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Polarization Modulation Design for Reduced RF Chain Wireless

Ibrahim A. Hemadeh, Member, IEEE, Pei Xiao, Senior Member, IEEE, Yasin Kabiri, Member, IEEE, Lixia Xiao, Member, IEEE, Vincent Fusco, Fellow, IEEE, and Rahim Tafazolli, Senior Member, IEEE

Abstract—In this treatise, we introduce a novel polarization modulation (PM) scheme, where we capitalize on the reconfigurable polarization antenna design for exploring the polarization domain degrees of freedom, thus boosting the system throughput. More specifically, we invoke the inherent properties of a dual polarized (DP) antenna for transmitting additional information carried by the axial ratio (AR) and tilt angle of elliptic polarization, in addition to the information streams transmitted over its vertical (V) and horizontal (H) components. Furthermore, we propose a special algorithm for generating an improved PM constellation tailored especially for wireless PM modulation. We also provide an analytical framework to compute the average bit error rate (ABER) of the PM system. Furthermore, we characterize both the discrete-input continuous-output memoryless channel (DCMC) capacity and the continuous-input continuous-output memoryless channel (CCMC) capacity as well as the upper and lower bounds of the CCMC capacity. The results show the superiority of our proposed PM system over conventional modulation schemes in terms of both higher throughput and lower BER. In particular, our simulation results indicate that the gain achieved by the proposed Q-dimensional PM scheme spans between 10dB and 20dB compared to the conventional modulation. It is also demonstrated that the PM system attains between 54% and 87.5% improvements in terms of ergodic capacity. Furthermore, we show that this technique can be applied to MIMO systems in a synergistic manner in order to achieve the target data rate target for 5G wireless systems with much less system resources (in terms of bandwidth and the number of antennas) compared to existing MIMO techniques.

Index Terms—5G, wireless networks, MIMO, dual-polarized, polarization modulation, index modulation, spatial modulation, polarization, MPSK, MQAM, practical implementations, channel modulation, hard-decision detection.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) techniques are capable of providing unprecedented improvements for wireless communication systems in terms of capacity [1], [2]. Explicitly, MIMO systems are capable of attaining an enhanced bit error rate (BER) performance as well as an improved throughput in comparison to single-antenna implementations, provided that each of the transmitted signals has a unique signature at each of the receive antenna elements (AEs). In the context of spatial transmission schemes, multiple AEs are spaced sufficiently apart in order to experience independent fading. Typically, array elements are placed 10λ apart from each other at the base station, where λ represents the carrier wavelength. However, it is often impractical to accommodate multiple AEs, especially in small hand-held devices [3]. One solution is to communicate at high frequency bands, such as the millimeter-wave (mmWave) band [4], which allows fitting a high number of AEs within a relatively small area, while still providing an independent fading. However, it would still be a challenging task to obtain a unique spatial signature of distinct AEs in a highly dense and closely spaced antenna arrays due to the dominant line-of-sight (LOS) component. An alternative way of overcoming this problem is to separate the transmitted signals over the polarization domain, which can be achieved by using dual-polarized AEs (DP-AEs) [5], [6]. In particular, by employing DP-AEs the number of transmit and receive AEs can be doubled in comparison to uni-polarized AEs (UP-AEs).

In a nutshell, a single DP-AE constitutes a pair of co-located and orthogonally-polarized vertical (V) and horizontal (H) components. These are typically referred to as the VH components and come in different shapes and forms [7]. The orthogonality of the V and H components offers a new means of spatial separation, namely over the polarization dimension, providing a near nil spatial correlation at both the transmitter and the receiver [8], [9]. By invoking the additional degrees-of-freedom (DoF) offered by cross-polarized components, the spectral efficiency of a MIMO system can be further enhanced [10]. Note that the communication between cross-polarized components instigates channel depolarization, which impacts the cross-channel gains. This can be measured by the cross-polar discrimination (XPD) [11].

Polarization [12] is a key element of defining the electromagnetic (EM) wave propagation in addition to the frequency, time, amplitude and phase elements [12]. It is characterized by the variations of the direction and the amplitude of an EM wave with respect to time.
TABLE I

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>ABER</td>
<td>Average bit error rate</td>
</tr>
<tr>
<td>AE</td>
<td>Antenna element</td>
</tr>
<tr>
<td>AR</td>
<td>Axial ratio</td>
</tr>
<tr>
<td>CCMC</td>
<td>Continuous-input continuous-output memoryless channel</td>
</tr>
<tr>
<td>DCMC</td>
<td>Discrete-input continuous-output memoryless channel</td>
</tr>
<tr>
<td>DP</td>
<td>Dual-input continuous-output memoryless channel</td>
</tr>
<tr>
<td>H</td>
<td>Horizontal</td>
</tr>
<tr>
<td>MUX</td>
<td>Multiplexing</td>
</tr>
<tr>
<td>PM</td>
<td>Polarization modulation</td>
</tr>
<tr>
<td>PolarSK</td>
<td>Polarization shift keying</td>
</tr>
<tr>
<td>RS</td>
<td>Random search</td>
</tr>
<tr>
<td>SM</td>
<td>Spatial modulation</td>
</tr>
<tr>
<td>TAR</td>
<td>Tilted AR</td>
</tr>
<tr>
<td>TTTO</td>
<td>Two-input two-output</td>
</tr>
<tr>
<td>UP</td>
<td>Uni-polarized</td>
</tr>
<tr>
<td>V</td>
<td>Vertical</td>
</tr>
<tr>
<td>XPD</td>
<td>Cross-polar discrimination</td>
</tr>
</tbody>
</table>

Several technologies have been long utilizing the concept of polarization, namely in optical fiber communications [13], satellite communications [14] as well as in radar applications [15], however it has recently started to gain some interest in wireless communications as presented by Shafi et al. in [16] and the references therein. For instance, the polarization effect was considered in the development of various technologies, such as for the 2D and 3D spatial channel model (SCM) for the third-generation partnership project (3GPP) and 3GPP2 model [17], [18], the indoor communications operating at the 60 GHz band [19] as well as for the mmWave channel models presented in [4], [20]. Moreover, several studies focused mainly on the polarization effect in DP-based MIMO systems [6], [21].

The effect of polarization on spatial multiplexing was investigated by Bolcskei et al. in [22], where a two-input two-output (TITO) (2 × 2)-element DP system was presented and a closed-form average BER (ABER) expression was formulated. The results showed that even with high spatial fading correlation, a DP implementation is capable of attaining enhanced multiplexing gain. This was later extended by Nabar et al. in [23] to include both transmit diversity as well as spatial multiplexing. In [24], Anreddy and Ingram suggested that the BER performance of antenna selection with DP-AE outperforms that with UP-AE.

Polarization shift keying (POLSK) was first theorized by Benedetto and Poggiofanni in [13] for optical communications and was later applied to wireless communications systems by Dhanasekaran in [25]. Here, information is transmitted by switching on and off the V and H components of a DP-AE. This approach was later combined with spatial modulation (SM) [26]–[28] by Zafari et al. in the DP-SM scheme [29], which has the advantage of using a single transmit RF chain and multiple DP-AEs. More specifically, DP-SM switches on a single DP-AE and activates one of its orthogonal components (V or H) for transmitting a single complex symbol. This allows DP-SM to implicitly convey the implicit information of the activated component index. It was shown in [30] that the DP-SM system outperforms the conventional UP-based SM scheme, while doubling the number of transmit antennas. DP-SM was later investigated again by Zafari et al. in [30] over correlated Rayleigh and Rician fading channels. In [31], Zhang et al. extended the philosophy of using a single RF chain with DP-AEs in the polarization shift keying (PolarSK) scheme. PolarSK employs a single transmit RF chain with an improved design for transmitting a single PolarSK symbol, which is a combination of complex symbols as well as a specific polarization angle. Furthermore, Park and Clerckx proposed utilizing DP-AEs for multi-user transmission in a massive MIMO structure [32], where by employing DP-AEs the number of transmitting ports is doubled.

In this treatise, we propose a novel polarization modulation (PM) scheme, which invokes the polarization characteristics of DP-AEs for transmitting an extra information over the polarization dimension in addition to a pair of complex symbols, while maintaining a reduced number of RF chains. In particular, at each DP-AE, the PM system selects one out of multiple polarization configurations that is jointly applied to the V and H components for shaping the transmitted signal’s polarization pattern. The polarization configurations applied are predefined at the transmitter and are known to the receiver. Accordingly, the transmitted signal conveys both the complex symbols and the polarization pattern applied. In fact, each polarization pattern can shape the signal carrying the complex symbols differently and hence, we refer to the polarization patterns as the space-polarization dispersion matrices.

In PM, a space-polarization dispersion matrix disperses a pair of complex symbols over the space and polarization dimensions, in a similar manner to space-time dispersion matrices [33], [34]. Space-polarization dispersion matrices are represented by (2 × 2)-element diagonal matrices, since they configure two orthogonal components (V and H) over a single time slot. Having used a matrix representation of the polarization configurations, space-polarization dispersion matrices can be generated based on a fixed criterion [35]–[37] for optimizing the performance of the PM system [38]–[40]. Against this background, the novel contributions of this treatise are as follows:

1) We propose the novel concept of polarization modulation, which invokes the polarization characteristics of DP-AEs (i.e. magnitude and angle) for achieving an improved transmission rate as well as an enhanced BER performance.

2) We formulate a closed-form generalized ABER expression of the PM system with Rayleigh fading as well as with Rician fading channels.

3) We characterize both the discrete-input continuous-output memoryless channel (DCMC) capacity and the continuous-input continuous-output memoryless channel (CCMC) capacity of our PM system. Furthermore, we provide the upper and lower bounds of CCMC capacity.
TABLE II

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DESCRIPTION</th>
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<tbody>
<tr>
<td>OB</td>
<td>Minor axis</td>
</tr>
<tr>
<td>OA</td>
<td>Major axis</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Tilt angle</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(|H\Delta|^2)</td>
</tr>
<tr>
<td>(A_i)</td>
<td>The (i)-th set of ({A_q}_{q=1}^Q)</td>
</tr>
<tr>
<td>(\mathcal{L})</td>
<td>APM constellation size, (l=1,\ldots,\mathcal{L})</td>
</tr>
<tr>
<td>(N_t/2)</td>
<td>Number of transmit DP-AEs</td>
</tr>
<tr>
<td>(N_r/2)</td>
<td>Number of receive DP-AEs</td>
</tr>
<tr>
<td>(A_q)</td>
<td>(q)-th polarization matrix</td>
</tr>
<tr>
<td>(N_t^i)</td>
<td>Number of transmit RF chains (n_t^i=1,\ldots,N_t^i)</td>
</tr>
<tr>
<td>(B_{PM})</td>
<td>Input bits to each PM encoder</td>
</tr>
<tr>
<td>(B)</td>
<td>Total number of input bits (B = N_t^i B_{PM})</td>
</tr>
<tr>
<td>(Q)</td>
<td>Number of space-polarization shape matrices</td>
</tr>
<tr>
<td>(X_{l,v,h})</td>
<td>Vector of two APM symbols ((l_v \text{ and } l_h))</td>
</tr>
<tr>
<td>(S_{(n_t^i)})</td>
<td>PM symbol at the (n_t^i)-th PM encoder</td>
</tr>
<tr>
<td>(S)</td>
<td>PM symbol</td>
</tr>
<tr>
<td>(A_{q,v})</td>
<td>The vertical polarization coefficient of (A_q)</td>
</tr>
<tr>
<td>(A_{q,h})</td>
<td>The horizontal polarization coefficient of (A_q)</td>
</tr>
<tr>
<td>(a_{q,v,h})</td>
<td>modulus of (A_{q,v,h})</td>
</tr>
<tr>
<td>(e^{j\theta_{q,v,h}})</td>
<td>argument of (A_{q,v,h})</td>
</tr>
<tr>
<td>(H)</td>
<td>Channel matrix</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(\alpha / (2N_0))</td>
</tr>
<tr>
<td>(\mathcal{X})</td>
<td>XPD, (0 \leq \mathcal{X} \leq 1)</td>
</tr>
<tr>
<td>(\mathcal{X}_{dB})</td>
<td>Inverse of the XPD in dBs</td>
</tr>
<tr>
<td>(K)</td>
<td>Rician factor</td>
</tr>
<tr>
<td>(N_0)</td>
<td>Noise power</td>
</tr>
<tr>
<td>(F_1(\cdot))</td>
<td>Hypergeometric function</td>
</tr>
<tr>
<td>(\Phi(\cdot))</td>
<td>Moment-generating function</td>
</tr>
<tr>
<td>(\chi^2_L)</td>
<td>Chi-square variable with (L) degrees of freedom</td>
</tr>
<tr>
<td>(Q(\cdot))</td>
<td>Q-function</td>
</tr>
</tbody>
</table>

4) We conceive an efficient space-polarization matrix optimization technique for optimizing the PM constellation. To be specific, the optimized matrix set is generated based on the random search method, which aims for minimizing the maximum achievable ABER as well as maximizing the DCMC capacity.

The remainder of the treatise is organized as follows. In Section II, we introduce our PM system, which includes both the transmission and detection mechanisms. Next, a DCMC and CCMC achievable capacities are presented and the lower and upper bounds of the CCMC capacity are developed in Section III. In Section IV, we derive the improved PM-constellation generation technique is introduced in Section V. Section VI provides the numerical results, while the conclusions are drawn in Section VII.

II. PROPOSED POLARIZATION MODULATION

In this contribution we consider an \((N_t \times N_r)\)-element MIMO system with \(N_t/2\) being the number of DP-AEs employed at the transmitter and \(N_r/2\) the number of DP-AEs employed at the receiver. The transmitter is equipped with \(N_t^i\) RF-chains, each of which is connected to a single DP-AE. A single DP-AE constitutes both a vertical and a horizontal component and hence, the number of transmit antennas \(N_t\) is twice that of \(N_t^i\). In what follows, we present our PM transmission scheme, which is capable of conveying information bits by invoking the polarization characteristics of multi-polarized AEs. This approach opens a new dimension for implicit information transfer, while maintaining traditional amplitude-phase modulation (APM) complex symbol communication.

A. The Concept of PM

Let us now consider the DP-AE depicted in Figure 1, which constitutes a pair of co-located horizontally-and vertically-polarized ports. The trace of the EM field polarization ellipse emitted by the DP-AE is shaped by the conjoint characteristics of its vertical and horizontal components, which could form a linear, circular and more generally an elliptic polarization, as shown Figure 1. The resultant radio wave ellipse can be represented both by the axial ratio \(AR\) and by the tilt angle \(\tau\).

The AR represents the major axis \((OA)\) to minor axis \((OB)\) ratio defined as

\[
AR = \frac{OA}{OB}, \quad (1)
\]

as seen in Figure 1. Furthermore, the major and minor axes of Equation (1) of the polarization ellipse can be expressed as [12], [41]

\[
OA = \sqrt{\frac{1}{2}} \left[ E_x^2 + E_y^2 + \sqrt{E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2\delta_L)} \right], \quad (2)
\]

and

\[
OB = \sqrt{\frac{1}{2}} \left[ E_x^2 + E_y^2 - \sqrt{E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2\delta_L)} \right], \quad (3)
\]

The AR represents the major axis \((OA)\) to minor axis \((OB)\) ratio defined as

\[
AR = \frac{OA}{OB}, \quad (1)
\]
respectively, where \( (E_x, E_y) \) define the EM field vector components with a time-phase difference angle \( \delta_L = \delta_x - \delta_y \). Likewise, the angle \( \tau \), which describes the tilt angle with respect to the principal axis, as depicted in Figure 1 is given by
\[
\tau = \frac{1}{2} \arctan \left( \frac{2E_x E_y}{E_x^2 - E_y^2} \cos (\delta_L) \right). \tag{4}
\]

In this regard, we adjust both the AR and \( \tau \) components of DP-AEs in order to produce \( Q \) distinct polarization traces (or shapes), which may be used for implicitly transferring \( \log_2 (Q) \) bits over each DP-AE, while still transmitting a pair of APM complex symbols at the V and H components.

It is worth mentioning here that \( Q \) is always an integer power of 2, which is comparable to the size of a conventional APM constellation \( L \). Hence, when a single polarization shape is applied (e.g., \( Q = 1 \) with all vertical, horizontal or slant), no information will be transmitted over the polarization domain. Furthermore, the maximum value of \( Q \) is not fixed and can be adjusted according to the system requirements. However, choosing the number of polarization shapes depends mainly on the antenna specifications, which is represented by its AR and tilt angle ranges.

To further illustrate the mechanism of our proposed PM scheme, let us consider the PM constellation depicted in Figure 2, which is formed of a 4PSK constellation as well as a \( Q = 4 \) polarization states. Given that a pair of QPSK symbols can be transmitted at the V and H components of the DP-AE, which conveys a total of 4 bits per channel use (bpcu), an additional \( \log_2 (Q) = 2 \) bits can be transmitted by switching between the four distinct polarization traces of Figure 2. This allows the system to apply a dual transmission mechanism, using the conventional APM symbols as well as the polarization information. In what follows, we detail our PM encoding scheme at the transmitter.

\section{PM System Model}

The PM transmitter block diagram is depicted in Figure 3. The \( B \)-sized input bit stream of Figure 3 is divided into \( N_t \) parallel \( B_{PM} \)-sized sub-streams, where the \( n^t_{l,v} \)-th sub-stream at the \( n^t_{l,v} \)-th RF chain of \( B_{PM} \) bits is fed into the \( n^t_{l,v} \)-th PM encoder for generating the \( n^t_{l,v} \)-th PM symbol transmitted at the \( n^t_{l,v} \)-th DP-AE, given that \( n^t_{l,v} = 1, \ldots, N_t \). The PM encoder of Figure 3 will be detailed further in Section II-E. In a nutshell, the \( B_{PM} \)-sized sub-stream constitutes the pair of information denoting the polarization information as well as the APM symbols information. More explicitly, the first \( \log_2 (Q) \) bits of \( B_{PM} \) are used to select one out of \( Q \) polarization configurations, which configures the V and H components of the \( n^t_{l,v} \)-th DP-AE, while the remaining \( \log_2 (L) \) bits are utilized to modulate a pair of \( L \)-PSK symbols. The total number of bits transmitted by a PM system equipped with \( N_t \) PM encoders is given by
\[
B = N_t^t \cdot \log_2 (L^2 Q) \cdot \text{(bits)} \tag{5}
\]

Now, the symbol \( S(n^t_{l,v}) \in \mathbb{C}^{2 \times 1} \) transmitted at the \( n^t_{l,v} \)-th DP-AE can be expressed as
\[
S(n^t_{l,v}) = A_{n^t_{l,v}}(n^t_{l,v}) X(l_{l,v}, h_{l,h}), \tag{6}
\]
where \( A_{n^t_{l,v}} = \begin{bmatrix} A_{q,v}^t(n^t_{l,v}) & 0 \\ 0 & A_{q,h}^t(n^t_{l,h}) \end{bmatrix} \in \mathbb{C}^{2 \times 2} \) denotes the polarization shaping matrix, which configures the \( n^t_{l,v} \)-th DP-AE polarization using the \( q \)-th polarization information selected from \( \{A_{q}\}_{q=1}^{Q} \). Moreover, \( A_{q,v} = a_{q,v} e^{j\theta_{q,v}} \) and \( A_{q,h} = a_{q,h} e^{j\theta_{q,h}} \) represent the V and the H polarization information, which are associated with moduli \( |a_{q,v}| \) and \( |a_{q,h}| \) as well as arguments \( \theta_{q,v} \) and \( \theta_{q,h} \), respectively.\(^1\) The polarization matrices \( \{A_{q}\}_{q=1}^{Q} \) are constructed under the power constraint of trace \( \text{tr}(A_{q}A_{q}^H) = 1 \). Furthermore, \( X(l_{l,v}, h_{l,h}) = \begin{bmatrix} x(l_{l,v}, h_{l,h}) \end{bmatrix}^T \in \mathbb{C}^{2 \times 1} \) is the APM symbol vector, where \( x(l_{l,v}, h_{l,h}) \) represents the pair of \( L \)-PSK symbols transmitted at the \( (2n^t_{l,v} - 1) \)-th V component and at the \( (2n^t_{l,h} - 1) \)-th H component of the \( n^t_{l,v} \)-th DP-AE, respectively, given that \( l = 1, \ldots, L \). Hence, the \( n^t_{l,v} \)-th PM symbol vector can be expressed as
\[
S(n^t_{l,v}) = \begin{bmatrix} A_{q,v}^t(n^t_{l,v}) & 0 \\ 0 & A_{q,h}^t(n^t_{l,h}) \end{bmatrix} \begin{bmatrix} x(l_{l,v}, h_{l,h}) \end{bmatrix} = \begin{bmatrix} a_{q,v}^t \cdot x(l_{l,v}, h_{l,v}) \\ a_{q,h}^t \cdot x(l_{l,h}, h_{l,h}) \end{bmatrix}, \tag{7}
\]
while the \( (N_t \times 1) \)-element PM symbol vector \( S \) has the following form:
\[
S = \begin{bmatrix} S(1) \cdots S(N_t) \end{bmatrix}^T. \tag{8}
\]
\(^1\)\(|a_{q,h}| \) and \(|a_{q,v}| \) are equivalent to \( E_x \) and \( E_y \) in Equations (1-4), respectively, while \( \theta_{q,h} \) and \( \theta_{q,v} \) characterize \( \delta_x \) and \( \delta_y \) of the difference angle \( \delta_L \) presented in Section II-A.
Observe in (7) that an additional means of information transmission is introduced by adjusting the joint configurations of the moduli and arguments of the diagonal vector of $\mathbf{A}_q^{(n_c)}$. Given that the coefficients of $\mathbf{A}_q^{(n_c)}$ constitute the polarization information, $\{\mathbf{A}_q^{(n_c)}\}_1^Q$ can be constructed using one of the three following modes:

- The AR mode, where the polarization information is explicitly transmitted over the AR component, which is represented by the moduli of $\mathbf{A}_q$, denoted by $|a_{q,v}|$ and $|a_{q,h}|$. In the AR mode, no information is conveyed over the tilt component (e.g. $\theta_{q,v} = \hat{\theta}_v$ and $\theta_{q,h} = \hat{\theta}_h$). Hence, PolarSK is a special case of our proposed in [31], namely when associated with the AR modulation, tilt modulation, tilted-AR modulation as well as the basic QAM/PSK multiplexing modulation without the AR modulation, tilt modulation, tilted-AR modulation as well as PM on the polarization dimension (e.g. $Q$ has a unique signature constituted by a specific combination of AR (i.e. $|a_{q,v}|$ and $|a_{q,h}|$) and tilt angles (i.e. $\theta_{q,v}$ and $\theta_{q,h}$).

The PM system may also reduce to the conventional spatial multiplexing (MUX) system [22], [42], when no information is transmitted over the polarization dimension (e.g. $Q = 1$).

In this treatise, we refer to a PM system as PM(AR/Tilt/TAR/MUX, $N_t^t$, $\Delta n$, $Q$, $L$-QAM/PSK) and to the PM encoder as PM(AR/Tilt/TAR/MUX, $Q$, $L$-QAM/PSK), where AR, Tilt, TAR and MUX represent the AR modulation, tilt modulation, tilted-AR modulation as well as the basic QAM/PSK multiplexing modulation without any polarization, respectively.

It should also be noted that by using the Tilt mode, where the polarization information is explicitly transmitted over the tilt component the system converges to the PolarSK system proposed in [31], namely when associated with $N_t^t = 1$ and the PSK modulation. Hence, PolarSK is a special case of our PM scheme.

Now, having generated the space-polarization block, the PM symbol vector $\mathbf{S}$ of (8) is transmitted over a frequency-flat and slow fading channel and received by the $2N$ DP-AEs at the receiver. In general, the vector-based system model can be expressed as

$$\mathbf{Y} = \mathbf{H} \mathbf{S} + \mathbf{V},$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix and $\mathbf{V} \in \mathbb{C}^{N_r \times 1}$ is the zero-mean additive white Gaussian noise (AWGN) vector, each of which obeys $\mathcal{CN}(0, N_0)$, given that $N_0$ is the noise power.

In this regards, $\mathbf{H}$ describes the DP channel matrix that combines both the spatial separations and the XPD depolarization effects and it is defined as [5], [6], [43]

$$\mathbf{H} = \begin{bmatrix} H_{1,1} & \cdots & H_{1,N_r^t} \\ \vdots & \ddots & \vdots \\ H_{N_r,1,n_c^t} & \cdots & H_{N_r,1,N_r^t} \end{bmatrix},$$

where $H_{n_r,1,n_c^t} \in \mathbb{C}^{2 \times 2}$ designates the TITO channel matrix between the $n_c^t$-th and $n_r$-th transmit and receive DP-AEs, respectively. In particular, each TITO channel model can be expressed as

$$H_{n_r,1,n_c^t} = \begin{bmatrix} h_{n_c,1,n_r}^v & \sqrt{\chi} h_{n_c,1,n_r}^h \\ \sqrt{\chi} h_{n_c,1,n_r}^v & h_{n_c,1,n_r}^h \end{bmatrix},$$

where $\chi$ denotes the XPD, which is a combination of the cross-polar ratio (XPR) and the cross-polar isolation (XPI) as defined in [6]. More specifically, $\chi$ parameter indicates the cross-attenuation between the co-polarized channels ($vv$, $hh$) and the cross-polarized channels ($vh$, $hv$). XPD is defined as the ratio of the power of co-polarized channels to the power of cross-polarized channels over $V$ and $H$, expressed as [44]

$$\varphi_v^{-1} = E \left[ |h_{1,j,k}^v|^2 \right] / E \left[ |h_{1,j,k}^h|^2 \right],$$

$$\varphi_h^{-1} = E \left[ |h_{1,j,k}^h|^2 \right] / E \left[ |h_{1,j,k}^v|^2 \right],$$

respectively, where $|h_{1,j,k}^h|^2$ denotes the channel fading coefficient including the cross-attenuation effect, $E \left[ |h_{1,j,k}^v|^2 \right] = 1$, $E \left[ |h_{1,j,k}^h|^2 \right] = \varphi_v$ and $E \left[ |h_{1,j,k}^h|^2 \right] = \varphi_h$. By assuming equal cross-attenuation [22] (e.g. $\varphi_v = \varphi_h = \varphi$ and $0 \leq \varphi \leq 1$), the XPD parameter can be expressed as $\chi = \varphi$. In what follows, we express the inverse of the XPD in dBs as $\chi^{-1} = -10 \log \chi$ dB.

To expound a little further on the channel model, the SISO channel presented in (11) can be defined as

$$H_{n_r,1,n_c^t} = \tilde{H} \odot \chi,$$

where $\chi = \begin{bmatrix} 1 & \sqrt{\chi} \\ \sqrt{\chi} & 1 \end{bmatrix}$, $\odot$ denotes the Hadamard element-by-element product and $\tilde{H}$ represents the UP-based channel, which can be defined as

$$\tilde{H} = \sqrt{\frac{K}{K+1}} H_{LOS} + \sqrt{\frac{1}{K+1}} H_{NLOS},$$

and hence

$$H_{n_r,1,n_c^t} = \sqrt{\frac{K}{K+1}} \chi \odot H_{LOS} + \sqrt{\frac{1}{K+1}} \chi \odot H_{NLOS},$$

given that $K$ is the K-Rician factor, $H_{LOS}$ is the LOS channel component and $H_{NLOS}$ is the NLOS Rayleigh fading channel.
D. Detection

Having generated the PM symbol vector \( S \), we now introduce the ML detector of our PM scheme. In an uncoded scenario, the PM detector aims to detect both the APM symbols as well as the polarization information of the transmit DP-AEs, where both \( \{ A_q \} \) and \( \{ x_i \} \) denoting the PM constellation \( S \) are available at the receiver.

The ML detector’s main function is to maximize the \textit{a posteriori} probability by invoking the conditional probability of receiving \( Y \) given that \( S_i \) is transmitted defined by [45]

\[
p(Y | S_i) = \frac{1}{(\pi N_0)^{N/2}} \exp \left( -\frac{|Y - HS_i|^2}{N_0} \right),
\]

where \( S_i \in S \) represents the transmitted symbol vector under the assumption that all symbols in set \( S \) are equi-probable with \( p(S_i) = 1/2^B \forall S_i \in S \). Hence, the ML detector may be formulated as

\[
\hat{q}, \hat{l} = \arg \min_{q,l} |Y - H S_i|^2
\]

\[
= \arg \min_{q,l} |Y - H A_q x_i|^2,
\]

\[
= \arg \min_{q,l} \left| Y - \sum_{n_i=1}^{N_i} H_{q,l} A_q n_i X_{n_i} \right|^2,
\]

with \( H_{n_i} \in \mathbb{C}^{N_i \times 2} \) being the \( n_i \)-th sub-channel between the \( n_i \)-th DP-AE and the \( N_i/2 \) receive AE, which denotes the \( n_i \) and \( n_i + 1 \) column vectors of \( H \). Furthermore, \( \hat{q} \) and \( \hat{l} \) denote the estimated values of \( q \) and \( l \), which designate the selected sets of \( q \) and \( l \) information, respectively.

E. Practical Considerations

In this section, we present a discussion on the feasibility of the PM system in practical implementations, namely in the context of the PM encoder design as well as of its hardware considerations. In order to invoke the polarization characteristics of a DP-AE, a phase-shifter and a power amplifier are required at its front-end. However, more complications may arise in the construction of the transmitter if maintaining a dual stream transmission per DP-AE were required. For instance, a straightforward approach is to implement two distinct RF chains; one for the V port and the other for the H port of each DP-AE, and hence a total of \( \left( \frac{2N}{L} \right) \) RF chains are required.

1) PM Encoder Design: In order to retain a dual data stream transmission with a reduced RF-chain implementation, we propose the PM encoder architecture of Figure 4. In this figure, the \( B_{PM} \) input bits are divided into three parts for constructing the PM symbol vector. More specifically, the first part is used to select the \( q \)-th phase-shifter combination \( \angle A_q = (\theta_q, \theta_h) \), while the second part is used to generate the phases of the APM symbols pair \( \angle L \rightarrow APM = (\phi_l, \phi_h) \), as shown in Figure 4. A multiplier is employed to combine both phases and generate the \( n_i \)-th PM symbol’s phase \( \angle S_{q,l} = (\phi_l, \theta_q + \phi_h) \). Furthermore, the third part is used to produce the \( (ql) \)-th power arrangement \( \langle |x_{q,l}|^2, |x_{q,h}|^2 \rangle \), which configures the variable power amplifiers to match the \( (ql) \)-th PM symbol’s moduli, as portrayed in Figure 4. Observe in Figure 4 that by entirely relying on phase-based modulation schemes, the two variable gain power amplifiers can be replaced with a single power amplifier connected at the front-end of the encoder, which improves the encoder’s power efficiency. This can be achieved with the aid of reconfigurable antennas, which are capable of continuously tuning both the AR and the tilt angle of the transmitted signal [46]. In what follows, we consider the PM encoder of Figure 4, which produces a pair of APM symbols amalgamated with the polarization information of the DP-AE.

2) Hardware Considerations: The PM encoder design requires the switching and DP-AE controlling units presented in Figure 4 for the sake of maintaining a dual-stream transmission, which increases the hardware complexity of the transmitter. This is one of the noticeable drawbacks of the PM encoder design, when compared to conventional RF implementations. However, by comparing the architecture of a single switching-aided RF-chain of Figure 4 to a pair of end-to-end RF chains, which are required to operate a couple of AEs (e.g. two DP-AE ports), the hardware requirements become less demanding. For instance, it has been shown in [47] that the most expensive component (in terms of cost and power consumption) in switch-aided transmitters, comparable to our PM design, is the RF chain (see [48] for details). This excludes the additional switching modules, serial-to-parallel (S/P) converters and the RF switches of our PM encoder. Nonetheless, the practical implementations of the PM system require further investigation, albeit the evident cost-power consumption and complexity design trade-off.

We note here that the design of Figure 4 may be relaxed by transmitting a single APM symbol rather than two symbols over the DP-AE ports. However, this would reduce the achievable throughput \( B \) of Equation (5) to \( (N_i \cdot \log_2 (LQ)) \) bits. The implementation of DP-AEs using the above-mentioned architecture is worthwhile investigating, hence in what follows we characterize both the capacity as well as the BER performance of the PM system.

III. PM System Capacity

In this section, we present both the DCMC capacity and the ergodic CCMC capacity of our PM system. Furthermore,
we formulate the upper and lower bounds of the ergodic CCMC capacity.

A. DCMC Capacity

The DCMC capacity of our PM system, which designates the mutual information expressing the number of error-free bits that can be decoded at the PM receiver, can be formulated as [49]

\[ C_{DCMC} = \max_{p(S)} I( S; Y ) \]

\[ = \max_{\{ p(S) \}_{q,t=1}} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p( Y | S ) p( S ) \log_2 \left( \sum_{\forall \bar{S}} p( Y | \bar{S} ) p( \bar{S} ) \right) dY, \]

which can be maximized by using equi-probable \( p( S ) \).

Next, by relying on the system’s conditional probability of Equation (17), the DCMC capacity can be now formulated as [49]

\[ C_{DCMC} = B - \epsilon_b \sum_{q,t} E \left[ \log_2 \left( \sum_{\forall \bar{S}} \exp \left( -\overline{\psi} | S \right) \right) \right], \]

where \( \epsilon_b = \frac{1}{\sqrt{2}} \) and \( \overline{\psi} \) is given as

\[ \overline{\psi} = -\| H ( S_i - S_i ) + V \|^2 + \| V \|^2, \]

with \( S_i \) being the transmitted symbol vector having \( \{ \bar{q}, \bar{l} \} \) indices. Unfortunately, there is no closed-form formulation available for Equation (22) and hence, we rely on numerical averaging procedures for evaluating the DCMC capacity.

B. Ergodic CCMC Capacity

On the other hand, the ergodic CCMC capacity of a MIMO system including our PM system is provided for maximizing the mutual information in a MIMO channel, which can be denoted as the maximum number of bits in an error-free continuous transmission and it is defined as [50]

\[ C_{CCMC} = \max_{p(S)} H( Y ) - H( Y | S ), \]

where \( H( Y ) \) and \( H( Y | S ) \) denote the destination entropy and the entropy of \( Y \) given \( S \), respectively, which can be written as

\[ C_{CCMC} = E \left[ \log_2 \left( I_{N_t} + \rho \frac{H( H^H Y )}{N_t} \right) \right], \]

At \( \lambda_{dB}^{-1} \to 0 \): The XPD provided in Equation (11) attains its maximum \( (\lambda=1) \) and the system transforms to a conventional UP-based MIMO system. Hence, closed-form of Equation (25) at \( \lambda=1 \) can be expressed as [51]

\[ C_{X_{dB}^{-1} \to 0} \geq \mu \log_2 \left( 1 + \frac{\rho}{N_t} \exp \left( \frac{1}{\mu} \sum_{j=1}^{K-1} \frac{1}{p} - \tilde{\gamma} \right) \right), \]

given that \( \mu = \min(N_t, N_r) \), \( K = \max(N_t, N_r) \) and \( \tilde{\gamma} \approx 0.577215 \) is Euler’s constant. This can be obtained by relying on

\[ E \left\{ \ln \left( \frac{1}{N_t} \left( H H^H \right) \right) \right\} = \sum_{j=1}^{N_r} E \left\{ \ln \Omega_j \right\} - N_r \ln N_t, \]

given that

\[ E \left\{ \ln \Omega_j \right\} = \psi(N_r - j - 1) = \sum_{p=1}^{N_r-1} \frac{1}{p} - \tilde{\gamma}, \]

where \( \Omega_j \sim \chi^2(N_r - j - 1) \).

Here, \( C_{X_{dB}^{-1} \to 0} \) represents the upper bound of the capacity \( C_{CCMC} \), since no cross polarization attenuation exists between the V and H components, and hence no degradation in the achievable capacity is incurred.

At \( \lambda_{dB}^{-1} \to \infty \): The cross V/H channels attenuation of (11) becomes infinitesimally low (i.e. \( \sqrt{\lambda} = 0 \)) and the row vectors \( h^v_{n_r/2} \) and \( h^h_{n_r/2} \) of \( H \) in (10) denoting the V and H receive AE channels at the \( n_r/2 \)-th received DP-AE, respectively, are then expressed as

\[ \begin{bmatrix} h^v_{n_r/2} \\ h^h_{n_r/2} \end{bmatrix} = \begin{bmatrix} \cdots \\ h^v_{n_r/2, n_r/2} \\ 0 \\ h^h_{n_r/2, n_r/2} \\ 0 \\ \cdots \end{bmatrix}. \]

Observe in (29) that the resultant power of \( h_{n_r/2}^v \) reduces by half, which transforms the Chi-squared variable \( \Omega_j \) of (27) into \( \Omega_j' \sim \chi^2(2(N_r-j+1)) \), where \( E \{ \ln \Omega_j' \} = \psi (\frac{2(N_r-j+1)}{2} - 1) \).

Hence, the ergodic capacity reduces to

\[ C_{X_{dB}^{-1} \to \infty} \geq \mu \log_2 \left( 1 + \frac{\rho}{N_t} \exp \left( \frac{1}{\mu} \sum_{j=1}^{K-1} \frac{1}{p} - \tilde{\gamma} \right) \right). \]

The capacity \( C_{X_{dB}^{-1} \to \infty} \) of (30) denotes the lower bound of the achievable capacity given a total V/H communication blockage. Therefore, the CCMC capacity at any XPD level is bounded by \( C_{X_{dB}^{-1} \to 0} \) and \( C_{X_{dB}^{-1} \to \infty} \) as

\[ C_{X_{dB}^{-1} \to \infty} \leq C_{X_{dB}^{-1} \to 0} \leq C_{X_{dB}^{-1} \to \infty}. \]

It is clearly seen in (31) that as the XPD attenuation increases towards infinity the achievable capacity \( C_{X_{dB}^{-1} \to \infty} \) decreases towards the lower bound (30). However, as the XPD attenuation approaches zero the achievable capacity
approaches its maximum level, which is equivalent to a \((N_t \times N_r)\)-element\(^3\) UP-based system.

It is worth noting that the DCMC capacity as seen in Equation (22) is affected by the design of the set of space-polarization dispersion matrices \(\{A_q^t\}_q\). However, the ergodic capacity provided in Equation (24) is only restricted by the transmit power, bandwidth as well as the XPD level.

**IV. ABER ANALYSIS**

The average BER for the PM system is generally formulated using the general MIMO upper-bounding technique given by [52]

\[
BER = \sum_{q=1}^{N_t} \sum_{\ell=1}^{N_r} \frac{D_h(q,\ell,q,\ell)}{\log_2(B)} P(S \rightarrow \hat{S}),
\]

(32)

where \(D_h(q,\ell,q,\ell)\) denotes the hamming distance between the bit-mapping of \((q,\ell)\) and \(\hat{S}\) and \(P(S \rightarrow \hat{S})\) is the average pairwise error probability (APEP). The APEP in fact is the average probability \(E\{P(S \rightarrow \hat{S} | \mathbf{H})\}\), which determines the probability that a PM symbol \(S\) is erroneously detected as \(\hat{S}\) given \(\mathbf{H}\) and can be expressed as [52], [53]

\[
P(S \rightarrow \hat{S} | \mathbf{H}) = P(\|\mathbf{H}(S - \hat{S}) + \mathbf{V}\| < \|\mathbf{V}\|) = Q\left(\sqrt{\frac{\|\mathbf{H}\Delta\|^2}{2N_0}}\right),
\]

(33)

where \(\Delta = S - \hat{S}\) and \(Q(\cdot)\) denotes the Q-function defined in [54] as

\[
Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2 \sin^2 \theta}\right) d\theta,
\]

(34)

and subsequently the PEP representation of (33) can now be expressed as

\[
P(S \rightarrow \hat{S} | \mathbf{H}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\gamma}{2 \sin^2 \theta}\right) d\theta,
\]

(35)

where \(\gamma\) is a legitimate range of the random variable \(\gamma\), the unconditional PEP can be formulated as [55]

\[
P(S \rightarrow \hat{S}) = \frac{1}{\pi} \int_0^{\pi/2} \Phi(\cdot) d\theta,
\]

(36)

where \(\Phi(\cdot)\) denotes the moment-generating function (MGF) of \(\gamma\).

In case of implementing UP-AEs, where no cross attenuation exists between \(V\) and \(H\) (\(X_{dB} = 0\ dB\)), our PM system reduces to an ordinary spatial multiplexing system, which can be evaluated based on Appendix B of [56]. However, when introducing DP-AEs, a new parameter \(\mathcal{X}\) denoting the DP-AE polarization effects arises and hence should be considered for the ABER formulation.

Let us consider \(\Delta_{nt} = S_{nt} - \hat{S}_{nt}\) the symbol difference at the \(n_t\)-th transmit DP-AE, which can be expressed as

\[
\Delta_{nt} = \left[\begin{array}{c}
\Delta_{nt,V} \\
\Delta_{nt,H}
\end{array}\right],
\]

(37)

where \(\Delta_{nt,V}\) and \(\Delta_{nt,H}\) denote the symbol difference at the vertical and horizontal components of the \(n_t\)-th transmit DP-AE, respectively. Given \(\alpha = \|\mathbf{H}\Delta\|^2\) and using Equation (37), \(\alpha\) can be rewritten as

\[
\alpha = \sum_{n_t=1}^{N_t} \sum_{n_r=1}^{N_r} \left| H_{n_r,n_t} \Delta_{nt} \right|^2,
\]

(38)

where \(H_{n_r,n_t}\) is the TITO sub-channel between the \(n_t\)-th transmit DP-AE and the \(n_r\)-th receive DP-AE defined in (11). Hence, \(\alpha\) appears in the following form

\[
\alpha = \sum_{n_t=1}^{N_t} \sum_{n_r=1}^{N_r} \left[ \left| h_{n_r,n_t}^{V} \right|^2 + \frac{\sqrt{\mathcal{X} h_{n_r,n_t}^{H}}}{\sqrt{h_{n_r,n_t}^{V}}} \left| \Delta_{nt,V} \right|^2 \right].
\]

(39)

Now, by using the norm representation of \(\|A_{I,X,J}\|^2 = \sum_{i=1}^I \sum_{j=1}^J |a_{i,j}|^2\), Equation (40) can be rewritten as [57]

\[
\alpha = \sum_{n_t=1}^{N_t} \left( \sum_{n_r=1}^{N_r} \left[ \left| \Delta_{nt,V} h_{n_r,n_t}^{V} \right|^2 + \sqrt{\mathcal{X}} \left| \Delta_{nt,H} \right|^2 \right] \right)^2
\]

\[
+ \sum_{n_r=1}^{N_r} \left| h_{n_r,n_t}^{H} \right|^2 \left( \left| \Delta_{nt,V} \right|^2 \right)^2.
\]

(41)

Each element of the MIMO channel matrix \(H\) of (10) is assumed to be an i.i.d random variable, and hence (41) can be reformulated as

\[
\alpha = \sum_{n_t=1}^{N_t} \left( \sum_{n_r=1}^{N_r} \left[ \left| \Delta_{nt,V} h_{n_r,n_t}^{V} \right|^2 + \sqrt{\mathcal{X}} \left| \Delta_{nt,H} \right|^2 \right] \right)^2 \gamma_n, \quad \gamma_n = \frac{1}{2N_0} (\gamma_{n_t} + \gamma_{n_r}),
\]

(42)

and

\[
\gamma = \frac{2\pi}{N_0} \left( \sum_{n_t=1}^{N_t} \gamma_n \right),
\]

(43)

with \(\gamma_{n_t} \sim \chi^2_{N_r}\), being a noncentral chi-squared random variable (RV)\(^4\) with \(N_r\) degrees of freedom and noncentrality parameter of \(K\).

\(^3\)It should be equipped with double the number of DP-AEs (i.e. \(2N_t \times 2N_r\)-element).

\(^4\)In NLOS (i.e. \(K = 0\)), \(\gamma_{n_t}\) reduces to a Chi-squared distributed random variable.
By substituting $\gamma$ of (43) into (35), the PEP can be formulated as

$$P(S \rightarrow \hat{S} | H) = \frac{1}{\pi} \int_0^{\pi/2} \exp \left( - \frac{(Y_{c_1}^2, N_r)}{4N_0 \sin^2 \theta} + \frac{(Y_N^2, N_r)}{4N_0 \sin^2 \theta} \right) d\theta,$$

and hence after averaging it over $[0, \infty]$, Equation (44) can be expressed as

$$P(S \rightarrow \hat{S}) = \frac{1}{\pi} \int_0^{\pi/2} \Phi_{Y_{c_1}^2, N_r} \left( \frac{1}{4N_0 \sin^2 \theta} \right) \Phi_{Y_N^2, N_r} \left( \frac{1}{4N_0 \sin^2 \theta} \right) d\theta.$$  \hspace{1cm} (45)

A. Rayleigh Fading, $K = 0$

In the case of considering a Rayleigh fading channel (e.g. $K = 0$), Equation (53) can be rewritten as [56]

$$P(S \rightarrow \hat{S}) = \frac{1}{\pi} \int_0^{\pi/2} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c_1} \right)^{N_r/2} d\theta,$$  \hspace{1cm} (46)

where $c_1 = \frac{2N_r}{N_0}$ and $c_2 = \frac{2N_r}{N_0}$ and the MGF of the chi-squared RV $\zeta_1^2$ is defined by

$$\Phi_{\zeta_1^2} (-s) = \left( 1 + 2as \right)^{-\frac{N_r}{2}}.$$  \hspace{1cm} (47)

The closed-form solution of (46) can be formulated using two approaches. Following the solution provided in Appendix 5A.9 in [58], the first closed-form solution of (46) can be expressed as

$$P(S \rightarrow \hat{S}) = \frac{1}{2} \left[ \phi \Pi_{k=1}^{L+1} \left( \frac{N_r}{N_0} - k \right)^{\frac{N_r}{2}} \right]_{x=-\frac{1}{2}}^{x=\infty},$$

given that

$$Jkl = \frac{\left( \frac{N_r}{N_0} - k \right)^{\frac{N_r}{2}} \Pi_{n=1}^{N_r} \left( \frac{c_l}{c_l + 1} \right)^{\frac{N_r}{2}}}{\left( \sin^2 \theta + c_l \right)^{\frac{N_r}{2}}}.$$ \hspace{1cm} (49)

For the special case of using a single DP-AE receiver (e.g. $\frac{N_r}{N_0} = 1$), Equation (48) reduces to

$$P(S \rightarrow \hat{S}) = \frac{1}{2} \sum_{l=1}^{L+1} \left[ 1 - \sqrt{c_l} \left( 1 + c_l \right)^{\frac{N_r}{2}} \right] \Pi_{j=0}^{N_r} \left( \frac{N_r}{N_0} - j \right)^{\frac{N_r}{2}} \left( \frac{c_l}{c_l + 1} \right)^{\frac{N_r}{2}} \left( \frac{c_l}{c_l + 1} \right)^{\frac{N_r}{2}}.$$ \hspace{1cm} (50)

In the second approach, the closed-form of the PEP given in (46) can be formulated as

$$P(S \rightarrow \hat{S}) = \frac{1}{2\pi^2} (c_1c_2) \beta(1/2, N_r + 1/2) \cdot F_1 \left( \frac{N_r + \frac{1}{2}N_r}{2}, 0, -c_1, -c_2 \right),$$

which is detailed in Appendix A, where $\beta(\cdot, \cdot)$ denotes the Beta function and $F_1(\cdot, \cdot)$ the confluent hypergeometric function of two variables (Equation (61)).

In the high SNR-regime (i.e. $N_0 \gg 1$), Equation (51) can be written as

$$P(S \rightarrow \hat{S}) \approx \frac{1}{2\pi} \left( \frac{Y_v Y_h}{16N_0^2} \right)^{\frac{N_r}{2}} \beta(1/2, N_r + 1/2),$$ \hspace{1cm} (52)

where $F_1(\cdot, \cdot)$ is defined at $c_1 \rightarrow \infty$ and $c_2 \rightarrow \infty$. Hence, the achievable diversity gain defined by the slope of $P(S \rightarrow \hat{S})$ is equivalent to $N_r$.

Note here that Equation (46) simplifies to Equation (36) when $N_{TB} = 0$ dB (i.e. $\gamma_v = \gamma_h$) and hence, Equation (46) can be solved using (56), Equation (64). Additionally, it can be seen in (52) that the XPD level does not have any effect on the achievable diversity order of the PM system.

B. Rician Fading, $K > 0$

When considering a Rician fading channel (e.g. $K > 0$), Equation (45) can be rewritten as [59]

$$P(S \rightarrow \hat{S}) = \frac{1}{\pi} \int_0^{\pi/2} \Phi_{\zeta_2^2, N_r} \left( \frac{Kc_l}{\sin^2 \theta + c_l} \right)^{\frac{N_r}{2}} d\theta,$$ \hspace{1cm} (53)

where the MGF of the noncentral chi-squared RV $\zeta_2^2$ is defined as [56]

$$\Phi_{\zeta_2^2} (-s) = \left( 1 + 2as \right)^{-\frac{N_r}{2}} \exp \left( -\frac{KN_r}{2} \cdot \frac{s}{1 + 2as} \right).$$ \hspace{1cm} (54)

There is no closed-form of Equation (53) and hence, it can be evaluated numerically. Note here that at $K = 0$ the problem reduces to Equation (46).

However, by using the Q-function approximation proposed in [60], the APEP of Equation (45) can be approximated as

$$P(S \rightarrow \hat{S}) \approx \frac{1}{12} \left( \Phi_{Y_{c_1}^2, N_r} \left( \frac{1}{4N_0} \right) \cdot \Phi_{Y_{c_2}^2, N_r} \left( \frac{1}{4N_0} \right) \right)^{\frac{N_r}{2}} + \frac{1}{4} \left( \Phi_{Y_{c_1}^2, N_r} \left( \frac{1}{3N_0} \right) \cdot \Phi_{Y_{c_2}^2, N_r} \left( \frac{1}{3N_0} \right) \right)^{\frac{N_r}{2}},$$ \hspace{1cm} (55)

which is detailed in Appendix B.
The PM system is comparable to a spatial multiplexing system, which suffers from a degraded performance in the presence of a LOS component, as a result of the correlation fading effect. To overcome this issue in a DP-based MIMO, we employ our PM system by relying on a single transmit DP-AE ($N_1^t=1$) at high XPDs, yielding $E\left(\hat{h}_{i,j}^{vh}\right)^2 \ll 1$ and $E\left(\hat{h}_{i,j}^{hv}\right)^2 \ll 1$.

V. SPACE-POLARIZATION IMPROVED CONSTELLATION

In this section, we introduce our PM improved-constellation generation procedure. Observe in Equation (6) that the polar-ization configuration matrix $A_q$ disperses the PSK/QAM complex symbols of $X_i$ over the spatial and polarization dimensions at a single time slot, in a conceptually similar manner to space-time dispersion matrices [33], [34], [37]. This opens a new prospect for designing the polarization shape of PM constellations.

In a nutshell, the polarization shaping matrices $\{A_q\}_1^Q$ may be randomly generated so that the performance of the system is improved. In this regard, the shaping matrices may be constructed so that the unconditional PEP of Equation (46) is minimized, while retaining the maximum achievable diversity order. Hence, the optimal set of $Q$ unit polarization vectors $A_{\text{opt}}$ can be constructed by conducting a Random Search (RS) that aims at minimizing the maximum PEP as

$$A_{\text{opt}} = \arg A_i \min \left\{ \max P(\tilde{S} \rightarrow \tilde{S}) \right\}, \quad (56)$$

which translates to

$$A_{\text{opt}} = \arg A_i \max \left\{ c_1 c_2 \right\} = \arg A_i \max \left\{ \min (\tilde{\nu}, \tilde{\tau}) \right\}, \quad (57)$$

which can be rewritten as

$$A_{\text{opt}} = \max \left\{ \min \left| \Delta \right| \right\}. \quad (58)$$

It is worth emphasizing here that the construction of $\{A_q\}_{1}^{Q}$, respectively, should fall within the polarization shaping capabilities of the DP-AE, namely its AR range (1) and its Tilt angle range (4). Additionally, multiple transmit AEs are spaced sufficiently far apart in order to experience independent fading hence, random search is performed using a single transmit DP-AE, where the $A_{\text{opt}}$ set produced is used at each DP-AE.

In what follows we present the generation process of $A_{\text{opt}}$ satisfying (58) using a TITO ($2 \times 2$)-element system. We first generate a random set of ($2 \times 2$)-element unit vectors denoting the diagonal vectors of the ($2 \times 2$)-element matrix set $A_i = \{A_i\}_1^Q$. The vector set generated should obey the Rank Criterion (i.e. $\text{rank} \left(\Delta^{\text{H}}\right) = 1 \forall q, \bar{q} \in Q$) in order to guarantee a normalized power space-polarization set. Next, we calculate the minimum Euclidean distance $d_{min} = \min \left\{ \left| \Delta \right| \right\}$. The random search continues by repeating both steps, while retaining the $A_i$ set having the maximum $d_{min}$. The algorithm presented above is summarized in Algorithm 1. Furthermore, an example is provided in Appendix C to ease understanding.

Note that by obtaining the minimum distance $d_{min} = \max \left\{ \min \left| \Delta \right| \right\}$ in (58) the PEP $P(\|H(S - \tilde{S}) + V\| \leq \|V\|)$ of (33) is minimized, and hence the DCMC exponent $\bar{\psi} = - \|H(S_i - S_i) + V\|^2 + \|V\|^2$ of (23) is subsequently minimized, which improves the achievable DCMC capacity.

Algorithm 1 Polarization Shaping Algorithm

minimum distance: $\kappa = 0$

initialize $A_{\text{opt}}$;

Start: ($i = 1 : 10^6$ loops)

Loop: Generate $Q$ random ($2 \times 1$)-element unit vectors $\{a_{q}\}_1^Q$

Compute $S$, $\tilde{S}$ and $\Delta q_i l_1 l_2$

if $\text{rank} \left(\Delta^{\text{H}}\right) = 1$

Compute $OA$, $OB$ and $\tau$ using $\{A_q\}_{q=1}^Q$

if $(OA, OB$ and $\tau$ doesn’t match the DP-AE range)

GOTO Loop

else GOTO Loop

Compute $d_{min} = \min \left\{ \left| \Delta \right| \right\}$

if ($d_{min} > \kappa$)

Apply $A_{\text{opt}} = A_i$

GOTO Loop

Return $A_{\text{opt}}$

End

VI. SIMULATION RESULTS

In this section, we present our Monte Carlo simulation results with a minimum of $10^6$ bits per SNR value as well as the theoretical analysis of our PM system. In our simulations we assume perfect CSI at the receiver side for invoking the ML optimum detector of Equation (18). Furthermore, multiple DP-AEs are spaced sufficiently far apart in order to experience independent fading. We choose the polarization shaping matrix set $\{A_q\}_{q=1}^Q$ by selecting several $AR$ and $\tau$ values based on the discussion presented in Section II-A. Particularly, Table III shows the main PM systems used in our simulations with $Q=4$ as follows:\footnote{Other systems with various $Q$ configuration are used.}: three AR systems (i.e. AR-1, ..., AR-3), two Tilt systems (i.e. Tilt-1, Tilt-2) and four TAR systems (i.e. TAR-1, ..., TAR-4). Additionally, all plots showing the performance of PM-systems associated with the RS-aided constellation presented in Section V are labeled as TAR-RS. The TAR-RS system used below is presented in Appendix C.

Note here that the tuning capabilities of DP-AEs over the AR and the tilt angle vary from one antenna to another. For instance, the reconfigurable DP-AE presented in [46] utilizes a maximum AR of 35 dB and a tilt angle spanning between $30^\circ$ and $100^\circ$.

A. Comparison Fairness

In this contribution we define fair comparison as follows: a fair performance comparison between a DP-based system and a UP-based system is attained by employing an equivalent number of AEs in both systems. To expound a little further,
consider a PM system that is equipped with a single transmit DP-AE. This system would require a single RF chain for transmitting a single PM symbol, and hence it is comparable to a UP-based system having a single UP-AE. By increasing the number of UP-AEs to match the number of ports in a single DP-AE (e.g. use UP-AEs) an additional RF chain is required, which negates fairness.

Furthermore, in MIMO implementations, AE spacing has to be on the order of ten wavelengths, in order to experience independent channel fading. In DP-based MIMOs, the V and H components of each DP-AE are separated over the polarization dimension, where \( N_t/2 \) AEs only require to be spaced far apart. However, adding \( N_t \) UP-AEs would require double the area of a DP-based system. In what follows, we refer to any simulated system as \((M \times N)\), where \( M \) and \( N \) denote the number of transmit and receive AEs (DP or UP), respectively.

**B. DCMC Capacity**

Based on the unified capacity metric provided in Equation (22), Figure 5 depicts the DCMC capacity curves of our PM system designed for achieving a normalized throughput of 4 bpcu. Here, we employed \((1 \times 1)\) DP-AEs with various PM configurations. More specifically, Figure 5 shows the DCMC curves of the AR-1-3, Tilt-1-2 and TAR-1-3 systems detailed in Table III as well as of TAR-RS and TAR-RS-1PSK, where TAR-RS-1PSK is a symbol-free RS-based PM (TAR, 1, 1, \( Q = 16 \), 1PSK) system (i.e. polarization information only). We also characterize the conventional \((1 \times 1)\) UP-AE-based 16QAM and 16PSK systems. It can be observed in Figure 5 that TAR-based PM systems outperform all the other PM configurations, while the RS-based systems achieve the highest throughput. For instance, TAR-RS outperforms PolarSK (i.e. Tilt-PM) by 2.8 dB and conventional 16QAM and 16PSK by 3.7 dB and 6 dB, respectively. This verifies the discussion presented in Section V, where constructing the optimal \( A_{opt} \) under the constraint of maximizing \( d_{min} = \{ \min \| \Delta \| \} \) could further improve the achievable capacity of the PM system.

In order to characterize the effect of the XPD on the PM system, Figure 6 portrays the 3D surface of the achievable capacity of a PM (TAR, 1, 1, \( Q = 4 \), BPSK) system with respect to XPD and SNR. Furthermore, the achievable throughput at \( \lambda_{dB} = 0 \) dB is projected onto the (SNR, Capacity)-plane for the sake of comparison. As seen in Figure 6, the achievable throughput degrades as the XPD increases, which can be clearly seen at high XPDs. To expound a little further, Figure 7 showcases the projected 3D surface of Figure 6 onto the (SNR, Capacity)-plane between \( \lambda_{dB}^{-1} = 0 \) dB and \( \lambda_{dB}^{-1} = 30 \) dB. It can be seen from the figure that a maximum degradation of 3.5 dB is observed in the DCMC capacity between \( \lambda_{dB}^{-1} = 0 \) dB and \( \lambda_{dB}^{-1} = 30 \) dB. However, the degradation in the achievable capacity becomes marginal at high XPDs, especially at \( \lambda_{dB}^{-1} > 15 \) dB.

**C. CCMC Capacity**

To investigate the ergodic CCMC capacity of our PM system, the capacities of three PM systems are illustrated by the 3D surfaces drawn in Figure 8, namely for the \((1 \times 1), (2 \times 2)\) and \((4 \times 4)\) DP-AEs MIMO arrangements. One can observe in Figure 8 that the CCMC capacity is affected both by the transmission power as well as the XPD level.

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**TABLE III**

**AR AND TILT ANGLES OF VARIOUS PM SYSTEMS DESIGNED FOR PROVIDING \( Q = 4 \) SPACE-POLARIZATION CONFIGURATIONS DENOTED BY \( \{a_{q,h}, a_{q,v}\} \) AND \( \{\theta_{q,h}, \theta_{q,v}\} \), RESPECTIVELY**

<table>
<thead>
<tr>
<th>PM</th>
<th>( {a_{q,h}, a_{q,v}}^Q )</th>
<th>( {\theta_{q,h}, \theta_{q,v}}^Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR-1</td>
<td>((1, 1), (1, 1.2), (1, 1.4), (1, 1.6))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>AR-2</td>
<td>((1, 1.09), (1, 1.6), (1, 2), (1, 2.5))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>AR-3</td>
<td>((1, 1), (1, 1.5), (1, 2), (1, 3.5))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>Tilt-1</td>
<td>((1, 1))</td>
<td>((0, 10), (0, -10), (0, 35), (0, -35))</td>
</tr>
<tr>
<td>Tilt-2</td>
<td>((1, 1))</td>
<td>((0, 35), (0, 65), (0, -35), (0, -65))</td>
</tr>
<tr>
<td>TAR-1</td>
<td>((1, 1.09), (1, 1.6), (1, 2), (1, 2.5))</td>
<td>((0, 10), (0, 35), (0, -10), (0, -35))</td>
</tr>
<tr>
<td>TAR-2</td>
<td>((1, 1), (1, 1.4), (1, 1.8), (1, 2.2))</td>
<td>((0, 35), (0, 65), (0, -35), (0, -65))</td>
</tr>
<tr>
<td>TAR-3</td>
<td>((1, 1), (1, 2), (1, 3), (1, 4))</td>
<td>((0, 1.2), (0, 0.74), (0, 37), (0, 110))</td>
</tr>
<tr>
<td>TAR-4</td>
<td>((1, 1.09), (1, 1.9), (1, 2.5), (1, 2.5))</td>
<td>((0, 1.2), (0, 45.8), (0, 1.8), (0, 45.8))</td>
</tr>
</tbody>
</table>

---

**Fig. 5.** DCMC capacity comparison between various PM systems attaining 4 bpcu by relying on the AR, Tilt and TAR configurations with different polarization shapes at an XPD of \( \lambda_{dB} = 10 \) dB.
Fig. 6. A 3D representation of the DCMC capacity of a PM(TAR, 1, 1, Q = 4, BPSK) system with respect to SNR and XPD.

Fig. 7. The 2D projection of Figure 6 onto the (SNR, DCMC)-plane.

Figures 9(a)-(c) depict the 2D projection of the 3D surfaces of Figure 8 onto the (SNR, CCMC)-plane at an XPD of $X^{-1}_{dB}=10$ dB. The theoretical upper and lower bounds of equations (26) and (30), respectively, are also shown in each figure. Furthermore, the capacity of an equivalent number of UP-AEs is shown for the sake of comparison, where the capacity improvement of DP-based systems is shown additionally by the red curve. It can be observed in Figure 9 that DP-AE implementations substantially boost the capacity of a MIMO system, achieving between 87.5% and 54% capacity improvement over an SNR range spanning between −10 dB and 40 dB, respectively, for all three systems considered. Moreover, we note that the DP-based capacity curves portrayed in Figure 9 are confined within the upper and lower bounds described in Section III, which are separated 3 dB apart. In fact, the simulated curves (through Monte Carlo) of the DP-based systems in Figures 9(a)-(c) and the lower bound analysis ($C_{X^{-1}_{dB} \to \infty}$) precisely match at $X^{-1}_{dB}=10$ dB.

To examine the effect of the XPD on the achievable CCMC capacity, let us assume that we project Figure 8 onto the (XPD, CCMC)-plane at SNR of 12 dB, 16 dB and 20 dB, as portrayed in Figure 10. The figure shows that as the XPD level increases the ergodic capacity decreases, however it remains relatively constant after a specific value of XPD, as for example at an XPD of $X^{-1}_{dB}=16$ dB at SNR of 12 dB. Furthermore, one can observe that at an even high XPD level, the maximum loss in CCMC capacity is less than 1.4 bps/Hz.

The novel polarization modulation technique presented in this paper constitutes a viable solution to significantly boost the data transmission rate for future wireless systems. In what follows, we present the BER performance of our PM system.

D. BER Simulation

In Figure 11 we compare the achievable BER performance of $(1 \times 1)$ and $(2 \times 2)$ PM systems,\(^6\) which achieve a throughput of 4 and 8 bpcu, respectively, at an XPD of $X^{-1}_{dB}=10$ dB and $K=0$. Moreover, the BER performance of their UP-AE $(1 \times 1)$ and $(2 \times 2)$ counterparts 16PSK are included for comparison,\(^7\) while the dashed curves represent the theoretical\(^{6}\)A $(1 \times 1)$-DP-AE implementation is equivalent to a $(2 \times 2)$ UP system, since the V and H components transmit over separate polarization dimensions.\(^7\)We use the same number of AEs for both systems (i.e. DP-AE and UP-AE) in order to maintain fairness. The $(1 \times 1)$ DP-AE system for instance has two input ports, while the $(1 \times 1)$ UP-AE has a single input port, while both require a single RF chain implementation. In case two UP-AEs are used to compare with the $(1 \times 1)$ DP-AE system, two RF chain are required, which leads to unfairness in the number of RF components as well as in the required transmitted power.
upper bounds developed in Section IV. Figure 11(a) shows the performance of PM(AR, \( Q = 4 \), BPSK), PM(Tilt, \( Q = 4 \), BPSK), PM(TAR, \( Q = 4 \), BPSK) and PM(TAR, \( Q = 4 \), BPSK)-RS systems associated with \((1 \times 1)\)-DP-AEs.

The RS here features the improved RS-aided constellation provided in Section V.

Here, \( \log_2(4) = 2 \) bits are used to activate one out of \( Q = 4 \) space-polarization matrices, while the remaining \( 2 \log_2(2) = 2 \) bits are modulated to a pair of BPSK symbols. The performance of the PM(TAR, \( Q = 16 \), 1PSK)-RS system is also shown in Figure 11(a), where the whole \( B_{PM} = \log_2(16) = 4 \) bits are used to switch between \( Q = 16 \) polarization shapes. It is shown in 11(a) that the PM system outperforms the conventional \((1 \times 1)\)-DP-AE by 10 dB, 15 dB and 19 dB at a BER of \( 10^{-5} \), when employing the AR, Tilt and TAR configurations, respectively. Furthermore, the improved constellation PM systems provide further BER enhancements of 2 dB and 4 dB by using the PM(TAR, \( Q = 16 \), 1PSK)-RS and PM(TAR, \( Q = 4 \), BPSK)-RS systems, respectively. Note here that the theoretical model presented in Section IV matches perfectly with the Monte Carlo simulations.

In Figure 11(b), the BER performance of the above-mentioned PM systems is shown with a \((2 \times 2)\)-element multiplexing system. As seen in the Figure, the PM system outperforms the conventional \((2 \times 2)\)-element multiplexing system by 0.5 dB, 5.4 dB, 9.5 dB, when employing the AR, Tilt and TAR configurations, and by 12.7 dB and 15 dB by using the PM(TAR, \( Q = 16 \), 1PSK)-RS and PM(TAR, \( Q = 4 \), BPSK)-RS systems, respectively.

In Figures 12(a)-(b), we show the performance of a PM(TAR, \( Q = 4 \), BPSK)-RS system (i.e. TAR-RS) transmitting over a Rician fading channel at 4 bpcu, while employing a single transmit DP-AE at a high XPD of \( X_{dB}^{-1} = 30 \) dB. Furthermore, Figures 12(a)-(b) include both the exact and approximated theoretical bounds presented in equations (53) and (55), respectively. In particular, in Figure 12(a) we investigate the effect of the Rician factor on the performance of the PM system at \( K = 0, 5, 10 \) and 15, when associated with \((1 \times 1)\)-element implementation. We notice in Figure 12(a)
that upon increasing $K$ the BER performance of the PM system improves. As indicated by Equation (53), the ABER improves exponentially with the value of $K$. Furthermore, the exact theoretical model of (53) matches perfectly with the Monte Carlo simulations, while the approximate model of (55) is marginally shifted at $K = 0$ and $K = 5$, which perfectly overlaps at low BER values. On the other hand, Figure 12(b) shows the performance of the simulated PM system with a different number of receive DP-AEs, namely $N_r/2 = 1, 2, 4$ and 8 at $K = 5$. We notice in the figure that the approximate theoretical bound developed in Section IV tends to accurately match both the exact bound and the Monte Carlo simulations as the number of receive AEs increase.

A comparison between multiple configurations of 4-level $(Q = 4)$ PM systems with $(1 \times 2)$-elements is illustrated in Figure 13, which all achieve a spectral efficiency of 4 bpcu at $X_{d_B}^{-1} = 10$ dB and $K = 0$. To expound further, the PM systems under study are: AR-1-3, Tilt-1-2 and TAR-1-4 as well as TAR-RS, as detailed in Table III. As observed in Figure 13, the achievable performance of all systems spans over 35 dBs of SNR at a BER of $10^{-5}$. In all cases, it can be observed that TAR-based systems exhibit the best BER performance compared to AR-based and Tilt-based systems. This can be attributed to the multi-dimensional structure of TAR-based PM, where polarization information is dispersed over both the AR and the tilt contrary to other configurations (e.g. AR and Tilt) that exploit either of them.

In Figure 14, we compare the BER performance of our PM system with its DP-AE-based counterparts. More specifically, we compare the BER performance of PM(TAR, 1, 1, $Q = 2$, BPSK) with that of a DP-SM(1, 1, QPSK) system [30] as well as that of a PolarSK($N_r/2 = 1$, $Q = 2$, BPSK) system [31], where each exhibits a transmission rate of 3 bpcu over Rayleigh fading channel (i.e. $K = 0$) at an XPD of $X_{d_B}^{-1} = 10$ dB. Figure 14 further shows the performance of the improved-constellation PM(TAR, 1, 1, $Q = 2$, BPSK)-RS and PM(TAR, 1, 1, $Q = 8$, 1PSK)-RS systems as well as the performance of UP-AE-based SM and Quadrature SM (QSM) systems associated with $N_t = 2$ AEs. It can be observed from Figure 14 that our PM system outperforms PolarSK, DP-SM and the conventional SM by 2 dB, 1.2 dB, 22 dB, respectively.

To elaborate further on the effect of the level of XPD on the BER performance, the BER performance of a $(2 \times 2)$-DP-AE PM system associated with an PM(TAR, $Q = 2$, BPSK) encoder at different XPD levels is presented in Figure 15. More specifically, we show the BER performance of the system at an XPD spanning between $X_{d_B}^{-1} = 0$ dB and $X_{d_B}^{-1} = 30$ dB with a step of 5 dB, where the theoretical boundaries are shown exclusively at $X_{d_B}^{-1} = 0$ dB and $X_{d_B}^{-1} = 30$ dB. Figure 15 demonstrates that the performance of the PM system is directly affected by the XPD level, where it improves as the...
The BER performance of a $(1 \times 2)$-DP-AE PM systems associated with $Q = 4$ AR, Tilt and TAE configurations, which achieve a throughput of 4 bpcu at an XPD of $\lambda_{dB}^{-1} = 10$ dB and $K = 0$.

Fig. 13.


Fig. 14.

XPD decreases due to the increased polarization diversity gain. However, it can be seen in the figure that the performance is marginally affected when $\lambda_{dB}^{-1} \geq 25$ dB. Furthermore, the theoretical boundaries presented in Figure 15 confirms the precision of the XPD parameter provided in equations (46) and (51).

It can be observed in Figures 11-15 that the theoretical boundaries provided in Section IV match the Monte Carlo simulations for all PM configurations, namely for the AR, Tilt, TAR and MUX configurations over various antenna arrangements. In what follows, we present our conclusion.

VII. CONCLUSION

In this treatise, we have introduced a novel modulation technique referred to as the polarization modulation, which invokes the polarization characteristics of a DP-AE for data transmission. More specifically, a block of information in a PM system is formed by dispersing a pair of PSK/QAM symbols into the space- and polarization- dimension with the aid of $Q$ polarization shaping matrices $\{A_q\}_{q=1}^Q$. The polarization shaping matrix may adjust the AR, Tilt or Tilted-AR of the EM matrix with the aid of a single RF-chain per DP-AE. The polarization shaping matrices can be selected empirically, however we have proposed a special algorithm for generating an improved-constellation tailored for the PM modulation. Furthermore, we provided a theoretical analysis for the DCMC and CCMC capacity as well as for the BER performance of the PM system. It has been shown that by invoking the polarization dimension, the ergodic capacity of a DP-based MIMO system can be improved by 54\% to 87.5\% compared to UP-based MIMO. Similarly, the DCMC capacity of our PM system was improved by up to 6 dB in comparison to systems relying on UP-AE. Furthermore, the simulation results indicated that the gain achieved by our proposed PM system relying on $Q$-state polarization levels spans between 10dB and 20dB over UP-AE-based conventional systems. Our simulation also showed that by utilizing the proposed improved-constellation algorithm the DCMC capacity and BER performance of our PM system have significantly improved.

APPENDIX A

The derivation of Equation (51) can be formulated by substituting $u = \sin^2(\theta)$ and $d\theta = \frac{du}{2\sqrt{u(1-u)}}$ into Equation (46),
yielding
\[ P \left( S \rightarrow \hat{S} \right) = \frac{1}{\pi} \int_{0}^{1} \left( \frac{u}{u + c_1} \right)^{-\frac{\nu}{2}} \left( \frac{u}{u + c_1} \right)^{-\frac{\nu}{2}} \frac{du}{2\sqrt{u(1-u)}}. \] (59)

and
\[ P \left( S \rightarrow \hat{S} \right) = \frac{N_{s}}{2\pi} \int_{0}^{1} u^{N_{s}-\frac{1}{2}} (1-u)^{-\frac{1}{2}} \left( 1 + \frac{1}{u} \right)^{-\frac{\nu}{2}} \left( 1 + \frac{1}{u} \right)^{-\frac{\nu}{2}}. \] (60)

Now, by relying on the confluent hypergeometric function of two variables given as (Section 9.18 [61])
\[ F_1(\alpha, \beta; \gamma; x, y) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{1} z^{\alpha-1} (1-z)^{\beta-1} (1-xz)^{-\beta} (1-yz)^{-\beta} dz, \] (61)

the closed-form expression of (60) can be expressed as shown in Equation (51), where \( \Gamma(\cdot) \) denotes the Gamma function.

**APPENDIX B**

Instead of using the Q-function defined in Equation (34), we can simply use the approximation defined by \([60]\) as
\[ Q(x) \approx \frac{1}{12} e^{-x^2} + \frac{1}{4} e^{-2x^2}. \] (62)

By plugging (62) into (33), we arrive at
\[ P \left( S \rightarrow \hat{S} | H \right) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-2x^2}. \] (63)

Given \( N_{s} \) receive AEs, the SNR of the \( n_{s}/2 \)-th channel denoting the channel received at the \( n_{s}/2 \)-th AE is given by \( \gamma_{n_{s}/2} = \frac{1}{2N_{s}} \left\{ \gamma_{c_{v}} + \gamma_{h} \right\}_{N_{s}=2} \), where \( \left\{ \gamma_{c_{v}} + \gamma_{h} \right\}_{N_{s}=2} \) is equivalent to Equation (42) with \( N_{s} = 1 \) and \( N_{s} = 2 \).

Now, the average of PEP can be expressed as
\[ P \left( S \rightarrow \hat{S} \right) \approx \prod_{n_{s}/2}^{N_{s}/2} \left[ \exp \left( -\frac{\gamma_{n_{s}/2}}{2} \right) f_{\gamma} \left( \gamma_{n_{s}/2} \right) \right] \times \exp \left( -\frac{2\gamma_{n_{s}/2}}{3} \right) f_{\gamma} \left( \gamma_{n_{s}/2} \right). \] (64)

By using the definition of the MGF function in [56], Equation (21)), the close-form expression of \( P \left( S \rightarrow \hat{S} \right) \) can be formulated as shown in Equation (55).

**APPENDIX C**

Here, we provide an example of an RS-based PM(TAR, \( Q = 4 \), BPSK) system using the technique presented in Section V, which is referred to as TAR-RS in Section VI. Consider a PM system that relies on a set of BPSK symbols \( X_i = \{-1, +1\} \) and on a randomly generated set \( \{ A_{i_j} \}_Q \) for data transmission, which can be formulated as follows:
\[ A_1 = \left[ -0.331952 + 0.686751i \right], \]
\[ A_2 = \left[ -0.853098 + 0.0743714i \right], \]
\[ A_3 = \left[ -0.493946 - 0.228332i \right], \]
\[ A_4 = \left[ -0.160197 - 0.557432i \right], \]
where \( q = 1, \ldots, Q = 4 \). By using Equations (1-4) These configurations can be translated to the following parameters:
\[ E_h = \{0.762771, 0.856334, 0.544168, 0.579994\}, \]
\[ E_q = \{0.646669, 0.516423, 0.838976, 0.814621\}, \]
\[ \theta_h = \{115.798, 175.017, -155.191, -106.034\}, \]
\[ \theta_e = \{167.461, 87.8153, 161.886, -122.54\}, \]
and finally,
\[ \tau = \{130.29, 121.088, 57.0821, 54.55\}, \]
and
\[ AR_{dB} = \{42.1995, 38.6012, 27.1086, 52.4841\}. \] (74)

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Polarization Modulation Design for Reduced RF Chain Wireless

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Abstract—In this treatise, we introduce a novel polarization modulation (PM) scheme, where we capitalize on the reconfigurable polarization antenna design for exploring the polarization domain degrees of freedom, thus boosting the system throughput. More specifically, we invoke the inherent properties of a dual polarized (DP) antenna for transmitting additional information carried by the axial ratio (AR) and tilt angle of elliptic polarization, in addition to the information streams transmitted over its vertical (V) and horizontal (H) components. Furthermore, we propose a special algorithm for generating an improved PM constellation tailored especially for wireless PM modulation. We also provide an analytical framework to compute the average bit error rate (ABER) of the PM system. Furthermore, we characterize both the discrete-input continuous-output memoryless channel (DCMC) and continuous-input continuous-output memoryless channel (CCMC) capacity as well as the upper and lower bounds of the CCMC capacity. The results show the superiority of our proposed PM system over conventional modulation schemes in terms of both higher throughput and lower BER. In particular, our simulation results indicate that the gain achieved by the proposed D-dimensional PM scheme spans between 10dB and 20dB compared to the conventional modulation. It is also demonstrated that the PM system attains between 54% and 87.5% improvements in terms of ergodic capacity. Furthermore, we show that this technique can be applied to MIMO systems in a synergistic manner in order to achieve the target data rate target for 5G wireless systems with much less system resources (in terms of bandwidth and the number of antennas) compared to existing MIMO techniques.

Index Terms—5G, wireless networks, MIMO, dual-polarized, polarization modulation, index modulation, spatial modulation, polarization, MPSK, MQAM, practical implementations, channel modulation, hard-decision detection.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) techniques are capable of providing unprecedented improvements for wireless communication systems in terms of capacity [1], [2]. Explicitly, MIMO systems are capable of attaining an enhanced bit error rate (BER) performance as well as an improved throughput in comparison to single-antenna implementations, provided that each of the transmitted signals has a unique signature at each of the receive antenna elements (AEs). In the context of spatial transmission schemes, multiple AEs are spaced sufficiently apart in order to experience independent fading. Typically, array elements are placed 10λ apart from each other at the base station, where λ represents the carrier wavelength. However, it is often impractical to accommodate multiple AEs, especially in small hand-held devices [3]. One solution is to communicate at high frequency bands, such as the millimeter-wave (mmWave) band [4], which allows fitting a high number of AEs within a relatively small area, while still providing an independent fading. However, it would still be a challenging task to obtain a unique spatial signature of distinct AEs in a highly dense and closely spaced antenna arrays due to the dominant line-of-sight (LOS) component. An alternative way of overcoming this problem is to separate the transmitted signals over the polarization domain, which can be achieved by using dual-polarized AEs (DP-AEs) [5], [6]. In particular, by employing DP-AEs the number of transmit and receive AEs can be doubled in comparison to uni-polarized AEs (UP-AEs).

In a nutshell, a single DP-AE constitutes a pair of co-located and orthogonally-polarized vertical (V) and horizontal (H) components. These are typically referred to as the VH components and come in different shapes and forms [7]. The orthogonality of the V and H components offers a new means of spatial separation, namely over the polarization dimension, providing a near nil spatial correlation at both the transmitter and the receiver [8], [9]. By invoking the additional degrees-of-freedom (DoF) offered by cross-polarized components, the spectral efficiency of a MIMO system can be further enhanced [10]. Note that the communication between cross-polarized components instigates channel depolarization, which impacts the cross-channel gains. This can be measured by the cross-polar discrimination (XPD) [11].

Polarization [12] is a key element of defining the electromagnetic (EM) wave propagation in addition to the frequency, time, amplitude and phase elements [12]. It is characterized by the variations of the direction and the amplitude of an EM wave with respect to time.
Several technologies have been long utilizing the concept of polarization, namely in optical fiber communications [13], satellite communications [14] as well as in radar applications [15], however it has recently started to gain some interest in wireless communications as presented by Shafi et al. in [16] and the references therein. For instance, the polarization effect was considered in the development of various technologies, such as for the 2D and 3D spatial channel model (SCM) for the third-generation partnership project (3GPP) and 3GPP2 model [17], [18], the indoor communications operating at the 60 GHz band [19] as well as for the mmWave channel models presented in [4], [20]. Moreover, several studies focused mainly on the polarization effect in DP-based MIMO systems [6], [21].

The effect of polarization on spatial multiplexing was investigated by Bolcskei et al. in [22], where a two-input two-output (TITO) $(2 \times 2)$-element DP system was presented and a closed-form average BER (ABER) expression was formulated. The results showed that even with high spatial fading correlation, a DP implementation is capable of attaining enhanced multiplexing gain. This was later extended by Nabar et al. in [23] to include both transmit diversity as well as spatial multiplexing. In [24], Anreddy and Ingram suggested that the BER performance of antenna selection with DP-AE outperforms that with UP-AE.

Polarization shift keying (POLSK) was first theorized by Benedetto and Poggiolini in [13] for optical communications and was later applied to wireless communications systems by Dhanasekaran in [25]. Here, information is transmitted by switching on and off the V and H components of a DP-AE. This approach was later combined with spatial modulation (SM) [26]–[28] by Zafari et al. in the DP-SM scheme [29], which has the advantage of using a single transmit RF chain and multiple DP-AEs. More specifically, DP-SM switches on a single DP-AE and activates one of its orthogonal components (V or H) for transmitting a single complex symbol. This allows DP-SM to implicitly convey the implicit information of the activated component index. It was shown in [30] that the DP-SM system outperforms the conventional UP-based SM scheme, while doubling the number of transmit antennas. DP-SM was later investigated again by Zafari et al. in [30] over correlated Rayleigh and Rician fading channels. In [31], Zhang et al. extended the philosophy of using a single RF chain with DP-AEs in the polarization shift keying (PolarSK) scheme. PolarSK employs a single transmit RF chain with an improved design for transmitting a single PolarSK symbol, which is a combination of complex symbols as well as a specific polarization angle. Furthermore, Park and Clerckx proposed utilizing DP-AEs for multi-user transmission in a massive MIMO structure [32], where by employing DP-AEs the number of transmitting ports is doubled.

In this treatise, we propose a novel polarization modulation (PM) scheme, which invokes the polarization characteristics of DP-AEs for transmitting an extra information over the polarization dimension in addition to a pair of complex symbols, while maintaining a reduced number of RF chains. In particular, at each DP-AE, the PM system selects one out of multiple polarization configurations that is jointly applied to the V and H components for shaping the transmitted signal’s polarization pattern. The polarization configurations applied are predefined at the transmitter and are known to the receiver. Accordingly, the transmitted signal conveys both the complex symbols and the polarization pattern applied. In fact, each polarization pattern can shape the signal carrying the complex symbols differently and hence, we refer to the polarization patterns as the space-polarization dispersion matrices.

In PM, a space-polarization dispersion matrix disperses a pair of complex symbols over the space and polarization dimensions, in a similar manner to space-time dispersion matrices [33], [34]. Space-polarization dispersion matrices are represented by $(2 \times 2)$-element diagonal matrices, since they configure two orthogonal components (V and H) over a single time slot. Having used a matrix representation of the polarization configurations, space-polarization dispersion matrices can be generated based on a fixed criterion [35]–[37] for optimizing the performance of the PM system [38]–[40]. Against this background, the novel contributions of this treatise are as follows:

1) We propose the novel concept of polarization modulation, which invokes the polarization characteristics of DP-AEs (i.e. magnitude and angle) for achieving an improved transmission rate as well as an enhanced BER performance.

2) We formulate a closed-form generalized ABER expression of the PM system with Rayleigh fading as well as with Rician fading channels.

3) We characterize both the discrete-input continuous-output memoryless channel (DCMC) capacity and the continuous-input continuous-output memoryless channel (CCMC) capacity of our PM system. Furthermore, we provide the upper and lower bounds of CCMC capacity.

### TABLE I

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
<th>DEFINITION</th>
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<tbody>
<tr>
<td>ABER</td>
<td>Average bit error rate</td>
</tr>
<tr>
<td>AE</td>
<td>Antenna element</td>
</tr>
<tr>
<td>AR</td>
<td>Axial ratio</td>
</tr>
<tr>
<td>CCMC</td>
<td>Continuous-input continuous-output memoryless channel</td>
</tr>
<tr>
<td>DCMC</td>
<td>Discrete-input continuous-output memoryless channel</td>
</tr>
<tr>
<td>DP</td>
<td>Dual-polarized</td>
</tr>
<tr>
<td>H</td>
<td>Horizontal</td>
</tr>
<tr>
<td>MUX</td>
<td>Multiplexing</td>
</tr>
<tr>
<td>PM</td>
<td>Polarization modulation</td>
</tr>
<tr>
<td>PolarSK</td>
<td>Polarization shift keying</td>
</tr>
<tr>
<td>RS</td>
<td>Random search</td>
</tr>
<tr>
<td>SM</td>
<td>Spatial modulation</td>
</tr>
<tr>
<td>TAR</td>
<td>Tilted AR</td>
</tr>
<tr>
<td>TITO</td>
<td>Two-input two-output</td>
</tr>
<tr>
<td>UP</td>
<td>Uni-polarized</td>
</tr>
<tr>
<td>V</td>
<td>Vertical</td>
</tr>
<tr>
<td>XPD</td>
<td>Cross-polar discrimination</td>
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</table>
TABLE II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>OB</td>
<td>Minor axis</td>
</tr>
<tr>
<td>OA</td>
<td>Major axis</td>
</tr>
<tr>
<td>α</td>
<td>Tilt angle</td>
</tr>
<tr>
<td>A_i</td>
<td>The i-th set of {A_q}_{q=1}</td>
</tr>
<tr>
<td>L</td>
<td>APM constellation size, l=1,...,L</td>
</tr>
<tr>
<td>N_t/2</td>
<td>Number of transmit DP-AEs</td>
</tr>
<tr>
<td>N_r/2</td>
<td>Number of receive DP-AEs</td>
</tr>
<tr>
<td>A_q</td>
<td>q-th polarization matrix</td>
</tr>
<tr>
<td>N_{t,c}</td>
<td>Number of transmit RF chains</td>
</tr>
<tr>
<td>B_{PM}</td>
<td>Input bits to each PM encoder</td>
</tr>
<tr>
<td>B</td>
<td>Total number of input bits</td>
</tr>
<tr>
<td>Q</td>
<td>Number of space-polarization shape matrices</td>
</tr>
<tr>
<td>X_{l,v,h}</td>
<td>Vector of two APM symbols (l_v and l_h)</td>
</tr>
<tr>
<td>S(\nu_{t,c})</td>
<td>PM symbol at the \nu_{t,c}-th PM encoder</td>
</tr>
<tr>
<td>S</td>
<td>PM symbol</td>
</tr>
<tr>
<td>A_{q,v}</td>
<td>The vertical polarization coefficient of A_q</td>
</tr>
<tr>
<td>A_{q,h}</td>
<td>The horizontal polarization coefficient of A_q</td>
</tr>
<tr>
<td>a_{q,v/h}</td>
<td>modulus of A_{q,v/h}</td>
</tr>
<tr>
<td>e^{j\theta_{q,v/h}}</td>
<td>argument of A_{q,v/h}</td>
</tr>
<tr>
<td>H</td>
<td>Channel matrix</td>
</tr>
<tr>
<td>γ</td>
<td>XPD, 0 ≤ γ ≤ 1</td>
</tr>
<tr>
<td>X^{-1}</td>
<td>Inverse of the XPD in dBs</td>
</tr>
<tr>
<td>K</td>
<td>Rician factor</td>
</tr>
<tr>
<td>N_0</td>
<td>Noise power</td>
</tr>
<tr>
<td>F_1(·)</td>
<td>Hypergeometric function</td>
</tr>
<tr>
<td>Φ(·)</td>
<td>Moment-generating function</td>
</tr>
<tr>
<td>χ^2_L</td>
<td>Chi-square variable with L degrees of freedom</td>
</tr>
<tr>
<td>Q(·)</td>
<td>Q-function</td>
</tr>
</tbody>
</table>

4) We conceive an efficient space-polarization matrix optimization technique for optimizing the PM constellation. To be specific, the optimized matrix set is generated based on the random search method, which aims for minimizing the maximum achievable ABER as well as maximizing the DCMC capacity.

The remainder of the treatise is organized as follows. In Section II, we introduce our PM system, which includes both the transmission and detection mechanisms. Next, a DCMC and CCMC achievable capacities are presented and the lower and upper bounds of the CCMC capacity are developed in Section III. In Section IV, we derive the closed-form ABER expression. Then, the improved PM-constellation generation technique is introduced in Section V. Section VI provides the numerical results, while the conclusions are drawn in Section VII.

II. PROPOSED POLARIZATION MODULATION

In this contribution we consider an \((N_t \times N_r)\)-element MIMO system with \(N_t/2\) being the number of DP-AEs employed at the transmitter and \(N_r/2\) the number of DP-AEs at the receiver. The transmitter is equipped with \(N_{t,c}\) RF-chains, each of which is connected to a single DP-AE. A single DP-AE constitutes both a vertical and a horizontal component and hence, the number of transmit antennas \(N_t\) is twice that of \(N_{t,c}\). In what follows, we present our PM transmission scheme, which is capable of conveying information bits by invoking the polarization characteristics of multi-polarized AEs. This approach opens a new dimension for implicit information transfer, while maintaining traditional amplitude-phase modulation (APM) complex symbol communication.

A. The Concept of PM

Let us now consider the DP-AE depicted in Figure 1, which constitutes a pair of co-located horizontally-and vertically-polarized ports. The trace of the EM field polarization ellipse emitted by the DP-AE is shaped by the joint characteristics of its vertical and horizontal components, which could form a linear, circular and more generally an elliptic polarization, as shown Figure 1. The resultant radio wave ellipse can be represented both by the axial ratio (AR) and by the tilt angle \(τ\). The AR represents the major axis \((OA)\) to minor axis \((OB)\) ratio defined as

\[
AR = \frac{OA}{OB},
\]

as seen in Figure 1. Furthermore, the major and minor axes of Equation (1) of the polarization ellipse can be expressed as [12], [41]

\[
OA = \sqrt{\frac{1}{2} \left[ E_x^2 + E_y^2 + \sqrt{E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2δ_L)} \right]},
\]

and

\[
OB = \sqrt{\frac{1}{2} \left[ E_x^2 + E_y^2 - \sqrt{E_x^4 + E_y^4 + 2E_x^2E_y^2\cos(2δ_L)} \right]},
\]

\[
\text{Fig. 1. Dual-polarized antenna element with an elliptic polarization state.}
\]
respective, where \((E_x, E_y)\) define the EM field vector components with a time-phase difference angle \(\delta_L = \delta_x - \delta_y\). Likewise, the angle \(\tau\), which describes the tilt angle with respect to the principal axis, as depicted in Figure 1 is given by
\[
\tau = \frac{1}{2} \arctan \left( \frac{2E_x E_y}{E_x^2 - E_y^2} \cos(\delta_L) \right). \tag{4}
\]

In this regard, we adjust both the AR and \(\tau\) components of DP-AEs in order to produce \(Q\) distinct polarization traces (or shapes), which can be used for implicitly transferring \(\log_2(Q)\) bits over each DP-AE, while still transmitting a pair of APM complex symbols at the V and H components.

It is worth mentioning here that \(Q\) is always an integer power of 2, which is comparable to the size of a conventional APM constellation \(L\). Hence, when a single polarization shape is applied (e.g., \(Q = 1\) with all vertical, horizontal or slant), no information will be transmitted over the polarization domain. Furthermore, the maximum value of \(Q\) is not fixed and can be adjusted according to the system requirements. However, choosing the number of polarization shapes depends mainly on the antenna specifications, which is represented by its AR and tilt angle ranges.

To further illustrate the mechanism of our proposed PM scheme, let us consider the PM constellation depicted in Figure 2, which is formed of a 4PSK constellation as well as a \(Q = 4\) polarization states. Given that a pair of QPSK symbols can be transmitted at the V and H components of the DP-AE, which conveys a total of 4 bits per channel use (bpcu), an additional \(\log_2(Q) = 2\) bits can be transmitted by switching between the four distinct polarization traces of Figure 2. This allows the system to apply a dual transmission mechanism, using the conventional APM symbols as well as the polarization information. In what follows, we detail our PM encoding scheme at the transmitter.

**B. PM System Model**

The PM transmitter block diagram is depicted in Figure 3. The \(B\)-sized input bit stream of Figure 3 is divided into \(N^t_c\) parallel \(B_{PM}\)-sized sub-streams, where the \(n^t_c\)-th sub-stream at the \(n^t_c\)-th RF chain of \(B_{PM}\) bits is fed into the \(n^t_c\)-th PM encoder for generating the \(n^t_c\)-th PM symbol transmitted at the \(n^t_c\)-th DP-AE, given that \(n^t_c = 1, \ldots, N^t_c\). The PM encoder of Figure 3 will be detailed further in Section II-E. In a nutshell, the \(B_{PM}\)-sized sub-stream constitutes the pair of information denoting the polarization information as well as the APM symbols information. More explicitly, the first \(\log_2(Q)\) bits of \(B_{PM}\) are used to select one out of \(Q\) polarization configurations, which configures the V and H components of the \(n^t_c\)-th DP-AE, while the remaining \(2\log_2(L)\) bits are invoked to modulate a pair of \(L\)-PSK symbols. The total number of bits transmitted by a PM system equipped with \(N^c_e\) PM encoders is given by
\[
B = N^t_c \cdot \log_2 (L^2 Q), \tag{5}\text{(bits)}
\]

Now, the symbol \(S(n^t_c) \in \mathbb{C}^{2 \times 1}\) transmitted at the \(n^t_c\)-th DP-AE can be expressed as
\[
S(n^t_c) = A_q^{(n^t_c)} X(n^t_c), \tag{6}
\]
where \(A_q^{(n^t_c)} = \begin{bmatrix} A_{q,v} & 0 \\ 0 & A_{q,h} \end{bmatrix} \in \mathbb{C}^{2 \times 2}\) denotes the polarization shaping matrix, which configures the \(n^t_c\)-th DP-AE polarization using the \(q\)-th polarization information selected from \(\{A_q\}_{q=1}^Q\). Moreover, \(A_{q,v} = a_{q,v} e^{j\theta_{q,v}}\) and \(A_{q,h} = a_{q,h} e^{j\theta_{q,h}}\) represent the V and the H polarization information, which are associated with moduli \(|a_{q,v}|\) and \(|a_{q,h}|\) as well as arguments \(\theta_{q,v}\) and \(\theta_{q,h}\), respectively.\(^1\) The polarization matrices \(\{A_q\}_{q=1}^Q\) are constructed under the power constraint of trace \((A_q A_q^H) = 1\). Furthermore, \(X(n^t_c) = [x_{1,v}^t, x_{1,h}^t]^T \in \mathbb{C}^{2 \times 1}\) is the APM symbol vector, where \(x_{1,v}^t\) and \(x_{1,h}^t\) represent the pair of \(L\)-PSK symbols transmitted at the \((2n^t_c - 1)\)-th V component and at the \((2n^t_c)\)-th H component of the \(n^t_c\)-th DP-AE, respectively, given that \(l = 1, \ldots, L\) is the \(n^t_c\)-th PM symbol vector can be expressed as
\[
S(n^t_c) = \begin{bmatrix} A_{q,v}^t & 0 \\ 0 & A_{q,h}^t \end{bmatrix} [x_{1,v}^t, x_{1,h}^t]^T = \begin{bmatrix} A_{q,v}^t \cdot x_{1,v}^t \\ A_{q,h}^t \cdot x_{1,h}^t \end{bmatrix}, \tag{7}
\]
while the \((N^t_c \times 1)\)-element PM symbol vector \(S\) has the following form:
\[
S = [S^{(1)} \ldots S^{(N^t_c)}]^T. \tag{8}
\]
\(^1\)|\(a_{q,h}^t|\) and \(|a_{q,v}^t|\) are equivalent to \(E_x\) and \(E_y\) in Equations (1-4), respectively, while \(\theta_{q,h}\) and \(\theta_{q,v}\) characterize \(\delta_{q,h}\) and \(\delta_{q,v}\) of the difference angle \(\delta_E\) presented in Section II-A.
Observe in (7) that an additional means of information transmission is introduced by adjusting the joint configurations of the moduli and arguments of the diagonal vector of $A_q^{(n_t)}$. Given that the coefficients of $A_q^{(n_t)}$ constitute the polarization information, $\{A_q^{(n_t)}\}_1^q$ can be constructed using one of the three following modes:

- **The AR mode**, where the polarization information is explicitly transmitted over the AR component, which is represented by the moduli of $A_q$ denoted by $|a_{q,v}|$ and $|a_{q,h}|$. In the AR mode, no information is conveyed over the tilt component (e.g. $\theta_{q,v} = \theta_v$ and $\theta_{q,h} = \theta_h$).\[\forall \{(A_q)_{q=1}^Q\}$, where $\theta_{v}$ and $\theta_{h}$ are constant angle values.

- **The Tilt mode**, where the polarization information is explicitly transmitted over the tilt component designated by the arguments $\theta_{q,v}$ and $\theta_{q,h}$ of $A_q$, while having static moduli (e.g. $|a_{q,h}| = \bar{a}_h$ and $|a_{q,v}| = \bar{a}_v$ $\forall \{(A_q)_{q=1}^Q\}$), where $\bar{a}_h$ and $\bar{a}_v$ are constant real numbers.

- **The tilted-AR mode**, where information is conveyed over an amalgam of both the tilt and the AR components, which is characterized by the general representation of $A_q^{(n_t)}$ in (7). In this mode, every polarization shaping matrix in $\{(A_q)_{q=1}^Q\}$ has a unique signature constituted by a specific combination of AR (i.e. $|a_{q,v}|$ and $|a_{q,h}|$) and tilt angles (i.e. $\theta_{q,v}$ and $\theta_{q,h}$).

The PM system may also reduce to the conventional spatial multiplexing (MUX) system [22], [42], when no information is transmitted over the polarization dimension (e.g. $Q = 1$).

In this treatise, we refer to a PM system as PM(AR/Tilt/TAR/MUX, $N_t^i$, $\frac{N_r}{2}$, $Q$, $\mathcal{L}$ — QAM/PSK) and to the PM encoder as PM(AR/Tilt/TAR/MUX, $Q$, $\mathcal{L}$ — QAM/PSK), where AR, Tilt, TAR and MUX represent the AR modulation, tilt modulation, tilted-AR modulation as well as the basic QAM/PSK multiplexing modulation without any polarization, respectively.

It should be also noted that by using the Tilt mode, where the polarization information is explicitly transmitted over the tilt component the system converges to the PolarSK system proposed in [31], namely when associated with $N_t^i = 1$ and the PSK modulation. Hence, PolarSK is a special case of our PM scheme.

Now, having generated the space-polarization block, the PM symbol vector $S$ of (8) is transmitted over a frequency-flat and slow fading channel and received by the $\frac{N_r}{2}$ DP-AEs at the receiver. In general, the vector-based system model can be expressed as

$$Y = HS + V,$$

where $H \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix and $V \in \mathbb{C}^{N_r \times 1}$ is the zero-mean additive white Gaussian noise (AWGN) vector, each element of which obeys $\mathcal{CN}(0, N_0)$, given that $N_0$ is the noise power.

$\textit{C. Channel Model}$

In this regards, $H$ describes the DP channel matrix that combines both the spatial separations and the XPD depolarization effects and it is defined as [5], [6], [43]

$$H = \begin{bmatrix} H_{1,1} & & \cdots & H_{1,N_t^i} \\ \vdots & & & \vdots \\ H_{N_r/2,N_t^i} & & \cdots & H_{N_r/2,N_t^i} \end{bmatrix}, \quad (10)$$

where $H_{n_r/2,n_t^i} \in \mathbb{C}^{2 \times 2}$ designates the TITO channel matrix between the $n_t^i$-th and $n_r/2$-th transmit and receive DP-AEs, respectively. In particular, each TITO channel model can be expressed as

$$H_{n_r/2,n_t^i} = \begin{bmatrix} h_{n_r/2,n_t^i}^{uv} & \sqrt{X} h_{n_r/2,n_t^i}^{vh} \\ \sqrt{X} h_{n_r/2,n_t^i}^{hv} & h_{n_r/2,n_t^i}^{hh} \end{bmatrix} \quad (11),$$

where $X$ denoting the XPD, which is a combination of the cross-polar ratio (XPR) and the cross-polar isolation (XPI) as defined in [6]. More specifically, the $X$ parameter indicates the cross-attenuation between the co-polarized channels ($vv$, $hh$) and the cross-polarized channels ($hv$, $vh$). XPD is defined as the ratio of the power of co-polarized channels to the power of cross-polarized channels over $V$ and $H$, expressed as [44]

$$\varphi_v^{-1} = E \left[ \left| h_{i,j}^{uv} \right|^2 \right] / E \left[ \left| h_{i,j}^{hh} \right|^2 \right], \quad (12)$$

$$\varphi_h^{-1} = E \left[ \left| h_{i,j}^{hv} \right|^2 \right] / E \left[ \left| h_{i,j}^{vh} \right|^2 \right], \quad (13)$$

respectively, where $h_{i,j}^{vh/hv}$ denotes the channel fading coefficient including the cross-attenuation effect, $E = E \left[ \left| h_{i,j}^{uv} \right|^2 \right] = E \left[ \left| h_{i,j}^{hh} \right|^2 \right] = 1$, $E \left[ \left| h_{i,j}^{vh} \right|^2 \right] = \varphi_v$ and $E \left[ \left| h_{i,j}^{hv} \right|^2 \right] = \varphi_h$. By approximating equal cross-attenuation [22] (e.g. $\varphi_v = \varphi_h = \varphi$ and $0 \leq \varphi \leq 1$), the XPD parameter can be expressed as $X = \varphi$. In what follows, we express the inverse of the XPD in dBs as $X^{-1}_{dB} = -10 \log X$ dB.

To expound a little further on the channel model, the SISO channel presented in (11) can be defined as

$$H_{n_r/2,n_t^i} = \tilde{H} \odot X, \quad (14)$$

where $X = \begin{bmatrix} \frac{1}{\sqrt{X}} & 0 \\ 0 & \frac{1}{\sqrt{X}} \end{bmatrix}$, $\odot$ denotes the Hadamard element-by-element product and $\tilde{H}$ represents the UP-based channel, which can be defined as

$$\tilde{H} = \sqrt{\frac{K}{K+1}} H_{LOS} + \sqrt{\frac{1}{K+1}} H_{NLOS}, \quad (15)$$

and hence

$$H_{n_r/2,n_t^i} = \sqrt{\frac{K}{K+1}} \odot H_{LOS} + \sqrt{\frac{1}{K+1}} \odot \tilde{H}_{NLOS}, \quad (16)$$

given that $K$ is the K-Rician factor, $H_{LOS}$ is the LOS channel component and $\tilde{H}_{NLOS}$ is the NLOS Rayleigh fading channel.
D. Detection

Having generated the PM symbol vector $S$, we now introduce the ML detector of our PM scheme. In an uncoded scenario, the PM detector aims to detect both the APM symbols as well as the polarization information of the transmit DP-AEs, where both $\{A_q\}_{q=1}^Q$ and $\{x_i\}_{i=1}^N$ denote the PM constellation $S$ available at the receiver.

The ML detector’s main function is to maximize the a posteriori probability by invoking the conditional probability of receiving $Y$ given $S$, is transmitted defined by [45]

$$p(Y|S_i) = \frac{1}{(\pi N_0)^{N_r}} \exp\left(-\frac{|Y - HS_i|^2}{N_0}\right),$$

where $S_i \in S$ represents the transmitted symbol vector under the assumption that all symbols in set $S$ are equi-probable with $p(S_i) =1/2^B \forall S_i \in S$. Hence, the ML detector may be formulated as

$$\langle q, l \rangle = \arg \min_{q, l} |Y - H S_i|^2$$

$$= \arg \min_{q, l} |Y - HA_i X_i|^2,$$

$$= \arg \min_{q, l} \|Y - \sum_{n_c=1}^{N_c} H_n^t A_{q,l}^{(k)} X^{(k)}\|,$$

with $H_n^t \in \mathbb{C}^{N_r \times 2}$ being the $n_c$-th sub-channel between the $n_c$-th DP-AE and the $N_r/2$ receive AE, which denotes the $n_c$ columns of $H_i$. Furthermore, $q$ and $l$ denote the estimated values of $q$ and $l$, which designate the selected sets of $q$ and $l$ information, respectively.

E. Practical Considerations

In this section, we present a discussion on the feasibility of the PM system in practical implementations, namely in the context of the PM encoder design as well as of its hardware considerations. In order to invoke the polarization characteristics of a DP-AE, a phase-shifter and a power amplifier are required at its front-end. However, more complications may arise in the construction of the transmitter if maintaining a dual stream transmission per DP-AE were required. For instance, a straightforward approach is to implement two distinct RF chains; one for the V port and the other for the H port of each DP-AE, and hence a total of $(2^L - 1)$ RF chains are required.

1) PM Encoder Design: In order to retain a dual data stream transmission with a reduced RF-chain implementation, we propose the PM encoder architecture of Figure 4. In this figure, the $B_{PM}$ input bits are divided into three parts for constructing the PM symbol vector. More specifically, the first part is used to select the $q$-th phase-shifter combination $\angle A_q = (\theta_{q,v}, \theta_{q,h})$, while the second part is used to generate the phases of the APM symbols pair $\angle \mathcal{L} - \mathcal{P} \leftarrow \mathcal{P}$, as shown in Figure 4. A multiplier is employed to combine both phases and generate the $n_c$-th PM symbol’s phase $\angle S_{q,v,h} = \langle \theta_{q,v} + \theta_{q,h} \rangle$, as shown in Figure 4. Furthermore, the third part is used to produce the $(q,l)$-th PM symbol’s phase $\angle S_{q,v,h} = \langle \theta_{q,v} + \theta_{q,h} \rangle$. Finally, the variable power amplifiers can be replaced with a single power amplifier connected at the front-end of the encoder, which improves the encoder’s power efficiency. This can be achieved with the aid of reconfigurable antennas, which are capable of continuously tuning both the AR and the tilt angle of the transmitted signal [46]. In what follows, we consider the PM encoder of Figure 4, which produces a pair of APM symbols amalgamated with the polarization information of the DP-AE.

2) Hardware Considerations: The PM encoder design requires the switching and DP-AE controlling units presented in Figure 4 for the sake of maintaining a dual-stream transmission, which increases the hardware complexity of the transmitter. This is one of the noticeable drawbacks of the PM encoder design, when compared to conventional RF implementations. However, by comparing the architecture of a single switching-aided RF-chain of Figure 4 to a pair of end-to-end RF chains, which are required to operate a couple of AEs (e.g. two DP-AE ports), the hardware requirements become less demanding. For instance, it has been shown in [47] that the most expensive component (in terms of cost and power consumption) in switch-aided transmitters, comparable to our PM design, is the RF chain (see [48] for details). This excludes the additional switching modules, serial-to-parallel (S/P) converters and the RF switches of our PM encoder. Nonetheless, the practical implementations of the PM system require further investigation, albeit the evident cost-power consumption and complexity design trade-off.

We note here that the design of Figure 4 may be relaxed by transmitting a single APM symbol rather than two symbols over the DP-AE ports. However, this would reduce the achievable throughput $B$ of Equation (5) to $(N_c \cdot \log_2(LQ))$ bits. The implementation of DP-AEs using the above-mentioned architecture is worthwhile investigating, hence in what follows we characterize both the capacity as well as the BER performance of the PM system.

III. PM System Capacity

In this section, we present both the DCMC capacity and the ergodic CCMA capacity of our PM system. Furthermore,
we formulate the upper and lower bounds of the ergodic CCMC capacity.

A. DCMC Capacity

The DCMC capacity of our PM system, which designates the mutual information expressing the number of error-free bits that can be decoded at the PM receiver, can be formulated as [49]

\[
C_{DCMC} = \max_{p(S)} I(S; Y) = \max_{p(S)} \sum_{\mathcal{S}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(Y | S_i) p(S_i) \log_2 \left( \frac{p(Y | S_i)}{\sum_{S \in \mathcal{S}} p(Y | S_i) p(S_i)} \right) dY, 
\]

(21)

which can be maximized by using equi-probable \( p(S_i) \).

Next, by relying on the system’s conditional probability of Equation (17), the DCMC capacity can be now formulated as [49]

\[
C_{DCMC} = B - \epsilon_b \sum_{q,t} E \left[ \log_2 \left( \sum_{\mathcal{S}} \exp \left( \bar{\psi} | S_i \right) \right) \right], 
\]

(22)

where \( \epsilon_b = \frac{1}{2} \log_2 e \) and \( \bar{\psi} \) is given as

\[
\bar{\psi} = -\|H(S_i - S_i) + V\|^2 + \|V\|^2, 
\]

(23)

with \( S_i \) being the transmitted symbol vector having \( (I, I) \) indices. Unfortunately, there is no closed-form formulation available for Equation (22) and hence, we rely on numerical averaging procedures for evaluating the DCMC capacity.

B. Ergodic CCMC Capacity

On the other hand, the ergodic CCMC capacity of a MIMO system including our PM system is provided for maximizing the mutual information in a MIMO channel, which can be denoted as the maximum number of bits in an error-free continuous transmission and it is defined as [50]

\[
C_{CCMC} = \max_{p(S)} H(Y) - H(Y | S), 
\]

(24)

where \( H(Y) \) and \( H(Y | S) \) denote the destination entropy and the entropy of \( Y \) given \( S \), respectively, which can be written as

\[
C_{CCMC} = E \left[ \log_2 \left( \frac{1}{N_t} \left( H(Y) \right) \right) \right]. 
\]

(25)

C. Ergodic Capacity Bounds

In order to clearly show the effect of XPD on the achievable capacity of the PM system, in what follows we examine the bounds of \( C_{CCMC} \) of (25) at the ultimate minimum XPD (i.e. \( \lambda_{dB}^{-1} \to 0 \)) and the ultimate maximum XPD (\( \lambda_{dB}^{-1} \to \infty \)), given \( K = 0 \).

At \( \lambda_{dB}^{-1} \to 0 \): The XPD provided in Equation (11) attains its maximum (\( \lambda = 1 \)) and the system transforms to a conventional UP-based MIMO system. Hence, closed-form of Equation (25) at \( \lambda = 1 \) can be expressed as [51]

\[
C_{\lambda_{dB}^{-1} \to 0} \geq \mu \log_2 \left[ 1 + \frac{\rho}{N_t} \exp \left( \frac{1}{\mu} \sum_{j=1}^{K} \sum_{p=1}^{1} \frac{1}{p} \right) \right], 
\]

(26)

given that \( \mu = \min(N_t, N_r) \), \( K = \max(N_t, N_r) \) and \( \bar{\gamma} \approx 0.577215 \) is Euler’s constant. This can be obtained by relying on

\[
E \left[ \ln \left( \frac{1}{N_t} \left( H(Y) \right) \right) \right] = \sum_{j=1}^{N_r} E \left[ \ln \Omega_j \right] - N_t \ln N_t, 
\]

(27)

given that

\[
E \left[ \ln \Omega_j \right] = \psi(N_t - j - 1) = \sum_{p=1}^{K-j} \frac{1}{p} - \bar{\gamma}, 
\]

(28)

where \( \Omega_j \sim \chi^2(N_t-j+1) \).

Here, \( C_{\lambda_{dB}^{-1} \to 0} \) represents the upper bound of the capacity \( C_{CCMC} \), since no cross polarization attenuation exists between the V and H components, and hence no degradation in the achievable capacity is incurred.

At \( \lambda_{dB}^{-1} \to \infty \): The cross V/H channels attenuation of (11) becomes infinitesimally low (i.e. \( \lambda = 0 \)) and the row vectors \( h_{tt/2}^v \) and \( h_{tt/2}^h \) of \( H \) in (10) denoting the V and H receive AE channels at the \( n_r/2 \)-th received DP-AE, respectively, are then expressed as

\[
\begin{bmatrix}
    h_{tv/2}^v \\
    h_{tv/2}^h \\
    \vdots \\
    h_{tn_r/2,t_n_r/2}^v \\
    h_{tn_r/2,t_n_r/2}^h \\
    \cdots \\
    h_{tn_r/2,t_n_r/2}^h \\
    \cdots \\
    0 \\
    0 \\
    \vdots \\
    \vdots \\
    0 \\
    \cdots \\
    0 \\
    \cdots
\end{bmatrix}
\]

(29)

Observe in (29) that the resultant power of \( h_{tv/2}^v \) reduces by half, which transforms the Chi-squared variable \( \Omega_j \) of (27) into \( \Omega_j' \sim \chi^2(N_t-j+1) \), where \( E \left[ \ln \Omega_j' \right] = \psi \left( \frac{j+1}{2} \right) - 1 \).

Hence, the ergodic capacity reduces to

\[
C_{\lambda_{dB}^{-1} \to \infty} \geq \mu \log_2 \left[ 1 + \frac{\rho}{N_t} \exp \left( \frac{1}{\mu} \sum_{j=1}^{K} \sum_{p=1}^{1} \frac{1}{p} - \bar{\gamma} \right) \right]. 
\]

(30)

The capacity \( C_{\lambda_{dB}^{-1} \to \infty} \) of (30) denotes the lower bound of the achievable capacity given a total V/H communication blockage. Therefore, the CCMC capacity at any XPD level is bounded by \( C_{\lambda_{dB}^{-1} \to 0} \) and \( C_{\lambda_{dB}^{-1} \to \infty} \) as

\[
C_{\lambda_{dB}^{-1} \to \infty} \leq C_{\lambda_{dB}^{-1} \to 0} \leq C_{\lambda_{dB}^{-1} \to \infty}. 
\]

(31)

It is clearly seen in (31) that as the XPD attenuation increases towards infinity the achievable capacity \( C_{\lambda_{dB}^{-1} \to \infty} \) decreases towards the lower bound (30). However, as the XPD attenuation approaches zero the achievable capacity
The average BER for the PM system is generally formulated using the general MIMO upper-bounding technique given by [52]

\[
\text{BER} = \sum_{q=1}^{N_t} \sum_{i=1}^{N_r} \sum_{l=1}^{N_r} \frac{D_h(q,l,q,l)}{\log_2(B)} P(S \rightarrow \hat{S}),
\]

where \(D_h(q,l,q,l)\) denotes the distance between the bit-mapping of \(S\) and \(\hat{S}\) and \(P(S \rightarrow \hat{S})\) is the average pairwise error probability (APEP). The APEP in fact is the average probability \(\mathbb{E}\{ P(S \rightarrow \hat{S} | H) \}\), which determines the probability that a PM symbol \(S\) is erroneously detected as \(\hat{S}\) given \(H\) and can be expressed as [52, 53]

\[
P(S \rightarrow \hat{S} | H) = P(\|H(S - \hat{S}) + V\| < \|V\|) = Q\left(\sqrt{\frac{\|H\Delta\|^2}{2N_0}}\right),
\]

where \(\Delta = S - \hat{S}\) and \(Q(\cdot)\) denotes the Q-function defined in [54] as

\[
Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2 \theta}\right) d\theta,
\]

and subsequently the PEP representation of (33) can now be expressed as

\[
P(S \rightarrow \hat{S} | H) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\gamma}{2\sin^2 \theta}\right) d\theta,
\]

Now, by averaging Equation (35) over \([0, \infty)\) the legitimate range of the random variable \(\gamma\), the unconditional PEP can be formulated as [55]

\[
P(S \rightarrow \hat{S}) = \frac{1}{\pi} \int_0^{\pi/2} \Phi(\cdot) \left(\frac{\gamma}{2\sin^2 \theta}\right) d\theta,
\]

where \(\Phi(\cdot)\) denotes the moment-generating function (MGF) of \(\gamma\).

In case of implementing UP-AEs, where no cross attenuation exists between \(V\) and \(H\) (\(\chi_{dB} = 0\) dB), our PM system reduces to an ordinary spatial multiplexing system, which can be evaluated based on Appendix B of [56]. However, when introducing DP-AEs, a new parameter \(\chi^2\) denoting the DP-AE polarization effects arises and hence should be considered for the ABER formulation.

Let us consider \(\Delta_{n_c} = \hat{S}(n_c) - \hat{S}(n_c)\) the symbol difference at the \(n_c\)-th transmit DP-AE, which can be expressed as

\[
\Delta_{n_c}^t = \begin{bmatrix} \Delta_{n_c,v} & \Delta_{n_c,h} \end{bmatrix},
\]

where \(\Delta_{n_c,v}\) and \(\Delta_{n_c,h}\) denote the symbol difference at the vertical and horizontal components of the \(n_c\)-th transmit DP-AE, respectively. Given \(\alpha = \|H\Delta\|^2\) and using Equation (37), \(\alpha\) can be rewritten as

\[
\alpha = \sum_{n_c=1}^{N_c} \sum_{n_c=1}^{N_r} H_{n_c,v} H_{n_c,h}^* \|\Delta_{n_c,v}\|^2 \|\Delta_{n_c,h}\|^2,
\]

where \(H_{n_c,v}\) is the TITO sub-channel between the \(n_c\)-th transmit DP-AE and the \(n_c\)-th receive DP-AE defined in (11). Hence, \(\alpha\) appears in the following form

\[
\alpha = \sum_{n_c=1}^{N_c} \sum_{n_c=1}^{N_r} \frac{\Delta_{n_c,v}^v}{\sqrt{\chi} \Delta_{n_c,h}^h} \frac{\Delta_{n_c,h}^h}{\sqrt{\chi} \Delta_{n_c,v}^v} \|\Delta_{n_c,v}\|^2 \|\Delta_{n_c,h}\|^2.
\]

Now, by using the norm representation of \(\|A_{l,x,j}\|^2 = \sum_{i=1}^{N_c} \sum_{j=1}^{N_r} |a_{i,j}|^2\), Equation (40) can be rewritten as [57]

\[
\alpha = \sum_{n_c=1}^{N_c} \frac{1}{2} \left( |\Delta_{n_c,v}|^2 + \chi |\Delta_{n_c,h}|^2 \right)^2 + \sum_{n_c=1}^{N_c} \frac{1}{2} \left( |\Delta_{n_c,h}|^2 + \chi |\Delta_{n_c,v}|^2 \right)^2.
\]

Each element of the MIMO channel matrix \(H\) of (10) is assumed to be an i.i.d random variable, and hence (41) can be reformulated as

\[
\alpha = \sum_{n_c=1}^{N_c} \frac{1}{2} \left( |\Delta_{n_c,v}|^2 + \chi |\Delta_{n_c,h}|^2 \right)^2 + \sum_{n_c=1}^{N_c} \frac{1}{2} \left( |\Delta_{n_c,h}|^2 + \chi |\Delta_{n_c,v}|^2 \right)^2.
\]
By substituting $\gamma$ of (43) into (35), the PEP can be formulated as

$$P \left( S \rightarrow \hat{S} \mid H \right) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( - \frac{\left( Y_v \varsigma^2_{N_v} \right) + \left( Y_h \varsigma^2_{N_h} \right)}{4N_0 \sin^2 \theta} \right) d\theta,$$

(44)

and hence after averaging it over $[0, \infty]$, Equation (44) can be expressed as

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{\pi} \int_{0}^{\pi/2} \Phi_{Y_v, \varsigma^2_{N_v}} \left( \frac{1}{4N_0 \sin^2 \theta} \right) \cdot \Phi_{Y_h, \varsigma^2_{N_h}} \left( \frac{1}{4N_0 \sin^2 \theta} \right) d\theta.$$  

(45)

A. Rayleigh Fading, $K = 0$

In the case of considering a Rayleigh fading channel (e.g. $K = 0$), Equation (53) can be rewritten as [56]

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{\pi} \int_{0}^{\pi/2} \prod_{l=1}^{\lfloor L/2 \rfloor} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c_l} \right)^{\frac{N_v}{2}} d\theta,$$

(46)

where $c_1 := \frac{1}{2N_0}$, $c_2 := \frac{1}{2N_0}$ and the MGF of the chi-squared RV $\varsigma^2_1$ is defined by

$$\Phi_{\varsigma^2_1} (-s) = \left( 1 + 2as \right)^{-\frac{s}{4}}.$$  

(47)

The closed-form solution of (46) can be formulated using two approaches. Following the solution provided in Appendix 5A.9 in [58], the first closed-form solution of (46) can be expressed as

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{2} \sum_{l=1}^{L/2} \sum_{k=1}^{N_v/2} J_{kl} \left[ 1 - \sqrt{\frac{c_l}{c_l + 1}} \cdot \prod_{j=0}^{k} \sqrt{\frac{2j}{j}} \cdot \frac{1}{\left[ 1 + \left( 1 + c_l \right) \right]^j} \right],$$

(48)

given that

$$J_{kl} = \left. \left( \frac{N_v - k}{2} \right) \prod_{n=1}^{L/2 - k} \left( \frac{N_v - 1}{2n} \right)^{\frac{N_v}{2}} \right|_{x = -\frac{1}{2}}.$$  

(49)

For the special case of using a single DP-AE receiver (e.g. $N_v = 1$), Equation (48) reduces to

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{2} \sum_{l=1}^{L/2} \left[ 1 - \sqrt{\frac{c_l}{c_l + 1}} \right] \cdot \prod_{n=1}^{L/2 - l} \left( \frac{2j}{j} \right) \left( \frac{c_l}{[c_l - e_n]} \right).$$

(50)

In the second approach, the closed-form of the PEP given in (46) can be formulated as

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{2\pi} \left( c_1 c_2 \right)^{-\frac{s}{4}} \cdot \beta \left( \frac{1}{2}, N_r + \frac{1}{2} \right) \cdot F_1 \left( N_r + \frac{1}{2}, \frac{N_r}{2}, \frac{N_r + 1; -c_1^{-1}, -c_2^{-1}}{2} \right),$$

(51)

which is detailed in Appendix A, where $\beta (\cdot, \cdot)$ denotes the Beta function and $F_1 (\alpha, \beta, \beta; \gamma; x, y)$ the confluent hypergeometric function of two variables (Equation (61)).

In the high SNR-regime (i.e. $N_0 \gg 1$), Equation (51) can be written as

$$P \left( S \rightarrow \hat{S} \right) \leq \frac{1}{2\pi} \left( \frac{Y_v Y_h}{16N_0^2} \right)^{\frac{s}{4}} \cdot \beta \left( \frac{1}{2}, N_r + \frac{1}{2} \right),$$

(52)

where $F_1 \left( N_r + \frac{1}{2}, \frac{N_r}{2}, \frac{N_r + 1; 0, 0}{} \right) = 1$ at $c_1 \rightarrow \infty$ and $c_2 \rightarrow \infty$. Hence, the achievable diversity gain defined by the slope of $P \left( S \rightarrow \hat{S} \right)$ is equivalent to $N_r$.

Note here that Equation (46) simplifies to Equation (36) when $N_v^2 = 0$ dB (i.e. $Y_v = Y_h$) and hence, Equation (46) can be solved using (56), Equation (64). Additionally, it can be seen in (52) that the XPD level does not have any effect on the achievable diversity order of the PM system.

B. Rician Fading, $K > 0$

When considering a Rician fading channel (e.g. $K > 0$), Equation (45) can be written as [59]

$$P \left( S \rightarrow \hat{S} \right) = \frac{1}{\pi} \int_{0}^{\pi/2} \prod_{l=1}^{\lfloor L/2 \rfloor} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c_l} \right)^{\frac{N_v}{2}} \cdot \exp \left( - \frac{K c_l}{\sin^2 \theta + c_l} \right)^{\frac{N_v}{2}} d\theta,$$

(53)

where the MGF of the noncentral chi-squared RV $\varsigma^2_1$ is defined as [56]

$$\Phi_{\varsigma^2_1} (-s) = \left( 1 + 2as \right)^{-\frac{s}{4}} \exp \left( - \frac{K N_r}{2} \cdot \frac{s}{1 + 2as} \right).$$

(54)

There is no closed-form of Equation (53) and hence, it can be evaluated numerically. Note here that at $K = 0$ the problem reduces to Equation (46).

However, by using the Q-function approximation proposed in [60], the APEP of Equation (45) can be approximated as

$$P \left( S \rightarrow \hat{S} \right) \approx \frac{1}{12} \left( \Phi_{Y_v, \varsigma^2_{N_v}} \left( \frac{1}{4N_0} \right) \cdot \Phi_{Y_h, \varsigma^2_{N_h}} \left( \frac{1}{4N_0} \right) \right)^{\frac{N_v}{2}} + \frac{1}{4} \left( \Phi_{Y_v, \varsigma^2_{N_v}} \left( \frac{1}{3N_0} \right) \cdot \Phi_{Y_h, \varsigma^2_{N_h}} \left( \frac{1}{3N_0} \right) \right)^{\frac{N_v}{2}},$$

(55)

which is detailed in Appendix B.
The PM system is comparable to a spatial multiplexing system, which suffers from a degraded performance in the presence of a LOS component, as a result of the correlation fading effect. To overcome this issue in a DP-based MIMO, we employ our PM system by relying on a single transmit DP-AE \((N'_c=1)\) at high XPDs, yielding \(E\left[\left|\bar{h}_{ij}^{v}\right|^2\right] \ll 1\) and

\[E\left[\left|\bar{h}_{ij}^{h}\right|^2\right] \ll 1.\]

V. SPACE-POLARIZATION IMPROVED CONSTELLATION

In this section, we introduce our PM improved-constellation generation procedure. Observe in Equation (6) that the polarization configuration matrix \(A_q\) disperses the PSK/QAM complex symbols of \(X_i\) over the spatial and polarization dimensions at a single time slot, in a conceptually similar manner to space-time dispersion matrices [33], [34], [37]. This opens a new prospect for designing the polarization shape of PM constellations.

In a nutshell, the polarization shaping matrices \(\{A_q\}_1^Q\) may be randomly generated so that the performance of the system is improved. In this regard, the shaping matrices may be constructed so that the unconditional PEP of Equation (46) is minimized, while retaining the maximum achievable diversity order. Hence, the optimal set of \(Q\) unit polarization vectors \(A_{opt}\) can be constructed by conducting a Random Search (RS) that aims at minimizing the maximum PEP as

\[A_{opt} = \arg_{A_q} \min \{\max P(S \rightarrow \tilde{S})\}, \quad (56)\]

which translates to

\[A_{opt} = \arg_{A_q} \max \{\min (c_1c_2)\} = \arg_{A_q} \max \{\min (\hat{\tau}_h\hat{\tau}_v)\}, \quad (57)\]

which can be rewritten as

\[A_{opt} = \max \{\min ||\Delta||\}. \quad (58)\]

It is worth emphasizing here that the construction of \(\{A_q,h,A_q,v\}\) designating the H and V configurations of \(\{A_q\}_1^Q\), respectively, should fall within the polarization shaping capabilities of the DP-AE, namely its AR range (1) and its Tilt angle range (4). Additionally, multiple transmit AEs are spaced sufficiently far apart in order to experience independent fading hence, random search is performed using a single transmit DP-AE, where the \(A_{opt}\) set produced is used at each DP-AE.

In what follows we present the generation process of \(A_{opt}\) satisfying (58) using a TITO \((2 \times 2)\)-element system. We first generate a random set of \((1 \times 2)\)-element unit vectors denoting the diagonal vectors of the \((2 \times 2)\)-element matrix set \(A_q=\{A_q\}_1^Q\). The vector set generated should obey the Rank Criterion (i.e. \(\text{rank}(\Delta^H) = 1 \forall q, \hat{q} \in Q\)) in order to guarantee a normalized power space-polarization set. Next, we calculate the minimum Euclidean distance \(d_{min} = \{\min ||\Delta||\}\).

The random search continues by repeating both steps, while retaining the \(A_q\) set having the maximum \(d_{min}\). The algorithm presented above is summarized in Algorithm 1. Furthermore, an example is provided in Appendix C to ease understanding.

Note that by obtaining the minimum distance \(d_{min} = \max\{\min ||\Delta||\}\) in (58) the PEP \(P(\|H(S - \tilde{S}) + V\| < \|V\|)\) of (33) is minimized, and hence the DCMC exponent \(\psi = -\|H(S_i - \bar{S}_i) + V\|^2 + \|V\|^2\) of (23) is subsequently minimized, which improves the achievable DCMC capacity.

Algorithm 1: Polarization Shaping Algorithm

minimum distance: \(\kappa = 0\)

initialize \(A_{opt}\);

\textbf{Start:} \(i = 1:10^6\) loops

\textbf{Loop:} Generate \(Q\) random \((2 \times 1)\)-element unit vectors \(\{a_q\}_1^Q\)

\(A_q = \{A_q = \text{diag}_2(a_q)\}_1^Q\)

compute \(S, \tilde{S}\) and \(\Delta q,q_i,q_j\)

if \((\text{rank}(\Delta^H) = 1)\)

Compute \(OA, OB\) and \(\tau\) using \(\{A_q\}_q=1\)

if \((OA, OB\) and \(\tau\) doesn’t match the DP-AE range) \(GOTO\) Loop

else \(GOTO\) Loop

Compute \(d_{min} = \min\{||\Delta||\}\)

if \((d_{min} > \kappa)\)

Apply \(A_{opt} = A_q\)

\textbf{GOTO} Loop

\textbf{End}

VI. SIMULATION RESULTS

In this section, we present our Monte Carlo simulation results with a minimum of \(10^6\) bits per SNR value as well as the theoretical analysis of our PM system. In our simulations we assume perfect CSI at the receiver side for invoking the ML optimum detector of Equation (18). Furthermore, multiple DP-AEs are spaced sufficiently far apart in order to experience independent fading. We choose the polarization shaping matrix set \(\{A_q\}_q=1\) by selecting several \(AR\) and \(\tau\) values based on the discussion presented in Section II-A. Particularly, Table III shows the main PM systems used in our simulations with \(Q=4\) as follows\(^5\): three AR systems (i.e. AR-1,..., AR-3), two Tilt systems (i.e. Tilt-1, Tilt-2) and four TAR systems (i.e. TAR-1,..., TAR-4). Additionally, all plots showing the performance of PM-systems associated with the RS-aided constellation presented in Section V are labeled as TAR-RS. The TAR-RS system used below is presented in Appendix C.

Note here that the tuning capabilities of DP-AEs over the AR and the tilt angle vary from one antenna to another. For instance, the reconfigurable DP-AE presented in [46] utilizes a maximum AR of 35 dB and a tilt angle spanning between 30° and 100°.

A. Comparison Fairness

In this contribution we define fair comparison as follows: a fair performance comparison between a DP-based system and a UP-based system is attained by employing an equivalent number of AEs in both systems. To expound a little further, \(\text{other systems with various } Q \text{ configuration are used.}\)
In a PM system designed for achieving a normalized throughput of 4 bpcu, here, we employed \((1 \times 1)\) DP-AEs with various PM configurations. More specifically, Figure 5 shows the DCMC curves of the AR-1-3, Tilt-1-2 and TAR-1-3 systems detailed in Table III as well as of TAR-RS and TAR-RS-1PSK, where TAR-RS-1PSK is a symbol-free RS-based PM (TAR, 1, 1, Q = 16, 1PSK) system (i.e. polarization information only). We also characterize the conventional \((1 \times 1)\) UP-ABASED 16QAM and 16PSK systems. It can be observed in Figure 5 that TAR-based PM systems outperform all the other PM configurations, while the RS-based systems achieve the highest throughput. For instance, TAR-RS outperforms PolarSK (i.e. Tilt-PM) by 2.8 dB and conventional 16QAM and 16PSK by 3.7 dB and 6 dB, respectively. This verifies the discussion presented in Section V, where constructing the optimal \(A_{opt}\) under the constraint of maximizing \(d_{min} = \max\{\min\parallel\Delta\parallel\}\) could further improve the achievable capacity of the PM system.

In order to characterize the effect of the XPD on the PM system, Figure 6 portrays the 3D surface of the achievable capacity of a PM (TAR, 1, 1, Q = 4, BPSK) system with respect to XPD and SNR. Furthermore, the achievable throughput at \(X_{dl} = 0\) dB is projected onto the (SNR, Capacity)-plane for the sake of comparison. As seen in Figure 6, the achievable throughput degrades as the XPD increases, which can be clearly seen at high XPDs. To expound a little further, Figure 7 shows the 3D surface of Figure 6 onto the (SNR, Capacity)-plane between \(X_{dl}^{-1} = 0\) dB and \(X_{dl}^{-1} = 30\) dB. It can be seen from the figure that a maximum degradation of 3.5 dB is observed in the DCMC capacity between \(X_{dl}^{-1} = 0\) dB and \(X_{dl}^{-1} = 30\) dB. However, the degradation in the achievable capacity becomes marginal at high XPDs, especially at \(X_{dl}^{-1} > 15\) dB.

### C. CCMC Capacity

To investigate the ergodic CCMC capacity of our PM system, the capacities of three PM systems are illustrated by the 3D surfaces shown in Figure 8, namely for the \((1 \times 1), (2 \times 2)\) and \((4 \times 4)\) DP-AEs MIMO arrangements. One can observe in Figure 8 that the CCMC capacity is affected both by the transmission power as well as the XPD level.

---

**TABLE III**

<table>
<thead>
<tr>
<th>PM</th>
<th>({a_{q,h}, a_{q,v}}_q^Q)</th>
<th>({\theta_{q,h}, \theta_{q,v}}_q^Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR-1</td>
<td>((1, 1), (1, 1.2), (1, 1.4), (1, 1.6))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>AR-2</td>
<td>((1, 1.09), (1, 1.6), (1, 2), (1, 2.5))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>AR-3</td>
<td>((1, 1), (1, 1.5), (1, 2), (1, 3.5))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>Tilt-1</td>
<td>((1, 1))</td>
<td>((0, 10), (0, -10), (0, 35), (0, -35))</td>
</tr>
<tr>
<td>Tilt-2</td>
<td>((1, 1))</td>
<td>((0, 35), (0, 65), (0, -35), (0, -65))</td>
</tr>
<tr>
<td>TAR-1</td>
<td>((1, 1.09), (1, 1.6), (1, 2), (1, 2.5))</td>
<td>((0, 10), (0, 35), (0, -10), (0, -35))</td>
</tr>
<tr>
<td>TAR-2</td>
<td>((1, 1), (1, 1.4), (1, 1.8), (1, 2.2))</td>
<td>((0, 35), (0, 65), (0, -35), (0, -65))</td>
</tr>
<tr>
<td>TAR-3</td>
<td>((1, 1), (1, 2), (1, 3), (1, 4))</td>
<td>((0, 1.2), (0, 74), (0, 37), (0, 110))</td>
</tr>
<tr>
<td>TAR-4</td>
<td>((1, 1.09), (1, 1.9), (1, 2.5), (1, 2.5))</td>
<td>((0, 1.2), (0, 45.8), (0, 1.8), (0, 45.8))</td>
</tr>
</tbody>
</table>

*Fig. 5. DCMC capacity comparison between various PM systems attaining 4 bpcu by relying on the AR, Tilt and TAR configurations with different polarization shapes at an XPD of \(X_{dl} = 10\) dB.*
Fig. 6. A 3D representation of the DCMC capacity of a PM(TAR, 1, 1, Q = 4, BPSK) system with respect to SNR and XPD.

Fig. 7. The 2D projection of Figure 6 onto the (SNR, DCMC)-plane.

Fig. 8. A 3D representation of the ergodic CCMC capacity of three PM systems in terms of SNR (dB) and XPD (dB), namely for the (1 × 1), (2 × 2) and (4 × 4) DP-AEs arrangements.

Figures 9(a)-(c) depict the 2D projection of the 3D surfaces of Figure 8 onto the (SNR, CCMC)-plane at an XPD of $X_{\text{dB}} = 10$ dB. The theoretical upper and lower bounds of equations (26) and (30), respectively, are also shown in each figure. Furthermore, the capacity of an equivalent number of UP-AEs is shown for the sake of comparison, where the capacity improvement of DP-based systems is shown additionally by the red curve. It can be observed in Figure 9 that DP-AE implementations substantially boost the capacity of a MIMO system, achieving between 87.5% and 54% capacity improvement over an SNR range spanning between −10 dB and 40 dB, respectively, for all three systems considered. Moreover, we note that the DP-based capacity curves portrayed in Figure 9 are confined within the upper and lower bounds described in Section III, which are separated 3 dB apart. In fact, the simulated curves (through Monte Carlo) of the DP-based systems in Figures 9(a)-(c) and the lower bound analysis ($C_{X_{\text{dB}} \to \infty}$) precisely match at $X_{\text{dB}} = 10$ dB.

To examine the effect of the XPD on the achievable CCMC capacity, let us assume that we project Figure 8 onto the (XPD, CCMC)-plane at SNR of 12 dB, 16 dB and 20 dB, as portrayed in Figure 10. The figure shows that as the XPD level increases the ergodic capacity decreases, however it remains relatively constant after a specific value of XPD, as for example at an XPD of $X_{\text{dB}} = 16$ dB at SNR of 12 dB. Furthermore, one can observe that at an even high XPD level, the maximum loss in CCMC capacity is less than 1.4 bps/Hz.

The novel polarization modulation technique presented in this paper constitutes a viable solution to significantly boost the data transmission rate for future wireless systems. In what follows, we present the BER performance of our PM system.

D. BER Simulation

In Figure 11 we compare the achievable BER performance of (1 × 1) and (2 × 2) PM systems, which achieve a throughput of 4 and 8 bpcu, respectively, at an XPD of $X_{\text{dB}} = 10$ dB and $K = 0$. Moreover, the BER performance of their UP-AE (1 × 1) and (2 × 2) counterparts 16PSK are included for comparison, while the dashed curves represent the theoretical

6A (1 × 1)-DP-AE implementation is equivalent to a (2 × 2) UP system, since the V and H components transmit over separate polarization dimensions.

7We use the same number of AEs for both systems (i.e. DP-AE and UP-AE) in order to maintain fairness. The (1 × 1) DP-AE system for instance has two input ports, while the (1 × 1) UP-AE has a single input port, while both require a single RF chain implementation. In case two UP-AEs are used to compare with the (1 × 1) DP-AE system, two RF chain are required, which leads to unfairness in the number of RF components as well as in the required transmitted power.
upper bounds developed in Section IV. Figure 11(a) shows the performance of PM(AR, Q = 4, BPSK), PM(Tilt, Q = 4, BPSK), PM(TAR, Q = 4, BPSK) and PM(TAR, Q = 4, BPSK)-RS systems associated with (1 × 1)-DP-AEs.

The RS here features the improved RS-aided constellation provided in Section V.

Here, $\log_2 (4) = 2$ bits are used to activate one out of $Q = 4$ space-polarization matrices, while the remaining $2 \log_2 (2) = 2$ bits are modulated to a pair of BPSK symbols. The performance of the PM(TAR, Q = 16, 1PSK)-RS system is also shown in Figure 11(a), where the whole $B_{PM} = \log_2 (16) = 4$ bits are used to switch between $Q = 16$ polarization shapes. It is shown in 11(a) that the PM system outperforms the conventional (1 × 1)-DP-AE by 10 dB, 15 dB, and 19 dB at a BER of $10^{-5}$, when employing the AR, Tilt and TAR configurations, respectively. Furthermore, the improved constellation PM systems provide further BER enhancements of 2 dB and 4 dB by using the PM(TAR, Q = 16, 1PSK)-RS and PM(TAR, Q = 4, BPSK)-RS systems, respectively. Note here that the theoretical model presented in Section IV matches perfectly with the Monte Carlo simulations.

In Figure 11(b), the BER performance of the above-mentioned PM systems is shown with a (2 × 2)-element multiplexing system by 0.5 dB, 5.4 dB, 9.5 dB, when employing the AR, Tilt and TAR configurations, and by 12.7 dB and 15 dB by using the PM(TAR, Q = 16, 1PSK)-RS and PM(TAR, Q = 4, BPSK)-RS systems, respectively.

In Figures 12(a)-(b), we show the performance of a PM(TAR, Q = 4, BPSK)-RS system (i.e. TAR-RS) transmitting over a Rician fading channel at 4 bpcu, while employing a single transmit DP-AE at a high XPD of $X_{dB}^{-1} = 30$ dB. Furthermore, Figures 12(a)-(b) include both the exact and approximated theoretical bounds presented in equations (53) and (55), respectively. In particular, in Figure 12(a) we investigate the effect of the Rician factor on the performance of the PM system at $K = 0, 5, 10$ and 15, when associated with (1 × 1)-element implementation. We notice in Figure 12(a)
that upon increasing $K$ the BER performance of the PM system improves. As indicated by Equation (53), the ABER improves exponentially with the value of $K$. Furthermore, the exact theoretical model of (53) matches perfectly with the Monte Carlo simulations, while the approximate model of (55) is marginally shifted at $K = 0$ and $K = 5$, which perfectly overlaps at low BER values. On the other hand, Figure 12(b) shows the performance of the simulated PM system with a different number of receive DP-AEs, namely $N_r/2 = 1, 2, 4$ and $8$ at $K = 5$. We notice in the figure that the approximate theoretical bound developed in Section IV tends to accurately match both the exact bound and the Monte Carlo simulations as the number of receive AEs increase.

A comparison between multiple configurations of 4-level $(Q = 4)$ PM systems with $(1 \times 2)$-elements is illustrated in Figure 13, which all achieve a spectral efficiency of 4 bpcu at $X_{dB}^{-1} = 10$ dB and $K = 0$. To expound further, the PM systems under study are: AR-1-3, Tilt-1-2 and TAR-1-4 as well as TAR-RS, as detailed in Table III. As observed in Figure 13, the achievable performance of all systems spans over 35 dBs of SNR at a BER of $10^{-5}$. In all cases, it can be observed that TAR-based systems exhibit the best BER performance compared to AR-based and Tilt-based systems. This can be attributed to the multi-dimensional structure of TAR-based PM, where polarization information is dispersed over both the AR and the tilt contrary to other configurations (e.g. AR and Tilt) that exploit either of them.

In Figure 14, we compare the BER performance of our PM system with its DP-AE-based counterparts. More specifically, we compare the BER performance of PM(TAR, 1, 1, $Q = 2$, BPSK) with that of a DP-SM(1, 1, QPSK) system [30] as well as that of a PolarSK($N_r/2 = 1, Q = 2$, BPSK) system [31], where each exhibits a transmission rate of 3 bpcu over Rayleigh fading channel (i.e. $K = 0$) at an XPD of $X_{dB}^{-1} = 10$ dB. Figure 14 further shows the performance of the improved-constellation PM(TAR, 1, 1, $Q = 2$, BPSK)-RS and PM(TAR, 1, 1, $Q = 8$, 1PSK)-RS systems as well as the performance of UP-AE-based SM and Quadrature SM (QSM) systems associated with $N_t = 2$ AEs. It can be observed from Figure 14 that our PM system outperforms PolarSK, DP-SM and the conventional SM by 2 dB, 1.2 dB, 22 dB, respectively.

To elaborate further on the effect of the level of XPD on the BER performance, the BER performance of a $(2 \times 2)$-DP-AE PM system associated with an PM(TAR, $Q = 2$, BPSK) encoder at different XPD levels is presented in Figure 15. More specifically, we show the BER performance of the system at an XPD spanning between $X_{dB}^{-1} = 0$ dB and $X_{dB}^{-1} = 30$ dB with a step of 5 dB, where the theoretical boundaries are shown exclusively at $X_{dB}^{-1} = 0$ dB and $X_{dB}^{-1} = 30$ dB. Figure 15 demonstrates that the performance of the PM system is directly affected by the XPD level, where it improves as the
Fig. 13. The BER performance of a \((1 \times 2)\)-DP-AE PM systems associated with \(Q = 4\) AR, Tilt and TAE configurations, which achieve a throughput of 4 bpcu at an XPD of \(\chi_{dB}^{-1} = 10\) dB and \(K = 0\).

XPD decreases due to the increased polarization diversity gain. However, it can be seen in the figure that the performance is marginally affected when \(\chi_{dB}^{-1} \geq 25\) dB. Furthermore, the theoretical boundaries presented in Figure 15 confirms the precision of the XPD parameter provided in equations (46) and (51).

It can be observed in Figures 11-15 that the theoretical boundaries provided in Section IV match the Monte Carlo simulations for all PM configurations, namely for the AR, Tilt, TAR and MUX configurations over various antenna arrangements. In what follows, we present our conclusion.

VII. CONCLUSION

In this treatise, we have introduced a novel modulation technique referred to as the polarization modulation, which invokes the polarization characteristics of a DP-AE for data transmission. More specifically, a block of information in a PM system is formed by dispersing a pair of PSK/QAM symbols into the space- and polarization-dimension with the aid of \(Q\) polarization shaping matrices \(\{A_q\}_{q=1}^Q\). The polarization shaping matrix may adjust the AR, Tilt or Tilted-AR of the EM matrix with the aid of a single RF-chain per DP-AE. The polarization shaping matrices can be selected empirically, however we have proposed a special algorithm for generating an improved-constellation tailored for the PM modulation. Furthermore, we provided a theoretical analysis for the DCMC and CCMC capacity as well as for the BER performance of the PM system. It has been shown that by invoking the polarization dimension, the ergodic capacity of a DP-based MIMO system can be improved by \(54\%\) to \(87.5\%\) compared to UP-based MIMO. Similarly, the DCMC capacity of our PM system was improved by up to \(6\) dB in comparison to systems relying on UP-AE. Furthermore, the simulation results indicated that the gain achieved by our proposed PM system relying on \(Q\)-state polarization levels spans between \(10\) dB and \(20\) dB over UP-AE-based conventional systems. Our simulation also showed that by utilizing the proposed improved-constellation algorithm the DCMC capacity and BER performance of our PM system have significantly improved.

APPENDIX A

The derivation of Equation (51) can be formulated by substituting \(u = \sin^2 (\theta)\) and \(d\theta = \frac{du}{2\sqrt{u(1-u)}}\) into Equation (46),

\[
\int_{u_1}^{u_2} \frac{1}{\sqrt{u(1-u)}} \, du
\]
yielding

\[ P \left( S \to \hat{S} \right) = \frac{1}{\pi} \int_0^1 \left( \frac{u}{u + c_1} \right)^{-\frac{\nu}{2}} \left( \frac{u}{u + c_1} \right)^{-\frac{\nu}{2}} du \]

and

\[ P \left( S \to \hat{S} \right) = \frac{c_1 c_2}{2\pi} \int_0^1 u^{\nu_r - \frac{1}{2}} \left( 1 - u \right)^{-\frac{1}{2}} \]

\[ \left( 1 + \frac{1}{c_1 u} \right)^{-\frac{\nu}{2}} \left( 1 + \frac{1}{c_2 u} \right)^{-\frac{\nu}{2}} du. \]

Now, by relying on the confluent hypergeometric function of two variables given as (Section 9.18 [61])

\[ F_1 (\alpha, \beta, \beta'; \gamma; x, y) = \frac{\Gamma (c)}{\Gamma (a) \Gamma (c - a)} \int_0^1 z^{\alpha-1} (1-z)^{-\gamma-\alpha-1} (1-xz)^{-\beta} (1-yz)^{-\beta'} dz, \]

the closed-form expression of (60) can be expressed as shown in Equation (51), where \( \Gamma (\cdot) \) denotes the Gamma function.

**APPENDIX B**

Instead of using the Q-function defined in Equation (34), we can simply use the approximation defined by [60] as

\[ Q (x) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{2x^2}{3}}. \]

By plugging (62) into (33), we arrive at

\[ P \left( S \to \hat{S} | H \right) \approx \frac{1}{12} e^{-\frac{x^2}{2}} + \frac{1}{4} e^{-\frac{2x^2}{3}}. \]

Given \( N_r \) receive AEs, the SNR of the \( n_r/2 \)-th channel denoting the channel received at the \( n_r/2 \)-th AE is given by

\[ \gamma_{n_r} = \frac{1}{2N_0} \left( \gamma_\nu + \gamma_\nu \right) N_r = 2, \]

where \( \left( \gamma_\nu + \gamma_\nu \right) N_r = 2 \) is equivalent to Equation (42) with \( N_r = 1 \) and \( N_r = 2 \).

Now, the average of PEP can be expressed as

\[ P \left( S \to \hat{S} \right) \approx \int_0^\infty \cdots \int_0^\infty \prod_{n = 1}^{N_r/2} \left( \exp \left( -\frac{\gamma_{n_r}}{2} \right) f_{\gamma} (\gamma n_r) \right) \]

\[ + \exp \left( -\frac{2\gamma_{n_r}}{3} \right) f_{\gamma} (\gamma n_r) \) d\gamma_1 \cdots d\gamma_{n_r}. \]

By using the definition of the MGF function in (56), Equation (21)), the close-form expression of \( P \left( S \to \hat{S} \right) \) can be formulated as shown in Equation (55).

**APPENDIX C**

Here, we provide an example of an RS-based PM(TAR, \( Q = 4 \), BPSK) system using the technique presented in Section V, which is referred to as TAR-RS in Section VI. Consider a PM system that relies on a set of BPSK symbols \( X_i = \{-1, +1\} \) and on a randomly generated set \( \{A_q\}_q \) for data transmission, which can be formulated as follows:

\[ A_1 = \left[ -0.331952 + 0.686751i, 0, -0.631246 + 0.140389i \right], \]

\[ A_2 = \left[ -0.853098 + 0.074374i, 0, 0.0196869 + 0.516047i \right], \]

\[ A_3 = \left[ -0.493946 - 0.228332i, 0, -0.797398 + 0.260841i \right], \]

\[ A_4 = \left[ -0.160197 - 0.557432i, 0, -0.43818 - 0.686735i \right], \]

where \( q = 1, \ldots, Q = 4 \). By using Equations (1-4) these configurations can be translated to the following parameters:

\[ \theta_h = \{0.762771, 0.896334, 0.544168, 0.579994\}, \]

\[ \theta_v = \{0.646669, 0.516423, 0.838976, 0.814621\}, \]

\[ \theta_\gamma = \{115.798, 175.017, -155.191, -106.034\}, \]

\[ \theta_\nu = \{167.461, 87.8153, 161.886, -122.54\}, \]

and finally,

\[ \tau = \{130.29, 121.088, 57.0821, 54.55\}, \]

\[ AR_{dB} = \{42.1995, 38.6012, 27.1086, 52.4841\}. \]

**REFERENCES**


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