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# Low Complexity Greedy Detection Method with Generalized Multicarrier Index Keying OFDM

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Abstract—Multicarrier Index Keying (MCIK) is a recently developed technique that modulates subcarriers but also indices of the subcarriers. In this paper a novel low-complexity detection scheme of subcarrier indices is proposed for an MCIK system and addresses a substantial reduction in complexity over the optimal maximum likelihood (ML) detection. For the performance evaluation, a closed-form expression for the pairwise error probability (PEP) of an active subcarrier index, and a tight approximation of the average PEP of multiple subcarrier indices are derived in closed-form. The theoretical outcomes are validated using simulations, at a difference of less than 0.1dB. Compared to the optimal ML, the proposed detection achieves a substantial reduction in complexity with small loss in error performance ( $\leq 0.6dB$ ).

# I. INTRODUCTION

In recent years multicarrier modulation schemes have become increasingly popular, in particular orthogonal frequency division multiplexing (OFDM) has been included in many wireless standards due to its robustness to multipath fading. The concept of multicarrier index keying (MCIK) was originally applied to OFDM in [1] and [2] where data is transmitted via the indices of subcarriers as well the subcarriers. This is similar to spatial modulation (SM) [3], and later generalized SM (GSM) [4], which were introduced as multi-antenna transmission schemes where data is not only transmitted via antenna but also via the indices of the transmitting antenna. The principle of both MCIK and SM comes from exploiting the indices of channel(s) over which the data is delivered.

The MCIK techniques provide a variety of benefits over the conventional OFDM such as reduced bit error rate (BER) and complexity, and increased energy efficiency. In particular, the OFDM with index modulation (OFDM-IM) [5] was proposed to combat the assumption of a perfect feed forward of excess subcarriers, which are needed to signal the subsequent mapping of subcarrier indices in [1] and [2]. OFDM-IM considered a method of separating the subcarriers of a system into clusters of an adjustable number of subcarriers called subblocks. Despite OFDM-IM presenting a better trade off of energy efficiency and system performance compared with the conventional OFDM, it still has limitations. OFDM-IM only performs marginally better than OFDM at high SNR regions with the same transmission rates. In order to improve the performance, OFDM with interleaved subcarrier index modulation (OFDM-ISIM) was introduced in [6]. The OFDM-

ISIM scheme incorporated a subblock interleaver which increased the Euclidean distance between received data symbols, resulting in performance improvement over OFDM-IM and OFDM. More recently the generalization of OFDM-IM is proposed in [7], named as OFDM with generalized index modulation (OFDM-GIM). This scheme differs from OFDM-IM in that the number of active subcarriers in a subblock is no longer fixed. By more flexibly selecting which subcarriers are to be active, the spectral efficiency can be further improved with a minor increase in BER.

In terms of performance analysis of the MCIK system, the error performance of the pairwise error probability (PEP) has been presented in [5] and more recently an expression for a tight upper bound on the BER of joint OFDM and MCIK is presented in [8].

In this work, we focus on developing a detection method for MCIK systems. To the best knowledge of the authors, the existing MCIK approaches are based on maximum-likelihood (ML) detection, which is highly complex to operate. The contribution of this paper is twofold. Firstly, a novel low complexity greedy detection method for a generalized MCIK-OFDM system is proposed. The generalization comes from the fact that the concerned MCIK is for a range of number of active subcarriers. The key advantage of the proposed greedy detection method is to significantly reduce the complexity over the optimal ML detection scheme [5] lending itself to applications such as: device to device (D2D) communications, bodycentric communications (BCC), or other applications with low complexity as a key design parameter. Secondly, the PEP for MCIK with greedy detection will be analyzed deriving closedform expressions. The performance of the MCIK-OFDM with greedy detection over the benchmark detection scheme is then analyzed for comparison. We show that the greedy detection achieves a negligible loss in performance with a substantial reduction in complexity.

Annotation: Bold, lower case and capital letters are used for vectors and matrices, respectively. S denotes the complex signal constellation.  $\lfloor \cdot \rfloor$  represents the floor function.  $\binom{n}{k}$  is the binomial coefficient for *n* choose *k*.

#### II. SYSTEM MODEL

Consider an OFDM system with N subcarriers. In the conventional OFDM system, a stream of M-QAM symbols is

first serial-to-parallel converted, where every N symbols are grouped into a vector  $\mathbf{s}_{OFDM} = [s_1, s_2, \dots, s_N]^T$  and each data symbol  $s_i \in S$  is used to modulate each subcarrier.

For every transmission in MCIK-OFDM, only a fraction of the N subcarriers are modulated to deliver data symbols in s, while the remaining N - K subcarriers are inactive. In particular, we consider an MCIK-OFDM, where only K among the N subcarriers are activated. K active subcarriers transmit data symbols, while the remaining N - K subcarriers are zero padded.

Let a set of K active subcarrier indices be denoted by  $\mathbf{I} = \{i_1, \ldots, i_K\}$ , where  $i_k \in [1, \ldots, N]$  and  $k = 1, \ldots, K$ . Accordingly, a block of data symbols is denoted by  $\mathbf{s} = [s(1), \ldots, s(K)]$  where  $s(k) \in S$ . Using both  $\mathbf{I}$  and  $\mathbf{s}$ , the OFDM block based on the MCIK is given by  $\mathbf{x} = [x(1)x(2)\cdots x(N)]^T$ , where  $x(\gamma) \in \{0, S\}$ , and  $\gamma = 1, \ldots, N$ . Unlike the conventional OFDM,  $\mathbf{x}$  has (N - K) zero elements whose indices help to carry additional data.

The total available number of active subcarrier index combinations is  $\binom{N}{K}$ ; but for simplicity in analysis and efficient mapping of the binary bits, we use  $c = 2^{\lfloor \log_2 \binom{N}{K} \rfloor}$  combinations. In every transmission,  $m_1 = \lfloor \log_2 \binom{N}{K} \rfloor$  bits are used to modulate the indices of the subcarriers that will deliver the data symbols. In addition,  $m_2 = K \log_2 M$  bits are transmitted via the data symbols on the subcarriers whose indices are modulated by  $m_1$  bits. Thus  $m = m_1 + m_2$  bits in total are transmitted per transmit interval. As a result, not only is information conveyed via complex data symbols but also through the indices of subcarriers.

The MCIK-OFDM block is transmitted over a frequencyselective Rayleigh fading channel. The channel impulse coefficients (CIR) can be defined as  $\mathbf{H} = \text{diag}(h(1), \ldots, h(N))$ where  $h(\gamma)$  for  $\gamma \in \mathbf{I}$  represent Rayleigh fading channel as an independent and identically distributed (i.i.d.) complex Gaussian random variable (RV) with zero mean and unit variance, i.e.,  $h(\gamma) \sim C\mathcal{N}(0, 1)$ , and  $h(\gamma)$  for  $\gamma \notin \mathbf{I}$  are zeros. We can then define the input-output relationship as

$$\mathbf{y} = \mathbf{s}\mathbf{H} + \mathbf{n} \tag{1}$$

where  $\mathbf{y} = [y(1), \ldots, y(N)]$ ,  $\mathbf{s} = [\mathbf{0}_{1 \times i_k - 1}, s(i_k), \mathbf{0}_{1 \times N - i_k}]$ for  $s(i_k) \in S$ , and  $\mathbf{n} = [n(1), \ldots, n(N)]$  is an independent additive white Gaussian noise (AWGN) vector where  $n(\gamma) \sim C\mathcal{N}(0, N_0), \forall \gamma$ . The signal to noise ratio is denoted by  $\rho = E_s/N_0$  where  $E_s$  is the average symbol power of the data symbol.

#### **III. GREEDY DETECTOR**

We propose a novel low-complexity greedy detector for the MCIK scheme. Note that we investigate the MCIK scheme as generalized with any number K of active subcarriers between 1 and (N-1). Unlike the conventional OFDM, the receiver must follow two part detection process where the indices of the active subcarriers and corresponding data symbols are detected separately. To first detect the active subcarrier indices, the proposed detection method involves the detection process involves finding the received signal power on each subcarrier. Among

the measured received powers, the criterion of detection is to choose subcarriers with K greatest received powers as the estimate of the active subcarriers. Here, subcarriers only under favorable channel fading are highly likely to be estimated as activated ones, and is referred to as greedy detection, hereafter. This energy based greedy detection does not require channel information. Secondly, the data symbols in s are detected, applying the ML decision individually to the estimated active subcarriers.

Specifically, the greedy detection rules are described in the following two step process (i.e., Step 1.1-1.2 and Step 2).

Step 1.1: Estimation of the indices of active subcarriers.

- 1) Let a residual vector  $\mathbf{z}_0 = \mathbf{y}$ . A demodulated vector  $\mathbf{r}_0$  is set to a zero vector, i.e.,  $\mathbf{r}_t = [r_t(1), \ldots, r_t(N)]$  with  $r_0(\gamma) = 0$ ,  $\forall \gamma$ , and the iteration count t = 0 where  $t = 1, \ldots, K$ .
- 2) The subcarrier with the greatest received power is estimated as one of the activated subcarrier and its index  $\hat{\gamma}$  is given by

$$\hat{\gamma} = \arg\max_{\gamma} |z_t(\gamma)|^2 \tag{2}$$

- 3) Let  $r_t(\hat{\gamma}) = z_t(\hat{\gamma})$  and  $z_t(\hat{\gamma}) = 0$ , and increment t by t = t + 1.
- 4) Repeat parts 2 and 3 until t = K.

Step 1.2: Recovery of  $m_1$  bits via a look-up table (LUT).

- 1) Set all non-zero elements in  $\mathbf{r}_K$  equal to 1.
- 2) Recover the  $m_1$  bits for the corresponding  $\mathbf{r}_K$  using an LUT. An example of an LUT used in the case where N = 4 and K = 2 can be seen in Table I.

LUT For $N = 4$ and $K = 2$			
	$\mathbf{r}_{K}$	$m_1$ bits	]
	1100	001	
	1010	010	
	1001	011	
	0110	100	
	0101	101	
	0011	110	]

Table I

*Step 2:* Finally we estimate s by using the following ML decision individually at each subcarrier  $\hat{\gamma}$  as

$$\hat{\mathbf{s}}\left(\hat{\gamma}\right) = \arg\min_{s\left(\hat{\gamma}\right)\in\mathcal{S}}\left|y\left(\hat{\gamma}\right) - h\left(\hat{\gamma}\right)s\left(\hat{\gamma}\right)\right|^{2}, \forall\hat{\gamma}.$$
(3)

# IV. PEP ANALYSIS

For the performance analysis, we focus only on the noisy estimate case when subcarrier indices are incorrectly detected. This is because the main difference between the greedy detection and the optimal ML detection is the way each method detects which subcarrier indices are activated.

To this end, let the pairwise error event (PEE) be the case when an active subcarrier index is incorrectly detected as another subcarrier index, i.e.,  $(\gamma \rightarrow \tilde{\gamma})$  where  $\gamma, \tilde{\gamma} \in \{1, \dots, N\}$ and  $\gamma \neq \tilde{\gamma}$ . With the channel information available at the receiver, we define the conditional pairwise error probability (PEP) as the probability of the PEE  $(\gamma \rightarrow \tilde{\gamma})$  given h, i.e.,  $P(\gamma \rightarrow \tilde{\gamma} | \mathbf{h})$ . Given the conditional PEP and h, the average pairwise error probability (APEP) can be obtained by a miss-detection of subcarrier indices, using the law of total probability, and is given by the weighted sum of

$$APEP \leq \frac{1}{N} \sum_{\gamma} \sum_{\tilde{\gamma} \neq \gamma} P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) P\left(\gamma\right), \qquad (4)$$

where  $P(\gamma)$  denotes the priori probability of uniformly activating subcarrier  $\gamma$ , i.e.,  $P(\gamma) = K/N, \forall \gamma$ .

### A. Conditional PEP expression

We first derive the conditional PEP. The probability that the subcarrier with the greatest received noise power is greater than the received power of an active subcarrier can be found using double integration of both probability density functions (PDF), given by

$$P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) = \int_{0}^{\infty} \int_{r_{\gamma}}^{\infty} p_{\tilde{\gamma}_{1:(N-K)}}\left(r_{\tilde{\gamma}}\right) p_{\gamma}\left(r_{\gamma}\right) dr_{\tilde{\gamma}} dr_{\gamma},$$
(5)

where the received signal powers from an inactive subcarrier and an active subcarrier are denoted by  $r_{\tilde{\gamma}} = \|n(\tilde{\gamma})\|^2$  and  $r_{\gamma} = \|h(\gamma) s(\gamma) + n(\gamma)\|^2$ , respectively,  $p_{\tilde{\gamma}_{1:(N-K)}}(r_{\tilde{\gamma}})$  is the PDF of the greatest received noise power of any inactive subcarriers, i.e.,  $\max_{\tilde{\gamma}} r_{\tilde{\gamma}}$ , and  $p_{\gamma}(r_{\gamma})$  is the PDF of the received power of active subcarriers. It is worth mentioning that  $r_{\tilde{\gamma}}$  is chi-square distributed with 2*f* degrees of freedom, i.e.,  $r_{\tilde{\gamma}} \sim \chi_{2f}^2$ , and  $r_{\gamma}$  is non-central chi-square distributed with 2*f* degrees of freedom with a non-centrality parameter  $\lambda$ , i.e.,  $r_{\alpha} \sim \chi_{2f}^2(2\lambda)$ . The non-centrality parameter  $\lambda$  represents the deterministic part of the received signal and can be written as  $2\delta$  where  $\delta$  is the instantaneous SNR, i.e.,  $\delta = |h(\gamma)|^2 \rho$ .

Thus, the PDF of max  $r_{\tilde{\gamma}}$  can be found by using the order statistic [9], and is given by

$$p_{\tilde{\gamma}_{1:(N-K)}}(r_{\tilde{\gamma}}) = (N-K) \left[ F_{\tilde{\gamma}}(r_{\tilde{\gamma}}) \right]^{N-K-1} p_{\tilde{\gamma}}(r_{\tilde{\gamma}}), \quad (6)$$

where  $F_{\tilde{\gamma}}(r_{\tilde{\gamma}})$  and  $p_{\tilde{\gamma}}(r_{\tilde{\gamma}})$  are the Cumulative Distribution Function (CDF) and the PDF of  $r_{\tilde{\gamma}}$ , respectively. Note that  $F_{\tilde{\gamma}}(r_{\tilde{\gamma}})$  and  $p_{\tilde{\gamma}}(r_{\tilde{\gamma}})$  are represented, as [10]

$$F_{\tilde{\gamma}}(r_{\tilde{\gamma}}) = 1 - e^{\left(-\frac{r_{\tilde{\gamma}}}{2N_0}\right)} \sum_{k=0}^{N_r - 1} \frac{1}{k!} \left(\frac{r_{\tilde{\gamma}}}{2N_0}\right)^k, \qquad (7)$$

$$p_{\tilde{\gamma}}(r_{\tilde{\gamma}}) = \frac{1}{\sqrt{N_0}^{2N_r} 2^{2N_r/2} \Gamma\left(\frac{2N_r}{2}\right)} r_{\tilde{\gamma}}^{2N_r/2-1} e^{\left(-\frac{r_{\tilde{\gamma}}}{2N_0}\right)}, \quad (8)$$

where  $\Gamma(\cdot)$  is the gamma function which can be found in [10], and  $N_r = f/2$  is the number of receive antennas of the system.

For simplicity of analysis and without the loss of generality, consider a single antenna system hereafter, i.e.,  $N_r (= f/2) =$  1. With the normalized  $N_0 = 1$ , (7) and (8) can be simplified to:

$$F_{\tilde{\gamma}}\left(r_{\tilde{\gamma}}\right) = 1 - e^{\left(-\frac{r_{\tilde{\gamma}}}{2}\right)},\tag{9}$$

$$p_{\tilde{\gamma}}\left(r_{\tilde{\gamma}}\right) = \frac{e^{\left(-\frac{r_{\tilde{\gamma}}}{2N_{0}}\right)}}{2}.$$
(10)

Substituting (9) and (10) into (6), we get

$$p_{\tilde{\gamma}_{1:(N-K)}}(r_{\tilde{\gamma}}) = (N-K) \left[1 - e^{\left(-\frac{r_{\tilde{\gamma}}}{2}\right)}\right]^{N-K-1} \frac{e^{\left(-\frac{r_{\tilde{\gamma}}}{2\sigma^{2}}\right)}}{2}.$$
(11)

Using (9)-(11), (5) can be written as

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$$P(\gamma \to \tilde{\gamma} \mid \mathbf{h}) = \int_0^\infty p_\gamma(r_\gamma) \left\{ 1 - \int_0^{r_\gamma} p_{\tilde{\gamma}_{1:(N-K)}}(r_{\tilde{\gamma}}) dr_{\tilde{\gamma}} \right\} dr_\gamma$$
(12)

$$P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) = 1 - \int_{0}^{\infty} F_{\tilde{\gamma}_{1:(N-K)}}\left(r_{\gamma}\right) p_{\gamma}\left(r_{\gamma}\right) dr_{\gamma}, \quad (13)$$

where  $F_{p_{\tilde{\gamma}_{1:(N-K)}}}(r_{\gamma}) = \left[1 - e^{\left(-\frac{r_{\gamma}}{2}\right)}\right]^{N-K}$ . When  $N_r \ge 1$ ,  $p_{\gamma}(r_{\gamma})$  in 5 is given by

$$p_{\gamma}(r_{\gamma}) = \frac{1}{2N_0} \left(\frac{r_{\gamma}}{2\lambda}\right)^{(2N_r - 2)/4} e^{\left(-\frac{2\lambda + r_{\gamma}}{2N_0}\right)} \times I_{\frac{2N_r}{2} - 1}\left(\sqrt{r_{\gamma}}\frac{\sqrt{2\lambda}}{N_0}\right), \quad (14)$$

where  $I_z(x)$  is the *z*th order modified Bessel function of the first kind, which is given by  $I_z(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{z+2k}}{k!\Gamma(z+k+1)}$ .

Similarly to (11), considering the single antenna case when  $N_r = 1$  with  $N_0 = 1$ , we can simplify (14) to

$$p_{\gamma}(r_{\gamma}) = \frac{1}{2}e^{\left(-\frac{2\lambda+r_{\gamma}}{2}\right)}I_0\left(\sqrt{r_{\gamma}2\lambda}\right).$$
 (15)

Inserting  $\left[1 - e^{\left(-\frac{r_{\gamma}}{2}\right)}\right]^{N-K}$  and (15) into (11), we therefore obtain the conditional PEP expression as

$$P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) = 1 - \frac{1}{2} \int_{0}^{\infty} \left[1 - e^{\left(-\frac{r_{\gamma}}{2}\right)}\right]^{(N-K)} \\ \times e^{\left(-\frac{2\lambda + r_{\gamma}}{2}\right)} I_{0}\left(\sqrt{r_{\gamma}2\lambda}\right) dr_{\gamma}.$$
 (16)

Looking at (16) it can be observed certain parameters have both desirable and undesirable effects on the PEP. As the number (N - K) of inactive subcarriers increases so does the PEP. As the SNR is increased, so does the non-centrality parameter  $\lambda$ , thus decreasing the PEP. These observation can be confirmed in Sections V and VI.

## B. Average PEP expression in closed-form

Using binomial expansion ,  $\left[1-e^{\left(-\frac{r_{\gamma}}{2}\right)}\right]^{(N-K)}$  in (16), can be given by

$$\left[1 - e^{\left(-\frac{r_{\gamma}}{2}\right)}\right]^{(N-K)} = \sum_{q=0}^{(N-K)} \binom{(N-K)}{q} (-1)^q e^{\left(-\frac{qr_{\gamma}}{2}\right)}$$
(17)

Substituting (17) into (16), we get

$$P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) = 1 - \frac{1}{2} e^{(-\lambda)} \sum_{q=0}^{(N-K)} \binom{(N-K)}{q} (-1)^{q} \\ \times \int_{0}^{\infty} e^{\left(-\frac{qr\gamma + r\gamma}{2}\right)} I_{0}\left(\sqrt{r_{\gamma}2\lambda}\right) dr_{\gamma}.$$
 (18)

Using (6.614-3) from [11], the integration in (18) can be expressed as

$$\frac{e^{\frac{\lambda}{2+2q}}}{\frac{1}{2}\sqrt{\lambda\left(1+q\right)}}\frac{\Gamma\left(v+1\right)}{\Gamma\left(2v+1\right)}M_{-\frac{1}{2},v}\left(\frac{\lambda}{1+q}\right),\quad(19)$$

where  $M_{p,q}(z) = e^{-\frac{1}{2}z} z^{\frac{1}{2}+q} M\left(\frac{1}{2}+q-p,1+2q,z\right)$  is the Whittaker function,  $M(a,b,z) = \sum_{s=0}^{\infty} \frac{(a)_s}{(b)_s s!} z^s$  is the Kummer function of the first kind,  $(x)_n = \frac{\Gamma(x+1)}{\Gamma(x-n+1)}$  is the Pochhammer Symbol.

We can simplify (19) to

$$\frac{2e^{\frac{\lambda}{q+1}}}{q+1}.$$
(20)

Now we can substitute (20) back into (18) to get the conditional PEP in closed-form, given as

$$P\left(\gamma \to \tilde{\gamma} \mid \mathbf{h}\right) = 1 - \frac{1}{2} \sum_{q=0}^{(N-K)} \binom{(N-K)}{q} \left(-1\right)^q \frac{2e^{\frac{\lambda}{q+1}-\lambda}}{q+1}.$$
(21)

Finally, substituting (21) into (4), an upper-bound expression for the APEP can be obtained as

$$\begin{aligned} APEP &\leq \frac{1}{N} \sum_{\gamma}^{N} \sum_{\substack{\gamma \neq \tilde{\gamma} = 1 \\ \gamma \neq \tilde{\gamma} = 1}}^{N} \\ &\times \left[ \left[ 1 - \frac{1}{2} \sum_{q=0}^{(N-K)} \binom{(N-K)}{q} (-1)^q \frac{2e^{\frac{\lambda}{q+1} - \lambda}}{q+1} \right] \left[ \frac{K}{N} \right] \right]. \end{aligned}$$

$$(22)$$

As seen in (22), the APEP is a function of both K and  $\lambda$ . That is, as K increases for a given N, the APEP also does.

# V. ASYMPTOTIC ANALYSIS

# A. Effects of N on the conditional PEP

Now consider an extreme case when N is very large for a very broadband wireless system. We aim to asymptotically investigate the effect of N on the conditional PEP. To this end, let N grows significantly while fixing K = 1. Notice that the summation within (21), i.e.,  $\sum_{q=0}^{(N-K)} {\binom{N-K}{q}} (-1)^q \frac{2e^{\frac{\lambda}{q+1}-\lambda}}{q+1}$ , decreases as N grows. This is due to the fact the binomial coefficient in (21) increases as N grows. It is important to note that the iteration term in the summation at q = 0 equals 2, i.e.,  $\binom{(N-K)}{0}(-1)^0 \frac{2e^{\frac{\lambda}{D+1}-\lambda}}{0+1} = 2$ . Considering the power term, i.e.,  $(-1)^q$ , the subsequent iterations will switch between negative and positive values. This leads to that the total value of the summation is limited to be no more than 2 when q = 0. As the iteration count q increases the summation count will only decrease. Thus, when  $N = \infty$  the summation will equal zero, and thus the PEP = 1. Therefore, for a large number N of subcarriers, the error performance can be stated asymptotically to increase. This behavior is counter-intuitive, as the error performance of a system tends to improve as diversity is increased.

Our analysis is verified by the computer simulations in Fig.2. The conditional PEP provides a tight upper bound  $(\ll 0.1 dB)$  to the computer simulations.

#### B. Effects of K on APEP

Consider the case when, for a given N, K tends to grow up to (N-1). In this case, we study the effect of K on the conditional PEP and thus on the APEP. However, for a fixed number N, the average PEP (22) increases as K tends to N/2. This can be explained by the double summation at the beginning of the APEP expression. On the other hand, when K > N/2 the APEP will plateau then begin to decrease as K tends to (N-1), as the value of the conditional PEP within the APEP becomes increasingly smaller. These asymptotic observations are counter-intuitive, as stated in [7] "the number of active subcarriers in each OFDM subblock is fixed to a value of K and the best performance in terms of BER and spectral efficiency is achieved when K = N/2 if BPSK is used". Therefore, we asymptotically observe that the APEP of MCIK with the greedy detection behaves concavely with K for a given N. However, the APEP is at its worst when  $K \cong N/2.$ 

Our analysis is verified by the computer simulations in Fig.1. The APEP provides a tight upper bound ( $\ll 0.1 dB$ ) to the computer simulation from  $PEP > 10^{-1}$ .

#### VI. COMPLEXITY DISCUSSION

In the following we briefly address the optimal ML detection which can be found in [5] and compare its complexity to that of the greedy detector. The optimal-ML detector performs a exhaustive search for all possible subcarrier index combinations and data symbol combinations in order to make a joint decision on them both, minimizing the following decision

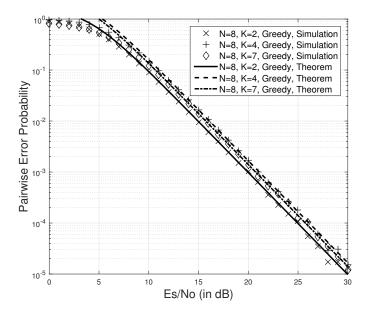


Figure 1. PEP performance of MCIK-OFDM with greedy detection and different configurations and varying  ${\cal K}$ 

$$(\hat{\mathbf{x}}) = \arg\min_{\mathbf{x}} |\mathbf{y} - \mathbf{h}\mathbf{x}|^2$$
(23)

If we consider the ML decision metric, the total number of metric combinations performed is  $cM^K$ , where  $c = 2^{\lfloor \log_2 \binom{K}{K} \rfloor}$  is the total number of active subcarrier index combinations, and  $M^K$  number of data symbol combinations. As a result, the optimal-ML detection becomes impractical for larger values of c and K due to the exponentially growing decoding complexity.

Unlike the ML, the sub-optimal greedy detection scheme performs the two-stage search which finds the active subcarrier index followed by the data symbol. As greedy detection does not perform an exhaustive search for the active subcarrier index combination, *the total number of metric combinations performed is reduced by a substantial factor of c* making it a low complexity alternative to the optimal-ML detection. These observations can be confirmed through the analysis of Section VII.

Consider an MCIK system with N = 16, K = 4, and M = 4. In this example, the number of metric combinations required for the ML detector is 262144, while that for greedy detector is only 256. The detection complexity is reduced by c = 1024 times from using the greedy over the ML.

#### VII. NUMERICAL EVALUATIONS AND SIMULATIONS

In this section, we present simulation results for different configurations of MCIK-OFDM in order to analyze the performance of the greedy detection scheme against the benchmark optimal-ML detection scheme. All simulations have been run over Rayleigh fading channels and a BPSK constellation (M = 2), has been employed. For simulation purposes we assume that there are a total of 128 subcarriers that are split

into clusters of size N. It is important to note that the PEP is not effected by the number of clusters.

#### A. Effects of N on the PEP

Fig. 2 depicts the conditional PEP of MCIK-OFDM with a varying N. Both greedy detection and optimal ML detection are employed for analysis. For a fair comparison, the number of active subcarriers remains fixed at K = 1. The PEP with greedy detection presents a very tight upper bound to that of the PEP with ML detection. The greedy detection scheme presents a negligible loss in error performance over ML detection (< 0.1dB) despite a substantial reduction of complexity.

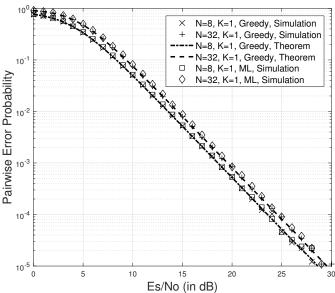


Figure 2. PEP performance of MCIK-OFDM with different configurations and varying  ${\cal N}$ 

# B. Effects of K on APEP

Fig. 3 depicts the Average PEP of MCIK-OFDM with a varying K. Both greedy detection and optimal ML detection are employed for analysis. The PEP of the ML detector outperforms that of the greedy detector for K > 1. Although, as N increases we observe the difference in error performance between ML and greedy detection decreases for K > 1. The largest difference in error performance between the two detection methods occurs when (N, K) = (4, 2), resulting in a  $\cong 0.6dB$  advantage with ML detection employed. However, this advantage decreases as N increases, when (N, K) = (8, 2) the difference in error performance reduces to  $\cong 0.2dB$ . It can be seen that the greedy detection scheme can be employed in a high data rate system with a minor reduction in error performance over the benchmark scheme.

#### C. Performance of MCIK versus OFDM

Fig. 4 depicts the symbol error rate (SER) of MCIK-OFDM with greedy detection and conventional OFDM. For a fair comparison a more general multitap frequency-selective

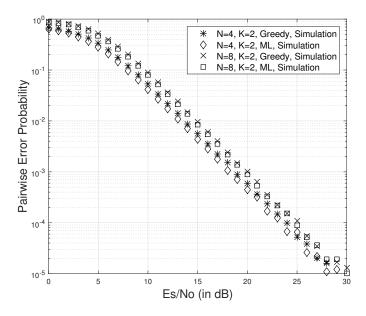


Figure 3. PEP performance of MCIK-OFDM with different configurations and varying  ${\cal N}$ 

Rayleigh fading channel is used. The SER of MCIK with greedy detection outperforms OFDM at higher regions of SNR. At a SER of  $10^{-3}$  MCIK achieves approximately 5 dB better SER performance over OFDM. The improvement in SER can be explained by the improved distance spectrum obtained from data carried by subcarrier indices. It is important to note that the above power gain is achieved despite the MCIK scheme operating at half of the transmission power of OFDM, as it is likely to be applied to low-power D2D communications. In addition, the cost is reduced by a factor of 50 percent, as half of the amount of subcarrier modulators and demodulators are needed.

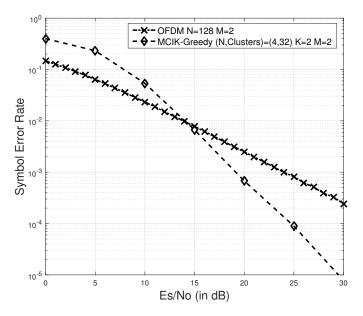


Figure 4. SER performance of MCIK-OFDM versus OFDM

## VIII. CONCLUSIONS

We proposed the novel low-complexity greedy detection method for the generalized MCIK system, which detects K subcarriers with the greatest received power without the exhaustive search. To measure the performance the conditional PEP and average PEP are derived in closed-form and analyzed for various cases. The greedy detection scheme provides a comparatively low complexity alternative to the benchmark ML detection scheme. Computer simulations and asymptotic analysis show that the proposed detection performed at negligible losses in error performance in comparison to the optimal-ML detection scheme. Such minor loss in the error performance of greedy detection is shown to decrease as Nincreases. It is clearly suggested that the greedy detection scheme can be a viable replacement of the benchmark ML detection scheme for any low-complexity MCIK systems with negligible losses in error performance. In the future, we will develop the greedy detection based MCIK systems, optimized to frequency selective fading channel.

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