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An Experimental Investigation into the Pressure Drop for Turbulent Flow in 90° Elbow Bends.

N M Crawford, G Cunningham and S W T Spence

Abstract

The prediction of the pressure drop for turbulent single-phase fluid flow around sharp 90° bends has proved difficult owing to the complexity of the flow arising from frictional and separation effects. Several empirical equations exist, which accurately predict the pressure loss due to frictional effects. More recently, Crawford et al. [1] proposed an equation for the prediction of pressure loss due to separation of the flow. This work proposes a new composite equation for the prediction of pressure drop due to separation of the flow, which incorporates bends with ratio $R/r < 2$. A new composite equation is proposed to predict pressure losses over the Reynolds number range $4 \times 10^3 - 3 \times 10^5$. The predictions from the new equation are within a range of -4 to +6 per cent of existing experimental data.

Introduction

Fluid flow in bends and the associated losses are relevant to many industrial applications such as inlet and exhaust systems in the automotive industry, HVAC duct fittings, fuel systems in the aerospace industry, and commercial pipelines. Losses in pipe flow are manifested as a drop in total pressure, assuming that there is no work or heat transfer. In straight pipes, this pressure drop is predominantly due to frictional effects, while in bends the pressure drop is due to a combination of frictional effects, secondary flows and separation. Existing empirical relationships hold for the region where the pressure loss is due to frictional effects. However, when the pressure loss is, at least in part, due to separation of the flow, the accuracy of empirical relationships diminishes. Bends are typically characterised by the ratio R/d , the radius of curvature divided by the diameter of the pipe, or R/r which uses the pipe radius r instead of the diameter. Both formats appear in the literature; for the sake of consistency, this report uses the ratio R/r throughout. Bends with R/r ratios above 20 are termed curved pipes, whilst those with $R/r < 20$ are referred to as bends (or elbows). The two are obviously interlinked but often in the former case the flow is fully developed, and upstream and downstream tangent effects are absent, while in the case of elbows, the approaching flow is

often not fully developed and the effects on the upstream and downstream tangents are significant.

The flow in curved pipes differs from that in a straight pipe principally through exhibiting secondary flow in planes normal to the main flow. Dean [2-4] produced approximate theoretical solutions for fully developed laminar flow in curved pipes. He showed that the presence of a steady secondary flow caused an increase in the frictional resistance over that in a straight pipe. Additional secondary vortices occur above some explicit Dean number, De

$$De = \text{Re} \left(\frac{r}{R} \right)^{0.5} \quad (1)$$

where r is the radius of the pipe bore. Hawthorne [5] undertook a theoretical analysis of the secondary flow for the important practical case of flow in a 90° bend and postulated that the secondary flow may become oscillatory in tighter bends with smaller R/r values.

Review papers by Ito [6], and more recently, Spedding et al. [7] present a detailed survey of the published work on curved bends in the laminar, transitional and turbulent flow regimes. The work presented in the remainder of this paper is concerned only with turbulent flow.

Ito [8] experimented with fully developed turbulent flow in a series of curved pipes, each with a different radius of curvature, R . He produced the following semi-empirical formula to calculate the frictional factor in smooth curved pipes, λ_c , which was within $\pm 1\%$ of his experimental data,

$$\lambda_c \left(\frac{R}{r} \right)^{0.5} = 0.029 + 0.304 \left[\text{Re} \left(\frac{r}{R} \right)^2 \right]^{0.25} \quad (2)$$

Ito showed that λ_c could be expressed in terms of λ_s , Blasius' equation for the friction factor of a straight smooth pipe, as follows,

$$\frac{\lambda_c}{\lambda_s} = \left[\text{Re} \left(\frac{r}{R} \right)^2 \right]^{0.05} \quad \lambda_s = 0.316 \text{Re}^{-0.25} \quad \text{Re} \left(\frac{r}{R} \right)^2 > 6 \quad (3)$$

However, expressing the friction factor for a curved pipe in this form gives a wider spread of errors. It is established that below values of $\text{Re}(r/R)^2 = 0.034$, the friction factor of a curved pipe practically coincides with that of a straight pipe.

Weske [9] made an important contribution to the analysis of flow in curved pipes. He measured the velocity profiles and both static and total pressure distributions of turbulent flow through a selection of pipe bends. The velocity distributions showed that the maximum axial velocity was near the outside wall with the slower moving fluid largely restricted to a thin ‘shedding layer’ that transported boundary layer fluid around the periphery of the pipe from the outer wall to the inner wall of the bend. Recently, these findings were corroborated by Taylor et al. [10] and Anwer et al. [11]. The radial pressure gradient created in the bend propagates upstream into the inlet pipe through the wall boundary layer. Ito [8] and Rowe [12] established that the upstream and downstream tangents greatly affected the loss coefficient.

Pressure loss data have been published for elbow bends with varying angles of curvature. The majority of the work has concentrated on the 180° U-bend [5, 9, 11-16], and much less attention has been given to the more widely used 90° elbow bend. Early investigations were made into turbulent flow in a 90° elbow bend by Hofmann [17]. He discovered that disturbances in the flow due to the bend disappeared between fifty and seventy diameters downstream of the bend. Consequently, for accurate results, pressure measurements should be taken at least this distance downstream. There is tremendous scatter in the results obtained by authors on the subject of pressure losses in 90° bends. This ambiguity could be due to a combination of varying surface roughness in the bends, lack of uniformity in the pipe section at the bend, eddies from imperfections at the joints, tangent effects and inaccurate piezometer readings. Piggot [18] experimented on bends with various pipe diameters and a known roughness value. He demonstrated that there are different types of roughness. Sand casting produces a different roughness to pipes that have been artificially roughened. Piggot showed that the bend roughness is altered during some manufacturing processes, with wrinkling

occurring on the inner wall, noticeably increasing the pressure loss. Bend pressure loss data were presented along with existing results. Widespread scatter was evident.

Reviews by Spedding et al. [7] and by Zanker and Brock [19] stated that the Ito data [8,14] was considered to be reliable and showed that the minimum pressure loss incurred when turning a flow through 90° corresponded to a ratio $R/r = 5$. Above this minimum, the value of pressure loss increased gradually, as the centreline length increased viscous losses. For R/r ratios below 5, the pressure loss is predominantly due to separation of the flow and increases rapidly with decreasing R/r values. Recently Crawford et al. [1] have shown that the pressure drop in an elbow bend is equivalent to the pressure drop in a curved pipe (Equation 2) plus an additional pressure loss component due to the separation of the flow. Equation 2 yields the frictional resistance in a curved pipe, λ_c , which can be written as an equivalent length for a curved pipe, as shown in Equation 4.

$$\left(\frac{l_e}{d}\right)_c = 2.4792\lambda_c \text{Re}^{0.25}\left(\frac{R}{r}\right) \quad (4)$$

Crawford et al. [1] found that the addition of Equation 4, the pressure drop due to a curved pipe, and Equation 5, the pressure drop due to separation of the flow, yielded the pressure drop in an elbow bend.

$$\left(\frac{l_e}{d}\right)_B = 22.2126\left[\text{Re}\left(\frac{r}{R}\right)^2\right]^{0.7888} \text{Re}^{-0.71438} \quad (5)$$

Ito [14] experimented on several bends of various pipe diameters and R/r values between 2.01 and 14.60. In addition, tests were also conducted on two commercial screw elbows of 45° and 90°. The bends were manufactured from brass castings and their inner surfaces were machined and polished with fine sand paper to ensure smooth and accurate geometries. The straight pipes, or tangents, which were connected to the bends, were made from smooth drawn copper tubing. The tests were performed using water from a surge tank, which passed through a honeycomb flow straightener and bellmouth, then into the test section. Ito calculated the following empirical formulae for the bend loss coefficient:

$$K = 0.00873\alpha f_c \theta \left(\frac{R}{r}\right) \quad \text{Re}\left(\frac{r}{R}\right)^2 < 91 \quad (6)$$

$$K = 0.00241\alpha \theta \text{Re}^{-0.17} \left(\frac{R}{r}\right)^{0.84} \quad \text{Re}\left(\frac{r}{R}\right)^2 > 91 \quad (7)$$

For $\theta = 90$

$$\alpha = 0.95 + 17.2 \left(\frac{R}{r}\right)^{-1.96} \quad \left(\frac{R}{r}\right) < 19.7 \quad (8)$$

$$\alpha = 1 \quad \left(\frac{R}{r}\right) > 19.7 \quad (9)$$

Ito showed that a screwed elbow of ratio $R/r = 1.628$ had approximately double the pressure drop of a smooth inner walled equivalent elbow and reported a significant circumferential pressure variation within the screwed elbow, much greater than for the corresponding smooth elbow. The pressure observed on the outer wall of the bend was high due to the impaction of the fluid, while at the inner wall of the bend separation of the boundary layer and subsequent formation of secondary circulation resulted in a low pressure. The growth of and separation of the boundary layer, particularly at the inner wall, grossly changed the velocity distribution in the bend as reported by Taylor et al. [10], Rowe [12], and Ward-Smith [20], and introduced oscillations in the flow with respect to the distance downstream of sharp bends, which was in agreement with the findings Hawthorne [5], Kirchbach [21] and Schubart [22]. This oscillation of the flow along the downstream tangent could be due to switching of the secondary flow patterns, explaining the inability of theoretical developments to treat elbow bends (Ito [14]).

Miller [23] presented total pressure loss coefficients for bends with various angles of deflection for a range of Reynolds number above approximately 10^5 , where it is accepted that the effects of Reynolds number on the pressure loss coefficients are negligible. The results were presented as smoothed performance charts, where the loss coefficients were plotted against geometrical variables and, where suitable, flow conditions.

In summary, for the practical applications of flow in bends, there remains a great deal of scatter in the existing data. The unsatisfactory correlation of the existing data is most probably due to manufacturing variations, which affect the surface finish and assembly of the components. The pressure measurement locations are also significant, as it is important to have adequate development lengths upstream and downstream of the geometry to minimise the effect of local flow disturbances. Differences in the data acquisition and analysis have also led to disagreement between experimenters. For example with bends, some investigators have considered the inlet and outlet tangents. Others compensate for the centreline distance or the centreline intersectional distance of the bend, thus arriving at a value of bend pressure loss, without any element of length within the system. What is certain is that the inner wall of the elbow and/or the body of the bend must be coincident with the inner wall of the pipe if excessive pressure drop is to be avoided. This paper presents the results of a series of experiments that were performed in order to determine accurate pressure loss data for various bends. The data is then compared to published work and used to determine an equation to predict the pressure drop due to separation of the flow in an elbow bend at very low R/r values.

Experiments

In this work experiments were carried out on three 90° elbow bends with ratio $R/r = 20$, 5 and 1.3 and a 90° T-junction (see Figure 1). These geometries were chosen as the pressure losses in the $R/r = 20$ bend would be predominantly due to friction, in the $R/r = 5$ bend they would be due to a combination of frictional effects and separation of the flow, and in the $R/r = 1.3$ bend the losses would be primarily due to separation of the flow. The test geometries were machined from aluminium blocks and had an internal diameter, $d = 25.4$ mm. The test pieces were manufactured in two halves and carefully polished using 1200-grade abrasive paper to ensure a smooth surface. The surface roughness achieved was assessed as having an Ra value of 0.2 micro-meters. Dowel pins were used to assemble the bends/junction and a leak tight geometry was created with silicon sealant and bolts. There was sufficient length upstream and downstream of the test geometry to ensure the flow was isolated from external disturbances ($140d$ upstream and $110d$ downstream). For the $R/r = 20$ bend (Figure 1(a)), pressure measurement locations were machined at 0°, 22.5°, 45°, 67.5°, and 90° on the outside and inside of the bend. For the $R/r = 5$ bend (b), the pressure measurement locations were 0°, 45°, and 90° on the outside and inside of the bend. For the $R/d = 1.3$ bend (c), the pressure measurements were recorded at 0°, 45°, and 90° on the outside and 0° and 90° on the inside

of the bend (only two measurement locations were possible on the inner wall due to the tight geometry of the bend). The diameter of the pressure measurement holes was 0.5 mm.

Figure 2 is a schematic of the experimental apparatus. The present study is restricted to the steady flow of a single-phase fluid, namely air, over a Reynolds number range of approx 19800 to 1.26×10^5 . A bellmouth was connected at the inlet of the upstream pipe, to introduce a smooth stream of air into the test section. Two PT100 temperature probes were mounted on the bellmouth and used to measure the total temperature. A honeycomb flow straightener was inserted into the pipe downstream of the bellmouth, eliminating swirl and aiding fully developed flow. A distance of $50d$ was allowed between the bellmouth and first pressure measurement location to avoid the disturbances present in entrance flow.

The numbered locations in Figure 2 represent the pressure measurement stations of the upstream and downstream pipes, given in Table 1. The static pressure at each of these stations was measured using a differential pressure transducer; each pressure measurement on the upstream and downstream pipes was taken relative to atmospheric pressure in the test laboratory. The pressure measurements recorded around the bend were also taken using a differential pressure transducer, although these readings were taken relative to the pressure tapping on the top of the pipe at station 4 ($5d$ upstream of the bend). This enabled the use of a lower range transducer for the bend and hence better measurement resolution. The measurements were recorded using a data-logger with a resolution of 0.02%. All pressure instrumentation was calibrated along with the data-logging system, either against a certified pressure calibration standard or, in the case of the lowest range transducers, against a Betz manometer with a quoted accuracy of ± 1 Pa. Two types of pressure transducers were used in the experimental study; strain gauge transducers were used for the higher range measurements and for the orifice plate, while dry capacitance cell transducers were used for the low range differential measurements.

Mass flow rate measurement was achieved using an orifice plate mounted in a 54.8 mm ID pipe. The orifice plate arrangement was in accordance with BS EN ISO 5167, with the corresponding pressure and temperature instrumentation being calibrated against traceable standards. The maximum uncertainty in the flow rate measurements was determined as 1.7%.

A gate valve was fitted downstream of the orifice plate and used to manually alter the mass flow rate. The test rig was connected to a 75 m³ vacuum vessel that was evacuated using three vane-type vacuum pumps, which were able to sustain a continuous vacuum. In all the tests, the ambient temperature of the air was in the range 295 - 300 K, and did not vary significantly during a test.

Table 1 Measurement stations and corresponding locations from junction.

Upstream pipe	Downstream pipe
1 = 90d	5 = 5d
2 = 50d	6 = 10d
3 = 10d	7 = 20d
4 = 5d	8 = 30d
	9 = 40d
	10 = 50d
	11 = 60d
	12 = 70d
	13 = 90d

Data Analysis

There are several ways to present the bend pressure loss as an equivalent length, l_e . This section outlines the method used in the current work. Figure 3 shows a schematic of the bend pressure loss, the numbered locations represent the pressure measurement stations. The static pressure drop per unit length between stations 1 - 2, 2 - 3 and 3 - 4 was calculated and averaged to produce a pressure loss per unit length for the upstream pipe, $\Delta p / \Delta L_{upstream}$. The downstream static pressure loss per unit length, $\Delta p / \Delta L_{downstream}$ was also obtained. However, as disturbances occur some distance downstream of the bend, the calculations were started at station 9 to avoid the downstream transition region.

The upstream gradient was projected downstream from station 1 for the length of the bend (using the bend centreline length) and the downstream tangent. The pressure difference was then calculated at stations 9 - 13 by subtracting the measured pressure from the projected

value. This yielded an average pressure difference based on the upstream gradient projected from station 1. The upstream gradient was also projected from stations 2 and 3 and an average pressure difference was calculated for each case. The three average pressure difference values were averaged to produce, Δp_B , the pressure loss due to the bend.

Three separate data sets were taken for each flow condition to assess variation in the measurements. The standard deviation of the different downstream pressure differences calculated by projecting each upstream gradient was less than 0.2% between the three different data sets. This value corresponds to the $R/r = 1.3$ bend at the highest Reynolds number condition of 1.21×10^5 , since this was found to be the case with most variation. The standard deviation of the overall bend pressure drop values between each of the data sets was less than 0.1%. The values of variation between the data sets indicate the level of repeatability achieved with the experimental measurements.

This method of systematically taking each measurement location and working out the pressure gradient had the advantage of highlighting any variations that occurred from station to station due to subtle flow phenomena, such as pressure oscillations in the downstream tangent, so that their influence on the calculated pressure difference could be avoided. For higher flow rate conditions where the oscillations stretched further downstream, the first downstream measurement stations (9, 10 etc.) were excluded from the calculations as was judged necessary on a case-by-case basis. It was not found necessary to exclude any measurement stations from $40d$ onwards for the lower flow conditions.

The pressure loss due to the bend is converted into an equivalent length, l_{eB} , using the following equation:

$$l_{eB} = \frac{\Delta p_B}{\left(\Delta p / \Delta L \right)_{upstream}} \quad (10)$$

The equivalent bend straight pipe length, l_{eBE} , is given by,

$$l_{eBE} = \frac{2\pi R_{centreline}}{4} \quad (11)$$

Thus the total equivalent length is given by,

$$l_{eBT} = l_{eBE} + l_{eB} \quad (12)$$

and non-dimensionalised using the pipe diameter, d .

$$\frac{l_{eBT}}{d} = \frac{l_e}{d} \quad (13)$$

This method of analysis has been used by Ito and has been shown to give rise to more consistent results than other methods [6,13].

The method used to calculate the loss coefficient is the same as that used by Piggot [17] and Ito [13] where,

$$K = \frac{\Delta p_{BT}}{\frac{1}{2} \rho U^2} \quad (14)$$

The equivalence between the two methods of presentation is given by Ito [13],

$$\frac{l_e}{d} = \frac{K}{4f} \quad (15)$$

Results and Discussion

In this section the pertinent results from the experimental work are presented. Initially, pressure values in the upstream and downstream tangents are presented along with detailed pressure measurements around the bends. Previously published measurements for losses around bends have been presented in either equivalent length format, l_e/d , or as loss coefficients, K . In order to facilitate comparisons with previous work, the experimental results from this study have been given in terms of both l_e/d , and K . Finally, an equation is presented to calculate the pressure loss due to separation of the flow in elbow bends with low R/r ratios.

Figure 4 shows the measured static pressure distribution along the upstream tangent, inner wall and downstream tangent of the $R/r = 20$, 5, and 1.3 bends at $Re = 1.21 \times 10^5$. The pressure measurement location at $-90d$ corresponds to 90 diameters upstream of the bend. Similarly, the measurement location at $+90d$ corresponds to 90 diameters downstream of the bend. In each bend, the pressure gradient of the upstream tangent changed before it reached the bend (approximately $5d$ upstream). This increase in pressure was caused by the effect of the bend on the velocity profile with the bend acting as a constriction. For the more gentle $R/r = 20$ bend, disturbances in the flow downstream of the bend were manifested as fluctuations in the pressure values, which dispersed before reaching approximately $40d$ downstream. At this location, the pressure gradient corresponded to that of the upstream tangent. For the $R/r = 5$ bend, the disturbances could still be detected at $50d$ downstream of the bend. Finally, for the $R/r = 1.3$ bend, the disturbances were more evident and could still be detected at $70d$ downstream of the bend, which is the upper limit of the distance of $50d - 70d$ reported by Hofmann [17]. These disturbances were not apparent at the $90d$ downstream station.

Oscillations with respect to distance along the tangent were present in the flow downstream of the $R/r = 1.3$ bend for the $Re = 1.21 \times 10^5$ case (Figure 5). This behaviour was damped out as the flow travelled along the downstream length and disappeared after approximately $70d$. Hawthorne [5], Kirchbach [21] and Schubart [22] also reported pressure variations downstream of the bend. A very weak oscillating pattern was present in the $Re = 1.98 \times 10^4$ case, but as the mass flow rate was increased this behaviour became more apparent. The magnitude of these pressure oscillations was lowest for the $R/r = 20$ and $R/r = 5$ bends respectively.

In Figure 6 the static pressure variation along the inner and outer walls of the bends is presented for $Re = 1.21 \times 10^5$. In the case of these plots the deviation of static pressure from that at the reference station $5d$ upstream of the bend, p_{ref} , has been non-dimensionalised using the dynamic pressure calculated at the reference station. The graph also displays pressure measurements $5d$ upstream and downstream of the bend. For the $R/r = 20$ bend, (Fig 6a), the outer gradient was constant from the inlet of the bend, where the flow was forced outwards, to the 22.5° position. From this location to the exit of the bend, the gradient was negative. The inner gradient was negative at the inlet and exit of the bend. The absence of positive gradient suggests that regions of deceleration and local separation of the flow did not occur on either the inner or outer walls, due to the more gentle curvature of this bend. For the $R/r =$

5 bend, (b), a positive gradient was observed on the outer wall and a negative gradient on the inner wall at the bend inlet, with the reverse at the bend exit. The static pressure variation around a $R/r = 5.6$ bend, reported by Enayet et al. [24], is also plotted for comparison. Although the data of Enayet et al were for a slightly different curvature and related to different locations, the values for the $R/r = 5$ bend measured in this investigation were found to fit well with the pressure profile measured by Enayet. For the $R/r = 1.3$ bend, (c), the pressure was greatest at the outer wall farthest from the centre of curvature and least at the inner wall where it was near to the centre of curvature. At the bend inlet the pressure gradient was positive on the outer wall and may be sufficiently strong to produce local separation. The gradient is negative on the inner wall and caused the flow to be accelerated. At the bend exit, the opposite occurred. The outer wall experienced a negative gradient in which the flow was accelerated and a positive gradient occurred at the inner wall, which caused local separation of the flow.

The pressure drop, expressed as an equivalent length, is given in Figure 7. The pressure loss increased with Reynolds number and the data show a clear trend, following an exponential relationship within this Reynolds number range. As expected, the greatest pressure drop occurred for the $R/r = 1.3$ bend and the lowest for the $R/r = 5$ bend.

The bend pressure loss is more commonly presented as a loss coefficient. Figure 8 plots the loss coefficients that were measured for the bends, compared with the values calculated using the empirical equations of Ito [14] (eqns 7, 8 and 9) to determine static pressure loss coefficient. Also shown is the Miller [23] pressure loss coefficient curve, with the appropriate Reynolds number and roughness corrections applied. The general trends of the graphs show the loss coefficient decreasing with increasing Reynolds number. The experimental data from this study show some scatter compared to the equations of Ito and Miller. However, it must be remembered that their equations are essentially empirical curve fits to their own data. Consequently, these lines represent smoothed experimental data from other studies, rather than predictions based on flow physics. It has already been noted that modest variations in surface finish and geometry of the experimental pipes and bends produce significant difference in measured pressure loss values.

For the $R/r = 20$ and $R/r = 5$ bends (Figs. 8a and 8b), the farthest deviation of the experimental loss coefficient from the Ito and Miller curves occurred at the lowest Reynolds

number. The pressure loss values determined at these low flow rates were the small difference between two larger measurements and were therefore highly sensitive to any measurement inaccuracies, although the repeatability of the different experimental data sets was found to be good. The differences between the Ito and Miller predictions were quite small for the $R/r = 20$ and $R/r = 5$ bends.

The loss coefficients determined from this study were generally higher than those of Ito or Miller. It must be noted that the same trend of loss coefficient with Reynolds number existed for all three bends that were tested. The loss coefficient is high at low Reynolds number ($< 2 \times 10^4$), falls sharply as the Reynolds number increases to 2.5×10^4 , but then rises to another peak between $4 \times 10^4 < Re < 8 \times 10^4$, before falling again.

Ito experimented on bends with $R/r > 2$. Consequently, the Ito curve shown for the $R/r = 1.3$ bend in Fig. 8c has been obtained by extrapolating Ito's equation to a lower radius ratio. While there is reasonable agreement between the experimental loss coefficients and Miller's curve, Ito's curve appears unsatisfactory when extrapolated for tighter bends with small R/r values.

The variation in pressure loss, expressed as an equivalent length, with R/r ratio, is shown in Figure 9 for the flow case corresponding to $Re = 1.00 \times 10^5$. The minimum pressure loss occurred at a $R/r = 5$, with the pressure loss gradually increasing with R/r ratio, in the region where the loss was predominantly due to friction, as shown by Ito [7] and Piggot [17]. As the R/r ratio was decreased below 5, there was a sharp increase in the pressure loss, which probably corresponds to the development of increasingly severe separation in the flow. Also plotted on the graph is the pressure loss data of a sharp-edged T-junction configuration that was tested with the flow in one leg blanked off, which although not exactly equivalent to a sharp elbow, effectively had a curvature of $R/r = 1$ and produced the maximum pressure loss of the geometries tested. The trend shown in Fig. 9 would benefit from additional geometries being tested, but it was only possible to consider three separate bends in the course of this study.

In this work it has been found that equation 16 can predict the pressure loss at low R/r values where the losses are likely to be dominated by separation of the flow:

$$\frac{l_e}{d} = 1.25 \left(\frac{R}{r} \right)^{-1.5} \text{Re}^{0.35} \quad (16)$$

The addition of equation (16) and the Ito equation (eqn. 4) gave the prediction of the bend pressure drop over the turbulent Reynolds number range $4 \times 10^3 - 3 \times 10^5$ within a spread of -4 to $+6\%$. Figure 10 shows the loss predicted by previous equations, and the proposed new composite equation, compared with experimental data. To maximise the range of comparison, the experimental points included in the plot comprise values from this work along with those published by Ito (14).

Figure 11a presents a comparison of the new composite equation and the previous composite equation of Spedding [19] to predict the pressure loss due to separation of the flow. Both equations show an excellent correlation with the experimental data for R/r values above 3. However, below this value, the accuracy of the Spedding equation is diminished and shows a $+13\%$ error for $R/r < 2$. This is more clearly shown in Fig. 11b which has an amplified x-axis scale. The proposed new composite equation yields a corresponding error of -2% . Overall, the spread of variation of the new composite equation is -4 to $+6\%$ from the existing experimental data, while the Spedding equation is within $+13$ to -2% . The improvement has arisen primarily from the more extensive experimental data that was available to calibrate the new composite equation.

Conclusions

An experimental investigation was conducted to determine the pressure losses in three different bends with curvatures of $R/r = 1.3, 5$ and 20 . The $R/r = 5$ bend resulted in the minimum pressure loss. Increasing the radius of curvature brought increased losses due to wall friction, while decreasing the radius of curvature led to higher losses that were probably due to separation. During the experimental tests, disturbances in the flow were observed up to a distance between $50d$ and $70d$ downstream of the bend, which was within the range reported by Hofmann [17].

For the $R/r = 20$ and $R/r = 5$ bends, the measured loss coefficient data showed similar trends but higher absolute values than predicted by the curves of Ito [14] and Miller [23]. For the $R/r = 1.3$ bend, the loss coefficient values were significantly above the equation of Ito, since that

equation had been extrapolated for this low R/r value. A new empirical equation has been proposed to predict the pressure loss in the region where separation is predominant. The combination of this new equation with Ito's equation was shown to effectively predict the loss coefficient over a wide range of R/r values, and in particular yielded improvements over existing methods at low R/r values.

Non-uniformity in the pressure profile was observed downstream of the bends, as previously reported by Hawthorne [5], Kirchbach [21] and Schubart [22]. These variations were greatest for the $R/r = 1.3$ bend and were likely to be intensified by the increased separation and turbulence that followed the sharper bend. However, although less evident, the same phenomenon was also noted for the $R/r = 20$ bend, where flow would have been unlikely to have separated from the inner wall through the bend.

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Nomenclature

C_p	Pressure recovery coefficient
d	Pipe diameter, m
De	Dean number
f	Fanning friction factor, $\frac{2\tau_w}{\rho U^2}$
K	Loss coefficient
l_e	Equivalent length, m
L	Pipe length, m
p	Static pressure, $\text{kgm}^{-1}\text{s}^{-2}$
$\dot{m}$	Mass flow rate, kg s^{-1}
r	Pipe radius, m
R	Radius of curvature, m
Re	Reynolds number, $\frac{\rho U d}{\mu}$
U	Mean velocity, ms^{-1}

Greek Symbols

α	Numerical factor
θ	Deflection angle, degrees
λ	Darcy friction factor, $4 \times f$
ρ	Density, kgm^{-3}
τ_w	Wall shear stress, $\text{kgm}^{-1}\text{s}^{-2}$

Subscripts

B	Bend
BE	Bend equivalent straight pipe length
BT	Bend total
c	Curved bend
ref	Reference
s	Straight pipe

Figure captions

Figure 1 Bend geometries for experimental tests

Figure 2 Experimental arrangement for testing

Figure 3 Bend pressure loss schematic showing measurement locations

Figure 4 Static pressure profiles of 90° bends at $Re = 1.21 \times 10^5$

Figure 5 Deviation from the average downstream pressure value on $R/r = 1.3$ bend at $Re = 1.21 \times 10^5$

Figure 6 Static pressure variation around 90° bends at $Re = 1.21 \times 10^5$

Figure 7 Bend pressure drop expressed as an equivalent length

Figure 8 Bend pressure drop expressed as a loss coefficient, K , at $Re = 1.21 \times 10^5$ for each of the three bends

Figure 9 Variation in bend pressure loss with R/r ratio at $Re = 1.00 \times 10^5$

Figure 10 Variation of pressure loss with R/r ratio for a 90° bend at $Re = 10^5$

Figure 11a Comparison of pressure loss equations for a 90° bend at $Re = 10^5$

Figure 11b Reproduction of Fig. 11a with magnified x-axis scale for low R/r values

An Experimental Investigation into the Pressure Drop for Turbulent Flow in 90° Elbow Bends.

N M Crawford, G Cunningham and S W T Spence

Fig. 1a $R/r = 20$ bend

Figure 1 Bend geometries for experimental tests

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Fig. 1b $R/r = 5$ bend

Figure 1 Bend geometries for experimental tests

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Fig. 1c $R/r = 1.3$ bend

Figure 1 Bend geometries for experimental tests

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Fig. 1d 90° T-junction

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Fig. 6a $R/r = 20$ bend

Figure 6 Static pressure variation around 90° bends at $Re = 1.21 \times 10^5$

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Fig. 6b $R/r = 5$ bend

Figure 6 Static pressure variation around 90° bends at $Re = 1.21 \times 10^5$

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Fig. 6c $R/r = 1.3$ bend

Figure 6 Static pressure variation around 90° bends at $Re = 1.21 \times 10^5$

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Figure 7 Bend pressure drop expressed as an equivalent length

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Fig. 8a $R/r = 20$ bend

Figure 8 Bend pressure drop expressed as a loss coefficient, K , at $Re = 1.21 \times 10^5$ for each of the three bends

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Fig. 8b $R/r = 5$ bend

Figure 8 Bend pressure drop expressed as a loss coefficient, K , at $Re = 1.21 \times 10^5$ for each of the three bends

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Fig. 8c $R/r = 1.3$ bend

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