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SVD-Aided Multi-Beam Directional Modulation Scheme Based on Frequency Diverse Array

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Abstract—With the assistance of singular value decomposition (SVD), a multi-beam directional modulation (DM) scheme based on symmetrical multi-carrier frequency diverse array (FDA) is proposed. The proposed DM scheme is capable of achieving range-angle dependent physical layer secure (PLS) transmissions in free space with much lower complexity than the conventional zero-forcing (ZF) method. Theoretical and simulated results about secrecy rate and complexity verify the improved computational efficiency and considerable memory savings despite a very small penalty of secrecy rate.

Index Terms—Directional modulation; frequency diverse array; physical layer security; singular value decomposition.

I. Introduction

DIRECTIONAL modulation (DM) has increasingly become a promising technique capable of achieving physical layer secure (PLS) communications.

The phased array (PA) was first utilized for DM synthesis in [1] by optimizing the phase shifters at the radio frequency (RF) frontend. A low-complexity PA-DM synthesis method was proposed in [2], which transferred the DM synthesis from RF frontend to baseband. Afterwards, multi-beam PA-DM synthesis methods were investigated in [3]-[6]. Specifically, the orthogonal vector synthesis approach was implemented in [3], while [4] proposed a robust multi-beam PA-DM scheme with imperfect direction knowledge. The multipath nature of the channel was exploited in [5] to create a multi-user DM transmissions. Artificial noise (AN) aided multi-beam DM system was studied in [6] using zero-forcing (ZF) criterion.

The above-mentioned PA-DM schemes [1]-[6] can only achieve angle-dependent secure transmissions, the security of which will fail when the eavesdropper is located along the same direction as the desired receiver. To address this problem, the frequency diverse array (FDA) with non-linear frequency increments was introduced to synthesize both range-angle dependent DM in [7]. The work in [8] proposed a random FDA-based range-angle dependent DM synthesis method, while the AN-aided FDA-DM communication over Nakagami-m fading channels was studied in [9].

Apart from the single-beam FDA-DM schemes [7]-[9], the multi-beam FDA-DM transmission was studied in [10] by jointly optimizing the frequency increments, the beamforming vector, and the orthogonal AN vector, of which the optimization problem was too complicated to solve practically. Moreover, the non-optimization ZF method in [6] was applied into multi-beam FDA-DM transmission in [11]. The ZF method, however, consumes too much time and space resources as well, of which the computational complexity and memory consumption remain to be reduced.

This paper is dedicated to reducing the complexity of multi-beam FDA-DM systems by means of singular value decomposition (SVD). The proposed SVD method differs from the state-of-the-art in two aspects: the first lies in the orthogonal matrix, which is obtained from SVD algorithm rather than zero forcing; the second depends on the AN vector, of which the size is much smaller. Beneficially, the proposed SVD method can improve computational efficiency and save much memory with only a small loss of secrecy rate, which makes it easier to achieve multi-beam FDA-DM transmissions.

Notations: The operators $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, and $(\cdot)^{\dagger}$ represent the transpose, Hermitian transpose, inverse, and Moore-Penrose inverse operations of a matrix, respectively.

II. System Model

As shown in Fig. 1, we consider a $(2N + 1)$-element symmetrical linear FDA with multiple carriers [12]. The
spacing $d$ between adjacent elements is designed as half central wavelength, and $L$ carriers are transmitted via each element. The frequency of the $l$-th ($l = 0, 1, \ldots, L - 1$) carrier for the $n$-th ($n = -N, 0, \ldots, N$) element of FDA is designed as

$$f_{n,l} = f_0 + \Delta f_{n,l} = f_0 + \Delta f \ln\left(\cev{(n+1)}(l+1)\right)$$  \hspace{1cm} (1)

where $f_0$ denotes the central carrier frequency, and $\Delta f$ is a fixed frequency offset satisfying $|\Delta f| \ll f_0$.

Let $r$ and $\theta$ represent the range between an arbitrary spatial position and the central FDA element, and the azimuth angle, respectively. The normalized steering vector radiated from the transmit antenna array at $(r, \theta)$ is a $(2N + 1) \times 1$ vector for such a system, which can be expressed as

$$\mathbf{h}(r, \theta) = \frac{1}{\sqrt{(2N + 1)L}} \begin{bmatrix} \mathbf{a}_N^\top(r, \theta) \cdots \mathbf{a}_1^\top(r, \theta) \end{bmatrix} \hspace{2cm} (2)$$

where $\mathbf{a}_n(r, \theta) = [e^{j \phi_{n,0}} \cdots e^{j \phi_{n,-1}}]^\top$ is an $L \times 1$ sub-steering vector caused by the $L$ carriers of the $n$-th antenna element [12]. The phase of the $l$-th entry of $\mathbf{a}_n(r, \theta)$ is

$$\phi_{n,l} = 2\pi \Delta f \ln\left(\cev{(n+1)}(l+1)\right) + \frac{f_0 + n \Delta f \sin \theta}{c}$$  \hspace{1cm} (3)

where $c$ is light speed, $d = \lambda/2$ denotes the spacing between adjacent elements, $t$ refers to the time variable, and $\lambda = c/f_0$ represents the wavelength of the central carrier.

The multi-beam DM model consists of a transmitter with a $(2N + 1)$-element FDA, $K$ stationary desired receivers, and $U$ passive eavesdroppers in different locations. The locations of the $K$ desired receivers are assumed to be perfectly estimated, of which the estimation disparities can be practically processed using the robust synthesis method in [4]. If we combine the $K$ desired receivers’ locations as a set, i.e.,

$$(\mathbf{Y}_d, \Theta_d) = \left\{ (r_{d,1}, \theta_{d,1}), (r_{d,2}, \theta_{d,2}), \ldots, (r_{d,K}, \theta_{d,K}) \right\}$$  \hspace{1cm} (4)

then a steering matrix $\mathbf{H}(\mathbf{Y}_d, \Theta_d)$ with size $(2N + 1) \times K$ can be obtained by combining the steering vectors $\mathbf{h}(r_{d,k}, \theta_{d,k})$ of the $K$ desired locations $(\mathbf{Y}_d, \Theta_d)$, i.e.,

$$\mathbf{H}(\mathbf{Y}_d, \Theta_d) = \begin{bmatrix} \mathbf{h}(r_{d,1}, \theta_{d,1}) & \mathbf{h}(r_{d,2}, \theta_{d,2}) & \cdots & \mathbf{h}(r_{d,K}, \theta_{d,K}) \end{bmatrix}$$  \hspace{2cm} (5)

Let $\mathbf{x}_d = [x_{d1}^\top \ x_{d2}^\top \ \cdots \ x_{dN}^\top]^\top$ denote the confidential baseband symbol vector for the $K$ desired receivers. The weighted transmit signal vector of the FDA is designed as

$$\mathbf{s} = \beta_1 \sqrt{P_s} \mathbf{P}_1 \mathbf{x}_d + \beta_2 \sqrt{P_s} \mathbf{P}_2 \mathbf{z}$$  \hspace{2cm} (6)

where $P_s$ is the total transmit power constraint, $\beta_1$ and $\beta_2$ are the power splitting factors satisfying $\beta_1^2 + \beta_2^2 = 1$, $\alpha$ denotes the normalization factor for the inserted AN, and $\mathbf{z}$ is the inserted complex AN vector with each entry having zero mean and variance $\sigma^2_z$. In order to obtain a desired standard modulation constellation at the desired receivers while distorting the received signals at other undesired receivers, the normalization matrix $\mathbf{P}_1$ and the orthogonal matrix $\mathbf{P}_2$ should satisfy the following criteria:

$$\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{P}_1 = \mathbf{I}_K, \hspace{1cm} \mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{P}_2 = \mathbf{0}$$  \hspace{1cm} (7)

In this paper, the normalization matrix is directly designed as the Moore-Penrose inverse matrix of the steering matrix $\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d)$, i.e.,

$$\mathbf{P}_1 = \mathbf{H}(\mathbf{Y}_d, \Theta_d) \left( \mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{H}(\mathbf{Y}_d, \Theta_d) \right)^{-1}$$  \hspace{1cm} (8)

which satisfies $\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{P}_1 = \mathbf{I}_K$, and the size of which is $(2N + 1)L \times K$.

It is known that the null space of a matrix can be derived from its SVD. Therefore, in order to design the orthogonal matrix $\mathbf{P}_2$, we perform the SVD operation on the steering matrix $\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d)$, which is now expressed as

$$\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\top = \mathbf{U} \mathbf{D} \mathbf{0} \mathbf{V}^\top$$  \hspace{1cm} (9)

Next, we can obtain a null space, that is, $\mathbf{V}_0 \in \mathbb{C}^{(2N+1) \times (2N+1-K)}$ for $\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d)$ from (9), which means $\mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{V}_0 = \mathbf{0}_{K \times (2N+1-K)}$. Therefore, the orthogonal matrix of the SVD method can be designed as

$$\mathbf{P}_2 = \mathbf{V}_0$$  \hspace{1cm} (10)

The normalized line-of-sight (LoS) channel is considered in this paper. In fact, the proposed method can also hold over fading channels like Nakagami-m fading [9], as long as the channel state information (CSI) is perfectly estimated. After passing through an LoS channel, the combined vector of the received signals at the desired receivers can be expressed as

$$\mathbf{y}(\mathbf{Y}_d, \Theta_d) = \mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{s} + \mathbf{w}_d = \beta_1 \sqrt{P_s} \mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{P}_1 \mathbf{x}_d$$

$$+ \alpha \beta_2 \sqrt{P_s} \mathbf{H}^\top(\mathbf{Y}_d, \Theta_d) \mathbf{P}_2 \mathbf{z} + \mathbf{w}_d$$  \hspace{2cm} (11)

where $\mathbf{w}_d \sim \mathcal{CN}(\mathbf{0}_{K \times 1}, \sigma^2_{w_d} \mathbf{I}_K)$ is the circularly symmetric complex additive white Gaussian noise (AWGN) vector with each entry having zero mean and variance $\sigma^2_{w_d}$.

Similarly, the received signal of an arbitrary eavesdropper at $(r_e, \theta_e)$, $(r_e, \theta_e) \neq (r_{d,k}, \theta_{d,k})$, can be expressed as

$$y(r_e, \theta_e) = \mathbf{h}^\top(r_e, \theta_e) \mathbf{s} + w_e = \beta_1 \sqrt{P_s} \mathbf{h}^\top(r_e, \theta_e) \mathbf{P}_1 \mathbf{x}_d$$

$$+ \alpha \beta_2 \sqrt{P_s} \mathbf{h}^\top(r_e, \theta_e) \mathbf{P}_2 \mathbf{z} + \mathbf{w}_e$$  \hspace{2cm} (12)

where $w_e \sim \mathcal{CN}(0, \sigma^2_{w_e})$ is the AWGN with zero mean and variance $\sigma^2_{w_e}$. It is worth emphasizing that the first term of (12) denotes distortion signal for the undesired receiver, and the second term is the inserted AN which cannot be eliminated at the undesired receiver due to the fact that the designed $\mathbf{P}_2$ is non-orthogonal to its steering vector $\mathbf{h}^\top(r_e, \theta_e)$. 

III. Performance Analysis

A. Secrecy Rate

We assume the $U$ eavesdroppers are located at $(r_u^u, \theta_u^u)$, $1 \leq u \leq U$, respectively, which satisfy $(r_u^u, \theta_u^u) \neq (r_d^k, \theta_d^k)$ for $\forall u \in \{1, 2, \ldots, U\}$ and $\forall k \in \{1, 2, \ldots, K\}$. The achievable rate of the link from the transmitter to the $k$-th desired receiver can be calculated by [9]

$$R(r_d^k, \theta_d^k) = \log_2 [1 + \gamma(r_d^k, \theta_d^k)]$$

where $\gamma(r_d^k, \theta_d^k)$ is the ratio of signal to interference plus noise (SNR) at the $k$-th desired receiver with the expression of

$$\gamma(r_d^k, \theta_d^k) = \frac{\beta_d^2 P_d R_h(r_d^k, \theta_d^k) P_1 P_2^H R_h(r_d^k, \theta_d^k)}{\sigma_w^2 + \alpha^2 \beta_d^2 P_d R_h(r_d^k, \theta_d^k) P_2 P_2^H R_h(r_d^k, \theta_d^k)}$$

(14)

Similarly, the achievable rate of the link from the transmitter to the $u$-th eavesdropper is expressed as

$$R(r_u^u, \theta_u^u) = \log_2 [1 + \gamma(r_u^u, \theta_u^u)]$$

(15)

where

$$\gamma(r_u^u, \theta_u^u) = \frac{\beta_u^2 P_u R_h(r_u^u, \theta_u^u) P_1 P_2^H R_h(r_u^u, \theta_u^u)}{\sigma_w^2 + \alpha^2 \beta_u^2 P_u R_h(r_u^u, \theta_u^u) P_2 P_2^H R_h(r_u^u, \theta_u^u)}$$

(16)

Therefore, the secrecy rate of the proposed multi-beam DM system can be defined as

$$R_s = \max_{k \in \{1, \ldots, K\}} \left( \min_{u \in \{1, \ldots, U\}} (R(r_d^k, \theta_d^k) - R(r_u^u, \theta_u^u)) \right)^+$$

(17)

where $[\cdot]^+ = \max\{0, \cdot\}$.

B. Time Complexity

Both the normalization matrices of the ZF method in [6][11] and the proposed SVD method are designed as $[H_h(Y_d, \Theta_d)]^\dagger$, so the complexity depends on the orthogonal matrix $P_2$ and the inserted AN $z$, as shown in Table I.

<table>
<thead>
<tr>
<th>Items</th>
<th>ZF method</th>
<th>SVD method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orthogonal matrix $P_2$</td>
<td>$P_2^{ZF} = I_{(2N+1)L} - [H_h(Y_d, \Theta_d)]^\dagger H_h(Y_d, \Theta_d)$</td>
<td>$P_2^{SVD} = U_0</td>
</tr>
<tr>
<td>Size of $P_2$</td>
<td>$(2N+1)L \times (2N+1)L$</td>
<td>$(2N+1)L \times (2N+1 - K)$</td>
</tr>
<tr>
<td>Artificial noise $z$</td>
<td>$z^{ZF} \in \mathbb{C}^{(2N+1)L \times 1}$</td>
<td>$z^{SVD} \in \mathbb{C}^{(2N+1-K) \times 1}$</td>
</tr>
<tr>
<td>Time complexity of calculating $P_2$</td>
<td>$O\left((2N+1)^2L^2K\right)$</td>
<td>$O\left((2N+1)LK^2\right)$</td>
</tr>
<tr>
<td>Space complexity of storing $P_2$ and $z$</td>
<td>$O\left((2N+1)^2L^2\right)$</td>
<td>$O\left((2N+1)L(2N+1 - K)\right)$</td>
</tr>
</tbody>
</table>

The SVD of $H_h(Y_d, \Theta_d)$. Since the size of $H_h(Y_d, \Theta_d)$ is $K \times (2N+1)L$, $(K < (2N+1)L)$, the time complexity of calculating $P_2^{SVD}$ can be obtained by $O\left(K^2(2N+1)L\right)$ [13].

Apart from $P_2^{SVD}$, the normalization matrix $P_4^{SVD}$ can also be easily acquired by (9), i.e., $P_4^{SVD} = \Sigma \Sigma^H$, since $\Sigma$ is a rectangular diagonal matrix and its Moore-Penrose inverse $\Sigma^H$ can be easily get by taking the reciprocal of each diagonal element [13], which further simplifies the calculation of the SVD method. By contrast, the normalization and orthogonal matrices of the ZF method have to be calculated independently.

C. Space Complexity

The sizes of the orthogonal matrix $P_2$ and the AN $z$ can impact the the memory consumption significantly. As shown in Table I, the size of the orthogonal matrix $P_2^{ZF}$ of the ZF method is $(2N+1)L \times (2N+1)L$, and the size of the inserted AN $z^{ZF}$ is $(2N+1)L \times 1$. By contrast, the size of the orthogonal matrix $P_2^{SVD}$ of the proposed SVD method is $(2N+1)L \times (2N+1 - K)$, and the size of the inserted AN $z^{SVD}$ is $(2N+1 - K) \times 1$.

We define a metric $\zeta$ as the ratio of the total memory consumed by $P_2^{SVD}$ and $z^{SVD}$ to that of the ZF method, i.e.,

$$\zeta = \frac{(2N+1)L \times (2N+1 - K) + (2N+1 - K)}{(2N+1)L + (2N+1 - K)} \times 100\%$$

$$= \frac{(2N+1)^2L - (2N+1)(LK - 1) - K}{(2N+1)^2L^2 + (2N+1)L} \times 100\%$$

(18)

It is worth noting that $\zeta \to 1/L$ when $K$ is determined and $N \to \infty$, which means the proposed SVD method can consume at most $1/L$ of the memory required by the ZF method, thereby reducing the amount of memory and lowering DC power consumption requirements.

IV. Simulation Results

Simulation parameters in this paper are set as $f_0 = 10$ GHz, $\Delta f = 2$ kHz, $\beta_1 = 0.9$, $P_1 = 1$, $N = 8$, $L = 7$, and $K = 3$, respectively. The $K$ desired receivers’ locations are $(r_1^d, \theta_1^d) = (150km, 50^\circ)$, $(r_2^d, \theta_2^d) = (180km, -40^\circ)$, and $(r_3^d, \theta_3^d) = (260km, 0^\circ)$, respectively. For the proposed DM system, an arbitrary modulation scheme based on amplitudes or/and phases such as ASK, PSK and QAM is applicable and different desired receivers can adopt...
different modulations. For convenience, $\pi/4$-QPSK is considered in our simulations.

Fig. 2(a) shows the secrecy rates versus SNR (dB) with AWGN eavesdroppers ($\sigma^2_w = \sigma^2_u$) and noiseless eavesdroppers, respectively, where the eavesdroppers’ locations are randomly selected in the simulations. It is observed that the SVD method requires slightly higher SNR (dB) than the ZF method in order to achieve the same secrecy rate. For example, only 0.5 dB of additional SNR is required for the SVD method to match the ZF method when $R_z = 8$ bits/s/Hz and $U = 2$. The secrecy loss is due to that the size of the inserted AN of the SVD method is smaller than that of the ZF method, which makes the SINRs of eavesdroppers a little higher than that of the ZF method. Moreover, a positive secrecy rate can always be achieved no matter the eavesdroppers are noiseless or not, which guarantees the transmission security.

Fig. 2(b) and (c) illustrate the simulated BER performances for ZF and SVD methods versus angle and range, respectively. In the angle dimension, the BER lobes of the ZF method are slightly narrower than the proposed SVD method around the desired receivers. In the range dimension, both ZF and SVD methods can achieve almost the same BER performances.

In order to illustrate the time complexity, we conducted $10^4$ Monte Carlo experiments to record the average time consumption of calculating $P_2$ using MATLAB\textsuperscript{1}. The result shows that the average time consumptions of ZF and SVD methods are 0.2258 and 0.1933 ms, respectively, which verifies that the computational efficiency of the proposed SVD method is better than that of the ZF method.

Moreover, Fig. 3 shows the total amount of memory required by the orthogonal matrix $P_2$ and the AN $z$, and the ratio $\zeta$ of SVD to ZF versus $N$, $K$, and $L$, respectively. From Fig. 3(a), the SVD method only consumes up to 14.29% of the memory required by the ZF method with $L = 7$ and $K = 3$, which is due to $\zeta \to 1/L$ when $N \to \infty$ as shown in (18). From Fig. 3(b) and Fig. 2(c), the ratio of SVD to ZF decreases as $L$ and $K$ increases. Therefore, the SVD method is more efficient than the ZF method with respect to memory required.

V. Conclusion

A new SVD-aided low-complexity range-angle dependent multi-beam DM scheme based on symmetrical multicarrier FDA was proposed. The proposed SVD method outperforms the ZF method in regard to the computational complexity and the amount of memory required for processing, while introducing only a small performance loss of secrecy rate. The SVD method opens a way to

\textsuperscript{1}Computer configurations: Intel(R) Xeon(R) CPU E5-1620 v2 @ 3.7 GHz; 8.0 GB RAM; 64-bit operating system. MATLAB version: R2016a.
reduce the implementing complexity and lower power consumption requirements for range-angle dependent multi-beam DM systems.

References


