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Frequency Diverse Array with Random Logarithmically Increasing Frequency Offset

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Abstract—The Frequency diverse arrays (FDAs) have been extensively investigated since it can provide a unique angle-range-dependent beampattern. This range-dependent characteristic can suppress the range ambiguous clutters, leading to many novel potential Radar applications. Two issues, however, exist in conventional FDA (CFDA) beampatterns, i.e., range-angle coupling and range periodicity. Therefore, for a CFDA Radar, target range information cannot be obtained directly from the beampattern peaks and target ambiguity occurs in the range domain. In this work, a new frequency diverse array (FDA) transmitter architecture with random logarithmically increasing frequency offset (log-RFDA) is presented. It is shown that the proposed strategy provides a non-periodic thumbtack-like beampattern, eliminating the ambiguity in range domain, suffered in the CFDA Radar. Furthermore, reduced side lobe and higher detection resolution can be achieved, compared with the previously reported log-FDA.

Index Terms—Frequency diverse arrays (FDAs), random logarithmically increasing frequency offset, non-periodic thumbtack-like beampattern, higher detection resolution

I. INTRODUCTION

Frequency diverse array (FDA) has gained increasing attention in recent year in Radar community [1–6] due to its distinct range-dependent beampattern that enables the capability of rejecting interference in range domain. A conventional FDA (CFDA) is constructed by applying a fraction of linear carrier frequency offset across the radiated array elements, named as linear-FDA hereafter. This linear frequency offset creates range-angle-time-dependent radiation patterns [2]. Beampattern peak occurs periodically in each of the three domains, i.e. angle, range and time. Ambiguity exists in range domain as infinite number of field peaks occur when the angle and time are fixed. Moreover, range information cannot be directly obtained from the beampattern peak due to the coupled range-angle

relationship. Some recent works have been reported in order to address these issues [3–7]. Specifically, in [3] instead of linear frequency offsets the logarithmically increasing frequency offset was employed to achieve a non-periodic beampattern with compromised resolution performance in range and angle domains. It was further extended to multi-carrier scenarios in [4]. In [5], a non-monotonically increasing increment used as logarithmic frequency offset was described, achieving improved range resolution at the price of larger bandwidth. Square and cubic increasing frequency offsets were also developed for the decoupling purpose in [6], which, however, can only eliminate the multiple maxima occurring in a specific area. Apart from altering the frequency offset profile, a random frequency diverse array (RFDA) was introduced in [7], providing a decoupled range-angle beampattern. Periodicity, however, still exists.

In this letter, the authors propose to combine the random characteristics with the logarithmically increasing frequency offsets in order to achieve a nonperiodic beampattern with a decoupled range-angle relationship. Additionally, this proposed log-RFDA with a single-carrier logarithmic offset enjoys an enhanced range resolution and sidelobe suppression as compared to the previously reported log-FDA.

The paper is organized as follows. In Section II, the Log-RFDA transmitter architecture is first described, followed by the elaboration of its transmit beampattern analysis. In Section III, the radiated pattern and range-angle localization simulations of the Log-RFDA are presented and compared with those other previously reported transmitters. Finally, conclusions are drawn in Section IV.

II. PROPOSED LOG-RFDA

A. Log-RFDA Transmitter

Considering a one-dimension (1D) array with uniformly spaced N identical isotropic radiating elements, a random logarithmic carrier frequency offset Δf_{C_n} is applied onto each antenna element,

$$f_n = f_0 + \Delta f_{C_n}, \quad (1)$$

$$\Delta f_{C_n} = \log(C_n)\delta, \quad (2)$$

where f_n denotes the radiated carrier frequency of the n^{th} element, and f_0 is a reference frequency satisfying $f_0 \gg \Delta f_{C_n}$. In (2), ‘ $\log(\cdot)$ ’ denotes the natural logarithm and δ represents a frequency offset parameter in the order of kHz. C_n is a random variable with its probability density function (PDF) $p(C_n)$. In the study, a discrete uniform PDF is assumed, i.e., $C_n \in \{1, 2, 3, \dots, C\}$ and $p(C_n) = 1/C$. The log-RFDA transmitter architecture is illustrated in Fig. 1.

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Fig. 1. Structure of proposed FDA with random logarithmically increasing frequency offset.

B. Log-RFDA transmit beampattern

In the proposed log-RFDA, the radiated signal by the n^{th} element can be written as

$$s_n(t) = W_n e^{-j2\pi f_n t} \quad 0 \leq t \leq T, \quad (3)$$

where $W_n = a_n \exp(j\varphi_n)$. a_n and φ_n respectively represent the excitation amplitude and phase at the time instant $t = 0$. Here uniform amplitude is considered, namely $a_n = 1$. T refers to one single pulse duration. As seen in Fig. 1, a far-field target A is assumed at (R_1, θ) with the first ($n = 1$) antenna element taken as the range reference and the array boresight set as the angle reference. The waveform S reaching the target A can be expressed as

$$S(t; R_1, \theta) = \sum_{n=1}^N \frac{1}{R_n} s_n \left(t - \frac{R_n}{c} \right), \quad (4)$$

where $R_n = R_1 - (n-1)d \sin \theta$ and the weighting coefficient $1/R_n \approx 1/R_1$ due to far-field operation. c is the speed of light in free space. Substituting (1), (2) and (3) into (4), we get

$$S(t; R_1, \theta) = \frac{1}{R_1} e^{-j2\pi f_0 \left(t - \frac{R_1}{c} \right)} \sum_{n=1}^N e^{j\varphi_n} \times e^{-j2\pi \left(\log(C_n) \delta \left(t - \frac{R_1}{c} \right) + \frac{(f_0 + \log(C_n) \delta)(n-1)d \sin \theta}{c} \right)}. \quad (5)$$

In (5), $\delta \ll f_0$, and $\log(C_n)$ is a number smaller than 3 when $C = 16$, thus, $\log(C_n) \delta \ll f_0$. Then (5) can be approximated as

$$S(t; R_1, \theta) = \frac{1}{R_1} e^{-j2\pi f_0 \left(t - \frac{R_1}{c} \right)} \sum_{n=1}^N e^{j\varphi_n} \times e^{-j2\pi \left(\log(C_n) \delta \left(t - \frac{R_1}{c} \right) + \frac{f_0(n-1)d \sin \theta}{c} \right)}. \quad (6)$$

In (6), the term $e^{j\varphi_n}$ is exploited to steer the radiation peak. For instance, $e^{j\varphi_n}$ can be designed as

$$e^{j\varphi_n} = e^{-j2\pi \left(\log(C_n) \delta \frac{R_X}{c} - \frac{f_0(n-1)d \sin \theta_X}{c} \right)} \quad (7)$$

to project the radiation pattern peak to a target at (R_X, θ_X) . Based on (6) and (7), the magnitude of transmit beampattern of the proposed log-RFDA at the target location (R_X, θ_X) can be expressed as

$$B(t; R, \theta) = \left| \sum_{n=1}^N e^{-j2\pi \left(\log(C_n) \delta \left(t - \frac{(R-R_X)}{c} \right) + \frac{f_0(n-1)d(\sin \theta - \sin \theta_X)}{c} \right)} \right|^2. \quad (8)$$

III. SIMULATION RESULTS AND COMPARISON ANALYSIS

Table 1 Parameters for various simulation schemes

Array name	Frequency offset of the n^{th} element	Number of elements N	Carrier frequency f_0	Element spacing d	Δf	δ	C	Maximum bandwidth
Log-RFDA	$\text{Log}(C_n) \delta$	16	8 GHz	0.015 m	$\times$	7.5 kHz	16	20.794 kHz
Log-FDA	$\text{Log}(n) \delta$	16	8 GHz	0.015 m	$\times$	7.5 kHz	16	20.794 kHz
RFDA	$C_n \Delta f$	16	8 GHz	0.015 m	1.5 kHz	$\times$	16	24 kHz
CFDA	$n \Delta f$	16	8 GHz	0.015 m	1.5 kHz	$\times$	$\times$	24 kHz
Square increasing frequency offset FDA	$n^2 \Delta f$	16	8 GHz	0.015 m	1.5 kHz	$\times$	$\times$	384 kHz
Cubic increasing frequency offset FDA	$n^3 \Delta f$	16	8 GHz	0.015 m	1.5 kHz	$\times$	$\times$	6.144 MHz

Fig .2. Normalised transmit beampattern versus range and angle at a target location ($R_X = 225$ km, $\theta_X = 30^\circ$) for (a) the proposed log-RFDA; (b) the log-FDA; (c) the CFDA; (d) the RFDA; (e) the square increasing frequency offset FDA; (f) the cubic increasing frequency offset FDA; (g) the averaged results of the proposed log-RFDA using 10,000 trials; and (h) the averaged results of RFDA using 10,000 trials.

To validate the efficacy of the proposed log-RFDA scheme, the transmit beampattern is simulated and compared with the CFDA, log-FDA, RFDA, square and cubic increasing frequency offset FDA schemes. Specifically, for the log-RFDA and RFDA schemes, the averaged transmit beampatterns with 10,000 Monte Carlo trials are shown as their frequency offsets are random variables. In addition, two examples of range-angle localization are presented by using multiple signal classification (MUSIC) algorithm for these transmitter schemes. The system parameters used in the simulations are listed in Table 1. For fair comparison, the signal bandwidth in the log-RFDA, log-FDA, RFDA and CFDA schemes are set to be approximately identical, and the other parameters are kept the same as those for the proposed log-RFDA.

A. Beampattern simulations and comparisons

Seen from Figs. 2(a), (g), the proposed log-RFDA exhibits a thumbtack-like beampattern with most of energy concentrated at the target location ($R_X = 225$ km, $\theta_X = 30^\circ$) and only a single maximum energy point occurs. Compared with Figs. 2(d), (h), a periodicity of thumbtack-like beampattern peaks results when RFDA is used, indicating that ambiguity exists in unlimited areas of the range domain when the travel time is fixed. In contrast, Fig. 2(b), the log-FDA shows a non-periodic beampattern peak. However, the energy distribution at the expected target location is not as much concentrated as log-RFDA or RFDA, e.g., the sidelobe energy levels at two locations, (211.9 km, 25°) and (236.4 km, 42°), are about the same, 50% and 46%, respectively, when compared with the energy level at (R_X, θ_X). By contrast, the two location energy levels are much lower in the log-RFDA scheme. Thus, the proposed log-RFDA achieves a better

Fig. 3. HPBW beampattern comparison of range and angle at target location ($R_x = 225$, $\theta_x = 30^\circ$) for (a) the proposed log-RFDA; (b) the log-FDA; (c) the RFDA; (d) the square increasing frequency offset FDA; and (e) the cubic increasing frequency offset FDA.

sidelobe suppression behavior. While in Fig. 2(c), CFDA a periodic transmit beampattern with range-angle coupling is shown, suggesting that the targets' range information cannot be directly obtained from the beampattern peak, meanwhile, multiple maxima beampattern characteristics exacerbates the capability of rejecting interference. In Figs. 2(e), (f), both the square and cubic frequency offset FDAs show a periodic beampattern with multiple maxima peaks. Especially, higher side lobe levels are achieved by the cubic frequency offset FDA.

In Fig. 3, the half power bandwidth (HPBW) beampattern shows the superiority of the proposed log-RFDA in respect of sidelobe suppression compared with log-FDA. Also, comparing Fig. 3(a) with Fig. 3(c), we can conclude that the log-RFDA can achieve the same array resolution as the RFDA scheme when their radiated signal bandwidths are identical. Meanwhile, Comparing Fig. 3(d), (e) with the others, the square and cubic increasing frequency offset FDAs show higher range resolutions because of the larger bandwidth.

B. Range-angle localization

Example 1: Single Target

Here, only one transmission pulse is used to obtain the joint range and angle estimation of targets by way of employing the MUSIC algorithm. In this example, it is assumed that a single target A is positioned at $(35^\circ, 120 \text{ km})$. Fig. 4 shows the simulation results of the target localization for the six transmitter array schemes. Seen from Figs. 4 (a), (b), it is shown that the target information can be successfully estimated in the range-angle dimension when the log-RFDA and log-FDA schemes are adopted, respectively. By contrast, for RFDA, square and frequency offset FDA schemes fake targets detection occurs, suggesting that target ambiguity exists in

these transmit schemes, see Figs. 4 (c), (e), (f). In Fig. 4 (c), for RFDA, ambiguity exists only in the range domain. Hence we can get the correct angle information of target A . While, ambiguity exists in the range and angle domains for square and cubic frequency offset FDA schemes, resulting in errors in the measured target position. Fig. 4(d) shows that the range-angle two-dimensional information cannot be obtained by the CFDA scheme.

Example 2: Two Target

In this example, to compare the detection resolutions between the log-FDA and the proposed log-RFDA, the two targets A and B are assumed to be positioned at $(35^\circ, 120 \text{ km})$ and $(37^\circ, 125 \text{ km})$, respectively. In Fig. 5(a), it is shown that target A and B can be detected and separated by the proposed log-RFDA scheme. In contrast, in Fig. 5(b), these two targets are entangled, leading to error occurs in locating the targets. Therefore, the proposed log-RFDA scheme can achieve a higher detection resolution than the log-FDA scheme.

IV. CONCLUSION

Frequency diverse random frequency offset combined with log-FDA was proposed in this letter. It has been shown that in the proposed scheme the characteristics of non-periodic beampattern achieved by log-FDA is preserved and that the sidelobe levels in the log-FDA scheme are suppressed. Meanwhile array resolution is improved. Therefore, the proposed log-RFDA using single carrier for each element integrates the benefits of both log-FDA and RFDA without consuming additional bandwidth.

Fig. 4. MUSIC spectra of example 1. (a) The proposed log-RFDA; (b) The log-FDA; (c) The RFDA; (d) The CFDA; (e) The square increasing frequency offset FDA; (f) The cubic increasing frequency offset FDA.

Fig. 5. MUSIC spectra of example 2. (a) The proposed log-RFDA; (b) The log-FDA.

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