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Adaptive Fuzzy Integral Sliding Mode Control for Robust Fault Tolerant Control of Robot Manipulators with Disturbance Observer

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Abstract—This paper develops a new strategy for robust fault tolerant control (FTC) of robot manipulators using an adaptive fuzzy integral sliding mode control and a disturbance observer (DO). First, an integral sliding mode control (ISMC) is developed for the FTC system. The major features of the approach are discussed. Then, to enhance performance of the system, a fuzzy logic system (FLS) approximation and a DO are introduced to approximate the unknown nonlinear terms, which include the model uncertainty and fault components, and estimates the compounded disturbance, respectively, and then integrated into the ISMC. Next, a switching term based on an adaptive two-layer super-twisting algorithm is designed to compensate the disturbance estimated error and guarantee stability and convergence of the whole system. The nominal controller of the ISMC is reconstructed using backstepping control technique to achieve the stability for the nominal system based on Lyapunov criteria. The computer simulation results demonstrate the effectiveness of the proposed approach.

Index Terms—Control of robots, backstepping control, fault tolerant control, integral sliding mode control, adaptive neural network, disturbance observer

I. INTRODUCTION

Robot manipulators have been widely utilized in industrial applications to increase the quality and quantity of the products. In order to increase the effectiveness of the robots for practical applications, beside increasing the tracking performance of the robots, safety issue is also significant, particularly when the robots work very close to or interact with human during operation. The failed robot could not only damage to the products but also generate dangerous to the human. Therefore, it is needed to counteract the failed operation of the robot. Due to this emergent need, fault diagnosis (FD) and fault tolerant control (FTC) have been developed for the robots [1]–[4]. The aim of the FD scheme is to detect the presence of faults, while the aim of the FTC is to tackle the effects of faults in the system so that the system can maintain the desired tracking performance despite the presence of the faults in the system [5].

Generally, FTC operation can be performed by using either active fault tolerant control (AFTC) or passive fault tolerant control (PFTC) [6]. In the AFTC approach, the control system is reconfigured according to the fault estimation, which is obtained from a fault diagnosis observer [7]–[9]. However, the design of the additional FD observer increases the computational load of the system. In the PFTC approach, faults are compensated by the robustness of a robust controller without requiring fault estimation [10], [11]. One of the most advantages of the PFTC is that it can compensate the faults’ effects quicker that helps the system recovered from the fault states quicker. However, since the PFTC needs to counteract the highest faulty effects, the nominal controller of the PFTC should have high robustness [12]. Due to its inherent high robustness property, sliding mode control (SMC) has been extensively studied for many FTC systems [13]–[16]. However, there exists two major shortcomings in the design of the conventional SMC: (i) it provides a reaching phase, and (ii) it provides big oscillation, which is known as chattering. In order to handle the first shortcoming of the SMC, an investigation based on integral sliding mode control (ISMC) has been made [17]–[20].

In general, the ISMC includes a nominal controller and a switching term. The nominal controller is used to control the nominal system, and the switching term is used to handle the uncertainty and disturbance and stabilize the whole system. The superior properties of the ISMC compared to its counterpart, i.e., the conventional SMC, has been discussed in [21]. Due to the impressive advantages, several FTC systems have been developed for many practical applications based on the ISMC [22]–[25]. Particularly, in [22], a FTC based on ISMC has been developed for linear time invariant (LTI) system. In [23], an analysis of ISMC for FTC of nonlinear systems has been carried out. In [24] and [25], ISMCs have been studied in the design of the FTC of attitude control of spacecraft. However, the ISMC generates undesired chattering like the drawback of the conventional SMC. The chattering is extremely undesired since it generates a lot of undesired mechanical oscillations in the system. To suppress the chattering phenomenon, many attempts have been made. First, the boundary method has been employed [22]. However, due to the effects of the boundary, the errors of the system in stable states are increased. An alternative method is to employ higher-order sliding mode control (HOSMC) [26]–[30]. Since the amount of chattering is proportional to the
magnitude of the switching gain, which is chosen to be bigger than the bounded value of the uncertainty and disturbance, one way to reduce the chattering is to reduce the effects of the uncertainty by using a continuous compensation term. In this approach, an estimation/approximation technique is designed to estimate/approximate the unknown function, i.e., uncertainty and disturbance, of the system. Then, the obtained estimation/approximation is used as a compensation term to compensate for the effects of the uncertainty in the system. Consequently, the effects of the uncertainty can be reduced and the sliding gain can be chosen as a smaller value, which will provide less chattering. Therefore, developing effective approximation/estimation techniques become necessary.

In the literature, the approximation capability of neural network (NN) and fuzzy logic system (FLS) have been extensively employed to approximate unknown nonlinear functions [31]–[38]. In [31], a great effort on stable adaptive NN framework has been spent. In [32], an advanced radial basis function neural network has been developed and integrated into the adaptive control of nonlinear system. In [33]–[35], fuzzy approximations have been utilized in the design of controller and observer of nonlinear systems. In [36], a neural approximation has been developed to approximate the unmodeled dynamics of nonlower triangular systems. Fuzzy approximations have also been developed for approximating the unknown dynamic model of robot manipulators [37], [38]. Basically, the learning technique provides good approximation only when the unknown function being approximated is a function with respect to the system states and/or control input. Otherwise, it usually provides big approximation error for unknown disturbances like external disturbance. To approximate disturbance, an estimation technique based on disturbance observer (DO) has been developed [39]. For example, in [40], a DO has been developed to estimate the disturbance and integrated into a SMC for a 3-DOF nanopositioning stage. In [41], a DO has been employed to observe the uncertainties of robot manipulators, and a SMC has been employed to compensate the fast changing component. A framework of DO-based controller has been developed for nonlinear systems in [42].

It can be seen that each individual approach like SMC, NN or DO has their own advantages and disadvantages, as pointed out in the aforementioned discussion. Therefore, the performance of the system might be reduced when applying the techniques individually into the practical applications. In order to enhance performance of the system, hybrid control methods, which combine the advantages while trying to eliminate the disadvantages of the individual components, have been developed. For example, due to the advantages of the ISMC and DO, a hybrid ISMC and DO has been developed in [43]–[45]. Hybrid approximation and estimation methods based on NN/FLS and DO have also been developed in [46]–[52]. On the other hand, due to the robustness property of the SMC and the approximation capability of NN/fuzzy logic, some hybrid controllers, which combine the merits of both parties, have been developed [53]–[56]. However, as aforementioned discussions, since the ISMC offers some major advantages over the SMC, it is desired to develop a hybrid controller, which can combine the merits of the ISMC and the NN/FLS. Unfortunately, very few efforts in the literature have been spent to realize this interesting control paradigm [57]. The reason may come from the fact that it is difficult to reconstruct the control input of the hybrid system such that the stability and convergence of the whole system can be guaranteed. In addition, in order to promote the system performance, it is desired that the system should take the advantages of the ISMC, learning technique and DO into account. This demand is particularly needed for the FTC system, where strong unknown nonlinear functions (uncertainty, disturbance and fault) are present and high robustness controller is required. This is one of the most motivations of this paper.

Inspired by the aforementioned desired features, in this paper, a new robust FTC strategy is suggested for robot manipulators based on an integration between the ISMC, FLS and DO. First, a FTC scheme based on a ISMC is developed. The efficiency and effectiveness of the ISMC for FTC system is analyzed thoroughly. Then, to compensate for the limitations of the ISMC, an adaptive FLS and a DO are developed and integrated into the ISMC. Since the effects of the uncertainties and disturbances are mostly compensated by the adaptive FLS and DO, the chattering generated by the ISMC is significantly reduced, and thus the performance of the system is massively increased. In addition, in order to further reduce the chattering and at the same time improve the tracking precision, an adaptive two-layer super-twisting algorithm [29], [30] is developed as a switching term of the ISMC. The nominal controller of the ISMC is reconstructed using backstepping control technique so that the stability of the nominal system can be guaranteed based on Lyapunov criteria. Finally, computer simulation is carried out based on a PUMAS60 robot to test the merit features of the proposed approach. The simulation results for this example verify the superior performance of the proposed approach when comparing with other state-of-the-art controllers for FTC system. In summary, the major expressive features of the proposed approach can be marked as below:

- An adaptive FLS and a disturbance observer is integrated to design a new integral sliding surface. Compared to the sliding surfaces used in the conventional SMC [13]–[16] and the conventional ISMC [17]–[20], the proposed sliding surface enhances the robustness and reduces the chattering of the system.
- Compared to the conventional chattering elimination techniques based on boundary method [22] or HOSM [26]–[28], the chattering elimination technique employed in this paper is an integration between the fuzzy approximation, disturbance observer and the super-twisting high-order sliding mode algorithm, so the chattering is almost eliminated in the system.
- Compared to the conventional fuzzy approximation [37], [38] or disturbance observer [39]–[41], a hybrid fuzzy approximation and disturbance observer is developed to fully estimating the effects of the uncertainties, disturbances and faults in the system.
The rest of this paper is organized as follows. Section II presents the problem formulation and some definitions. The ISMC for FTC of robot manipulators are initially studied in Section III. The design of the adaptive FLS, DO and the hybrid ISMC, FLS and DO are described in Section IV. Section V presents simulation results for a PUMAS60 robot. Conclusions are given in Section VI.

II. PROBLEM STATEMENT AND DEFINITIONS

A. Problem statement

Without loss of generality, the following robot dynamics is considered:

\[
\ddot{q} = M^{-1}(q)(\tau - C(q, \dot{q})\dot{q} - F(q) - G(q) - \tau_d)
+ \gamma(t - T_f)\phi(q, \dot{q}, \tau)
\]  

(1)

where \(q \in \mathbb{R}^n\), \(\dot{q} \in \mathbb{R}^n\) and \(\ddot{q} \in \mathbb{R}^n\) represents the position, velocity and acceleration of the robot, respectively, \(\tau \in \mathbb{R}^n\) is the actuator inputs. The inertia matrix \(M(q) \in \mathbb{R}^{nxn}\) is positive and definite. \(C(q, \dot{q}) \in \mathbb{R}^{nxn}\) consists of the Coriolis and centripetal forces, \(F(q) \in \mathbb{R}^n\) is the friction matrix, \(\tau_d \in \mathbb{R}^n\) denotes the vector of gravity terms. \(\phi(q, \dot{q}, \tau) \in \mathbb{R}^n\) indicates the fault components in the system. \(\gamma(t - T_f)\) is the time profile of the faults, in which \(T_f\) is the time of occurrence of the faults.

The considered robot dynamics in (1) is assumed to satisfy the following standard property:

\[
0 < \lambda_m\{M(q)\} \leq \|M(q)\| \leq \lambda_M\{M(q)\} \leq \chi, \chi > 0
\]  

(2)

In the above equation, \(\lambda_m\{M\}\) and \(\lambda_M\{M\}\) indicates the minimum and maximum eigenvalues of matrix \(M\), respectively.

The matrix \(\gamma(t)\) is configured as

\[
\gamma(t - T_f) = \text{diag}\{\gamma_1(t - T_f), \gamma_2(t - T_f), \ldots, \gamma_n(t - T_f)\}
\]  

(3)

where \(\gamma_i\) represents the fault component existing in the \(i\)th state equation.

The time profile of each state equation is introduced by [8]:

\[
\gamma_i(t - T_f) = \begin{cases} 
0, & \text{if } t < T_f \\
1 - e^{-\xi_i}(t - T_f), & \text{if } t \geq T_f
\end{cases}
\]  

(4)

where \(\xi_i > 0\) indicates the developing of the fault. When the value of \(\xi_i\) is small, the incipient fault is assumed. In contrast, when the value of \(\xi_i\) is large, the abrupt fault exists.

The model (1) can be rearranged as follows:

\[
\ddot{q} = M^{-1}(q)(\tau - C(q, \dot{q})\dot{q} - F(q) - G(q))
+ M^{-1}(q)(-F(q) - \tau_d) + \gamma(t - T_f)\phi(q, \dot{q}, \tau)
\]  

(5)

In this paper, the actuator fault is considered because it is the most serious failure and often occurs in the system [58]–[60]. There are many types of faults that may occur in the actuator, among them gain fault and bias fault are the most often ones. To generally represent both the gain fault and bias fault, the control input in (5) is described as

\[
\tau_c = \Gamma \tau + \Delta \tau
\]  

(6)

where \(\Gamma \in \mathbb{R}^n\) and \(\Delta \tau \in \mathbb{R}^n\) represent the gain fault and bias fault, respectively, \(\tau_c\) and \(\tau\) are the actual and the desired control value, respectively. In this condition, the fault function \(\phi(q, \dot{q}, \tau)\) in (5) can be re-described as

\[
\phi(q, \dot{q}, \tau) = M^{-1}(q)(\Gamma - I)\tau + M^{-1}(q)\Delta \tau
\]  

(7)

The following state space model can be obtained from (5) by introducing \(x_1 = q\) and \(x_2 = \dot{q}\):

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \Lambda u + f(x_1, x_2) + \delta(x_1, x_2, u) + \Xi
\end{align*}
\]  

(8)

where \(\Lambda = M^{-1}(q), f(x_1, x_2) = M^{-1}(q)(-C(q, \dot{q})\dot{q} - G(q))\) denotes the lumped known component and \(\delta(x_1, x_2, u) = M^{-1}(q)(-F(q)) + \gamma(t - T_f)\phi(q, \dot{q}, \tau)\) denotes the lumped uncertainty and \(\Xi = M^{-1}(q)\tau_d\) denotes the lumped disturbance component in the system. \(u = \tau\) is the control input.

The objective of this paper is to design a FTC input such that the system can maintain its high tracking precision despite the existing of the uncertainties, disturbance and faults.

B. Definitions

For a vector of input, i.e., \(x = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n\) and an output variable, i.e., \(y = f(x) \in \mathbb{R}\), the fuzzy logic system (FLS) is developed based on a set of If-Then rules to generate a map between the input and output [33]–[35]. The jth If-Then rule can be designed as follows:

\[
\text{Rule } j: \text{If } x_1 \text{ is } A^j_1 \text{ and } ... \text{ and } x_n \text{ is } A^j_n \text{ then } y = B^j
\]  

(9)

where \(A^j_1, A^j_2, ..., A^j_n\) and \(B^j\) represent fuzzy sets. Then, fuzzy output with a singleton fuzzifier can be provided as

\[
y = \sum_{j=1}^{h} \frac{\prod_{i=1}^{n} \mu_{A^j_i}(x_i)}{\sum_{j=1}^{h} \prod_{i=1}^{n} \mu_{A^j_i}(x_i)} \mu_{B^j}(y)
\]  

(10)

where \(h\) is the number of If-Then rules, and \(\mu_{A^j_i}(x_i)\) denotes the membership function value of fuzzy variables \(x_i\). \(w = [w_1, w_2, ..., w_h]^T\) stands for the adjustable weight matrix, and \(\psi(x) = [\psi_1(x), \psi_2(x), ..., \psi_h(x)]^T\) is a fuzzy basis vector, in with \(\psi_j(x)\) is defined as

\[
\psi_j(x) = \frac{\prod_{i=1}^{n} \mu_{A^j_i}(x_i)}{\sum_{j=1}^{h} \prod_{i=1}^{n} \mu_{A^j_i}(x_i)}
\]  

(11)

Lemma 1 [33]–[35]: For any given real continuous function \(f(x)\) on a compact set \(\Omega \subset \mathbb{R}^n\) and an arbitrary \(\theta > 0\), there exists a fuzzy logic system, i.e., \(w^T \Psi(x)\), such that

\[
\sup_{x \in \Omega} \left| f(x) - w^T \Psi(x) \right| \leq \theta
\]  

(12)

where \(\Psi(x) = [\psi_1(x), \psi_2(x), ..., \psi_h(x)]^T\) / \(\sum_{j=1}^{h} \psi_j(x)\) is the basis function vector, \(w = [w_1, w_2, ..., w_h]^T\) is the weight vector with \(h > 1\) being the number of the fuzzy rules and \(\psi_j(x)\) is chosen as the following form:

\[
\psi_j(x) = \exp \left( -\frac{(x - \mu_j)^T(x - \mu_j)}{\sigma_j^2} \right)
\]  

(13)
where $s_j$ is the width of the Gaussian function and $\mu_j = [\mu_{j1}, \mu_{j2}, ..., \mu_{jn}]^T$ is the center vector. And, based on the property of the fuzzy logic described above, we have
\[
||\Psi(x)|| \leq v
\] (14)
where $v$ is an unknown positive constant.

**Lemma 2** [50]–[52]: Consider a nonlinear system $\dot{x} = f(x)$.
Suppose that there exists a smooth positive definite function $V(x) > 0$ such that
\[
\dot{V}(x) \leq -aV(x) + b
\] (15)
where, $a > 0$ and $b > 0$. Then, the solution $x(t)$ is uniformly bounded.

### III. Design of Fault Tolerant Control Using Integral Sliding Mode Control

In (8), the nominal system can be defined as $\Omega = M^{-1}(q)u + f(x_1, x_2)$, and the unknown component can be defined as $\Psi = \delta(q, \dot{q}, \tau) + \Xi$ with $||\Psi|| \leq \rho$. From (7), we can see that the first fault component $(\Gamma - I)\tau$ is always smaller than the control input $\tau$, i.e., $(\Gamma - I)\tau < \tau$. It means the control input $u = \tau$ can be increased freely up to its maximum value without any bounds. The remain issue is to get the bound of the bias fault $\Delta \tau$. The assumption on the bound of the bias fault $\Delta \tau$ is generic and has been widely utilized in the design of FTC (see [58]–[60]). This condition states that the FTC is designed for the situation where the system is not "exploding". It means the Lipschitz condition is practically satisfied in the considered operational region (see [61]).

Let $e = x_1 - x_d$ be the tracking error of the system, where $x_d$ the desired trajectory of the system. First, an accumulated error signal is defined as
\[
s = \dot{e} + \lambda e
\] (16)
where $\lambda$ is a design parameter. Integrating the derivative of (16) and the result in (8) yields
\[
\dot{s} = \dot{\dot{e}} + \lambda \dot{e}
\] (17)
\[
= \Lambda u + f(x_1, x_2) + \delta(q, \dot{q}, \tau) + \Xi + \lambda \dot{e} - \ddot{x}_d
\]
The proposed integral sliding surface has a form below
\[
\sigma(t) = s(t) - s(0) - \int_{t=0}^{t} (\Lambda u_0 + f(x_1, x_2) + \lambda \dot{e} - \ddot{x}_d)dt
\] (18)
where $s(0)$ is the value of $s(t)$ at $t = 0$. The term $-s(0)$ is employed here to get the desired feature that is $\sigma(0) = 0$. The nominal controller, $u_0$, is used to stabilize the nominal system (the system without the lumped uncertainty and disturbance).

The derivative of (18) according to time domain is obtained as
\[
\dot{\sigma} = (\Lambda u + f(x_1, x_2) + \delta(q, \dot{q}, \tau) + \Xi + \lambda \dot{e} - \ddot{x}_d)
\] (19)
\[-(\Lambda u_0 + f(x_1, x_2) + \lambda \dot{e} - \ddot{x}_d)]

To stabilize the system (19), the following control input is suggested:
\[
u = u_0 + u_s
\] (20)
where $u_s$ is used to compensate the effects of unknown component, and is selected as
\[
u = -\Lambda^{-1}(\rho + \nu)\text{sign}(\sigma)
\] (21)
where $\nu$ is a small positive constant and $\rho$ was defined above.

Accumulating the composite controllers (20) and (21) into (19), yields
\[
\dot{\sigma} = -(\rho + \nu)\text{sign}(\sigma) + (\delta(q, \dot{q}, \tau) + \Xi)
\] (22)
Consider a Lyapunov function candidate $V = (1/2)\sigma^T \sigma$. The following result can be achieved based on the derivative of the Lyapunov function and (22):
\[
\dot{V} = \sigma^T \dot{\sigma}
\] (23)
\[= \sigma^T(-\rho - \nu)\text{sign}(\sigma) + (\delta(q, \dot{q}, \tau) + \Xi)\sigma
\] (24)
\[< 0
\]
The above inequality verifies the stability of the system according to Lyapunov criteria.

### IV. Design of Adaptive Fuzzy Integral Sliding Mode Control for Fault Tolerant Control System Based on Disturbance Observer

The design of the ISMC in the previous section needs a big sliding gain to cope with the effects of the unknown component. Unfortunately, this leads to big chattering in the system. In order to circumvent this shortcoming, an ISMC based on a FLS approximation and a disturbance observer is developed in this section.

#### A. Design of Fuzzy Integral Sliding Mode Control

First, according to Lemma 1, the lumped uncertainty can be introduced by the output of an fuzzy logic system:
\[
\delta(x_1, x_2, u) = W^T \Psi(Z) + \zeta
\] (24)
where $W$, $\Psi(Z)$, $Z = [x_1, x_2, u_f]^T$ are the weight vector, the basis function and the input vector of the FLS, respectively. $\zeta$ is the approximation error. $u_f \approx u$ is the filtered signal and defined as
\[
u = H_L(s)u
\] (25)
where $H_L(s)$ is the Butterworth low-pass filter (LPF). The parameters of this LPF are selected from [62]. The goal of the filtered signal is to circumvent algebraic loop problems.

Using the result in (24), (17) can be rewritten as
\[
\dot{s} = \Lambda u + f(x_1, x_2) + W^T \Psi(Z) + \lambda \dot{e} - \ddot{x}_d + \zeta + \Xi
\] (26)
From (26), the nominal component includes $\Sigma = f(x_1, x_2) + W^T \Psi(Z) + \lambda \dot{e} - \ddot{x}_d$. Based on (26), a new form of sliding surface is suggested as
\[
\sigma(t) = s(t) - s(0) - \int_{t=0}^{t} (\Lambda u_0 + \Sigma)dt
\] (27)
From (27):
\[
\dot{\sigma} = (\Lambda u + \Sigma + \zeta + \Xi)
\] (28)
\[-(\Lambda u_0 + f(x_1, x_2) + W^T \Psi(Z) + \lambda \dot{e} - \ddot{x}_d)\]
To stabilize the system (28), the following control law is suggested
\[ u = u_0 + u_s \]  
(29)
where \( u_0 \) is the nominal controller, and \( u_s \) is given by:
\[ u_s = -\Lambda^{-1}(\vartheta + \nu)\text{sign}(\sigma) \]  
(30)
where \( \vartheta \) is chosen such that \( \|\zeta + \Xi\|\leq \vartheta \).

From the results in (29), (30) and (28), we have
\[ \dot{\sigma} = -(\vartheta + \nu)|\sigma| + (\zeta + \Xi) \]  
(31)

Define a Lyapunov function candidate as \( V = (1/2)\sigma^T\sigma \). The following result can be obtained based on the derivative of the Lyapunov function and the result in (31):
\[ \dot{V} = \sigma^T\dot{\sigma} \]
\[ = \sigma^T(-\vartheta - \vartheta + (\zeta + \Xi)) \]
\[ = -(\vartheta + \nu)|\sigma| + (\zeta + \Xi)|\sigma| \]
\[ < 0 \]

The inequality (32) demonstrates the stability and convergence of the system via Lyapunov criteria.

Due to approximation capability of the FLS, the FLS’s approximation error, i.e., \( \zeta \), is much smaller than the function being approximated, i.e., \( \delta(q, \dot{q}, \tau) \). Therefore, the bounded value \( \vartheta \) is much smaller than the bounded value \( \rho \). Consequently, the controller (30) provides much lower chattering compared to the controller (21). Hence, we can conclude that the employment of the FLS can improve the performance of the system.

**Remark 1:** As stated in Lemma 1, the FLS can provide good approximation for continuous function. In this paper, the FLS is used to estimate the unknown function \( \delta(x_1, x_2, u) \), which aggregates the friction force and fault. To get a good estimation output of the FLS, the function \( \delta(x_1, x_2, u) \) should be a continuous function. In practical applications, friction forces may become highly nonlinear and discontinuous functions. Moreover, abrupt change and intermittent faults may have discontinuous time behaviour. For these conditions, the FLS will provide big estimation error. However, these errors will be compensated by the disturbance observer and the integral sliding mode term to always maintain the high tracking precision of the system. This is the main advantage of the proposed approach.

**B. Design of disturbance observer**

For the sake of presenting the DO, let \( \Gamma(x_1, x_2, u, \dot{e}, \ddot{x}_d) = \Lambda u + f(x_1, x_2) + W^T\Psi(Z) + \lambda \dot{e} - \ddot{x}_d \), which denotes the new lumped known function. The system (26) can be rewritten as
\[ \dot{s} = \Gamma(x_1, x_2, u, \dot{e}, \ddot{x}_d) + \Delta(t) \]  
(33)
where \( \Delta = \zeta + \Xi \) denotes a new lumped disturbance in the system. The derivative of the lumped disturbance in the system is assumed to be bounded by \( \|\Delta\|\leq \xi \), where \( \xi \) is an unknown positive constant.

The disturbance observer for estimating the unknown time-varying disturbances \( \Delta(t) \) in (33) is designed as follows
\[ \hat{\Delta}(t) = p(t) + K_o s \]
\[ \dot{p}(t) = -K_o \left( \Gamma(x_1, x_2, u, \dot{e}, \ddot{x}_d) + \Delta(t) \right) \]  
(34)
where \( K_o = K_0^T \in \mathbb{R}^{n \times n} \) is a positive definite design matrix.
\[ \Gamma(x_1, x_2, u, \dot{e}, \ddot{x}_d) = \Lambda u + f(x_1, x_2) + W^T\Psi(Z) + \lambda \dot{e} - \ddot{x}_d, \]
where \( W \) is the estimate of the weight \( W \), which will be defined later.

Define the disturbance estimation error \( \tilde{\Delta} \) of the observer as
\[ \tilde{\Delta} = \hat\Delta - \Delta \]  
(35)

From (34) and (35), we have
\[ \hat{\Delta} = -K_o \left( \hat{\Gamma}(x_1, x_2, u, \dot{e}, \ddot{x}_d) + \hat\Delta \right) + K_o \hat{s} \]
\[ = -K_o (\Delta - \hat{\Delta} + (\hat{W} - W)^T\Psi(Z)) \]
\[ = -K_o (\hat{\Delta} + \hat{W}^T\Psi(Z)) \]  
(36)

To facilitate the stability analysis of the DO, we assume that the estimate weight \( \hat{W} \) of FLS converges to the optimal weight \( W \), i.e., \( \hat{W} = W \). The time derivative of (35) along (36) is
\[ \dot{\tilde{\Delta}} = -K_o \Delta - \Delta \]  
(37)

Consider a Lyapunov function candidate as
\[ V_\Delta = \frac{1}{2} \tilde{\Delta}^T \tilde{\Delta} \]  
(38)

From (37) and (38) and using Young’s inequality, we have
\[ V_\Delta = \tilde{\Delta}^T \tilde{\Delta} \]
\[ = -\Delta^T K_o \Delta - \tilde{\Delta}^T \Delta \]
\[ \leq -[\lambda_{\min}(K_o) - \frac{1}{2}] \tilde{\Delta}^T \tilde{\Delta} + \frac{1}{2} \Delta^T \Delta \]  
(39)
\[ \leq -[\lambda_{\min}(K_o) - \frac{1}{2}] \tilde{\Delta}^T \tilde{\Delta} + \frac{1}{2} \xi^2 \]
\[ = -a_1 V_\Delta + b_1 \]
where \( a_1 = \lambda_{\min}(K_o) - \frac{1}{2} \) with \( \lambda_{\min}(\cdot) \) denoting the minimum eigenvalue of a matrix, \( b_1 = \frac{1}{2} \xi^2 \), and \( K_o \) satisfies
\[ \lambda_{\min}(K_o) > \frac{1}{2} \]  
(40)

Since \( V_\Delta \) is ultimately bounded as \( t \to \infty \) as can be seen from the inequality above. Thus, \( \Delta \) is bounded according to Lemma 2.

**Remark 2:** According to (39), the designed disturbance observer provides higher estimation accuracy when the bounded value \( \xi \) is small. It means the designed disturbance observer is suitable for constant or slow time varying disturbances. For fast time varying disturbances, the disturbance estimation error is bigger. The disturbance estimation error will be compensated by the switching term of the integral sliding mode control, which will be presented in the following section.
C. Design of Adaptive Fuzzy Integral Sliding Mode Control using Disturbance Observer

Based on the approximated FLS and estimated disturbance, the sliding surface (26) can be rewritten as

\[ \dot{s} = \Lambda u + f(x_1, x_2) + \hat{W}^T \Psi(Z) + \lambda \hat{e} - \hat{x}_d + \hat{\Delta} + \epsilon \]  \hspace{1cm} (41)

In (41), the new compounded nominal component is \( \Pi = f(x_1, x_2) + \hat{W}^T \Psi(Z) + \lambda \hat{e} - \hat{x}_d + \Delta \), and \( \epsilon = \Delta - \hat{\Delta} \) is the disturbance estimation error, and \(|\epsilon| \leq \tau\), where \( \tau \) is an unknown constant. This value is supposed very small due to the approximation capability of the FLS and the effectiveness of the disturbance observer.

Based on (41), a new form of integral sliding surface is suggested as:

\[ \sigma(t) = s(t) - s(0) - \int_{t=0}^{t} (\Lambda u_0 + \Pi) \, dt \]  \hspace{1cm} (42)

Combining the derivative of (42) with the result in (41), we obtain

\[ \dot{\sigma}(t) = \dot{s}(t) - (\Lambda u_0 + \Pi) = \Lambda (u - u_0) + \epsilon \]  \hspace{1cm} (43)

Based on (43), the proposed controller can be designed as

\[ u_{FDO} = u = u_0 + u_s \]  \hspace{1cm} (44)

where,

\[ u_s = -\Lambda^{-1}(\tau + \nu)\text{sign}(\sigma) \]  \hspace{1cm} (45)

From (44), (45) and (43), we have

\[ \dot{\sigma}(t) = -(\tau + \nu)\text{sign}(\sigma) + \epsilon \]  \hspace{1cm} (46)

Consider a Lyapunov function candidate \( V = (1/2) \sigma^T \sigma \). Adding the result in (46) to the derivative of the Lyapunov function, we achieve

\[ \dot{V} = \sigma^T \dot{\sigma} = \sigma^T (-(\tau + \nu)\text{sign}(\sigma) + \epsilon) = -(\tau + \nu)|\sigma| + \epsilon \sigma \]  \hspace{1cm} (47)

< 0

Therefore, the stability of the system is established based on Lyapunov criteria.

Due to the effectiveness of the disturbance observer, the disturbance estimation error, i.e., \( \epsilon \), is much smaller than the compounded disturbance, i.e., \( \Delta \). Therefore, the chattering generated by the controller (45) will be smaller than that of the controller (30). Hence, we can conclude that, theoretically, the hybrid ISMC, FLS and DO is better than the hybrid ISMC and FLS, and much better than the ISMC alone.

In the controller (46), the switching term was designed based on the conventional SMC, which provides big chattering. In addition, the sliding gain was selected based on the bounded value \( \tau \), which might not be obtained in advance in practical applications. To eliminate the chattering and to relax the assumption, an adaptive dual-layer super-twisting algorithm [29], [30] is employed in this paper. Therefore, the switching term in (45) can be re-designed as

\[ u_s = -\Lambda^{-1}(\mu_1 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma) + \xi) \]  \hspace{1cm} (48)

where \( \mu_1 \) is a positive constant, and the dual layer adaptive scheme is given by:

\[ \delta_s(t) = k_s(t) - \frac{1}{v_1} |w_{eq}(t)| \]  \hspace{1cm} (49)

\[ \dot{k}_s(t) = -(q_0 + q_s(t)) \text{sign}(\delta_s(t)), \quad q_0 > 0 \]

\[ \dot{q}_s = \beta_s |\delta_s(t)|, \quad \beta_s > 0 \]

where \( w_{eq}(t) \) obtained by low-pass filtering (LPF) of \( k_s(t) \text{sign}(s) \), and the gain \( \mu_1 > 0 \) is to be selected large enough, and \( v_0, v_1, q_0, \beta_s > 0 \) are control parameters.

Integrating the composite controller (44) and (48) into (43):

\[ \dot{s}(t) = -\mu_1 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma) - \xi + \epsilon \]

\[ \dot{x} = k_s(t)\text{sign}(\sigma) \]  \hspace{1cm} (50)

As a similar analysis as in [29], [30] for the system (50), it is quite straightforward to verify that the sliding surface, i.e., \( \sigma \), is stable and convergent to zero in a finite time under the input of the composite controller (44) and (48) and the adaptive law (49).

Remark 3: The interested readers are encouraged to refer to [29], [30] for the stability proof of the system (50).

Remark 4: In this paper, the adaptive fuzzy approximation integrated with the disturbance observer is employed to reduce the magnitude of the gain \( k_s(t) \). Therefore, the sliding gain \( k_s(t) \) can be selected based on the bounded value \( \epsilon \) rather than the original unknown component \( (\delta(q, q, \tau) + \hat{\Xi}) \), as shown in (22). Therefore, suppose that the estimated error \( \epsilon \) is small, the chattering in this paper is almost eliminated since the adaptation gain \( k_s(t) \) can be selected as a smaller value.

D. Design of nominal controller for the nominal system

In the previous subsections, the obtained FLS approximation and disturbance estimation were added into the dynamic model of the system as a nominal component. In this section, a step-by-step design of backstepping control method is derived to stabilize the nominal system.

According to the aforementioned analysis, the nominal system in (8) can be rewritten as below:

\[ \dot{x}_1 = x_2 \]

\[ \dot{x}_2 = \Lambda u + f(x_1, x_2) + \hat{W}^T \Psi(Z) + \hat{\Delta} \]  \hspace{1cm} (51)

The following error signals are defined for the system (51):

\[ e_1 = x_1 - x_d \]

\[ e_2 = x_2 - \alpha_1 \]  \hspace{1cm} (52)

where \( \alpha_1 \) is a virtual control input to be designed later.

Step 1: From (52), we have

\[ \dot{e}_1 = \dot{x}_1 - \dot{x}_d = e_2 + \alpha_1 - \dot{x}_d \]  \hspace{1cm} (53)

Select the virtual control input as

\[ \alpha_1 = -K_1 e_1 + \dot{x}_d \]  \hspace{1cm} (54)
where \( K_1 \) is a positive definite matrix.

**Step 2:** From \( e_2 = x_2 - \alpha_1 \), it can be deduced that
\[
\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_1 = \dot{\alpha}_u + f(x_1, x_2) + W^T \Psi(Z) + \dot{\Delta} - \dot{\alpha}_1
\]  
(55)

**Step 3:** In this step, the following Lyapunov function candidate is selected:
\[
V = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i + \frac{1}{2} \Delta^T \Delta
\]  
(56)

where \( \Gamma_i \) (\( i = 1, 2, ..., n \)) is a symmetric positive definite constant matrix. \( \Delta = \Delta - \dot{\Delta} \) is the disturbance estimation error. 

Taking the derivative of the Lyapunov function with respect to time, one obtains
\[
\dot{V} = e_1^T \dot{e}_1 + e_2^T \dot{e}_2 + \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i + \dot{\Delta}^T \dot{\Delta}
\]
(57)

Adding the results in (36) into (57), we obtain
\[
\dot{V} = e_1^T (e_2 + \alpha_1 - \dot{x}_d)
\]
\[
+ e_2^T (\dot{\alpha}_u + f(x_1, x_2) + W^T \Psi(Z) + \dot{\Delta} - \dot{\alpha}_1)
\]
\[
+ \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i + \dot{\Delta}^T \dot{\Delta}
\]
\[
\leq e_1^T \left( \dot{e}_2 + \alpha_1 - \dot{x}_d \right)
\]
\[
+ e_2^T \left( \dot{\alpha}_u + f(x_1, x_2) + W^T \Psi(Z) + \dot{\Delta} - \dot{\alpha}_1 \right)
\]
\[
+ \sum_{i=1}^{n} \tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i + \dot{\Delta}^T (\dot{\Delta} - \dot{\Delta})
\]
\[
(58)
\]

The nominal control input is selected as
\[
u_0 = \Lambda^{-1}(-f(x_1, x_2) - W^T \Psi(Z) - \dot{\Delta} - \dot{\alpha}_1 - e_1 - K_2 e_2)
\]  
(59)

The adaptive law of FLS is selected as
\[
\dot{\hat{\Psi}}_i = \Gamma_i [e_2 \Psi_i(Z) - 2\gamma \hat{W}_i]
\]  
(60)

Inserting the control input (59) and the adaptive law (60) into (58), one obtains
\[
\dot{V} \leq -e_1^T K_1 e_1 - e_2^T K_2 e_2 - \dot{\Delta}^T K_0 \dot{\Delta}
\]
\[
+ \sum_{i=1}^{n} 2\gamma \hat{W}_i^T \hat{W}_i + \dot{\Delta}^T \Delta - \dot{\Delta}^T K_0 W^T \Psi(Z) + \frac{1}{2} \Delta^T \Delta
\]  
(61)

Using Young’s inequality, we obtain
\[
2\gamma \hat{W}_i^T \hat{W}_i \leq -\gamma \hat{W}_i^T \hat{W}_i + \gamma W_i^T W_i
\]
\[
\Delta^T \Delta \leq \frac{1}{2} \Delta^T \Delta + \frac{1}{2} \Delta^T \Delta
\]  
(63)

\[
\Delta^T K_0 W^T \Psi(Z) \leq \frac{\Delta^T \Delta}{2\gamma} + \varsigma \frac{\|K_0\|^2 \|\Psi(Z)\|^2 \|\hat{W}\|^2}{2}
\]  
(64)

\[
\|\Psi(Z)\|^2 \leq \nu^2
\]  
(65)

Inserting the results in (62), (63), (64) and (65) into (61), we have
\[
\dot{V} \leq -e_1^T K_1 e_1 - e_2^T K_2 e_2 - (K_0 + \frac{1}{2\gamma} - \frac{1}{2}) \dot{\Delta}^T \Delta
\]
\[
- (\gamma + \varsigma \frac{\|K_0\|^2 \nu^2}{2}) \sum_{i=1}^{n} \hat{W}_i^T \hat{W}_i + \frac{1}{2} \gamma \sum_{i=1}^{n} W_i^T W_i
\]
\[
+ \frac{\Delta^T \Delta}{2}
\]
\[
\leq -a_2 V + b_2
\]  
(66)

where \( a_2 = \min(2\lambda_{\min}(K_1), 2\lambda_{\min}(K_2), 2\lambda_{\min}(K_0 + \frac{1}{\nu^2} - \frac{1}{2}), 2\lambda_{\min}(\frac{\gamma + \varsigma \frac{\|K_0\|^2 \nu^2}{\nu^2}}{\lambda_{\max}(\sum_{i=1}^{n} \Gamma_i^{-1})}) \), \( b_2 = \frac{1}{2} + \gamma \sum_{i=1}^{n} W_i^T W_i + \frac{\Delta^T \Delta}{2} \).

\( \lambda_{\min}() \) represents the minimum eigenvalues of a matrix.

Solve the inequality (66), we get:
\[
V \leq (V(0) - \frac{b_2}{a_2}) e^{-a_2 t} + \frac{b_2}{a_2}
\]  
(67)

Since \( V \) is ultimately bounded as \( t \to \infty \) as can be seen from the inequality above. Thus, \( e_1, e_2, \dot{q}, \dot{\dot{q}} \) are also bounded according to Lemma 2. This completes the proof.

The overall structure of the proposed fault tolerant controller is illustrated in Fig. 1.

**Remark 5:** The proposed method presumes full measurement of the system states, i.e., \( q \) and \( \dot{q} \). This is one of the limitations of the proposed method. However, in the case that the velocity measurement is not available, the robust exact differentiator method proposed in [63] can be employed to eliminate this limitation.

V. RESULTS AND DISCUSSIONS

In order to verify the effectiveness of the proposed control method, we employ it for the fault tolerant control of a PUMA560 robot. The PUMA560 is a well-known robot in industrial applications, and it has been widely utilized as a benchmark robot platform in research [7], [8], [12]. The PUMA560 robot has 6-DOF (degree-of-freedom). In this paper, we use the first three joints of the robot only for the sake of presentation of the results.

The dynamic model of the PUMA robot can be described as in (1), where the nominal parameters of the robot are
taken from [64]. Because it is difficult to obtain the exact mathematical model of the system in practical applications, we assumed that the following friction and disturbance are present.

$$F(q, \dot{q}) = \begin{bmatrix}
0.5\dot{q}_1 + \sin(3q_1) + 0.5\sin(\dot{q}_1) \\
1.3\dot{q}_2 - 1.8\sin(2q_1) + 1.1\sin(\dot{q}_2) \\
-1.8\dot{q}_3 - 2\sin(q_3) + 0.15\sin(\dot{q}_3)
\end{bmatrix}$$

(68)

$$\tau_d = \begin{bmatrix}
2\cos(t) + 2\sin(t) \\
1.5\cos(t)^2 + 2\cos(t) \\
3\sin(t) - 2\sin(t)\cos(t)
\end{bmatrix}$$

(69)

Remark 6: There are no particular reason to assume the values of the disturbance $\tau_d$ used in (69). They are hypothetical and can be replaced by other reasonable function. In a robot system, $\tau_d$ represents the unknown load disturbance matrix, so it can be represented by a time varying function with a reasonable magnitude.

The robot is commanded to track the following trajectory

$$x_d = [\cos(t/5\pi) - 1, \cos(t/5\pi + \frac{\pi}{2}), \sin(t/5\pi + \frac{\pi}{2}) - 1]^T$$

(70)

In order to verify the performance of the proposed controller, i.e., AFISMC-DO, we compare it with other state-of-the-art controllers such as PID, computed torque control (CTC), nonsingular fast terminal sliding mode controller (NFTSMC), which has been recently proposed for FTC systems [8], [24]. The design of the NFTSMC is described in Appendix A. The design of the CTC and PID are omitted in this paper since they are well-known methods and can be easily found in the literature. TABLE I depicts the selection of parameters of the controllers. It is noticed that most of the gains of the proposed AFISMC-DO, i.e., $K_i$, $K_1$, $K_2$, $\lambda$, $\Gamma$, $\gamma$, are proportional gains, so the effects of these parameters are quite obvious. For instance, if bigger values of the gains are selected, the convergence rate of the system is quicker, but the oscillatory is bigger, and vice versa. The parameters used in this simulation are selected based on trial and error procedure or based on experience to obtain a compromise between the convergence speed and oscillatory. The set of membership functions of the fuzzy logic used in the AFISMC-DO are selected as follows:

$$\mu_{A_i} = \exp\left(-\frac{(x_i + 7)^2}{4}\right), \mu_{A_2} = \exp\left(-\frac{(x_i + 5)^2}{4}\right), \mu_{A_3} = \exp\left(-\frac{(x_i + 3)^2}{4}\right)$$

where $i = 1, 2, 3$.

The above membership functions was selected based on heuristic trial-and-error procedure.

To show the performance of the controllers, we consider the robot in two working scenarios.

First, we consider the robot working in normal operation. The effectiveness of the designed disturbance observer in (37) is analyzed at first. To facilitate in verifying the performance of the disturbance observer, we assume that the fuzzy logic system provides good estimation of the unknown function $\delta$, and thus, the effects of $\delta$ can be removed. The target of the disturbance observer in this condition is to precisely approximate the disturbance component $\tau_d$. The time history of the real disturbances $\tau_d$ and the outputs of the disturbance observer are illustrated in Fig. 2. From Fig. 2, it can be seen that the disturbance observer provides high estimation precision. The trajectory tracking performances and the tracking errors of these controllers are shown in Figs. 3 and 4, respectively. TABLE II reports the root-mean-square-error (RMSE) of the tracking errors of the controllers. From Figs. 3 and 4, it can be observed that the CTC does not provide good tracking performance for the system when the high uncertainty and disturbance are present. The PID provides better tracking performance than the CTC. Due to the robustness against the uncertainty and disturbance of the SMC, the NFTSMC and the AFISMC-DO provide much lower tracking error than the CTC and the PID controller. On the other hand, the proposed AFISMC-DO has better tracking performance than the NFTSMC. Particularly, as shown in TABLE II, the tracking errors of the CTC on three joints are 0.0395, 0.0237 and 0.1259, respectively. The corresponding tracking errors of the PID are 0.0139, 0.0174 and 0.0116, respectively. The high tracking performance of the NFTSMC and the AFISMC-DO are clearly shown in TABLE II, where the tracking errors of the NFTSMC are 0.0043, 0.0053 and 0.0051, respectively, while the tracking errors of the proposed method, i.e., AFISMC-DO, are 0.0023, 0.0019 and 0.0019, respectively.

In the next, we consider the tracking performance of these controllers when faults occur in the system. To do this, the following fault function is assumed:

$$\phi(q, \dot{q}, \tau) = \begin{bmatrix}
(20\sin(q_1q_2) + 14\cos(q_1q_2)) \\
+12\cos(q_1q_2)) \\
-0.85\tau_2, \\
-0.3\tau_3
\end{bmatrix}, \begin{array}{c}
t \geq 20 \\
t \geq 30 \\
t \geq 30
\end{array}$$

(71)

It means that the fault $\phi_1 = 20\sin(q_1q_2) + 14\cos(q_1q_2) + 12\cos(q_1q_2)$ occurs in the first actuator from the time $t = 20s$, and the faults $\phi_2 = -0.85\tau_2$ and $\phi_3 = -0.3\tau_3$ occur in the second and third actuator (the second and third actuator loss 85% and 30% their effectiveness, respectively), respectively.
The tracking errors of the system in the presence of the faults under the inputs of the controllers are shown in Fig. 5, while TABLE III illustrates the corresponding RMSE of the controllers. From Fig. 5, we can see that the CTC has very low robustness against the effects of fault; the stability of the system are almost broken down when the faults occurred at the time $t = 20s$ and $t = 30s$. The NFTSMC and the PID controllers provide much better robustness and transient response against the effects of faults than the CTC. As our expectation, the proposed AFISMC-DO provides very good transient response and robustness for the system despite the effects of fault. Consequently, the performance of the proposed AFISMC-DO is much better than the CTC, the PID and the NFTSMC in terms of tracking error and convergence speed, as shown in Fig. 3. Particularly, according to Table II, the RMSEs of the proposed AFISMC-DO for three joints are $0.0024$, $0.0020$ and $0.0020$, respectively. In contrast, the corresponding RMSEs of the CTC, the PID and the NFTSMC are ($0.3088$, $0.2003$ and $0.1944$), ($0.0297$, $0.0560$ and $0.0139$) and ($0.0053$, $0.0086$ and $0.0055$), respectively. One more interesting point that can be picked out from Fig. 4, Fig. 5, TABLE II and TABLE III is that no big difference between the system in normal operation and fault operation in the performance of the proposed AFISMC-DO received. This means that the proposed method tackles the effects of the faults very well.

To further verify the fast transient response and convergence of the proposed AFISMC-DO, we compare the variations of the adaptive sliding gains of the NFTSMC (in (78)) and the proposed AFISMC-DO ($k_e(t)$ in (49)). The variations of the adaptive gains of the NFTSMC are shown in Fig. 6, while those for the AFISMC-DO are shown in Fig. 7. As the results shown in Figs. 6 and 7, we can see that, generally, the sliding gains are increased when the faults occur in the system to compensate for the effects of faults. However, the proposed AFISMC-DO uses much smaller gain values to compensate for the assumed uncertainty, disturbance and faults than that of the NFTSMC.

The control inputs of the controllers, i.e., CTC, PID, NFTSMC and AFISMC-DO, are shown in Fig. 8. Obviously, the control inputs of the PID and the CTC controllers provide smooth control efforts. Otherwise, by using the effective chattering elimination method, the chattering of the NFTSMC and the AFISMC-DO are almost eliminated. In addition, considering the efforts of the controllers when the fault occurs (at the time $t = 30s$), as shown in Fig. 8, the CTC uses less control efforts than other controllers. Although the PID uses higher control efforts than the NFTSMC and the AFISMC-DO, however, as aforementioned discussions, the tracking error of the PID was significantly higher. The control efforts of the AFISMC-DO and the NFTSMC are quite comparable. However, as analyzed above, the tracking performance of the AFISMC-DO was higher than the NFTSMC. Therefore, when considering the performance of the FTC system in terms of tracking error, transient response, fast convergence and control efforts, the proposed method, i.e., AFISMC-DO, is superior.
TABLE I
PARAMETERS USED IN THE SIMULATION OF THE CONTROLLERS

<table>
<thead>
<tr>
<th>Controller</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC</td>
<td>$K_p, K_d$</td>
<td>200, 10</td>
</tr>
<tr>
<td>PID</td>
<td>$K_p, k_1, K_d$</td>
<td>200, 100, 10</td>
</tr>
<tr>
<td>NFTSMC</td>
<td>$k_1, k_2, \lambda, p, q, K$</td>
<td>10, 5, 1.4, 9.7</td>
</tr>
<tr>
<td></td>
<td>$k, a$ (in (78)), $\varphi$</td>
<td>$1/3, 0.01, 0.1$</td>
</tr>
<tr>
<td>AFISMC-DO</td>
<td>$\lambda$ (in the sliding surface(16))</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>$\mu_1, v_0, v_1, p_0, \beta$ in (48) and (49)</td>
<td>10, 3, 1.99, 3, 5</td>
</tr>
<tr>
<td></td>
<td>$K_1$ (the gain of DO in (54))</td>
<td>$\text{diag}(5, 5, 5)$</td>
</tr>
<tr>
<td></td>
<td>$K_2$ (in the nominal controller(59))</td>
<td>$\text{diag}(3, 3, 3)$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma, \gamma$ (in adaptive law of FLS(60))</td>
<td>2, 1</td>
</tr>
</tbody>
</table>

TABLE II
TRACKING ERROR OF THE SYSTEM IN NORMAL OPERATION

<table>
<thead>
<tr>
<th>Error</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC</td>
<td>0.0395</td>
<td>0.0237</td>
<td>0.1259</td>
</tr>
<tr>
<td>PID</td>
<td>0.0139</td>
<td>0.0174</td>
<td>0.0116</td>
</tr>
<tr>
<td>NFTSMC</td>
<td>0.0043</td>
<td>0.0052</td>
<td>0.0051</td>
</tr>
<tr>
<td>AFISMC-DO</td>
<td>0.0023</td>
<td>0.0019</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

Remark 7: The values of antecedent-vertex of FLS used in this simulation are selected based on experiments and human knowledge about the system. To analyze the generality of the proposed method, different pairs of antecedent-vertex have been checked in this simulation. The experimental results shown that the tracking accuracies for different pairs of antecedent-vertex were not much different. This was because the FLS approximation errors would have been eventually compensated by the disturbance observer, as discussed in section VI-B. In the literature, some papers have discussed the use of tensor product (TP) to obtain the generality of the use of FLS, for example [65]. Employing the TP to get generality of the use of FLS will be investigated in our future works.

Remark 8: In this simulation study, to reduce the length of the paper, we consider the effects of abrupt faults only since the effects of abrupt faults is much heavier than the incipient faults. Since the proposed method tackles the effects of the abrupt faults well, it is believed to tackle the incipient faults very well.

Remark 9: In this paper, the effects of faults are treated as the effects of an additional disturbance, and the robust fault tolerant control is developed to tackle the effects of the additional disturbance. Therefore, the proposed method can be considered as a strong robustness controller, which can be applied for wide practical applications to robust against the effects of uncertainties and disturbances.

VI. CONCLUSION

A new robust FTC is developed for robot manipulators using an adaptive fuzzy integral sliding mode control (AFISMC) and a disturbance observer (DO) (AFISMC-DO). The design procedure is started by analyzing the features of the ISMC for FTC system. Then, in order to enhance the performance of the system, an adaptive fuzzy logic system and a disturbance observer are developed and integrated into the nominal controller of the ISMC. In addition, an adaptive two-layer super-twisting algorithm is employed into the hybrid system to eliminate the chattering and enhance the tracking precision of the system. The nominal controller of the proposed method is built based on the backstepping control technique so that the adaptive
other advanced control methods. The comparison results verify the effectiveness of the proposed strategy.

In this paper, the parameters of the proposed control method were chosen based on experiences or trial and error procedure. Therefore, the selected parameters were not the optimal values. In addition, the constraints of the control inputs were not considered (the output constraints of the system could be guaranteed already due to the property of the ISMC, as aforementioned discussion). In the future work, we will investigate algorithms/methods to obtaining the optimal parameters of the system and study the effectiveness of the proposed controller for the system under the input constraints.

APPENDIX A

In this appendix, the design of the NFTSMC is presented. Define $e = x_1 - x_d$ as the trajectory tracking error. To obtain a finite time convergence, the sliding surface of NFTSMC can be chosen as follows [8], [24]:

$$\sigma = e + k_1 e^{[λ]} + k_2 e^{[p/q]}$$

(72)

where $σ$ is the sliding variable, $k_1 = diag(k_{11}, k_{12}, ..., k_{1n}) \in \mathbb{R}^{nxn}$ and $k_2 = diag(k_{21}, k_{22}, ..., k_{2n}) \in \mathbb{R}^{nxn}$ are positive definite matrices, respectively, $p$ and $q$ are positive odd numbers satisfying the relation $1 < p/q < 2$ and $λ > p/q$.

Differentiating (72) with respect to time, we have

$$\dot{\sigma} = \dot{e} + k_1 λ | e |^{λ-1} \dot{e}^{[p/q]}$$

$$+ k_2 \frac{p}{q} | e |^{(p/q)-1} \left( \Lambda u + f(x_1, x_2) + δ(q, ˙q, τ) + Ξ \right)$$

(73)

According to [8], [24] and based on (73), the NFTSMC can be designed as follows:

$$u = u_{eq} - u_s$$

(74)

where $u_{eq}$ is designed as:

$$u_{eq} = Λ^{-1}\left( -\frac{p}{k_2q} | e |^{[p/q]} + k_1 λ | e |^{λ-1} \dot{e}^{[p/q]} \right)$$

$$- f(x_1, x_2) + \ddot{x}_d$$

(75)

And, $u_s$ is as below

$$u_s = Λ^{-1}(ρ + ν) sign(σ)$$

(76)

The controller (76) is designed based on the assumption that the bounded value $ρ$ can be obtained in advance. In order to relax the assumption, the adaptive law can also be applied. The reaching law (76) can be modified as

$$u_s = Λ^{-1}(\hat{ρ} + ν) sign(σ)$$

(77)

where $\hat{ρ}$ is the estimation of $ρ$, and this value is adapted as follows:

$$\dot{\hat{ρ}} = \begin{cases} 0, & \text{if} \|σ\| < α \\ \frac{1}{k} \|σ\|, & \text{if} \|σ\| \geq α \end{cases}$$

(78)

In order to eliminate the chattering, the boundary method below is applied to replace the $sign$ function in (77).

$$u_s = -Λ^{-1}(\hat{ρ} + ν) \frac{σ}{|σ| + ϱ}$$

(79)

where $ϱ$ is the small positive constant.


[55] A. Levant, “Robust exact differentiation via sliding mode technique,” Automatica, vol. 34, DOI https://doi.org/10.1016/S0005-1098(97)00209-