An Adaptive Backstepping Nonsingular Fast Terminal Sliding Mode Control for Robust Fault Tolerant Control of Robot Manipulators


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Abstract—This paper develops a novel control methodology for tracking control of robot manipulators based on a novel adaptive backstepping nonsingular fast terminal sliding mode control (ABNFTSMC). In this approach, a novel backstepping nonsingular fast terminal sliding mode controller (BNFTSMC) is developed based on an integration of integral nonsingular fast terminal sliding mode surface and a backstepping control strategy. The benefits of this approach are that the proposed controller can preserve the merits of the integral nonsingular fast terminal sliding mode control in terms of high robustness, fast transient response, and finite time convergence, and of backstepping control strategy in terms of globally asymptotic stability based on Lyapunov criterion. However, the major limitation of the proposed BNFTSMC is that its design procedure is dependent on the prior knowledge of the bound value of the disturbance and uncertainties. In order to overcome this limitation, an adaptive technique is employed to approximate the upper bound value; yielding an adaptive backstepping nonsingular fast terminal sliding mode control (ABNFTSMC) is recommended. The proposed controller is then applied for tracking control of a PUMA560 robot and compared with other state-of-the-art controllers, such as computed torque controller (CTC), PID controller, conventional PID-based sliding mode controller (PID-SMC), and nonsingular fast terminal sliding mode control (NFTSMC). The comparison results demonstrate the superior performance of the proposed approach.

Index Terms—Control of robots, backstepping control, fault tolerant control, terminal sliding mode control.

I. INTRODUCTION

Tracking control of robot manipulators, which is required to provide high accuracy, stability and safety in some applications such as industrial robotics, surgical robotics, assistive robotics, in the presence of huge uncertainties, disturbance and faults has been a critical issue in both academic and industrial applications [1-3]. How to improve the tracking performance and transient response for robot manipulators, particularly in the presence of external disturbance and possible actuation failures is still a challenge in the research community [4-6].

In order to increase the tracking performance and reliability of the robot, fault tolerant control (FTC) strategies have been developed [7-9]. Generally, FTC approaches can be categorized into two classes: passive FTC (PFTC) and active FTC (AFTC). In AFTC, the control input is adapted according to the fault estimation feedback, which is obtained from a fault diagnosis observer [10-12]. This approach, therefore, requires a design of an additional observer, which will increase the computational burden of the system. In addition, due to the delay of the fault information feedback from the fault diagnosis observer, the fault compensational time of the system is delayed and consequently, the system may become unstable. In contrast, in PFTC approach, one robust controller is designed for both normal and fault operation without requiring any feedback from the fault diagnosis (FD) observer [13-14]. Since the PFTC does not need to wait for the fault information feedback, it can compensate the fault effects much faster. However, since the fault effects imposed on the PFTC system are heavier than that of the active approach, the PFTC requires a control technique with higher robustness.

In the literature, several methodologies have been developed to improve the tracking performance of the robot manipulators in the presence of disturbance, uncertainties and faults. In the initial approaches, PID or PD controller [15-16], computed torque controller (CTC) [6], intelligent and learning control [17-20], optimal control [21-22] and robust control [23-25], etc., have been developed. Among them, robust control techniques have been shown to provide higher robustness and disturbance and fault rejection capability. Sliding mode control (SMC) is a well-known robust control technique and has been widely utilized in real applications due to its strong robustness against uncertainties and disturbance [26]. This great feature of SMC has been applied in the design of FTC systems [27-29]. However, the traditional SMC still has four major drawbacks that limit its performance in real applications: 1) it does not provide a finite time convergence, 2) although it possesses a good transient response in normal operation, it is worst to tackle the fast variation effects of the disturbance

and/or faults, 3) it generates a big chattering in the system, and 4) the design procedure requires a prior knowledge of the upper bound value of the uncertainties, disturbance and faults.

Due to the benefits and limitations of the SMC, many great efforts have been spent on finding effective solutions to preserve the benefits and reduce or eliminate the drawbacks of the conventional SMC. First, in order to obtain a finite time convergence, terminal sliding mode control (TSMC) has been investigated [30-32]. However, the conventional TSMC has a drawback that provides a slow convergence speed and singular problem. In order to tackle the problems of SMC, fast terminal sliding mode control (FTSMC) [33-35] and nonsingular terminal sliding mode control (NTSMC) [36-40] have been developed separately. Unfortunately, the individual approaches based on FTSMC or NTSMC have just only solved one aspect and ignored the other problems of the conventional SMC. In order to obtain both fast finite time convergence and singular elimination, nonsingular fast terminal sliding mode control (NFTSMC) has been proposed [41-43]. Second, in order to enhance the transient response of the conventional SMC, integral sliding mode control (ISMС) or PID-based SMC (PID-SMC) has been developed [44-47]. The basic idea behind this approach is to mimic the property of the integral component in the PID controller to enhance the transient response of the system [48]. In order to obtain both fast transient response and finite time convergence, integral terminal sliding mode control (ITSMC), which combines the benefits of the ISMC and TSMC, have been developed [49-50]. However, the conventional ITSMC used the conventional TSMC in the design, and hence the aforementioned weaknesses of the SMC were still present. Third, in order to eliminate the chattering, several approaches have been developed by using either boundary method [51-53] or disturbance observer [54-55] or high-order sliding mode control (HOSMC) [56-60]. Among them, HOSMC is preferred to be used in wide applications since it provides both higher tracking accuracy and chattering elimination. In summary, each drawback of the conventional SMC has been tackled by a corresponding suitable approach, however, there are no approaches in the literature that intended to design a single controller that considers all the drawbacks of SMC together and solve them simultaneously. That is, for the FTC system, the design of SMC needs to guarantee three major demands: fast transient response, finite time convergence and chattering elimination.

Motivated by the above issues, this paper develops a PFTC to tackle the effects of the faults for uncertain robot manipulators using a novel adaptive backstepping nonsingular fast terminal sliding mode control (ABNFTSMC). Starting from the selection of an integral nonsingular fast terminal sliding surface (INFTS), the second-order dynamics model of the robot manipulators based on the position variable is transferred into a third-order dynamics model based on the selected sliding surface. In this way, the control output can be reconstructed such that it can provide a continuous control output like a manner to HOSMC. Since the proposed approach used the integral nonsingular fast terminal sliding mode control and the control output can be reconstructed in a similar manner to the HOSMC, it can preserve the merits of the ITSMC, NFTSMC and HOSMC simultaneously. Therefore the proposed approach will provide a finite time convergence without singular problem, fast transient response, high tracking precision and less chattering. In addition, the proposed design topology follows the design procedure of backstepping control technique [61-64], and thus the globally asymptotic stability of the system can be guaranteed based on the Lyapunov criterion. In summary, the main contributions of this paper are as follows:

- A novel integral nonsingular fast terminal sliding mode surface is proposed.
- A third-order dynamics model of the system is reconstructed based on the selected sliding surface.
- A backstepping design procedure is employed to derive the control output of the third-order dynamics system so that it can achieve a finite time convergence, fast transient response, low steady state error and globally asymptotic stability.
- In comparison with other state-of-the-art methods, such as CTC, PID, PID-SMC and conventional NFTSMC, the proposed approach provides a superior performance.

The remainder of this paper is organized as follows. Section II introduces the problem. Section III presents the robust fault tolerant control based on an adaptive backstepping nonsingular fast terminal sliding mode control (ABNFTSMC). The performance of the controller is verified in Section IV. Section V provides conclusions.

**Remark 1:** Beside the references cited and discussed above, there are several other approaches in the literature that were developed for fault tolerant control systems for different applications using different control techniques. For example, in [61-68], FTCs have been established using intelligent learning techniques such as neural network or fuzzy logic. However, the approaches require a huge computational burden due to the weight learning procedure of neural network or fuzzy logic. In [69], an adaptive FTC technique has been developed for uncertain nonlinear time delay systems. In [70], a prescribed performance FTC has been developed. In [71], a FTC scheme has been developed for uncertain linear system over networks with quantization, and so on. However, the focuses of these approaches are out of or far away from the scope of this paper. The focus of this paper is to develop a robust PFTC scheme for uncertain robot manipulators using an advanced robust sliding mode control.

**II. Problem Statement**

**A. Problem statement**

Consider the robot dynamics described by [10, 42]:

```
\( \dot{q} = M^{-1}(q)(\tau - H(q, \dot{q})\dot{q} - F(\dot{q}) - G(q) - \tau_d) \\
+ \gamma(t - T_f)\phi(q, \dot{q}, \tau) \quad (1) \)

where \( q \in \mathbb{R}^n \), \( \dot{q} \in \mathbb{R}^n \) and \( \ddot{q} \in \mathbb{R}^n \) denotes the position, velocity and acceleration, respectively. \( \tau \in \mathbb{R}^n \) stands for the actuator inputs. \( M(q) \in \mathbb{R}^{nn} \) is the positive and definite inertia matrix. \( H(q, \dot{q}) \in \mathbb{R}^n \) includes the Coriolis and centripetal forces. \( F(\dot{q}) \in \mathbb{R}^n \) is the friction matrix. \( \tau_d \) is a load disturbance matrix and \( G(q) \in \mathbb{R}^n \) is the vector of gravity terms. \( \phi(q, \dot{q}, \tau) \in \mathbb{R}^n \) represents the fault components affecting on the system. \( \gamma(t - T_f) \in \mathbb{R}^n \) represents the time profile of the faults, and \( T_f \) is the time of occurrence of the faults.

The following standard property is satisfied for the robot dynamics (1):

\[
0 < \lambda_M \{M(q)\} \leq \|M\| \leq \lambda_m \{M(q)\} \leq \chi, \quad \chi > 0
\]

where \( \lambda_M \) and \( \lambda_m \) are the maximum and minimum eigenvalues of matrix \( M \).

The term \( \gamma(t) \) is a diagonal matrix having the form below

\[
\gamma(t - T_f) = \text{diag} \{ \gamma_1(t - T_f), \gamma_2(t - T_f), \ldots, \gamma_n(t - T_f) \}
\]

where \( \gamma_i \) indicates the effects of fault on the \( i \)th state equation due to a fault function.

The model of the fault function is introduced by [10, 42]:

\[
\gamma_i(t - T_f) = \begin{cases} 
0 & \text{if } t < T_f \\
1 - e^{-\alpha_i}(t - T_f) & \text{if } t \geq T_f
\end{cases}
\]

where \( \alpha_i > 0 \) represents the unknown fault evolution rate. The fault is called as incipient fault when the value of \( \alpha_i \) is small. The fault becomes abrupt fault when the value of \( \alpha_i \) is large.

Rearranging the dynamics model (1), we have:

\[
\ddot{q} = M^{-1}(q)(\tau - H(q, \dot{q})\dot{q} - F(\dot{q}) - G(q)) \\
+ M^{-1}(q)(-F(\dot{q}) - \tau_d) + \gamma(t - T_f)\phi(q, \dot{q}, \tau)
\]

Define \( x_1 = q \) and \( x_2 = \dot{q} \), the equation (5) can be transformed into a second-order state space model as follows:

\[
x_1 = x_2 \\
x_2 = M^{-1}(q)u + f(x_1, x_2) + \Delta
\]

where \( f(x_1, x_2) = M^{-1}(q)(-H(q, \dot{q})\dot{q} - G(q)) \) denotes the lumped known component and \( \Delta = M^{-1}(q)(-F(\dot{q}) - \tau_d) + \gamma(t - T_f)\phi(q, \dot{q}, \tau) \) denotes the lumped unknown component in the system dynamics.

The objective of this paper is to design a control input \( u \) such that the system can provide high tracking performance despite the existence of the uncertainties, disturbance and the presence of faults.

### III. DESIGN OF ROBUST FAULT TOLERANT CONTROL BASED ON ADAPTIVE BACKSTEPPING NONSINGULAR FAST TERMINAL SLIDING MODE CONTROL

**A Design of robust fault tolerant control based on backstepping nonsingular fast terminal sliding mode control**

In the design of sliding mode control, the selection of the sliding surface has a significant impact on the performance of the system. The sliding surface is chosen such that when it converges to zero, the desired performance of the system can be satisfied. Let \( e = x_1 - x_d \) be the trajectory tracking error, where \( x_d \) denotes a desired trajectory. In order to obtain a fast transient response and finite time convergence without singular problem, an integral nonsingular fast terminal sliding mode control (INFTSM) surface is selected as follows:

\[
\sigma_1 = \int \left( e + k_1 e^{[\lambda]} + k_2 e \right) ds
\]

where \( \sigma_1 \) represents the sliding variable, \( k_1 = \text{diag} \{ k_{11}, k_{12}, \ldots, k_{1n} \} \in \mathbb{R}^{nn} \) and \( k_2 = \text{diag} \{ k_{21}, k_{22}, \ldots, k_{2n} \} \in \mathbb{R}^{nn} \) are positive definite matrices, respectively, \( p \) and \( q \) are positive odd numbers satisfying the relation \( 1 < p / q < 2 \) and \( \lambda > p / q \).

**Remark 2:** The proposed sliding surface (7) is proposed to combine the properties of the FTSMC [33] and NTSMC [36] such that the system can obtain fast finite time convergence without singular problem. In addition, the integral component is employed for the sliding surface (7) so that the system can possess the merits of both ITSMC and FTSMC, which increases the transient response and reduces the steady state error of the system. When the sliding surface (7) converges to zero, we have:

\[
e + k_1 e^{[\lambda]} + k_2 e \right) = 0
\]

The definite finite time \( t_c \) that is taken to travel from \( e(t_c) \neq 0 \) to \( e(t_c + t_c) = 0 \) is given by [40]:

\[
t_c = \frac{q k_1}{k_2 (p - 1)} \int_{t_c}^{t_c + t_c} \frac{p - 1}{q} \left( \frac{p - 1}{q} - 1 \right) ds
\]

where \( \Omega \) denotes Gauss’ hypergeometric function. The interested readers can refer to [40] for more detail.

The first and second derivative of the sliding surface can be computed as follows:

\[
\sigma_1 = e + k_1 e^{[\lambda]} + k_2 e
\]

\[
\sigma_1 = \dot{e} + k_1 \lambda e^{[\lambda - \dot{e}] + k_2 \rho e}^{[\lambda / \rho - 1] + \dot{e}}
\]

From (6) and (9)-(10), the second-order state space model of the system dynamics based on the state variable can be transformed into a third-order state space model based on the selected sliding surface, as below:
Based on (12), a backstepping design procedure is introduced. The following change of coordinates is introduced.

\[ V_1 = \frac{1}{2} \sigma_1^2 \]

where \( \sigma_1(1,2) \) is the virtual control in the \( i \)th step.

**Step 1:**
Differentiating \( \sigma_1 \) in (13) with respect to time, we have

\[ \dot{\sigma}_1 = \dot{\sigma}_2 + \alpha_1 \]

Define a positive definite Lyapunov function \( V_1 = \frac{1}{2} \sigma_1^2 \).

Based on \( V_1 \), virtual control \( \alpha_1 \) can be chosen as

\[ \alpha_1(t) = -\xi_1 \sigma_1(t) \]

where \( \xi_1 \) is a positive design parameter. Differentiating \( V_1 \) with respect to time, we have

\[ \dot{V}_1 = \dot{\sigma}_1 (\dot{\sigma}_2 + \alpha_1) \]

\[ = \dot{\sigma}_1 \sigma_2 + \alpha_1 \sigma_1 = -\xi_1 |\dot{\sigma}_1|^2 + \dot{\sigma}_1 \sigma_2 \]

From (18), we can see that \( \dot{V}_1 = -\xi_1 |\dot{\sigma}_1|^2 \) once \( \dot{\sigma}_1 = 0 \), and consequently, \( \dot{\sigma}_1 \) will be asymptotically stable.

**Step 2:**
Differentiating \( \dot{\sigma}_2 \) in (14) with respect to time, we have:

\[ \dot{\sigma}_2 = \dot{\sigma}_2 - \alpha_1 \]

\[ = \dot{\sigma}_2 + \alpha_2 - \alpha_1 \]

Consider the following Lyapunov function:

\[ V_2 = V_1 + \frac{1}{2} \sigma_2^2 \]

Based on (19), the virtual control input is chosen as

\[ \alpha_2(t) = -\xi_2 \sigma_2 - \dot{\sigma}_1 - \xi_1 \sigma_2 \]

Differentiating the Lyapunov function \( V_2 \) with respect to time, we have

\[ \dot{V}_2 = \dot{V}_1 + \dot{\sigma}_2 \sigma_2 \]

\[ = -\xi_1 |\dot{\sigma}_1|^2 + \dot{\sigma}_1 \sigma_2 + \sigma_2 \dot{\sigma}_2 \]

\[ = -\xi_1 |\dot{\sigma}_1|^2 - \xi_2 |\dot{\sigma}_2|^2 + \dot{\sigma}_2 \sigma_2 \]

From (22), \( \dot{V}_2 = -\xi_1 |\dot{\sigma}_1|^2 - \xi_2 |\dot{\sigma}_2|^2 \) can be obtained once \( \dot{\sigma}_2 = 0 \) is satisfied. Consequently, the states \( \dot{\sigma}_1 \) and \( \dot{\sigma}_2 \) will be asymptotically stable.

**Step 3:**
Consider the following Lyapunov function

\[ V_3 = V_2 + \frac{1}{2} \sigma_3^2 \]

Differentiating \( V_3 \) with respect to time and combining with the results in (12) and (15), we have:

\[ V_3 = V_2 + \frac{d}{dt} \sigma_3 \]

\[ = -\xi_1 |\dot{\sigma}_1|^2 - \xi_2 |\dot{\sigma}_2|^2 + \dot{\sigma}_2 \sigma_2 + \sigma_3 \dot{\sigma}_3 \]

Hence, a backstepping nonsingular fast terminal sliding mode control is designed as

\[ u(t) = \Xi(\dot{e})M(x(t))(u_{ref}(t) - u_e(t)) \]

where

\[ u_{ref}(t) = -\Xi(\dot{e})f(x(t)) - \Psi(\dot{e}, \dot{x}) + \alpha_2 - \int(\xi_3 \sigma_3 + \sigma_2) \]

and

\[ u_e(t) = (\lambda + \zeta)\text{sign}(\sigma_3) \]

where \( \Xi = \frac{d}{dt}(\Xi(\dot{e}) \Delta) \) and \( \Delta \) is selected such that \( |\dot{V}| \leq \Delta \). \( \zeta \) is a positive small constant.

Adding the proposed controller in (25)-(27) into (24), we obtain:

\[ V_3 = -\xi_1 |\dot{\sigma}_1|^2 - \xi_2 |\dot{\sigma}_2|^2 - \xi_3 |\dot{\sigma}_3|^2 - \lambda |\sigma_3| + \gamma \sigma_3 \]

Then, \( V_3 \leq 0 \), \( \dot{V}_3 \) is negative semidefinite, which implies that \( \sigma_1(t) \), \( \sigma_2(t) \) and \( \sigma_3(t) \) will converge to zero in a finite time. Therefore, it can be concluded that the proposed backstepping nonsingular fast terminal sliding mode system is asymptotically stable despite the existing of uncertainties, disturbance and faults.

**B. Design of robust fault tolerant control based on adaptive backstepping nonsingular fast terminal sliding mode control**

In the proposed controller (25)-(27), the design procedure is based on the assumption that the upper bound value \( \Lambda \) of the unknown function \( \Psi \) can be estimated in advance. However, this assumption is too conservative in practical engineering applications since it is difficult to estimate the bound value of
the uncertainty, disturbance and fault parameters in advance. It is required to resolve this limitation, a simple adaptive law is developed in this subsection, and an adaptive backstepping nonsingular fast terminal sliding mode control (ABNFTSMC) is proposed as a result.

Steps 1 and 2 of the ABNFTSMC are designed as the same procedure as the BNFTSMC. However, in Step 3, an adaptive control law \( u_{ad} \) is used to replace the control law \( u_s \) in (27), which requires an upper bound value of the disturbance and faults. Thus:

\[
u(t) = \Xi(\dot{e})^{-1}M(x_t)(u_n(t) - u_{ad}(t))
\]

(29)
where \( u_n(t) \) is designed just like in (26), and the adaptive reaching law can be designed as

\[
u_{ad}(t) = (\hat{\Lambda} + \zeta)\text{sign}(\hat{d}_3)
\]

(30)
where \( \hat{\Lambda} \) is used to approximate the upper bound value \( \Lambda \). It is adapted using the following law:

\[
\hat{\Lambda} = \frac{1}{\delta}\|\hat{d}_3\|
\]

(31)
where \( \delta \) is a user defined constant.

Let \( \hat{\Lambda} = \Lambda - \hat{\Lambda} \) is the estimation error. We consider the following Lyapunov function for Step 3:

\[
V_4 = V_3 + \frac{1}{2}\delta \hat{\Lambda}^2
\]

(32)
In combination with the result in (24), the derivative of the Lyapunov function (32) can be obtained as

\[
\dot{V}_4 = \dot{V}_3 + \delta(\hat{\Lambda} - \Lambda)\dot{\Lambda}
\]

(33)
\[\begin{align*}
\dot{\hat{\Lambda}} &= -\zeta_1|\hat{\Lambda}_3|^2 - \zeta_2|\hat{d}_2|^2 - \zeta_3|\hat{d}_3|^2 + \phi_2\phi_3 + \delta(\hat{\Lambda} - \Lambda)\dot{\Lambda}
\end{align*}\]

Adding the controller (29)-(31) into (33), we obtain

\[
\dot{V}_4 = -\zeta_1|\hat{\Lambda}_3|^2 - \zeta_2|\hat{d}_2|^2 - \zeta_3|\hat{d}_3|^2 + \phi_2\phi_3 + \delta(\hat{\Lambda} - \Lambda)\dot{\Lambda}
\]

(34)
Then, \( \dot{V}_4 \leq 0 \), \( \dot{V}_4 \) is negative semidefinite, which implies that \( \hat{\Lambda}_4(t) \), \( \hat{\Lambda}_2(t) \) and \( \hat{\Lambda}_3(t) \) will converge to zero in a finite time. Therefore, it can be concluded that the proposed backstepping nonsingular fast terminal sliding mode control is asymptotically stable despite the existing of uncertainties, disturbance and faults.

Remark 3: In order to eliminate the parameter drift problem when implementing the adaptive law (31), the following dead-zone technique is employed:

\[
\phi_2 = \begin{cases} 
0 & \text{if } |\hat{d}_2| \leq \epsilon \\
\frac{1}{\delta}|\hat{d}_2| & \text{if } |\hat{d}_2| > \epsilon
\end{cases}
\]

(35)
where \( \epsilon \) is the dead zone size.

Remark 4: The design of the proposed controller requires the measurements of position, velocity and acceleration. However, in practical robots, only position is measurable based on the encoder information. To estimate the velocity and acceleration measurement, the second-order exact differentiation (SOED) [57] is employed in this paper:

\[
\begin{align*}
\omega_0 &= \kappa_0 \\
\kappa_0 &= -m_1|\omega_0 - \omega_1|^{2/3}\text{sign}(\omega_0 - \omega_1) + \omega_1 \\
\omega_1 &= \kappa_1 \\
\kappa_1 &= -m_2|\omega_1 - \omega_0|^{1/2}\text{sign}(\omega_1 - \omega_0) + \omega_2 \\
\omega_2 &= -m_3\text{sign}(\omega_2 - \kappa_1).
\end{align*}
\]

Then, the estimates of the position, velocity and acceleration can be obtained as

\[
\omega_0 = x_1, \omega_1 = x_2, \omega_2 = \dot{x}_2
\]

(37)

Remark 5: The design procedure of the proposed controller is based on the assumption that the derivative of the unknown component \( \gamma \) is bounded by an unknown constant. However, this assumption may not be obtained in some conditions when the change of fault are fast, and thus the system will be unstable. Fortunately, due to the faster transient response of the proposed control method, which combines the backstepping and integral nonsingular fast terminal sliding mode control, the unstable condition may occur only for a very short time and the system will be stable quickly after then. This symptom will be illustrated in section IV.

Remark 6: The approximation learning techniques based on neural network and fuzzy logic can be used to approximate the unknown component \( \gamma \) as a similar design procedure to [59-66]. However, the design of approximation techniques based on neural network or fuzzy logic are computational expensive.

IV. RESULTS AND DISCUSSIONS

In order to verify the performance of the proposed approach, we employ it for a PUMA560 robot [72]. For simplicity in analyzing and presenting the results, only the first three joints of the robot are used. The dynamic and kinematic model of the PUMA560 robot are described as in [72].

The friction and disturbance term are modeled as

\[
F(q) + \tau_d = \begin{bmatrix}
0.5\dot{q}_1 + \sin(3\dot{q}_1) + 0.5\sin(q_1) \\
1.3\dot{q}_1 - 1.8\sin(2\dot{q}_1) + 1.1\sin(q_1) \\
-1.8\dot{q}_3 + 2\sin(3\dot{q}_3) + 0.15\sin(q_3)
\end{bmatrix}
\]

(38)
The desired trajectory of the system is selected as follows:

\[ x_d = \begin{bmatrix} \cos(t / 5 \pi) - 1, \cos(t / 5 \pi + \pi / 2), \sin(t / 5 \pi + \pi / 2) \end{bmatrix} \]  

(39)

The Matlab/Simulink environment using Runge-Kutta algorithm are used to conduct experiments. The sampling time is set as 0.005s .

In this paper, we consider the performance of the proposed method in both normal and fault operations. In order to demonstrate the superior performance of the proposed ABNFTSMC controller, we compare it with the state-of-the-art controllers, which have been proposed for improving trajectory tracking performance of robot manipulators such as computed torque control (CTC) [6], PID controller [15-16], conventional PID-based SMC controller (PID-SMC) [48], and conventional nonsingular fast terminal sliding mode control (NFTSMC) [41- 43]. The design of the CTC, PID, PID-based SMC (PID-SMC) and NFTSMC are represented in Appendix A, B, C and D, respectively. The selected parameters of these controllers are reported in Table I.

First, we assume that the system is operated in normal working condition with the assumed uncertainties and disturbance. The tracking performance of the system in Cartesian space under the controllers are shown in Fig. 2. For an easier comparison, the joint tracking errors \( e = x_1 - x_d \) of the system are shown in Fig. 3. From Figs. 2 and 3, we can see that the CTC provides worst tracking performance for the system in case of the presence of the disturbance and uncertainties. The PID controller provides better tracking performance compared to the CTC. However, the convergence times and steady state errors of these controllers are very large. Due to the robustness property of the SMC against the effects of the disturbance and uncertainties, the PID-SMC, NFTSMC and ABNFTSMC provide better performance compared to the CTC and PID controllers. Interestingly, due to the benefits of the integral component in the design, the PID-SMC and ABNFTSMC provide lower steady state error and faster transient response than the NFTSMC. In addition, the ABNFTSMC provides the best performance since it possesses low steady state error and fast transient response properties of the NFTSMC and ITSVM.

Then, we consider the fault tolerant capability of these controllers. To model the effects of faults in the system, we assume that the following fault function exits in the system:

\[ \phi(q, \dot{q}, \tau) = \begin{cases} 
30\sin(q_1q_2) + 4\cos(q_1q_2) \\
+15\cos(q_1q_2) \\
-0.8u_2 \\
0 
\end{cases} \quad \text{with } T_f \geq 20 \]  

(40)

It means that we assume an abrupt fault \( \phi_1 = 30\sin(q_1q_2) + 4\cos(q_1q_2) + 15\cos(q_1q_2) \) occurs in the first actuator from the time \( t \geq 20s \), while the second actuator losses 80% its effectiveness from the time \( t \geq 30s \). The third actuator is assumed to be healthy. The tracking errors of the system under the controllers are reported in Fig. 4. For an easier comparison, the averaged tracking error defined in (41)

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**Table I**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC</td>
<td>( K_p, K_d )</td>
<td>200,10</td>
</tr>
<tr>
<td>PID</td>
<td>( K_p, K_i, K_d )</td>
<td>200, 100, 10</td>
</tr>
<tr>
<td>PID-SMC</td>
<td>( K_p, K_i, K_d )</td>
<td>200, 100, 10</td>
</tr>
<tr>
<td>NFTSMC</td>
<td>( \lambda, \lambda, \nu )</td>
<td>20, 1, 0.1</td>
</tr>
<tr>
<td>ABNFTSMC</td>
<td>( k_1, k_2, \lambda, p, q )</td>
<td>10, 5, 14, 9, 7</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Error</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>ET(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC</td>
<td></td>
<td>0.4580</td>
<td>0.5938</td>
<td>0.2696</td>
<td>2.50</td>
</tr>
<tr>
<td>PID</td>
<td></td>
<td>0.2417</td>
<td>0.3698</td>
<td>0.1768</td>
<td>2.04</td>
</tr>
<tr>
<td>PID-SMC</td>
<td></td>
<td>0.0593</td>
<td>0.0478</td>
<td>0.0472</td>
<td>3.83</td>
</tr>
<tr>
<td>NFTSMC</td>
<td></td>
<td>0.1119</td>
<td>0.2453</td>
<td>0.0768</td>
<td>4.07</td>
</tr>
<tr>
<td>ABNFTSMC</td>
<td></td>
<td>0.0248</td>
<td>0.0219</td>
<td>0.0118</td>
<td>4.85</td>
</tr>
</tbody>
</table>

---

![Fig. 2. Desired position trajectory and actual position trajectory of the system under (a) CTC, (b) PID, (c) PID-based SMC, (d) NFTSMC and (e) ABNFTSMC controllers.](image-url)


\[ E_j = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \| e_j(t) \|^2 \right)} \quad j = 1, 2, 3 \]  

where \( N \) is the number of simulation step, and the elapsed time (ET) defined in Remark 7 are also reported in TABLE II.

From the results shown in Fig. 4 and TABLE II, we can see that the CTC and PID provide worst tracking performances for the system in the presence of fault; the system becomes unstable immediately when the faults occur. Even though the NFTSMC provides better tracking performance, the system is easily to be unstable due to the effects of faults, as shown in Fig. 4b). In the presence of faults, the benefits of the integral component are crucially significant, as shown in the performance of the PID-SMC and ABNFTSMC; using the PID-SMC and ABNFTSMC, the system provides very fast transient response and low tracking error. In addition, as shown in Fig. 4, the proposed ABNFTSMC provides superior performance compared to the PID-SMC. However, as shown in TABLE II, the elapsed time (ET) of the PID (2.04), CTC (2.50) and PID-SMC (3.83) is smaller compared to the NFTSMC (4.07) and ABNFTSMC (4.85). Fortunately, compared to the elapsed time of the PID, which is known as the simplest control method, the computational time of the proposed method is not significantly higher. Therefore, with the current advance hardware technologies, it is possible to apply the proposed technique in practical applications.

The control efforts of the NFTSMC, PID-SMC and ABNFTSMC are shown in Fig. 5. From Fig. 5(c), we can see that the proposed ABNFTSMC provides a continuous control input. As shown in Fig. 5(a) and (b), the PID-SMC and NFTSMC controllers also provide continuous control input due to the used boundary function (49). However, these approaches, as mentioned above, reduce the robustness and increase the steady state error of the system. The variation of the adaptive gains of the proposed ABNFTSMC for normal and fault operations are shown in Figs. 6 and 7, respectively. From the Figs. 6 and 7 we can see that the adaptive gains are variant due to the change of the effects of the uncertainties, disturbance and faults, and they will converge to some stable values when the systems are stable. In addition, the comparison results between Fig. 6 and Fig. 7 show that the adapted gains do not change much when the faults occur; the gains are just slightly increased when the faults occur. This is because the fault effects have been compensated by the gain component of the backstepping control \( (\alpha_2 - \left( \xi_3 + \beta_2 \right) ) \) in the design. This is also one of the major merits of the proposed approach.

From the above results we can conclude that the proposed controller, ABNFTSMC, provides the best performance compared to other controllers such as CTC, PID, SMC-PID and NFTSMC in terms of tracking accuracy, transient response, chattering elimination, and low steady state error. 

Remark 7: The elapsed time shown in TABLE II in this paper is the time that the Matlab has used to complete 50second simulation time. This may not reflect the true computational burden of the controllers but it can be used to provide a general idea about the comparison in computational time among the controllers.
Remark 8: From the developed control law, i.e., ABNFTS, in (26) and (29-31) and the selected control parameters in TABLE 1, we can see that the system’s performance is dependent on the selection of the sliding surface parameters $k_1, k_2, \lambda, \rho, q$, the adaptive gain parameters $\delta, \varepsilon$ and the backstepping gains $\xi_1, \xi_2, \xi_3$. While the sliding surface parameters and the adaptive gain parameters have been discussed widely in the literature, the backstepping gains are new defined parameters in this paper and their effects on the system performance are significant. Therefore, these parameters need to be effectively selected. In this paper, the backstepping gain parameters are chosen based on experiments. However, the values chosen based on the experiments are usually not optimal. Hence, there is a need to investigate a tuning mechanism to tune the parameters such that the system can get good approximation response. This will be studied in our future works.

Remark 9: From the proof for the stability of the system described in Section III, the stability of the system can be guaranteed when the sliding gain is selected such that its value is bigger than the value $\Lambda$, which is the bound value of the unknown function $Y$. Therefore, the stability of the system can be guaranteed when the adaptive gain is adapted to be bigger than the value $\Lambda$. In Figs. (6) and (7) we can see that the adaptive sliding gains of joints 2 and 3 converge after some transient time. The sliding gain of joint 1 is increased very slightly after some transient time. This symptom is caused due to the parameter drift problem as discussed in Remark 2. The convergence speed of the adaptive parameters can be obtained faster if we increase the dead zone size $\varepsilon$. However, there is a tradeoff between the steady state error and the convergence speed of the system when the dead zone size $\varepsilon$ is changed. When the value of $\varepsilon$ is high, the convergence speed is fast but the steady state error is high, and vice versa.

Remark 10: In this simulation study, we simulate the effects of abrupt faults only since its effects in the system are heavier than the incipient faults. Since the proposed method can compensate the abrupt faults well, it would absolutely be able to tackle the effects of the incipient faults well.

Remark 11: In this paper, we design a robust fault tolerant control to compensate for the effects of actuator or component faults only. The proposed method may not be able to compensate for the sensor faults. The effects of the measurement noise and the sensor faults to the performance of the system will be studied in our future work.

Remark 12: From the viewpoint of control engineering, it would be good to use experimental results to verify the effectiveness of the proposed control scheme in practical applications. However, simulation of different fault effects in the practical experiments are difficult, dangerous and possibly damage the system. Therefore, almost every approaches for fault tolerant control systems in the literatures [59-62] and the work in this paper used the simulation environment to verify the effectiveness of the methods. However, verifying the proposed approach in practical experiments by using effective manners without damaging the system and the environment surrounds is necessary and this will be investigated in our future works.
The system controller is designed based on the backstepping control procedure, and thus it can provide globally asymptotic stability based on Lyapunov criteria. The proposed controller is then applied for a PUMA560 robot and compared with other state-of-the-art controllers such as CTC, PID, PID-SMC, and NFTSMC. The simulation results verify the superior performance of the proposed method in terms of uncertainties and disturbance rejection in normal operation and fault compensation in fault operation.

For future work, the effects of the measurement noises and sensor faults to the system control performance will be studied. Tuning mechanisms will also be developed to obtain the optimal values for the major parameters of the proposed controller.

APPENDIX A

The following is the design procedure of the computed torque controller:

The computed torque controller can be designed as

\[ u = M(x_1) \left( \dot{x}_d + K_p (x_d - x_1) + K_d (\dot{x}_d - x_2) \right) + f(x_1, x_2) \]

where \( K_p \) is the proportional gain matrix and \( K_d \) is the derivative gain matrix.

APPENDIX B

The following is the design procedure of the PID controller:

Let \( e = x_1 - x_d \) is the tracking error. Then, the PID controller law is designed as

\[ u = K_p e + K_d \dot{e} + K_i \int_0^t e(t) dt \]

where \( K_p \) is the proportional gain matrix, \( K_d \) is the derivative gain matrix and \( K_i \) is the integral gain matrix.

APPENDIX C

The following is the design procedure of the PID-based SMC (PID-SMC) controller:

Let \( e = x_1 - x_d \) is the tracking error. Then, the sliding surface is selected as

\[ \sigma = K_p e + K_d \dot{e} + K_i \int_0^t e(t) dt \]

where \( K_p \) is the proportional gain matrix, \( K_d \) is the derivative gain matrix and \( K_i \) is the integral gain matrix.

The derivative of the sliding surface is obtained as

\[ \dot{\sigma} = K_p \dot{e} + K_d \ddot{e} + K_i \dot{e} \]

\[ = K_p \dot{e} + K_d \left( M^{-1} (x_1) u + f(x_1, x_2) + \Delta - \ddot{x}_d \right) + K_i \dot{e} \]

From (45), the PID-SMC is selected as
\[
\mathbf{u}(t) = \mathbf{u}_{eq}(t) - \mathbf{u}_s(t)
\]  
(46)

where, the equivalent control law is reconstructed when \( \sigma = 0 \) and \( \Lambda = 0 \):

\[
\mathbf{u}_{eq} = M(\mathbf{x}_1) \left( -\frac{\mathbf{k}_1}{\mathbf{k}_d} \mathbf{e} - \frac{\mathbf{k}_2}{\mathbf{k}_d} \dot{\mathbf{e}} - \left( f(\mathbf{x}_1, \mathbf{x}_2) - \dot{x}_d \right) \right)
\]  
(47)

and, the reaching law is designed as

\[
\mathbf{u}_s = M(\mathbf{x}_1) \left( \mathbf{\Lambda} + \mathbf{\zeta} \right) \text{sign}(\sigma)
\]  
(48)

where \( \mathbf{\Lambda} \) is chosen such that the unknown component is bounded by \( \|\mathbf{\Lambda}\| \leq \Lambda \).

In order to alleviate the chattering phenomenon due to the use of \( \text{sign}(s) \) function. The continuous function below is used instead:

\[
\mathbf{u}_s(t) = M(\mathbf{x}_1) \left( \mathbf{\Lambda} + \mathbf{\zeta} \right) \frac{s}{\|\mathbf{s}\| + \nu}
\]  
(49)

where \( \nu \in \mathbb{R} \) is a small positive scalar.

**APPENDIX D**

The following is the design procedure of the nonsingular fast terminal sliding mode controller (NFTSMC).

Let \( e = x_1 - x_2 \) is the tracking error. Then, the NFTSMC sliding surface is selected as

\[
\sigma = c + k_1 e^{[p]} + k_2 e^{[q]}
\]  
(50)

where \( \sigma \) is the sliding variable, \( k_1 = \text{diag}(k_{11}, k_{12}, \ldots, k_{1n}) \in \mathbb{R}^{n \times n} \) and \( k_2 = \text{diag}(k_{21}, k_{22}, \ldots, k_{2n}) \in \mathbb{R}^{n \times n} \) are positive definite matrices, respectively, \( p \) and \( q \) are positive odd numbers satisfying the relation \( 1 < p/q < 2 \) and \( \lambda > p/q \).

The first derivative of the sliding surface can be computed as follows:

\[
\dot{\sigma} = \dot{e} + k_1 \lambda e^{(p/q) - 1} \dot{e} + k_2 p q e^{(p/q) - 1} \frac{M^{-1}(\mathbf{x}_1) \dot{\mathbf{u}} + f(\mathbf{x}_1, \mathbf{x}_2) + \dot{x}_d}{M^{-1}(\mathbf{x}_1) \dot{e} - \dot{y}_d}
\]  
(51)

Based on (51), the NFTSMC controller is designed as

\[
\mathbf{u}(t) = \mathbf{u}_{eq}(t) - \mathbf{u}_s(t)
\]  
(52)

where, the equivalent control law is reconstructed when \( \sigma = 0 \) and \( \Lambda = 0 \):

\[
\mathbf{u}_{eq} = M(\mathbf{x}_1) \left( -\frac{1}{k_2} \dot{\mathbf{e}}^{2 - p/q} + k_2 \lambda \dot{\mathbf{e}}^{[p/q] - 1} \dot{\mathbf{e}}^{2 - p/q} \right)
\]  
(53)

and the reaching law is designed as

\[
\mathbf{u}_s = M(\mathbf{x}_1) \left( \mathbf{\Lambda} + \mathbf{\zeta} \right) \text{sign}(\sigma)
\]  
(54)

In order to reduce the chattering, the boundary method in (49) is also used to replace the (54).

**Remark 13:** The stability proofs for the controllers in the Appendixes are omitted to reduce the length of the paper.

**REFERENCES**


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N. M. Dehkhordi, N. Sadati and M. Hamzeh, “A Robust Backstepping High-Order Sliding Mode Control Strategy for Grid-Connected DG


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