$ and $\alpha = 0.8, \beta = 0.6, \gamma = 1$, which satisfy $\alpha + \gamma = 1.8$ and $\beta + \gamma = 1.6$. As predicted, the sum SE converges to zero for too aggressive power down-scaling. Moreover, the gaps between the two sets of curves narrow down with M and eventually vanish. This indicates that as long as $\alpha + \gamma$ and $\beta + \gamma$ are the same, the asymptotic sum SE remains unchanged. Now, let us focus on the two curves associated with $N = 5$. Interestingly, we see that the curve associated with $\gamma = 0.5$ yields better performance in the finite antenna regime, despite the fact that the user or relay power is over-reduced compared to the $\gamma = 1$ case, which suggests that the channel estimation accuracy is crucially important for the system. The same behavior appears for the unbounded SE scenario where $\alpha + \gamma = 0.9$ and $\beta + \gamma = 0.8$.

C. Power allocation

Fig. 6 illustrates the impact of the optimal power allocation scheme on the sum SE. The different large-scale fading parameters are arbitrarily generated by $\beta_{AR,i} = z_i (r_{AR,i}/r_0)^\alpha$ and $\beta_{RB,i} = z_i (r_{RB,i}/r_0)^\alpha$, where z_i is a log-normal random



(a) Case I, II, and III



(b) Zero limit and unbounded

Fig. 4: Sum SE versus the number of relay antennas M for $p_p = 10$ dB, $p_u = E_u/M^\alpha$ with $E_u = 10$ dB, and $p_r = E_r/M^\beta$ with $E_r = 20$ dB.

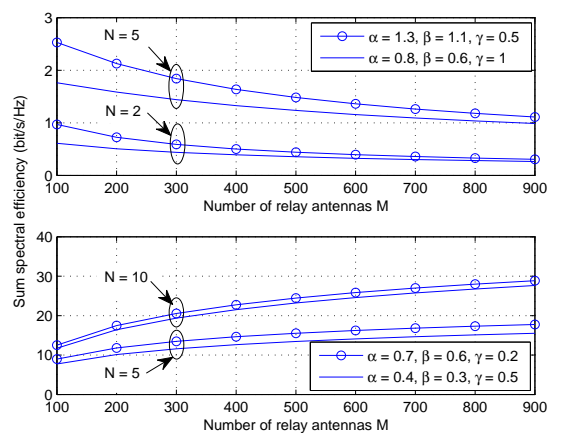


Fig. 5: Sum SE versus the number of relay antennas M for $p_u = E_u/M^\alpha$ with $E_u = 10$ dB, $p_r = E_r/M^\beta$ with $E_r = 15$ dB, and $p_p = E_p/M^\gamma$ with $E_p = 0$ dB.

variable with standard deviation 8 dB, $r_{AR,i}$ and $r_{RB,i}$ are

the locations of $T_{AR,i}$ and $T_{RB,i}$ from the relay, $\alpha = 3.8$ is the path loss exponent, and r_0 denotes the guard interval which specifies the nearest distance between the users and the relay. The relay is located at the center of a cell with a radius of 1000 meters and $r_0 = 100$ meters. We choose $P = 10$ dB, $p_p = 10$ dB, $N = 5$, $p_0 = P/2N$, $p_1 = P$, $\beta_{AR} = [0.2688, 0.0368, 0.00025, 0.1398, 0.0047]$, and $\beta_{RB} = [0.0003, 0.00025, 0.0050, 0.0794, 0.0001]$. The optimal power allocation curves are generated by Algorithm 1. Also, we plot the sum SE with uniform power allocation as a benchmark scheme for comparison. As can be observed, the optimal power allocation policy provides 23.1% SE enhancement when $M = 300$. Moreover, by focusing on the case where every user has the same transmit power, i.e., $p_{A,i} = p_{B,i} = p_u$, we can see that the optimal user transmit power is a decreasing function with respect to the number of user pairs N , which aligns with Theorem 7.

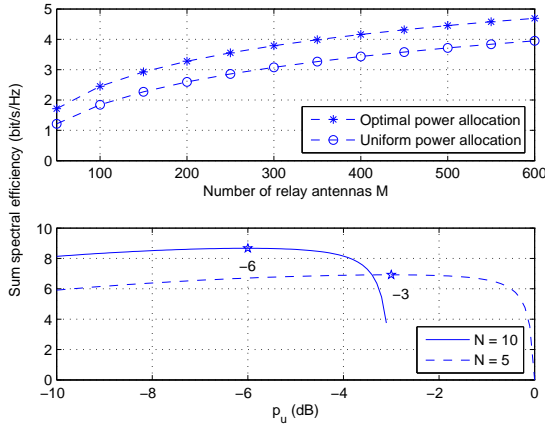


Fig. 6: Sum SE for a given power budget $P = 10$ dB.

VII. CONCLUSION

We have investigated the sum SE of a multipair two-way half-duplex relaying system employing the MR scheme taking into account of realistic CSI. In particular, a closed-form expression and a large-scale approximation were derived for the sum SE. Moreover, different power scaling laws of the system were characterized, which showed that the transmit powers of user, relay, and pilot symbol can be scaled down inversely proportional to the number of relay antennas. In addition, it was revealed that there exists a fundamental tradeoff between the user/relay transmit power and pilot symbol power, which provides great flexibility for the design of practical systems. Finally, the transmit powers of each user and the relay were optimized to enhance the sum SE.

APPENDIX A

PROOF OF THEOREM 1

The end-to-end SINR given in (11) consists of five terms: 1) desired signal power A_i ; 2) estimation error B_i ; 3) residual self-interference C_i ; 4) inter-user interference D_i ; 5) compound noise E_i .

1) Compute A_i :

$$\begin{aligned} & \mathbb{E} \{ \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} \} \\ &= \mathbb{E} \{ | \hat{\mathbf{g}}_{AR,i}^H \hat{\mathbf{g}}_{RB,i} |^2 + \| \hat{\mathbf{g}}_{AR,i} \|^2 \| \hat{\mathbf{g}}_{RB,i} \|^2 \} \\ &= M(M+1) \sigma_{AR,i}^2 \sigma_{RB,i}^2. \end{aligned} \quad (51)$$

Consequently, we obtain

$$A_i = p_{B,i} M^2 (M+1)^2 \sigma_{AR,i}^4 \sigma_{RB,i}^4. \quad (52)$$

2) Compute B_i :

$$\begin{aligned} & \mathbb{E} \{ | \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} |^2 \} \\ &= \mathbb{E} \left\{ \sum_{n=1}^N \sum_{l=1}^N \mathbf{g}_{AR,i}^T \mathbf{C}_n \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \mathbf{C}_l^H \mathbf{g}_{AR,i}^* \right\}, \end{aligned} \quad (53)$$

where $\mathbf{C}_n = (\hat{\mathbf{g}}_{RB,n}^* \hat{\mathbf{g}}_{AR,n}^H + \hat{\mathbf{g}}_{AR,n}^* \hat{\mathbf{g}}_{RB,n}^H)$, which can be decomposed into three different cases:

a) for $n \neq l \neq i$, we have $\mathbb{E} \{ | \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} |^2 \} = 0$.

b) for $n = l \neq i$, we have

$$\begin{aligned} & \mathbb{E} \{ | \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} |^2 \} \\ &= 2M(M+1) \beta_{AR,i} \beta_{RB,i} \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2. \end{aligned} \quad (54)$$

c) for $n = l = i$, we have

$$\begin{aligned} & \mathbb{E} \{ | \mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,i} |^2 \} \\ &= \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{RB,i}^* \hat{\mathbf{g}}_{AR,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{AR,i} \hat{\mathbf{g}}_{RB,i}^T \mathbf{g}_{AR,i}^* \} \\ &+ \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{RB,i}^* \hat{\mathbf{g}}_{AR,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{RB,i} \hat{\mathbf{g}}_{AR,i}^T \mathbf{g}_{AR,i}^* \} \\ &+ \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{AR,i}^* \hat{\mathbf{g}}_{RB,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{AR,i} \hat{\mathbf{g}}_{RB,i}^T \mathbf{g}_{AR,i}^* \} \\ &+ \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{AR,i}^* \hat{\mathbf{g}}_{RB,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{RB,i} \hat{\mathbf{g}}_{AR,i}^T \mathbf{g}_{AR,i}^* \}. \end{aligned} \quad (55)$$

The first term in (55) becomes

$$\begin{aligned} & \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{RB,i}^* \hat{\mathbf{g}}_{AR,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{AR,i} \hat{\mathbf{g}}_{RB,i}^T \mathbf{g}_{AR,i}^* \} \\ &= \mathbb{E} \{ | \tilde{g}_{AR,i} |^4 \| \hat{\mathbf{g}}_{RB,i} \|^4 \} + \tilde{\sigma}_{AR,i}^2 \mathbb{E} \{ | \tilde{g}_{AR,i} |^2 \| \hat{\mathbf{g}}_{RB,i} \|^4 \} \\ &+ \tilde{\sigma}_{RB,i}^2 \mathbb{E} \{ | \tilde{g}_{RB,i} |^2 \| \hat{\mathbf{g}}_{AR,i} \|^4 \} \\ &+ \tilde{\sigma}_{RB,i}^2 \mathbb{E} \{ | \hat{\mathbf{g}}_{RB,i}^H \mathbf{e}_{AR,i} |^2 \} \mathbb{E} \{ \| \hat{\mathbf{g}}_{AR,i} \|^2 \}, \end{aligned} \quad (56)$$

where $\tilde{g}_{AR,i} \triangleq \frac{\hat{\mathbf{g}}_{AR,i}^H \hat{\mathbf{g}}_{RB,i}}{\| \hat{\mathbf{g}}_{RB,i} \|^2}$ and $\tilde{g}_{RB,i} \triangleq \frac{\hat{\mathbf{g}}_{AR,i} \hat{\mathbf{g}}_{RB,i}^H}{\| \hat{\mathbf{g}}_{AR,i} \|^2}$. The vector $\tilde{g}_{AR,i}$ is a Gaussian random variable with zero mean and variance $\sigma_{AR,i}^2$ which is independent of $\hat{\mathbf{g}}_{RB,i}$, whereas $\tilde{g}_{RB,i}$ is a Gaussian random variable with zero mean and variance $\sigma_{RB,i}^2$ which is independent of $\hat{\mathbf{g}}_{AR,i}$. Therefore, (56) can be calculated as

$$\begin{aligned} & \mathbb{E} \{ \mathbf{g}_{AR,i}^T \hat{\mathbf{g}}_{RB,i}^* \hat{\mathbf{g}}_{AR,i}^H \mathbf{g}_{RB,i} \mathbf{g}_{RB,i}^H \hat{\mathbf{g}}_{AR,i} \hat{\mathbf{g}}_{RB,i}^T \mathbf{g}_{AR,i}^* \} \\ &= M(M+1) \sigma_{AR,i}^2 \sigma_{RB,i}^2 (\beta_{AR,i} \sigma_{RB,i}^2 + \beta_{RB,i} \sigma_{AR,i}^2) \\ &+ M^2 \sigma_{AR,i}^2 \sigma_{RB,i}^2 \tilde{\sigma}_{AR,i}^2 \tilde{\sigma}_{RB,i}^2. \end{aligned} \quad (57)$$

Following the same procedure, the last three terms in (55) can also be derived. Finally, combining a), b), and c), we obtain B_i .

3) Compute C_i : By utilizing the same technique as in the derivation of B_i , we obtain C_i .

4) Compute D_i :

$$\begin{aligned} & \mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} \\ &= \mathbb{E} \left\{ \sum_{n=1}^N \sum_{l=1}^N \mathbf{g}_{AR,i}^T \mathbf{C}_n \mathbf{g}_{AR,j} \mathbf{g}_{AR,j}^H \mathbf{C}_l^H \mathbf{g}_{AR,i}^* \right\}, \end{aligned} \quad (58)$$

which can be decomposed into six different cases:

a) for $n \neq l \neq i, j$ ($j \neq i$), we have $\mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} = 0$.

b) for $n = l \neq i, j$ ($j \neq i$), we have

$$\begin{aligned} & \mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} \\ &= 2M(M+1) \beta_{AR,i} \beta_{AR,j} \sum_{n \neq i, j} \sigma_{AR,n}^2 \sigma_{RB,n}^2. \end{aligned} \quad (59)$$

c) for $n = l = i$ ($j \neq i$), we have

$$\begin{aligned} & \mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} \\ &= M \beta_{AR,j} \sigma_{AR,i}^2 \sigma_{RB,i}^2 (M+1)(M+3) \sigma_{AR,i}^2 \\ &+ M \beta_{AR,j} \sigma_{AR,i}^2 \sigma_{RB,i}^2 2(M+1) \tilde{\sigma}_{AR,i}^2. \end{aligned} \quad (60)$$

d) for $n = l = j$ ($j \neq i$), we have

$$\begin{aligned} & \mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} \\ &= M \beta_{AR,i} \sigma_{AR,j}^2 \sigma_{RB,j}^2 (M+1)(M+3) \sigma_{AR,j}^2 \\ &+ M \beta_{AR,i} \sigma_{AR,j}^2 \sigma_{RB,j}^2 2(M+1) \tilde{\sigma}_{AR,j}^2. \end{aligned} \quad (61)$$

e) for $n = i$ and $l = j$ ($j \neq i$), we have $\mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} = 0$.

f) for $n = j$ and $l = i$ ($j \neq i$), we have $\mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} = 0$.

Altogether, $\mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\}$ is given by

$$\begin{aligned} & \mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{AR,j}|^2 \right\} \\ &= 2M(M+1) \beta_{AR,i} \beta_{AR,j} \sum_{n \neq i, j} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \\ &+ M \beta_{AR,j} \sigma_{AR,i}^2 \sigma_{RB,i}^2 (M+1)(M+3) \sigma_{AR,i}^2 \\ &+ M \beta_{AR,j} \sigma_{AR,i}^2 \sigma_{RB,i}^2 2(M+1) \tilde{\sigma}_{AR,i}^2 \\ &+ M \beta_{AR,i} \sigma_{AR,j}^2 \sigma_{RB,j}^2 (M+1)(M+3) \sigma_{AR,j}^2 \\ &+ M \beta_{AR,i} \sigma_{AR,j}^2 \sigma_{RB,j}^2 2(M+1) \tilde{\sigma}_{AR,j}^2. \end{aligned} \quad (62)$$

Following the same technique as in deriving (62), we can obtain $\mathbb{E} \left\{ |\mathbf{g}_{AR,i}^T \mathbf{F} \mathbf{g}_{RB,j}|^2 \right\}$ and finally derive D_i .

5) Compute E_i :

(a) Compute $\mathbb{E} \left\{ \|\mathbf{g}_{AR,i}^T \mathbf{F}\|^2 \right\}$:

Again, using the same technique as in the derivation of (62), we obtain

$$\begin{aligned} & \mathbb{E} \left\{ \|\mathbf{g}_{AR,i}^T \mathbf{F}\|^2 \right\} \\ &= 2M(M+1) \beta_{AR,i} \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \\ &+ M \sigma_{AR,i}^2 \sigma_{RB,i}^2 (M+1)(M+3) \sigma_{AR,i}^2 \\ &+ M \sigma_{AR,i}^2 \sigma_{RB,i}^2 2(M+1) \tilde{\sigma}_{AR,i}^2. \end{aligned} \quad (63)$$

(b) Compute ρ^2 :

$$\begin{aligned} & \mathbb{E} \left\{ \|\mathbf{F}\|_F^2 \right\} = \mathbb{E} \left\{ \text{tr} \left(\mathbf{A} \mathbf{B}^T \mathbf{B}^* \mathbf{A}^H \right) \right\}, \\ &= \text{tr} \left(\mathbb{E} \left\{ \hat{\mathbf{G}}_{AR} \hat{\mathbf{G}}_{RB} \hat{\mathbf{G}}_{RB}^* \hat{\mathbf{G}}_{AR}^H \right\} \right) \\ &+ \text{tr} \left(\mathbb{E} \left\{ \hat{\mathbf{G}}_{AR} \hat{\mathbf{G}}_{RB} \hat{\mathbf{G}}_{AR}^* \hat{\mathbf{G}}_{RB}^H \right\} \right) \\ &+ \text{tr} \left(\mathbb{E} \left\{ \hat{\mathbf{G}}_{RB} \hat{\mathbf{G}}_{AR} \hat{\mathbf{G}}_{RB}^* \hat{\mathbf{G}}_{AR}^H \right\} \right) \\ &+ \text{tr} \left(\mathbb{E} \left\{ \hat{\mathbf{G}}_{RB} \hat{\mathbf{G}}_{AR} \hat{\mathbf{G}}_{AR}^* \hat{\mathbf{G}}_{RB}^H \right\} \right), \\ &= 2M(M+1) \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2. \end{aligned} \quad (64)$$

Combining $\mathbb{E} \left\{ \|\mathbf{F} \mathbf{g}_{AR,i}\|^2 \right\}$, $\mathbb{E} \left\{ \|\mathbf{F} \mathbf{g}_{RB,i}\|^2 \right\}$, and (64), ρ^2 is expressed as

$$\rho^2 = \frac{p_r}{\sum_{i=1}^N (a_i + b_i) + 2M(M+1) \sum_{n=1}^N \sigma_{AR,n}^2 \sigma_{RB,n}^2}, \quad (65)$$

where

$$\begin{aligned} a_i &= 2(M+1) (\tilde{\sigma}_{AR,i}^2 p_{A,i} + \tilde{\sigma}_{RB,i}^2 p_{B,i}) \\ &+ M \sigma_{AR,i}^2 \sigma_{RB,i}^2 (M+1)(M+3) (\sigma_{AR,i}^2 p_{A,i} + \sigma_{RB,i}^2 p_{B,i}), \end{aligned} \quad (66)$$

$$b_i = 2M(M+1) (\beta_{AR,i} p_{A,i} + \beta_{RB,i} p_{B,i}) \sum_{n \neq i} \sigma_{AR,n}^2 \sigma_{RB,n}^2 \quad (67)$$

We arrive at the desired result E_i by combining (63) and (65).

APPENDIX B PROOF OF THEOREM 6

For a given p_u , the objective function of the optimization problem \mathcal{P}_3 is an increasing function with respect to p_r , while for a given p_r , this function is an increasing function with respect to p_u ; hence, the objective function is maximized when $2Np_u + p_r = P$ [38].

Now, focusing on $\tilde{R}_{A,i}$ and substituting $2Np_u + p_r = P$ into $\tilde{R}_{A,i}$, we have

$$\tilde{R}_{A,i} = \frac{1}{2} \log_2 \left(1 + \frac{1}{a + \frac{b}{p_u} + \frac{c}{d-p_u}} \right), \quad (68)$$

where $a = \frac{1}{M} \left(\frac{\beta_{RB,i} + 4\beta_{AR,i}}{\sigma_{RB,i}^2} + \frac{\beta_{AR,i}}{\sigma_{AR,i}^2} \right) + \frac{1}{M} \sum_{j \neq i} \left(\frac{\beta_{AR,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^2 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right) + \frac{1}{M} \sum_{j \neq i} \left(\frac{\beta_{RB,j}}{\sigma_{RB,i}^2} + \frac{\sigma_{AR,j}^2 \sigma_{RB,j}^4 \beta_{AR,i}}{\sigma_{AR,i}^4 \sigma_{RB,i}^4} \right)$, $b = \frac{1}{M \sigma_{RB,i}^2}$, $c = \frac{1}{2MN \sigma_{AR,i}^4 \sigma_{RB,i}^4} \sum_{n=1}^N (\sigma_{AR,n}^2 \sigma_{RB,n}^2 (\sigma_{AR,n}^2 + \sigma_{RB,i}^2))$, and $d = \frac{P}{2N}$.

Taking the second derivative with respect to p_u yields

$$\begin{aligned} \frac{\partial^2 \tilde{R}_{A,i}}{\partial p_u^2} = & -\frac{b^2 \left(2cd^2 + (1+2a)(d-p_u)^3 \right) (d-p_u)}{2 \ln 2 f^2 g^2} \\ & -\frac{c(c+2ac+2a(1+a)(d-p_u)) p_u^4}{2 \ln 2 f^2 g^2} \\ & -\frac{bp_u \left(c^2 d^2 + a(1+a)(d-p_u)^4 \right)}{\ln 2 f^2 g^2} \\ & -\frac{bp_u(1+2a)c(d-p_u)(d^2-dp_u+p_u^2)}{\ln 2 f^2 g^2} < 0, \end{aligned}$$

where $f = b(d-p_u) + (c+a(d-p_u))p_u$ and $g = f + (d-p_u)p_u$. Thus, $\tilde{R}_{A,i}$ is a strictly concave function with respect to p_u . Since nonnegative weighted sums preserve convexity [39], the objective function $\frac{\tau_c - \tau_p}{\tau_c} \sum_{i=1}^N \left(\tilde{R}_{A,i} + \tilde{R}_{B,i} \right)$ is also a strictly concave function with respect to p_u . Recall that the constraints of the optimization problem \mathcal{P}_3 are all affine functions, and hence \mathcal{P}_3 is a convex optimization problem.

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