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Simulation of failure in laminated polymer composites: building-block validation

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Abstract

The development of numerical tools to complement the experimental determination of structural design parameters is of key importance to hasten the certification process of new materials and structures. In this work, a methodology to simulate elastic and inelastic deformation of composite laminates at the subcomponent level based on finite element analysis is proposed. A modified version of a continuum damage model proposed in the literature combined with a frictional cohesive zone model is used to capture the intralaminar and interlaminar damage and failure of composite laminates in general loading conditions. The methodology is validated for three aerospace-grade carbon fibre reinforced (epoxy) polymer composite material systems and coupon configurations with increasing level of complexity, including unnotched tension/compression, open-hole tension/compression and filled hole compression. The predictions obtained are in good agreement with the experimental results for all the test cases, oftentimes within standard error of the tests, with maximum relative error of 13%.

Keywords: A. Composite laminates, B. Continuum damage mechanics, C. Finite element analysis

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1. Introduction

The design of composite structures is based on testing and simulation of composite coupons that represent structural details under simple loading scenarios, following an approach known as the building block approach [1]. This process ensures that a deep understanding of the structural behaviour under simple loading conditions is gained at the early stages of the design process, consequently mitigating the risk associated with the design of complex structures that usually have strict certification and safety requirements. The experimental determination of these design parameters is an inherently costly and time-consuming process, and consequently delays both the introduction of new materials and the design of new structures, two aspects that are critical for the competitiveness of the aeronautical industry. The development of numerical models to complement, or potentially replace, the purely experimental determination of the design parameters is, therefore, of key importance. Numerical solutions based on analytical [2–8] and finite elements models at the meso and macro scale [9–21] have been proposed; however, their use is still not widespread since the models are generally not implemented in commercially available software and/or their validation has not gained widespread acceptance for material systems of interest.

Finite element models to simulate failure of composite laminates using the ply as a building block have been proposed by several authors [9–20]. These are particularly powerful solutions given their flexibility to estimate the material behaviour of different laminates, geometries and loading conditions. However, their efficient use relies not only on the accuracy of the predictions of the ultimate failure loads, but also on the time frame in which these predictions are obtained. The time required to run the numerical models should be compatible with the requirements from industry. Moreover, it is important that the models rely on physically based material properties that can be determined following well-defined test methods and rely as little as possible on numerical calibration.

The objective of this work is to develop and implement a methodology to simulate composite laminates at coupon level compatible with industrial requirements. The model should be able to predict the strength of different structural details without changing its formulations or required material properties. The model is based on the continuum damage model proposed by Maimí et al. [16–18] and the cohesive zone model proposed by Turon et al. [22] and Alfano et al. [23]. The models were modified (as described in later sections) from their original versions to improve their ability to accurately predict damage initiation and evolution under general loading conditions

while maintaining the model complexity and, therefore, the computational cost. The current work serves as basis for modelling more complex geometries [24–26] and can potentially be used for the generation of statistically based design allowables for the most simple test cases when allied with powerful statistical tools.

2. Modelling strategy

The modelling strategy proposed in this work is presented in the following sections. The continuum damage model for the ply and respective improvements proposed are detailed in section 2.1 and the cohesive zone model for the interface used here is presented in section 2.2.

2.1. Continuum damage model for the ply

Maimí et al. [16–18] proposed a continuum damage model to predict the onset and accumulation of intralaminar damage mechanisms in laminated composites. Generally speaking, the formulation of the continuum damage model starts by the definition of a potential (the complementary free energy density) as a function of damage variables associated to the different failure modes. It is also necessary to define the damage activation functions, i.e. the conditions that lead to the onset of inelastic response, and the damage evolution functions. The model was developed assuming that the out-of-plane stresses are too small to promote damage and, therefore, damage is activated only by the in-plane components of the stress tensor. This assumption is not suitable for test cases where the triaxiality of the stresses is not negligible. In this work, the model was modified to improve its ability to accurately predict damage initiation and evolution under general load conditions without increasing the model complexity and, therefore, the computational cost. These modifications are reported in the following sections, with full details of the original model are available in Refs. [16–18].

2.1.1. Constitutive model

The scalar function corresponding to the ply complementary free energy density, accounting for the different longitudinal Young's moduli in tension and compression reads:

$$\mathcal{G} = \frac{\langle \sigma_{11} \rangle^2}{2(1-d_1)E_1} + \frac{\langle -\sigma_{11} \rangle^2}{2(1-d_1)E_{1c}} - \nu_{12}\sigma_{22} \left(\frac{\langle \sigma_{11} \rangle}{E_1} + \frac{\langle -\sigma_{11} \rangle}{E_{1c}} \right) + \frac{\sigma_{22}^2}{2(1-d_2)E_2} + \frac{\sigma_{12}^2}{2(1-d_6)G_{12}} + (\alpha_{11}\sigma_{11} + \alpha_{22}\sigma_{22})\Delta T + (\beta_{11}\sigma_{11} + \beta_{22}\sigma_{22})\Delta M \quad (1)$$

Figure 1: Longitudinal nonlinear ply behaviour in tension and compression.

where σ_{ij} ($i, j = 1, 2$) are the stress components, d_i ($i = 1, 2, 6$) are the scalar damage variables, E_1 and E_{1c} are respectively the longitudinal tensile and compressive Young's moduli, E_2 is the transverse Young's modulus, G_{12} is the shear modulus, ν_{12} is the Poisson's ratio, α_{ii} ($i = 1, 2$) are the coefficients of thermal expansion, β_{ii} ($i = 1, 2$) are the coefficients of hygroscopic expansion, ΔT is the temperature variation, ΔM is the variation in moisture content, and $\langle \bullet \rangle$ stands for the Macaulay brackets, which return the argument $\bullet$ if positive, and zero otherwise. The previous equation results in a non-linear behaviour in tension and compression if $E_1 \neq E_{1c}$ — see Fig. 1.

The strain tensor reads:

$$\epsilon = \frac{\partial \mathcal{G}}{\partial \sigma} = \mathbf{H} : \sigma + \alpha \Delta T + \beta \Delta M \quad (2)$$

where $\mathbf{H}$ is the lamina compliance tensor:

$$\mathbf{H} = \frac{\partial^2 \mathcal{G}}{\partial \sigma^2} = \begin{bmatrix} \frac{1}{(1-d_1)|\sigma_{11}|} \left(\frac{\langle \sigma_{11} \rangle}{E_1} + \frac{\langle -\sigma_{11} \rangle}{E_{1c}} \right) & -\frac{\nu_{12}}{|\sigma_{11}|} \left(\frac{\langle \sigma_{11} \rangle}{E_1} + \frac{\langle -\sigma_{11} \rangle}{E_{1c}} \right) & 0 \\ -\frac{\nu_{12}}{|\sigma_{11}|} \left(\frac{\langle \sigma_{11} \rangle}{E_1} + \frac{\langle -\sigma_{11} \rangle}{E_{1c}} \right) & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix} \quad (3)$$

To account for closure of longitudinal and transverse cracks under load reversal, four damage variables associated with longitudinal (d_{1+} and d_{1-}) and transverse (d_{2+} and d_{2-}) damage are considered to define the d_1 and d_2 :

$$d_1 = d_{1+} \frac{\langle \sigma_{11} \rangle}{|\sigma_{11}|} + d_{1-} \frac{\langle -\sigma_{11} \rangle}{|\sigma_{11}|} \quad (4)$$

$$d_2 = d_{2+} \frac{\langle \sigma_{22} \rangle}{|\sigma_{22}|} + d_{2-} \frac{\langle -\sigma_{22} \rangle}{|\sigma_{22}|} \quad (5)$$

Since shear damage represents transverse cracks which do not close under shear stresses, only one variable is considered for shear damage (d_6).

2.1.2. Damage activation functions

The continuum damage model predicts four intralaminar failure mechanisms: longitudinal tensile failure (F_{1+}), longitudinal compressive failure (F_{1-}), transverse failure where the crack plane is perpendicular to the mid-plane of the laminate (F_{2+}), and transverse failure where the crack plane is not perpendicular to the mid-plane of the laminate (F_{2-}) [17]. The respective activation functions are defined as:

$$\begin{aligned} F_{1+} &= \phi_{1+} - r_{1+} \leq 0 \quad ; \quad F_{1-} = \phi_{1-} - r_{1-} \leq 0 \\ F_{2+} &= \phi_{2+} - r_{2+} \leq 0 \quad ; \quad F_{2-} = \phi_{2-} - r_{2-} \leq 0 \end{aligned} \quad (6)$$

The four loading functions, ϕ_i ($i = 1+, 1-, 2+, 2-$), define the failure surfaces, and are established in terms of strain tensor, using an approximation of the LaRC03-04 failure criteria [27] that will be discussed later in section 2.1.4.1. For the sake of completeness, the loading functions are reported below but the reader is referred to Refs. [16–18] for full details on their definition and their physical meaning. The material properties of the ply are defined in Appendix A. The loading functions read:

$$\phi_{1+} = \frac{E_1}{X_T} \varepsilon_{11} \quad (7a)$$

$$\phi_{2+} = \begin{cases} \sqrt{\left(1 - \frac{G_{2+}}{G_6}\right) \frac{\tilde{\sigma}_{22}}{Y_T} + \left(\frac{G_{2+}}{G_6}\right) \left(\frac{\tilde{\sigma}_{22}}{Y_T}\right)^2 + \left(\frac{\tilde{\sigma}_{12}}{S_L}\right)^2} & \text{if } \tilde{\sigma}_{22} \geq 0 \\ \frac{1}{S_L} \langle |\tilde{\sigma}_{12}| + \eta^L \tilde{\sigma}_{22} \rangle & \text{if } \tilde{\sigma}_{22} < 0 \end{cases} \quad (7b)$$

$$\phi_{1-} = \frac{\langle |\tilde{\sigma}_{12}^R| + \eta^L \tilde{\sigma}_{22}^R \rangle}{S_L} \quad (7c)$$

$$\phi_{2-} = \sqrt{\left(\frac{\tilde{\tau}_{\text{eff}}^T}{S_T}\right)^2 + \left(\frac{\tilde{\tau}_{\text{eff}}^L}{S_L}\right)^2} \quad \text{if } \tilde{\sigma}_{22} < 0 \quad (7d)$$

The effective (undamaged) stress tensor, $\tilde{\sigma}$, is a function of the undamaged stiffness tensor, $\mathbf{C}_0$, and of the elastic strains $\boldsymbol{\varepsilon}_e$:

$$\tilde{\sigma} = \mathbf{C}_0 \boldsymbol{\varepsilon}_e \quad (8)$$

The components of the effective stress tensor in the coordinate system associated with the rotation of the fibres are calculated as:

$$\tilde{\sigma}_{22}^R = \tilde{\sigma}_{11} \sin^2 \varphi^C + \tilde{\sigma}_{22} \cos^2 \varphi^C - 2 |\tilde{\sigma}_{12}| \sin \varphi^C \cos \varphi^C \quad (9a)$$

$$\tilde{\sigma}_{12}^R = (\tilde{\sigma}_{22} - \tilde{\sigma}_{11}) \sin \varphi^C \cos \varphi^C + |\tilde{\sigma}_{12}| (\cos^2 \varphi^C - \sin^2 \varphi^C) \quad (9b)$$

where the misaligned angle at failure of a UD ply subjected to uniaxial longitudinal compression, φ^C , is given by:

$$\varphi^C = \arctan \left(\frac{1 - \sqrt{1 - 4 (S_L/X_C + \eta^L) S_L/X_C}}{2 (S_L/X_C + \eta^L)} \right) \quad (10)$$

The effective shear strengths, $\tilde{\tau}_{\text{eff}}^T$ and $\tilde{\tau}_{\text{eff}}^L$ are given by:

$$\tilde{\tau}_{\text{eff}}^T = \langle -\tilde{\sigma}_{22} \cos(\alpha_0) (\sin(\alpha_0) - \eta^T \cos(\alpha_0) \cos(\theta)) \rangle \quad (11a)$$

$$\tilde{\tau}_{\text{eff}}^L = \langle \cos(\alpha_0) (|\tilde{\sigma}_{12}| + \eta^L \tilde{\sigma}_{22} \cos(\alpha_0) \sin(\theta)) \rangle \quad (11b)$$

where the longitudinal and transverse frictional coefficients, η^L and η^T , respectively, read:

$$\eta^L \approx -\frac{S_L \cos(2\alpha_0)}{Y_C \cos^2 \alpha_0} \quad ; \quad \eta^T = \frac{1}{\tan(2\alpha_0)} \quad (12)$$

the transverse shear strength, S_T is approximated by:

$$S_T = Y_C \cos(\alpha_0) \left[\sin(\alpha_0) + \frac{\cos(\alpha_0)}{\tan(2\alpha_0)} \right] \quad (13)$$

and the sliding angle, θ is given by:

$$\theta = \arctan \left(\frac{-|\tilde{\sigma}_{12}|}{\tilde{\sigma}_{22} \sin(\alpha_0)} \right) \quad (14)$$

The elastic domain thresholds, r_i ($i = 1+, 1-, 2+, 2-$) are related to the damage variables by the damage evolution laws. r_i take the initial value 1 when the material is undamaged and increase as damage evolves. The elastic domain thresholds read:

$$r_{1+} = \max \left\{ 1, \max_{s=0,t} \{\phi_{1+}^s\}, \max_{s=0,t} \{\phi_{1-}^s\} \right\} \quad (15a)$$

$$r_{1-} = \max \left\{ 1, \max_{s=0,t} \{\phi_{1-}^s\} \right\} \quad (15b)$$

$$r_{2+} = \max \left\{ 1, \max_{s=0,t} \{\phi_{2+}^s\}, \max_{s=0,t} \{\phi_{2-}^s\} \right\} \quad (15c)$$

$$r_{2-} = \max \left\{ 1, \max_{s=0,t} \{\phi_{2-}^s\} \right\} \quad (15d)$$

2.1.3. Uniaxial response

To accurately capture the mechanical response of a unidirectional (UD) composite lamina before and after initiation of the different failure modes, a different stress-strain response is considered for each damage mechanism (Fig. 2):

- Under longitudinal tensile loading, a linear-elastic response followed by a bi-linear softening law after longitudinal tensile damage initiation is assumed — Fig. 2A.
- Following the experimental findings of Moran et al. [28], under longitudinal compressive loading the initially linear-elastic response (preceding kink band formation) is followed by linear softening, corresponding to the propagation of a kink band, until reaching a plateau, corresponding to kink band broadening at constant stress — Fig. 2B.
- The transverse tensile and compressive stress-strain relations are represented by a linear softening law following an initially linear-elastic response — Fig. 2C.
- Finally, under in-plane shear, a nonlinear elasto-plastic response is assumed, represented by a reduced slope in the stress-strain curve to account for the inelastic response caused by plastic deformation, followed by a linear softening law — Fig. 2D.

Figure 2: Uniaxial response in A) longitudinal tension, B) longitudinal compression, C) transverse tension or compression and D) in-plane shear.

To ensure correct energy dissipation during damage propagation and to avoid mesh-size dependency of the numerical solution, the slopes of the softening laws are determined as a function of the fracture toughness $\mathcal{G}_c$ associated with each failure mechanism ($\mathcal{G}_{1+}$, $\mathcal{G}_{1-}$, $\mathcal{G}_{2+}$ / $\mathcal{G}_{2-}$, and $\mathcal{G}_6$ — see Fig. 2), and as a function of the characteristic length l^* of the finite elements [29]. Since quadrangular elements are used, a damage mode independent element characteristic length is defined.

2.1.4. Effect of through-the-thickness stress

The three modifications related to the effect of pressure on damage onset and propagation proposed in this work are reported hereafter. A damage activation function for longitudinal compression that accounts for the effect of through-the-thickness stresses based on the 3D failure criteria proposed by Camanho et al. [30] is proposed in section 2.1.4.1 and engineering solutions to account for ii) the effect of pressure on the longitudinal compression and iii) in-plane shear fracture toughness are proposed in sections 2.1.4.2 and 2.1.4.3, respectively.

2.1.4.1 Damage activation function for longitudinal compression

The model proposed in Refs. [16–18] assumed that the out-of-plane stress components are negligibly small to promote damage, and therefore only the in-plane stress components of the stress tensor activate damage. However, this assumption is not suitable for test cases where the triaxiality of the stress state is not negligible. Therefore, to improve the ability to accurately predict damage initiation and evolution in 3D test cases, the model initially proposed in Refs. [16–18] was modified to include a 3D invariant-based failure criterion for fibre kinking (Fig. 3) [30]. This failure criterion is used to define an artificial in-plane shear strength S_L^{ef} (see Fig. 4), which represents the effect of hydrostatic pressure on the shear response of the polymer matrix [31–33]. S_L^{ef} is determined imposing that, at failure, the 2D damage activation function [17] is equal to the activation function from the 3D invariant-based failure criterion [30]:

$$\frac{\langle |\tilde{\sigma}_{12}^R| + \eta^L \tilde{\sigma}_{22}^R \rangle}{S_L} = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 \quad (16)$$

and solving for the in-plane shear strength, S_L . The 2D damage activation function (left-hand side of Eq. (16)), is defined in section 2.1.2. The 3D invariant-based failure criterion for fibre kinking (right-hand side of Eq. (16)) is defined by the set of invariants:

$$I_1 = \frac{1}{2} \text{tr} (\tilde{\sigma}^p)^2 - \mathbf{a} (\tilde{\sigma}^p)^2 \mathbf{a} \quad (17a)$$

$$I_2 = \mathbf{a} (\tilde{\sigma}^p)^2 \mathbf{a} \quad (17b)$$

$$I_3 = \text{tr} \tilde{\sigma} - \mathbf{a} \tilde{\sigma} \mathbf{a} \quad (17c)$$

assuming a decomposition of the effective stress tensor $\tilde{\sigma}$ in plasticity inducing stresses $\tilde{\sigma}^p$ and reaction stresses $\tilde{\sigma}^r$:

Figure 3: Fibre kinking kinematics and effect of through-thickness stresses predicted by a 3D invariant-based failure criterion [30].

$$\tilde{\sigma}^p = \tilde{\sigma} - \tilde{\sigma}^r \quad (18a)$$

$$\tilde{\sigma}^r = \frac{1}{2} (\text{tr } \tilde{\sigma} - \mathbf{a} \tilde{\sigma} \mathbf{a}) \mathbf{1} - \frac{1}{2} (\text{tr } \tilde{\sigma} - 3 \mathbf{a} \tilde{\sigma} \mathbf{a}) \mathbf{A} \quad (18b)$$

where $\mathbf{1}$ is the identity tensor and $\mathbf{A} = \mathbf{a} \otimes \mathbf{a}$ is the structural tensor that represents the intrinsic characteristic direction of the transversely isotropic material. The preferred direction $\mathbf{a}$, in the case of UD fibre-reinforced composites, coincides with the fibre direction. For the rotated fibres in the misalignment frame of a kink band, this preferred direction reads [30]:

$$\mathbf{a} = \begin{bmatrix} \cos \varphi \\ \cos \psi \sin \varphi \\ \sin \psi \sin \varphi \end{bmatrix} \quad (19)$$

where φ is the misalignment angle and ψ is the angle of the kinking plane (see Fig. 3). The parameters α_i (with $i = 1, 2, 3, 32$) are simple functions of the transverse and shear ply strengths that depend on the direction of the applied normal load through the sign of I_3 to account for the tensile/compressive strengths asymmetry (for more details, the reader is referred to Ref. [30]).

2.1.4.2 Softening law for longitudinal compression

In this work, the 3D invariant-based failure criterion for fibre kinking included in the damage model is also used to scale the fracture toughness for longitudinal compression ($\mathcal{G}_{1-}$) and the longitudinal compressive strength ratio at the inflection point (f_{XC}) as a function of the applied pressure (Fig. 5). The fracture toughness is scaled proportionally to the increase of longitudinal compressive strength:

Figure 4: Effect of hydrostatic pressure on the longitudinal in-plane shear response.

Figure 5: Effect of hydrostatic pressure on the longitudinal compressive response.

$$\mathcal{G}_{1-}^{ef} = \mathcal{G}_{1-} \left| \frac{\bar{\sigma}_{11}}{X_C} \right|, \quad \text{if } \left| \frac{\bar{\sigma}_{11}}{X_C} \right| > 1 \quad (20)$$

where X_C is the uniaxial longitudinal compressive strength and $\bar{\sigma}_{11}$ is the compressive strength predicted by the 3D invariant-based failure criterion for multiaxial stress states (see Fig. 6). The effective longitudinal compressive strength ratio at the inflection point (f_{XC}^{ef}) is:

$$f_{XC}^{ef} = \min \{1, f_{XC} (1 - f_{fxc} \langle -I_3 \rangle)\} \quad (21)$$

where $I_3 = \sigma_{22} + \sigma_{33}$ is the third invariant of the 3D invariant-based failure criterion [30] and f_{fxc} is a frictional parameter that needs to be calibrated using the experimental curves from a bearing, filled-hole compression or open-hole compression test.

Figure 6: UD ply failure by fibre kinking under multiaxial stress states.

Figure 7: Effect of transverse compressive stresses on the longitudinal in-plane shear response.

2.1.4.3 Fracture toughness for in-plane shear

It has been shown experimentally that through-thickness compression increases the apparent interlaminar mode II fracture toughness and delays delamination onset [34–36]. Cui et al. [37] suggested that this strengthening effect can be represented by an effective fracture toughness, G_{IIc}^{ef} :

$$G_{IIc}^{ef} = G_{IIc} (1 - \eta_G \langle -\sigma_{33} \rangle) \quad (22)$$

where G_{IIc} is the mode II fracture toughness, η_G is a material dependent empirically derived enhancement factor, σ_{33} is the through-thickness stress and $\langle x \rangle$ is the Macaulay operator [37, 38]. It can be postulated that, at the ply level, applying transverse compression stress should have a similar strengthening effect since a similar fracture process occurs for transverse cracking and, therefore, the effective fracture toughness for in-plane shear can be expressed as:

$$G_6^{ef} = G_6 (1 - \eta_G \langle -\sigma_{22} \rangle) \quad (23)$$

The resulting effect of this modification is shown in Fig. 7.

2.1.5. *In-situ strengths*

It is clear that a proper model formulation is fundamental to accurately predict damage onset and propagation in composite laminates, however the material properties used to populate the model also have to be accurately determined. It is particularly important to take into account that some of the ply strengths are *in-situ* properties, i.e. are a function of the ply thickness in a multidirectional laminate. This effect was first detected experimentally for transverse tension and in-plane shear by Parvizi et al. [39], and further analysed by other authors [40–49]. Camanho et al. [48] proposed an analytical model to predict the *in-situ* transverse tension, Y_T^{is} , and in-plane shear, S_L^{is} , strengths using Fracture Mechanics models that relate the *in-situ* properties with the fracture toughness of the material.

The calculation of the in-situ shear strength relies on the approximation of the non-linear shear response. The shear response is usually approximated by either the Tsai-Hahn or the Ramberg-Osgood laws. Different approximations result in different predictions of the in-situ strengths since, as currently derived, they are highly dependent on the shape of the stress-strain behaviour prior to failure and, therefore, highly dependent on the law used to approximate the shear response [50]. Given the high dependence of the *in-situ* shear strengths on the shear response law, adopting contradictory assumptions for the shear behaviour to derive the *in-situ* strengths and to the material constitutive model should be avoided [50]. To maintain consistency, the *in-situ* strengths should, therefore, be derived assuming the bilinear shear response implemented in the model and shown in Fig. 2.D. The derivation of the *in-situ* strengths was carried out and implemented during this work, and are presented in Appendix B. These properties are used in the failure criteria.

2.2. *Cohesive zone model for the interfaces between plies*

Accurate simulation of failure of composite laminates requires not only the prediction of intralaminar damage, but also the prediction of delamination onset and propagation. In the framework of this work, some common problems regarding the prediction of delamination were identified. Firstly, most of the formulations are developed for pure mode I and II loading and then extended for mixed-mode loading and are, therefore, poorly validated for mixed mode loading conditions. Secondly, in the formulation of cohesive zone models, friction is usually not accounted for. The effect of these issues can be minimized by using engineering solutions proposed by several authors [37, 38, 51, 52]; however a rigorous cohesive zone model to simulate

delamination under mixed mode loading and that accounts for the effect of friction is adopted here. Based on their previous work [51], Turon et al. [22] recently proposed a cohesive zone model that accurately predicts damage propagation under mixed mode loading by including a mode-dependent penalty stiffness. Alfano et al. [23] proposed an approach to combine interface damage with friction considering linear elastic behaviour for the undamaged part and the damage evolution law proposed by Crisfield et al. [53] combined with a simple Coulomb friction law. However, any cohesive zone model and friction law can be used. The cohesive zone model proposed by Turon et al. [22] was combined with the Coulomb friction law by the authors [36] and is used in this work to simulate delamination onset and propagation.

2.2.1. Constitutive model

Alfano et al. [23] proposed that the homogenized interface stress over the representative element area, τ , is given by:

$$\tau = (1 - d) \begin{bmatrix} K_{sh} & 0 & 0 \\ 0 & K_{sh} & 0 \\ 0 & 0 & K_n \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_n \end{bmatrix} + d \begin{bmatrix} K_{sh} & 0 & 0 \\ 0 & K_{sh} & 0 \\ 0 & 0 & K_n \end{bmatrix} \begin{bmatrix} \Delta_1 - \Delta_1^{di} \\ \Delta_2 - \Delta_2^{di} \\ -\langle -\Delta_n \rangle \end{bmatrix} \quad (24)$$

where d is the damage variable, $\mathbf{K}$ is the stiffness matrix, Δ is the relative displacement vector and Δ^{di} is the inelastic sliding displacement. The following friction function was introduced:

$$\phi = \mu \tau_n^d + \tau_{sh}^d \quad (25)$$

where τ_{sh}^d is given by

$$\tau_{sh}^d = \sqrt{(\tau_1^d)^2 + (\tau_2^d)^2} \quad (26)$$

The evolution of Δ^{di} is assumed to be governed by the following nonassociative relationship:

$$\dot{\Delta}^{di} = \dot{\lambda} \begin{bmatrix} \frac{\partial \phi}{\partial \tau_1^d} \\ \frac{\partial \phi}{\partial \tau_2^d} \\ 0 \end{bmatrix} = \dot{\lambda} \begin{bmatrix} \frac{\tau_1^d}{\tau_{sh}^d} \\ \frac{\tau_2^d}{\tau_{sh}^d} \\ 0 \end{bmatrix} \quad (27)$$

with the Kuhn-Tucker conditions $\dot{\lambda} \geq 0$, $\phi(\tau) \leq 0$ and $\dot{\lambda}\phi(\tau) = 0$.

2.2.2. Mode dependent penalty stiffness

Most of the formulations are developed for pure mode loading and then extended for mixed-mode loading and are, therefore, poorly validated for mixed mode loading conditions. Turon et al.

[52] concluded that, to avoid erroneous calculation of the energy dissipation during mixed mode loading and to guarantee accurate prediction of damage propagation under mixed mode loading conditions, the relationship between the normal and shear interface stiffness (K_n and K_{sh}), the mode I and mode II fracture toughness ($\mathcal{G}_{Ic}$ and $\mathcal{G}_{IIc}$) and the normal and shear interlaminar strengths (τ_n^0 and τ_{sh}^0) should be given as:

$$\frac{K_{sh}}{K_n} = \frac{\mathcal{G}_{Ic}}{\mathcal{G}_{IIc}} \left(\frac{\tau_{sh}^0}{\tau_n^0} \right)^2 \quad (28)$$

In their previous work, Turon et al. [52] assumed that the shear and normal penalty stiffness were equal $K_n = K_{sh}$ and therefore, the relation was imposed considering that the shear strength should not be a fully independent material property but instead a function of the fracture toughness and the normal strength:

$$\tau_{sh}^0 = \tau_n^0 \sqrt{\frac{\mathcal{G}_{IIc}}{\mathcal{G}_{Ic}}} \quad (29)$$

Note that the interface stiffness (K_n and K_{sh}) are parameters that ensure a stiff connection between the crack surfaces before crack propagation, but the fracture toughness ($\mathcal{G}_{Ic}$ and $\mathcal{G}_{IIc}$) and the interlaminar strengths (τ_n^0 and τ_{sh}^0) are properties that can be measured experimentally. For this reason, Turon et al. [22] reformulated the model assuming a dependent interface stiffness, $K_n \neq K_{sh}$, instead of dependent shear strength, $\tau_{sh}^0 = f(\tau_n^0, \mathcal{G}_{Ic}, \mathcal{G}_{IIc})$:

$$K_{sh} = K_n \frac{\mathcal{G}_{Ic}}{\mathcal{G}_{IIc}} \left(\frac{\tau_{sh}^0}{\tau_n^0} \right)^2 \quad (30)$$

This engineering solution required the reformulation of the cohesive zone model to include the mode-dependent penalty stiffness.

To formulate the damage evolution law, the mixed-mode norms of the tractions, τ and the displacement jumps λ have to be defined. Following Refs. [54, 55], Turon et al. [22] defined the mixed-mode traction as a function of the Euclidean norm of the individual tractions along the 1, 2 and 3 directions accounting for different penalty stiffness. Under these assumptions, the mixed-mode norm of the displacement jump is redefined as:

$$\lambda = \frac{K_{sh}\Delta_{sh}^2 + K_n\Delta_n^2 - K_n\langle -\Delta_n \rangle^2}{\sqrt{K_{sh}^2\Delta_{sh}^2 + K_n^2\Delta_n^2 - K_n^2\langle -\Delta_n \rangle^2}} \quad (31)$$

2.2.3. Damage evolution law

The damage activation function is given by [22]:

$$F(\Delta) = H(\Delta) - r_d \leq 0 \quad (32)$$

where $H(\Delta)$ is a monotonic loading function, and r_d is the threshold function given, respectively, by:

$$H(\Delta) = \min \left(\frac{\lambda - \Delta^o}{\Delta^f - \Delta^o}, 1 \right) \quad (33)$$

$$r_d = \max(1, \max_s [H(\Delta)]) \quad 0 < s < t \quad \forall t \quad (34)$$

being Δ^o and Δ^f the displacement jumps corresponding to delamination onset and propagation under mixed-mode conditions, respectively. The Benzeggagh and Kennane criterion [56] is used to define these parameters:

$$\Delta^o = \sqrt{\frac{K_n(\Delta_n^o)^2 + [K_{sh}(\Delta_{sh}^o)^2 - K_n(\Delta_n^o)^2] \beta^\eta}{K_\beta}} \quad (35)$$

$$\Delta^f = \frac{K_n \Delta_n^o \Delta_n^f + [K_{sh} \Delta_{sh}^o \Delta_{sh}^f - K_n \Delta_n^o \Delta_n^f] \beta^\eta}{K_\beta \Delta^o} \quad (36)$$

where K_β is a mode-dependent interfacial stiffness defined as:

$$K_\beta = K_n(1 - \beta) + \beta K_{sh} \quad (37)$$

and β is the local mixed mode ratio defined as:

$$\beta = \frac{K_{sh} \Delta_{sh}^2}{K_{sh} \Delta_{sh}^2 + K_n (\Delta_n^o)^2} \quad (38)$$

The damage variable is given by:

$$d = \frac{r_d \Delta^f}{r_d \Delta^f + (1 - r_d) r_d \Delta^o} \quad (39)$$

3. Numerical Results

Both the continuum damage model developed to simulate intralaminar damage onset and propagation in composite laminates presented in Section 2.1, and the cohesive zone model to simulate delamination presented in Section 2.2, were developed with the goal of increasing the ability of the model to accurately predict damage propagation when through-thickness stresses are not negligible, and/or under compressive loading conditions. In fact, when combined, the previous versions of the models [16–18, 55] were able to predict the in-plane mechanical behaviour of quasi isotropic and symmetric composite laminates with good accuracy in test cases where the triaxility of stresses was fairly negligible, such as open-hole tension and open-hole compression. Camanho et al. [57] used the continuum damage model proposed in Refs. [16–18] to simulate

the damage propagation in five open-hole tension tests of IM7/8552 carbon/epoxy system and obtained a maximum error of 10.5% and a mean error of 4.2%. Bessa [58] predicted the open-hole tensile and compressive strength of IM7/8552 carbon/epoxy material system by combining the continuum damage model [16–18] and the cohesive zone model presented in Ref. [55]. For the open-hole tensile strengths, a maximum error of 11.5% and a mean error of 5.0% were obtained; for the open-hole compressive strengths, the maximum error and the mean error were of 16.5% and 11.6%, respectively. Note that relative errors between the mean strength measured experimentally and the one predicted numerically are usually considered acceptable when below 10%, hence, the predictions obtained by Bessa [58] and Camanho et al. [57] for tensile loading are particularly accurate in both studies while, the open-hole compressive strengths were slightly over-predicted [58]. This could be attributed to the fact that compressive strengths are more difficult to predict because the damage mechanisms are more complex; however, since they are remarkably consistent, i.e. the errors across the different geometries are similar, it was attributed to the inaccuracy of the experimental value available for the fracture toughness for longitudinal compression. As will be detailed in Section 3.4, the original versions of the models failed to deliver accurate predictions for more complex cases with significant compressive through-thickness stresses, simply because the material model for intralaminar damage was developed assuming that the components of the out-of-plane stresses are too small to affect damage, and also because the cohesive zone model was not developed accounting for frictional effects.

To validate the proposed modelling strategy, experimental results of unnotched tension/compression, open-hole tension/compression and filled hole compression of multidirectional laminates are compared with finite element results. Open-hole tension and compression tests were simulated to ensure that the modifications reported in Sections 2.1 and 2.2 do not affect the predictions previously obtained for these test cases [57, 58]. Filled-hole compression tests are considered the main test case to evaluate the accuracy of the modelling strategy since the pressure applied by the bolt can be significant, and therefore, accounting for the effect of friction and the effect of the through-thickness stresses in the formulation becomes more important. Filled-hole compression simulations are, therefore, analysed with more detail.

Both open-hole tension and compression Finite Element (FE) models used one 8-node linear brick reduced integration element (C3D8R) per ply thickness, t , and the plies are connected by 0.01mm thick COH3D8 user material cohesive elements. The laminate is clamped on one end while on the other a displacement (through the definition of an appropriate smooth step amplitude) is applied to all nodes at the boundary. A mesh of $0.5 \times 0.5 \times t \text{ mm}^3$ is used for ply

elements and $0.5 \times 0.5 \times 0.01 \text{ mm}^3$ for the cohesive elements (see Fig. 8). A similar strategy was used to model the unnotched tension and compression specimens, however, in these cases, two linear elastic regions were modelled respectively on the upper and bottom side of the specimen and were connected to the laminate in order to avoid premature failure of the elements where the boundary conditions are applied. For the unnotched tension and compression models, a mesh of $0.25 \times 0.25 \times t \text{ mm}^3$ is used for ply elements and $0.25 \times 0.25 \times 0.01 \text{ mm}^3$ for the cohesive elements (see Fig. 9).

The filled hole compression model is composed by i) one bolt using of C3D8R elastic elements, ii) an upper and a lower bushing using C3D8R elastic elements, iii) the laminate. One C3D8R finite element per ply is used to simulate intralaminar damage and the plies are connected by 0.01mm thick COH3D8 user material cohesive elements. The pre-loading of the bolt is simulated by increasing the temperature of the upper bushing. The temperature applied to the upper bushing was calibrated so that the preload of the bolt matches the experimental values. The laminate is clamped on the lower end and a smooth step is applied to the upper end of the laminate. A mesh of $0.47 \times 0.47 \times t \text{ mm}^3$ is used for ply elements and $0.47 \times 0.47 \times 0.01 \text{ mm}^3$ for the cohesive elements (see Fig. 10). The material properties of the three material systems used are presented in Tables 1-5. Nominal values without standard deviation are provided for the experimental inputs to the models. In Appendix A, the material properties are defined and methods for their experimental determination are given.

Figure 8: Finite element models for open-hole tension and compression.

Figure 9: Finite element models for unnotched tension and compression.

Figure 10: Finite element models for filled-hole compression.

Table 1: Ply elastic properties of IM7/8552, T800/M21 and T700/M21 carbon/epoxy systems.

<i>Elastic</i>		IM7/8552		T800/M21		T700/M21	
E_1	(MPa)	171420	Ref. [7, 57]	172000	Ref. [59]	171200	Ref. [60]
E_{1c}	(MPa)	137136	Note (1)	137600	Note (1)	137000	Note (1)
E_2	(MPa)	9080	Ref. [7, 57]	8900	Ref. [59]	8500	Ref. [60]
G_{12}	(MPa)	5290	Ref. [7, 57]	5000	Ref. [59]	4500	Ref. [60]
ν_{12}	-	0.32	Ref. [7, 57]	0.32	Ref. [59]	0.32	Ref. [60]

(1) Assumed to be 80% of the Young's Modulus for longitudinal tension ($E_{1c} = 0.8E_1$)

Table 2: Ply strengths properties of IM7/8552, T800/M21 and T700/M21 carbon/epoxy systems.

<i>Strengths</i>		IM7/8552		T800/M21		T700/M21	
X_T	(MPa)	2323.5	Ref. [7, 57]	3039	Ref. [61]	2000	Ref. [60]
f_{XT}	(-)	0.4	Ref. [61]	0.4	Ref. [61]	0.4	Note (1)
X_C	(MPa)	1200.1	Ref. [7, 57]	1051	Ref. [62]	1300	Ref. [60]
f_{XC}	(-)	0.2	Note (2)	0.2	Note (2)	0.2	Note (3)
Y_T	(MPa)	62.3	Ref. [7, 57]	75	Ref. [61]	46	Ref. [63]
Y_C	(MPa)	253	Ref. [7, 57]	250	Ref. [61]	260	Ref. [63]
Y_{BT}	(MPa)	38.7	Ref. [19, 30]	46.5	Note (4)	28.5	Note (4)
Y_{BC}	(MPa)	600.0	Ref. [19, 30]	592.5	Note (5)	616.2	Note (5)
S_L	(MPa)	92.3	Ref. [7, 57]	95	Ref. [61]	81	Ref. [63]
S_L^P	(MPa)	66.9	Note (6)	66.9	Note (6)	50	Note (6)
K_p	(-)	0.08	Note (6)	0.09	Note (6)	0.29	Note (6)

(1) Assumed from the IM7/8552 material system

(2) Inversely identified using one OHC simulation

(3) Inversely identified using one FHC simulation

(4) The ratio Y_{BT}/Y_T was assumed to be constant and equal to $Y_{BT}/Y_T = 38.7/62.3 = 0.62$

(5) The ratio Y_{BC}/Y_C was assumed to be constant and equal to $Y_{BC}/Y_C = 600.0/253.0 = 2.37$

(6) Best fitting from the transverse shear stress-strain curve of the material

Table 3: Ply fracture toughness of IM7/8552, T800/M21 and T700/M21 carbon/epoxy systems.

<i>Fracture</i>		IM7/8552		T800/M21		T700/M21	
$\mathcal{G}_{2+}$	(kJ/m ²)	0.28	Ref.[7, 57]	0.228	Ref. [61]	0.35	Ref. [63]
$\mathcal{G}_{2-}$	(kJ/m ²)	1.31	Note (1)	1.0	Note (1)	2.0	Note (1)
$\mathcal{G}_6$	(kJ/m ²)	0.79	Ref.[7, 57]	0.652	Ref. [61]	1.2	Ref. [63]
$\mathcal{G}_{1+}$	(kJ/m ²)	133.3	Ref. [58]	340	Ref. [61]	133.3	Note (2)
f_{GT}	(-)	0.3	Note (3)	0.52	Ref. [61]	0.52	Note (4)
$\mathcal{G}_{1-}$	(kJ/m ²)	61	Ref. [64]	60	Note (5)	95	Note (6)

(1) $\mathcal{G}_{2-} = \mathcal{G}_6 / \cos(\alpha_0)$ where $\mathcal{G}_6$ is the mode II fracture toughness and α_0 is the fracture angle of the UD ply subjected to uniaxial transverse compression (approximately 53°)

(2) Assumed from the IM7/8552 material system

(3) Inversely identified using one OHT simulation

(4) Assumed from the T800/M21 material system

(5) Inversely identified using one OHC simulation

(6) Inversely identified using one FHC simulation

Table 4: Ply geometric and friction parameters of IM7/8552, T800/M21 and T700/M21 carbon/epoxy systems.

<i>Geometrical</i>		IM7/8552		T800/M21		T700/M21	
α_0	(rad)	0.925	-	0.925	-	0.925	-
t	(mm)	0.125	-	0.125/0.184	-	0.250	-
<i>Friction</i>							
η_G	(MPa ⁻¹)	0.0035	Note (1)	0.0035	Note (1)	0.0035	Note (1)
f_{fxc}	(MPa ⁻¹)	0.01	Note (2)	0.01	Note (2)	0.01	Note (2)

(1) Assumed to be equal to the η parameter that accounts for the increase in interlaminar mode II fracture toughness as a function of applied through-thickness compressive stress determined for the IM7/8552 carbon/epoxy material system in Ref. [36]

(2) Inversely identified from a OHC or FHC simulation

Table 5: Interlaminar material properties.

<i>Elastic</i>		IM7/8552		T800/M21		T700/M21	
K_n	(N/mm ³)	10 ⁶	Ref. [65]	10 ⁶	Ref. [65]	10 ⁶	Ref. [65]
<i>Strengths</i>							
τ_N	(MPa)	62.3	Ref. [7, 57]	75	Ref. [61]	46	Ref. [63]
τ_{sh}	(MPa)	92.3	Ref. [7, 57]	95	Ref. [61]	81	Ref. [63]
<i>Fracture</i>							
$\mathcal{G}_{IC}$	(kJ/m ²)	0.28	Ref. [7, 57]	0.228	Ref. [61]	0.35	Ref. [63]
$\mathcal{G}_{IIC}$	(kJ/m ²)	0.79	Ref. [7, 57]	0.652	Ref. [61]	1.2	Ref. [63]
η_{B-K}	(-)	1.45	Ref. [66, 67]	1.45	Note (1)	2.0	Ref. [63]
<i>Friction</i>							
μ	(-)	0.1	Ref. [36]	0.1	Ref. [36]	0.1	Ref. [36]

(1) Assumed from the IM7/8552 carbon/epoxy material system

3.1. Unnotched Strength

Figure 11 shows the experimental results for unnotched tensile and unnotched compressive strengths and the predictions obtained with the proposed modelling strategy for a quasi-isotropic [90/0/-45/45]_{3s} IM7/8552 laminate. The relative error for unnotched tensile strength is -13.1% and -4.7% for unnotched compressive strength. As shown in Figs. 12 and 13, modelled damage localizes at the 0° plies causing catastrophic failure of the specimen.

3.2. Open-hole tension

Figures 14-16 show the experimental results for open-hole tensile strengths and the predictions obtained with the proposed modelling strategy for a [90/0/-45/45]_{3s} IM7/8552 laminate and two quasi-isotropic T800/M21 laminates: [90/45/0/-45]_{3s} (L1) and [90₂/0₂/45₂/-45₂/90/0/45/-45]_s (L2). For the IM7/8552 material system, the maximum relative error is 7.9% for hole diameter $d = 8\text{mm}$ while the mean error is 5.3%. For T800/M21 material system, the relative errors for the two layups are below 10.7% and the mean error is 3.9%. The predictions for laminate L1 are more accurate than the ones for laminate L2. Both layups are quasi-isotropic, but L2 has plies with the same orientation blocked together. Having thicker 0° plies in a laminate promotes fibre splitting, which blunts the notch and improves the notched strength [68–71], resulting in an apparent increase of the fracture toughness of the laminate. However, the modelling strategy used, which

Figure 11: Comparison between the numerical and experimental results for the unnotched tensile and compressive strengths of an IM7/8552 carbon/epoxy quasi-isotropic laminate. The standard error related to the experimental test results is presented.

Figure 12: Damage extent at ultimate load in the unnotched tension simulation: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° (d_2), C) interface damage (d). The specimen presented in this figure corresponds to the $[90/0/-45/45]_{3s}$ IM7/8552 laminate.

is not based on structured meshes, is likely not able to accurately predict fibre splitting in the 0° plies and to distinguish blocked 0° plies of single 0° plies. An accurate prediction of splitting would require a more refined mesh, [accounting for large deformation/rotations in the model formulation \[72\]](#), and the use of structured, fibre-aligned meshes where the edges of the elements follow the fibre direction [73].

In general, at the point of maximum load the fibre damage in the 0° plies localizes in the vicinity of the hole and propagates perpendicular to the applied load (Fig. 17.A); the matrix

Figure 13: Damage extent at ultimate load in the unnotched compression simulation: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° (d_2), C) interface damage (d). The specimen presented in this figure corresponds to the $[90/0/-45/45]_{3s}$ IM7/8552 laminate.

damage in the 90° plies extends across the width of the specimen (Fig. 17.B); and interface damage is limited to the ligaments next to the hole (Fig. 17.C).

Figure 14: Comparison between the numerical and experimental results for the open-hole tensile strength of an IM7/8552 carbon/epoxy quasi-isotropic laminate [57]. The hole diameter-to-width ratio is $2R/W = 1/6$. The standard error related to the experimental test results is presented.

3.3. Open-hole compression

Figures 18-20 show the experimental results for open-hole compressive strengths and the predictions obtained with the proposed modelling strategy for the same material systems previously considered. For IM7/8552, the relative errors are below 4.4% and the mean error is 1.9%. For T800/M21 material system, the relative errors for the two layups are below 12.8% and the mean

Figure 15: Comparison between the numerical and experimental results for the open-hole tensile strength of an T800/M21 carbon/epoxy quasi-isotropic laminate (L1) [70]. The hole diameter-to-width ratio is $2R/W = 1/4$. The standard error related to the experimental test results is presented.

Figure 16: Comparison between the numerical and experimental results for the open-hole tensile strength of a T800/M21 carbon/epoxy quasi-isotropic laminate (L2) [70]. The hole diameter-to-width ratio is $2R/W = 1/4$. The standard error related to the experimental test results is presented.

error is 7.6%. The predictions for IM7/8552 material system are significantly improved compared with the predictions obtained by Bessa [58] (maximum error of 16.5% and mean error of 11.6%). Here, the fracture toughness for longitudinal compression used was determined by Catalanotti et al. [64]. This value is lower than the one used by Bessa [58] which explains why the errors obtained are, on average, around 10% lower. The experimental results for open-hole compression for the L2 T800/M21 layup that uses thicker ply blocks shows a pronounced size effect that is not predicted very accurately by the model. This lower consistency in the predictions may be due to the fact that the material was not fully characterized and therefore, some assumptions on the

Figure 17: Typical damage extent at ultimate load in an open-hole tension simulation: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° ply (d_2), C) interface damage (d). The specimen presented in this figure corresponds to the $[90/0/-45/45]_{3s}$ IM7/8552 laminate with a hole diameter of $d = 6\text{mm}$.

material properties had to be made by the authors, particularly on the longitudinal compressive strength and fracture toughness for longitudinal compression of the unidirectional ply.

Across the materials and layups considered, the OHC response has the general features that, at the point of maximum load, damage localizes at the vicinity of the hole as shown in Fig. 21: fibre damage in the 0° plies extends across the width of the specimen (Fig. 21.A), matrix damage in the 90° plies localizes in the vicinity of the hole and propagates perpendicularly to the applied load (Fig. 21.B), and interface damage is limited to the area of the hole (Fig. 21.C).

3.4. Filled-hole compression

Filled-hole compression simulations are analysed with more detail since the pressure applied by the bolt can be significant, and therefore, accounting for frictional effects and the effect of the through-thickness stress in the formulation becomes critical. It is important to note that, even though the previous versions of the models were not particularly suitable for test cases where the triaxiality of stresses is not negligible, some engineering solutions proposed by several authors [37, 38, 51, 52] can be used to improve the accuracy in the presence of through-thickness stresses. It is, therefore, interesting to compare four modelling strategies (S1 through S4). Note that S3 and S4 are based on the work presented in this paper:

- **S1** - Intralaminar damage is simulated using the user-written material subroutine described in Refs. [16–18] and interlaminar damage is simulated using built-in ABAQUS cohesive

Figure 18: Comparison between the numerical and experimental results for the open-hole compressive strength of an IM7/8552 carbon/epoxy quasi-isotropic laminate [58]. The hole diameter-to-width ratio is $2R/W = 1/6$. The standard error related to the experimental test results is presented.

Figure 19: Comparison between the numerical and experimental results for the open-hole compressive strength of an T800/M21 carbon/epoxy quasi-isotropic laminate (L1) [70]. The hole diameter-to-width ratio is $2R/W = 1/4$. The standard error related to the experimental test results is presented.

elements [74].

- **S2** - Intralaminar damage is simulated using the user-written material subroutine described in Refs. [16–18] and interlaminar damage is simulated using built-in ABAQUS cohesive elements [74]. Engineering solutions proposed by Turon and co-authors. [51, 52] and by Cui et al. [37] are used to calculate some of the properties of the cohesive elements to ensure proper energy dissipation in mixed-mode loadings and to artificially account for the effects of pressure and friction. These solutions are summarized in Appendix C.

Figure 20: Comparison between the numerical and experimental results for the open-hole compressive strength of a T800/M21 carbon/epoxy quasi-isotropic laminate (L2) [70]. The hole diameter-to-width ratio is $2R/W = 1/4$. The standard error related to the experimental test results is presented.

Figure 21: Typical damage extent at ultimate load in an open-hole compression simulation: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° ply (d_2), C) interface damage (d). The specimen presented in this figure corresponds to the $[90/0/-45/45]_{3s}$ IM7/8552 laminate with a hole diameter of $d = 4\text{mm}$.

- **S3** - Intralaminar damage is simulated using the user-written material subroutine described in Section 2.1 and interlaminar damage is simulated using built-in ABAQUS cohesive elements [74]. The same engineering solutions [37, 51, 52] used in S2 are used here. These solutions are also summarized in Appendix C.
- **S4** - Intralaminar damage is simulated using the user-written material subroutine described in Section 2.1 and interlaminar damage is simulated using the user-written material subroutine described in Section 2.2, i.e., the S4 is the application of the new formulation presented

in this paper.

Two simulations are performed with two different T700/M21 layups: 44/44/11 where 44.4(4)%, 44.4(4)% and 11.1(1)% of the plies are 0° , $\pm 45^\circ$ and 90° , respectively, and 11/44/44 where 11.1(1)%, 44.4(4)% and 44.4(4)% of the plies are 0° , $\pm 45^\circ$ and 90° , respectively. Note that 11/44/44 stands for a 90° rotation of 44/44/11. The stacking sequences of the laminates, the experimental filled-hole compression strengths and load-displacement curves are Airbus proprietary and are therefore not presented in absolute terms. However, their absence does not affect the key observations and conclusions of this section. The stress-displacement relations and the errors between the experimental and the normalized numerical notched strengths obtained are presented in Fig. 22. To better understand the consequences of each modelling strategy and to assess the most suitable one, the results of each simulation are analysed in detail. Given the stacking sequence, laminate 44/44/11 will be referred to as the "hard laminate" and laminate 11/44/44 will be referred to as the "soft laminate".

Figure 22: Filled-hole compression numerical stress-displacement curves for a) the hard laminate (44/44/11) and b) the soft laminate (11/44/44) for the four modelling approaches S1 through S4. The legend provides the % difference between the model and experimental ultimate strength. Note that the x- and y-axes in A and B are not the same.

- **S1:** Using the original modelling strategy, the ultimate strength for the hard and soft laminates are both slightly underpredicted by -9% and -12% respectively (see Fig. 22). More importantly, the model is not able to capture the post peak response and predicts that an abrupt, catastrophic failure will occur as shown in Fig. 22. In the hard laminate, failure is dominated by fibre failure (Fig. 23a.A and 24a.A) and, in the soft laminate, delamination develops around the hole and quickly propagates through the width of the specimen as shown in Fig. 23a.C.

- **S2:** Using the modelling strategy 2, the mode II fracture toughness of the cohesive elements is increased as a function of the bolt pre-load. However, the pressure is not constant over all elements. In fact, the through-thickness stress is only negative under the bushings as shown in Fig. 25 and therefore, increasing the fracture toughness in all cohesive elements might result in an underprediction of the extent of delamination. In this case, failure is dominated by fibre compressive failure (figs. 23b.A and 24b.A) and delamination plays a less relevant role than in S1 (Figs. 23b.C and 24b.C).
- **S3:** Using the modelling strategy 3, the predicted filled-hole compressive ultimate strengths increase towards the experimental values. Similarly to modelling strategy 2, the fracture toughness of the cohesive elements is equally increased and delamination might, therefore, be over-suppressed. Failure is progressive for two reasons. Firstly because delamination is partly suppressed, and secondly because the [softening law](#) for longitudinal compression was altered based on the experimental findings of Moran et al. [28], and therefore, even though damage in the 0° is extensive, the 0° plies do not fail completely as shown in Figs. 23c.A and 24c.A.
- **S4:** Finally, using the modelling strategy proposed in this work, the strengthening effect of pressure is more accurately represented since it is friction induced and not simply represented based on an empirical engineering solution. In the hard laminate, damage extension is similar to the one predicted using modelling strategy 3 because delamination does not play an important role in the failure of the specimen (Figs. 23c.C and 23d.C). However, for the soft laminate, unlike in S3, delamination propagates through the width of the specimen and its extent is similar to the one predicted using modelling strategy 1 (Figs. 24a.C and 24d.C). It should be noticed that, even though the elements might be fully damaged, they are still able to carry load through friction.

To further explore the predictive capabilities of modelling strategy S4, other laminates were simulated using the same methodology. Figs. 26 and 27 show the experimental results for filled hole compressive strengths and the predictions obtained with the proposed modelling strategy for six multidirectional T700/M21 laminates and one multidirectional T800/M21 laminate, respectively. The relative errors for all seven layups are below 6.8% and the mean error is 2.7%. The propagation of damage and eventual final failure is a complex combination of different failure mechanisms and interactions between matrix cracking, fibre failure and delamination. Note that the set of laminates tested include four hard and three soft laminates with different percentages

of 0° , 45° and 90° plies and that failure of each laminate is dominated by different failure modes. The load displacement curves were not available, and therefore, it was not possible to verify the accuracy of the simulations in predicting the evolution of applied stress as a function of the applied displacement. Nonetheless, the ability of the model to accurately predict the filled-hole compression ultimate strength is remarkable.

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Figure 23: Predicted damage at ultimate load in 44/44/11: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° ply (d_2), C) interface damage (d).

Figure 24: Predicted damage at ultimate load in 11/44/44: A) fibre damage in a 0° ply (d_1), B) matrix damage in a 90° ply (d_2), C) interface damage (d).

Figure 25: σ_{33} (in MPa) after bolt preload on the laminate of a filled hole compression specimen. σ_{33} vs x is plotted in red.

Figure 26: Comparison between the numerical and experimental results for the filled-hole compressive strength of six T700/M21 carbon/epoxy laminates.

Figure 27: Comparison between the numerical and experimental results for the filled-hole compressive strength of one T800/M21 carbon/epoxy laminate.

4. Conclusions

In this work, a methodology to simulate composite laminates at the coupon level based on modifications implemented in the continuum damage model proposed by Maimí et al. [16–18] and in the cohesive zone models based on the work of Turon et al. [22] and Alfano et al. [23] was proposed. The continuum damage model proposed by Maimí et al. [16–18] was enriched with a bilinear [softening law](#) for longitudinal compression more suitable to represent kinking failure, based on the experimental findings by Moran et al. [28]. Additionally, a damage activation function for longitudinal compression that accounts for the effect of through-thickness stresses based on the 3D failure criteria proposed by Camanho et al. [30] and engineering solutions to account for the effect of pressure on the longitudinal compression and in-plane shear fracture toughness were proposed. A frictional cohesive model proposed by the authors [36] was used in this work to more accurately represent delamination propagation and load carrying capability in test cases where the effect of through-thickness pressure cannot be neglected.

The methodology was thoroughly validated for various material systems and coupons with increasing level of complexity: from unnotched tension and compression, open-hole tension and compression and filled-hole compression of both soft and hard laminates. A good agreement was found between the predictions and the experimental ultimate strengths for all the test cases, oftentimes within standard error of the tests, with maximum relative error of 13%.

It is worth noting that the models developed follow the requirements typically used by the aeronautical industry, in terms of element size and mesh structure. Additionally, all the predictions were obtained using a common set of material properties and material models. These facts indicate that the work presented here is a step towards the simulation of failure of composite structures using a standard modelling strategy. Hence, this work serves as basis for modelling more complex geometries and can potentially be used for the generation of statistically-based design allowables for the most simple test cases when allied with powerful statistical tools.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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Appendix A. Material properties and their determination

The ply and interface properties required to populate the model proposed and suggested methods for their determination are provided in tables A.6 and A.7. Note that some properties, despite representing physical phenomena, are particularly hard to determine experimentally. To clarify punctual assumptions done in this work, the estimation of some of the material parameters is highlighter in this section.

Even though solutions to determine the cohesive laws for longitudinal tension and compression have been proposed throughout the years [75–79], they are particularly challenging to determine experimentally and there is up to date, no standard procedure to do so. In this work, the inflection points for the softening laws for longitudinal tension and compression (f_{XT} , f_{GT} and f_{XC}) which represent fibre bridging and crushing, respectively, were inversely identified using an open-hole or filled-hole simulation, since there was no experimental data available.

Similarly, the biaxial tensile and compressive strengths (Y_{BT} and Y_{BC}) are also impractical to determine, since the specimens are generally expensive to manufacture and the required complicated test rigs. A summary of test methods for composites under multiaxial and out-of-plane loading conditions is given in Ref. [80]. Micromechanical models can also be used to numerically determine the material response under multiaxial stress states [81, 82]. Y_{BT} and Y_{BC} were only available for IM7/8552 [19, 30] and were assumed from the remaining material systems by considering that the ratio Y_{BT}/Y_T and Y_{BC}/Y_C is constant and equal to the ratios available for the IM7/8552 material system.

The parameter, f_{xc} , is a frictional parameter that represents the strengthening effect of pressure on the longitudinal compressive behaviour of the ply. The concept was first introduced here and therefore, no procedure was defined to determine it. For this reason, in this work, the parameters was inversely identified from an open-hole or filled-hole simulation, depending on the material system.

Table A.6: Intralaminar material parameters and suggested method for their determination.

Material parameter		Methodology
<i>Ply Elastic Properties</i>		
E_1	Young's modulus for longitudinal tension	ASTM D3039 [83]
E_{1c}	Young's modulus for longitudinal compression	Ref. [84]
E_2	Young's modulus for transverse tension	ASTM D3039 [83]
G_{12}	In-plane shear modulus	ASTM D3518 [85]
ν_{12}	Major Poisson's coefficient	ASTM D3039 [83]
<i>Ply strengths</i>		
X_T	Longitudinal tensile strength	ASTM D3039 [83]
f_{X_T}	$f_{X_T} X_T$ is the longitudinal tensile stress at inflection point	Note (1)
X_C	Longitudinal compressive strength	Ref. [84]
f_{X_C}	$f_{X_C} X_C$ is the longitudinal compression stress at inflection point	Note (1)
Y_T	Transverse tensile strength (2)	ASTM D3039 [83]
Y_C	Transverse compressive strength (2)	Ref. [86]
Y_{BT}	Biaxial transverse tensile strength	Note (1)
Y_{BC}	Biaxial transverse compressive strength	Note (1)
S_L	In-plane shear strength (2)	ASTM D3518 [85]
S_{LP}	Shear stress that activates plastic flow	Note (3)
K_P	Shear incremental stiffness under plastic flow	Note (3)
<i>Fracture Toughness</i>		
$\mathcal{G}_{1+}$	Fracture toughness for long. tension	Ref. [87]
f_{GT}	Portion of $\mathcal{G}_{X_T}$ dissipated up to the inflection point	Note (1)
$\mathcal{G}_{1-}$	Fracture toughness for longitudinal compression	Ref. [64]
$\mathcal{G}_{2+}$	Mode I fracture toughness for transverse tension	ASTM D5528 [88]
$\mathcal{G}_{2-}$	Mode I fracture toughness for transverse compression	Calculated (4)
$\mathcal{G}_6$	Mode II fracture toughness	ASTM D7905 [89]
<i>Geometrical</i>		
t	Element thickness	Ply thickness
α_0	Fracture angle of the UD ply subjected to uniaxial transverse comp.	$\approx 53^\circ$ [90]
<i>Friction</i>		
f_{fxc}	Internal frictional parameter under long. compression	Note (1)
η_G	Internal frictional parameter under in-plane shear	Ref. [36]

(1) No standard methodology to determine this material parameter is available. The parameter is assumed or inversely identified.

(2) The references given provide methodologies to determine the UD strengths of the ply. The in-situ properties should be calculated in function of the ply thickness and the position in the laminate proposed in Appendix B

(3) Best fitting from the in-plane shear stress-strain curve of the ply

(4) $\mathcal{G}_{2-} = \mathcal{G}_6 / \cos(\alpha_0)$

Table A.7: Interlaminar material parameters and suggested method for their determination.

	Material parameter	Methodology
<i>Penalty Stiffness</i>		
K	Penalty stiffness	Recommended value: 10^6 N/mm ³ [65]
<i>Strengths</i>		
τ_N	Strength under pure mode I	ASTM D3039 [83]
τ_{sh}	Strength under pure mode II	ASTM D3518 [85]
<i>Fracture Toughness</i>		
$\mathcal{G}_{Ic}$	Mode I fracture toughness	ASTM D3039 [83]
$\mathcal{G}_{IIc}$	Mode II fracture toughness	ASTM D7905 [89]
η	Mixed-mode interaction parameter	ASTM D6671 [91]
<i>Friction</i>		
μ	Friction coefficient	Note (1)

(1) The frictional parameter can be inversely identified following Ref. [36]

Appendix B. Derivation of the *in-situ* properties

Appendix B.1. Transverse tensile and in-situ shear strengths

Camanho et al. [48] considered three ply configurations to calculate the transverse tensile and in-plane shear strengths: thick, thin inner and thin outer plies.

Appendix B.1.1. Thick embedded plies

In a thick ply embedded in a multidirectional laminate, a slit crack such as the one shown in Fig. B.1 will propagate firstly in the transverse direction (T), and therefore the mode I and mode II energy release rates read:

$$\mathcal{G}_I(T) = \frac{\pi a_0}{2} \Lambda_{22}^o \sigma_{22}^2 \quad (\text{B.1})$$

$$\mathcal{G}_{II}(T) = \frac{\pi a_0}{2} \chi(\gamma_{12}) \quad (\text{B.2})$$

where $2a_0$ is the slit crack length along the thickness, $\chi(\gamma)$ is given by:

$$\chi(\gamma_{12}) = 2 \int_0^{\gamma_{12}} \sigma_{12} d\gamma_{12} \quad (\text{B.3})$$

and Λ_{22}^o is given by [92]:

$$\Lambda_{22}^o = 2 \left(\frac{1}{E_2} - \frac{\nu_{12}^2}{E_1} \right) \quad (\text{B.4})$$

Figure B.1: Thick embedded ply [48]

Dvorak and Laws [92] proposed that the transverse tensile *in-situ* strength can be obtained by solving equation B.1 for Y_T^{is} , yielding

$$Y_T^{is} = \sqrt{\frac{2\mathcal{G}_{Ic}(T)}{\pi a_0 \Lambda_{22}^0}} \quad (\text{B.5})$$

In the Continuum Damage model presented here and implemented in the ABAQUS user subroutine, the shear response of a given laminated composite is approximated by:

$$\sigma_{12} = \begin{cases} G_{12}\gamma_{12} & , \gamma_{12} \leq S_{LP}/G_{12} \\ K_1\gamma_{12} + K_2 & , \gamma_{12} > S_{LP}/G_{12} \end{cases} \quad (\text{B.6})$$

where $K_1 = \frac{KpG_{12}}{1+Kp}$ and $K_2 = S_{LP} \left(1 - \frac{Kp}{1+Kp}\right)$, G_{12} is in-plane shear modulus, S_{LP} is the shear stress at which the plastic flow is activated and Kp is the shear incremental stiffness under plastic flow. Having defined σ_{12} , the mode II fracture toughness can be calculated replacing (B.6) in equation (B.2), yielding:

$$\begin{aligned} \mathcal{G}_{IIc}(T) &= \pi a_0 \left[\int_0^{\frac{S_{LP}}{G_{12}}} G_{12}\gamma_{12} d\gamma_{12} + \int_{\frac{S_{LP}}{G_{12}}}^{\gamma_{12}^{is}} (K_1\gamma_{12} + K_2) d\gamma_{12} \right] = \\ &= \pi a_0 \left[(\gamma_{12}^{is})^2 \frac{K_1}{2} + \gamma_{12}^{is} K_2 - \frac{S_{LP}^2}{2G_{12}(1+Kp)} \right] \end{aligned} \quad (\text{B.7})$$

Dvorak and Laws [92] also proposed that the a unidirectional laminate can be considered a special case of a thick ply with unconstrained outer surfaces and, therefore, the *in-situ* strengths of a thick ply can be related to those of an unidirectional thick ply. Using the classical solutions for stress intensity factors of surface cracks in unidirectional laminates [93], the mode I and mode II components of fracture toughness can also be calculated as

$$\mathcal{G}_{Ic}(T) = 1.12^2 \pi a_0 \Lambda_{22}^0 (Y^T)^2 \quad (\text{B.8})$$

and

$$\mathcal{G}_{IIc}(T) = 2\pi a_0 \left[\gamma_L^2 \frac{K_1}{2} + \gamma_L K_2 - \frac{S_{LP}^2}{2G_{12}(1 + Kp)} \right] \quad (B.9)$$

where $\gamma_L = \frac{S_L - K_2}{K_1}$.

Combining equations (B.5) and (B.8), the transverse tensile in-situ strength for a thick ply yields:

$$Y_T^{is} = 1.12\sqrt{2}Y_T \quad (B.10)$$

Combining equations (B.7) and (B.9), the in-plane shear strength can be obtained solving the following equation for γ_{12}^{is} :

$$(\gamma_{12}^{is})^2 \frac{K_1}{2} + \gamma_{12}^{is} K_2 = \gamma_L^2 K_1 + 2\gamma_L K_2 - \frac{S_{LP}^2}{2G_{12}(1 + Kp)} \quad (B.11)$$

Knowing γ_{12}^{is} the in-plane shear strength can be obtained solving the second member of equation (B.6) for σ_{12} :

$$S_L^{is} = \sqrt{2S_L^2 - S_{LP}K_2} \quad (B.12)$$

Appendix B.1.2. Thin embedded plies

In thin plies embedded in a multidirectional laminate, a slit crack such as the one shown in Fig. B.2 will propagate in the longitudinal direction (L) since it already extended through the ply thickness and, therefore, the mode I and mode II energy release rate read [92]:

$$\mathcal{G}_{Ic}(L) = \frac{\pi t}{8} \Lambda_{22}^0 \sigma_{22}^2 \quad (B.13)$$

$$\mathcal{G}_{IIc}(L) = \frac{\pi t}{8} \chi(\gamma_{12}) \quad (B.14)$$

The transverse tensile *in-situ* strength can be obtained by solving equation B.13 for Y_T^{is} yielding

$$Y_T^{is} = \sqrt{\frac{8\mathcal{G}_{Ic}}{\pi t \Lambda_{22}^0}} \quad (B.15)$$

Replacing equation (B.3) in equation (B.14) yields

$$\mathcal{G}_{IIc} = \frac{\pi t}{4} \left[(\gamma_{12}^{is})^2 \frac{K_1}{2} + \gamma_{12}^{is} K_2 - \frac{S_{LP}^2}{2G_{12}(1 + Kp)} \right] \quad (B.16)$$

Figure B.2: Thin embedded ply [48]

which can be solved for γ_{12}^{is} . Knowing γ_{12}^{is} the in-plane shear strength can be obtained solving the second member of equation B.6 for σ_{12} :

$$S_L^{is} = \sqrt{\frac{8\mathcal{G}_{IIc}K_1}{\pi t} + S_{LP}K_2} \quad (\text{B.17})$$

Appendix B.1.3. Thin outer plies

A thin outer ply is a special case of the thin ply for which the energy release rate is larger because the slit crack is closer to the surface of the laminate (Fig. B.3). The mode I and mode II energy release rate read:

$$\mathcal{G}_{Ic} = 1.22^2 \frac{\pi t}{4} \Lambda_{22}^0 \sigma_{22}^2 \quad (\text{B.18})$$

$$\mathcal{G}_{IIc} = \frac{\pi t}{4} \chi(\gamma_{12}) \quad (\text{B.19})$$

The transverse tensile *in-situ* strength can be obtained by solving equation B.18 for Y_T^{is} yielding

$$Y_T^{is} = 1.78 \sqrt{\frac{\mathcal{G}_{Ic}}{\pi t \Lambda_{22}^0}} \quad (\text{B.20})$$

Replacing equation (B.3) in (B.19), yields:

$$\mathcal{G}_{IIc} = \frac{\pi t}{2} \left((\gamma_{12}^{is})^2 \frac{K_1}{2} + \gamma_{12}^{is} K_2 - \frac{S_{LP}^2}{2G_{12}(1 + Kp)} \right) \quad (\text{B.21})$$

which can be solved for the γ_{12}^{is} . Knowing γ_{12}^{is} the in-plane shear strength can be obtained solving the second member of equation B.6 for σ_{12} :

$$S_L^{is} = \sqrt{\frac{4\mathcal{G}_{IIc}K_1}{\pi t} + S_{LP}K_2} \quad (\text{B.22})$$

Figure B.3: Thin outer ply [48]

Appendix B.2. Transverse compressive strength

The *in-situ* transverse compressive stress Y_C^{is} is calculated imposing that the slope in the σ_{22} - σ_{12} failure envelope when $\sigma_{22} = 0$, η_L , is equal to the slope of the envelope obtained with the *in-situ* properties (see Fig. B.4). To maintain consistency between the assumptions made to derive the *in-situ* strengths and the assumed behavior of the material, the criterion used in both cases is the LaRC03-04 failure criterion [27]. The coefficient of longitudinal influence η_L can be determined experimentally but in the absence of biaxial test data, it can be estimated as [27, 90]

$$\eta_L = \frac{-S_L^{UD} \cos(2\alpha_0)}{Y_C^{UD} \cos^2(\alpha_0)} \quad (B.23)$$

and therefore, the *in-situ* transverse compressive strength Y_C^{is} can be estimated as

$$\eta_L = \eta_{L,is} \Leftrightarrow Y_C^{is} = \frac{S_L^{is} Y_C^{UD}}{S_L^{UD}} \quad (B.24)$$

Figure B.4: σ_{22} - σ_{12} failure envelope [30]

Appendix B.3. General expression for the *in-situ* strengths

The general expression for the *in-situ* strengths are presented in this section.

For an embedded ply:

- the *in-situ* transverse tensile strength is the maximum between the transverse tensile strength of a thin embedded ply and a thick embedded ply, i.e.:

$$Y_T^{is} = \max\left\{\sqrt{\frac{8\mathcal{G}_{Ic}}{\pi t \Lambda_{22}^o}}, \quad 1.12\sqrt{2}Y_T^{UD}\right\} \quad (\text{B.25})$$

- the *in-situ* in-plane shear strength is the maximum between the in-plane shear strength of a thin embedded ply and a thick embedded ply, i.e.:

$$S_L^{is} = \max\left\{\sqrt{\frac{8\mathcal{G}_{IIc}K_1}{\pi t} + S_{LP}K_2}, \quad \sqrt{2S_L^2 - S_{LP}K_2}\right\} \quad (\text{B.26})$$

- the *in-situ* transverse compressive strength Y_C^{is} is given by:

$$Y_C^{is} = \frac{S_L^{is}Y_C^{UD}}{S_L^{UD}} \quad (\text{B.27})$$

For an outer ply:

- the *in-situ* transverse tensile strength is the maximum between:

$$Y_T^{is} = \max\left\{1.78\sqrt{\frac{\mathcal{G}_{Ic}}{\pi t \Lambda_{22}^o}}, \quad Y_T^{UD}\right\} \quad (\text{B.28})$$

- The *in-situ* in-plane shear strength is the maximum between the in-plane shear strength of a thin outer ply and an UD ply, i.e.:

$$S_L^{is} = \max\left\{\sqrt{\frac{4\mathcal{G}_{IIc}K_1}{\pi t} + S_{LP}K_2}, \quad S_L^{UD}\right\} \quad (\text{B.29})$$

- the *in-situ* transverse compressive strength Y_C^{is} is given by:

$$Y_C^{is} = \frac{S_L^{is}Y_C^{UD}}{S_L^{UD}} \quad (\text{B.30})$$

Appendix C. Engineering solutions used in the modelling strategies S2 and S3

Cohesive finite elements require very fine meshes: the fracture process zone should include at least three cohesive elements so that delamination is accurately simulated. Turon et al. [51]

proposed an engineering solution to avoid the use of such refined meshes that consists of lowering the cohesive strengths whilst keeping the fracture toughness constant to enlarge the cohesive zone and to allow a better representation of the softening behaviour at the vicinity of the crack tip. Following Refs. [51, 52], the cohesive properties should therefore, be determined as follows:

1. Take the penalty stiffness, K , equal to 10^6 N/mm³ [65]
2. Determine the transverse tensile strength Y_T^{UD} (ASTM D3039 [83])
3. Determine the mode I fracture toughness $\mathcal{G}_{Ic}$ (ASTM D5528 [88])
4. Determine the mode II fracture toughness $\mathcal{G}_{IIc}$ (ASTM D7905 [89])
5. Select the element size in the direction of crack propagation, l_e [51]
6. Select the number of elements in the cohesive zone, N_e , which should be at least 3 [51]
7. Calculate the strength in pure mode I as: $\bar{\tau}_N = \sqrt{\frac{9\pi E_2 \mathcal{G}_{Ic}}{32 N_e l_e}}$ [51]
8. Calculate the effective strength in pure mode I as [51]

$$\tau_N = \min\{Y_T^{UD}, \bar{\tau}_N\} \quad (C.1)$$

It was shown that through-thickness compression increases the interlaminar shear strength of laminates and delays delamination [34–36]. Cui et al. [37] suggests that this strengthening effect is a result of an increase on mode II fracture toughness under through-thickness compression that can be expressed as:

$$\mathcal{G}_{IIc}^{ef} = \mathcal{G}_{IIc} (1 - \eta_G \langle -\sigma_{33} \rangle) \quad (C.2)$$

where $\mathcal{G}_{IIc}$ is the mode II fracture toughness, η_G a material dependent empirically derived enhancement factor, σ_{33} is the through-thickness stress and $\langle x \rangle$ is the Mccaulay operator. [37, 38]. In situation where the applied pressure is fairly constant (e.g. in bolt bearing and filled-hole compression tests), equation C.2 can be used to artificially account for the strengthening effect on the interfaces caused by friction under shear.

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