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Improper Gaussian Signaling for Integrated Data and Energy Networking

H. Yu, H. D. Tuan, T. Q. Duong, Y. Fang, and L. Hanzo

Abstract—The paper considers the problem of beamforming design for a multi-cell network of downlink users, who either harvest energy or decode information or do both by receiving signals from the multi-antenna base station (BS) within a time slot and over the same frequency band. Our previous contributions have showed that the time-fraction based energy and information transmission, under which first the energy is transferred within the initial fraction of time and then the information is transferred within the remaining fraction, is the most efficient design alternative both in terms of its practical implementation and network performance. However, at the time of writing, both energy and information beamforming has only been implemented for proper Gaussian signaling (PGS), which has limited the network’s throughput. Although the network throughput could be improved in some specific scenarios by using non-orthogonal multi-access (NOMA), this may compromise the user secrecy. In order to circumvent the above implementations, we conceive improper Gaussian signaling (IGS) for information beamforming, which enables the network to substantially improve its throughput in any scenario without jeopardizing the user secrecy despite its low-complexity signal processing at the user end. A simpler subclass of IGS is also considered, which also outperforms NOMA PGS and works under any arbitrary scenario.

Index Terms—Improper Gaussian signaling, multi-cell system, energy-harvesting, information throughput, nonconvex optimization, energy-harvesting constraint.

I. INTRODUCTION

The Internet-of-things (IoT) further broadens the challenges imposed on wireless communications by demanding wireless access for not only information but also for energy [1]. An access point may provide an information service or energy service, or both. In terms of base stations (BSs), it is expected that they are able to transfer not only information but also energy, requiring both high information throughput and substantial harvested energy. In fact, signal processing conceived for high information throughput aims for mitigating the interference at the receiver end, whilst interference actually can be beneficial for harvesting energy.

At the time of writing, there are two popular techniques of transferring information and energy over the same wireless medium within a time slot. The first one is the so-called simultaneous wireless information and power transfer (SWIPT) [2]–[6], which splits the received signal into two components, namely one for energy-harvesting (EH) and one for information decoding (ID) either by power splitting or time-switching (TS). Its practical implementation requires a sophisticated variable power-splitter [4]. From a signal processing perspective, it would be counterproductive to design a common beamformer to optimize the conflicting targets of information and energy beamforming at the same time. The second approach is the so-called time-fraction-based information and power transfer (TFIPT) relying on separate fractions of the time-slot [7]–[9],1 which may be conveniently implemented in practice and it is capable of outperforming SWIPT. Under this approach, the EH is improved by energy beamformers, while the information throughput is improved by information beamformers.

To improve the information throughput, which suffers from the network’s ambition to provide EH service, we may invoke non-orthogonal multiple access (NOMA) (see e.g. [10], [11]), in order to compensate for the EH-induced throughput loss, when supporting multiple users. It was also shown in [11] that NOMA-based TFIPT outperforms its SWIPT counterpart. Since the main factor limiting the network throughput is multi-user interference, under NOMA the users of better channel conditions access and decode the information intended for users of poorer channel conditions to subtract it from their received signal before decoding their own information. However, this procedure degrades the secrecy of the users of poorer channel conditions. Moreover, the information throughput gain by NOMA is only substantial enough when the users channel conditions are strongly differentiated. Otherwise, conventional

1One should not confuse this with SWIPT, which splits the received signal using time-switching.
orthogonal multi-access (OMA) is still preferred in terms of both its information throughput and user secrecy.

Proper Gaussian signaling (PGS) relying on circularly symmetric complex Gaussian (proper) signals is popular owing to its ease of analysis and design, but it requires the multi-user interference (MUI) to be completely suppressed [12]. This requirement may be eliminated by Improper Gaussian signaling (IGS) [13], [14], which was shown to exhibit supremacy over PGS in diverse practical scenarios, for example in single-input single-output (SISO) networks [15]–[22] or in MIMO interference networks [23]–[27] of multiple unicast transmitter-receiver pairs, as well as in broadcast networks [28]–[30] and in cognitive networks [31]–[33] relying on PGS for the primary users and IGS for the secondary users. Most recently, NOMA-PGS and NOMA-IGS was designed for multi-user multi-cell networks in [34]. In contrast to proper Gaussian signals having arbitrary covariance, improper Gaussian signals are characterized by the so-called augmented covariance of double size associated with a special structure involving its covariance and pseudo-covariance [13]. As such, in contrast to PGSs which are generated by linearly beamforming proper Gaussian sources, IGSs are generated by the widely linear beamforming of Gaussian sources, which are determined by a pair of correlated beamforming vectors. The design of beamforming vectors for IGS is more complex than for PGS not only because it involves twice the number of decision variables, but more importantly, the throughput functions are log-determinant log det(.) even for multi-input single output (MISO) networks. Hence their optimization problem is much more computationally challenging than that of the optimization of the logarithmic PGS throughput. However, as mentioned above, NOMA PGS requires additional processing at the users of better channel conditions to decode the information intended for the users of poorer channel conditions, and thus jeopardizes the secrecy of weaker users. By contrast, IGS improves the users’ throughput without the above-mentioned extra NOMA-processing at the receiver end.

Against the above background, this paper proposes IGS for energy-harvesting aided networks with the following main contributions:

- We conceive and generate improper Gaussian signals by applying widely linear beamforming to proper Gaussian sources to improve the information throughput subject to EH constraints. The corresponding beamforming optimization problem becomes nonconvex, which involves log determinant functions, and thus it is computationally challenging. Hence a path-following computational procedure is proposed for this nonconvex problem, which iterates between improved feasible points and converges at least to a locally optimal solution.

- Additionally, we then develop a simplified IGS (s-IGS), which still improves the information throughput by applying linear beamforming to improper Gaussian sources. The resultant reduced-complexity beamforming optimization problem is then solved by a new path-following procedure.

- The simulation results provided demonstrate that both IGS and s-IGS outperform NOMA PGS. Hence, the information throughput can be improved without any additional signal processing at the user end and yet the user secrecy is preserved.

The paper is organized as follows. Beamforming optimization problems for IGS and s-IGS are addressed in Sections II and Section III, respectively, while the simulations demonstrating their advantage over NOMA are provided in Section IV. Section V concludes the paper, which is followed by the Appendix. The Appendix develops new fundamental matrix inequalities, which were used for developing the path-following algorithms in Sections II and III.

**Notation.** Bold-faced lower-case (upper-case, resp.) letters, e.g. $x$ (X, resp.), are used for vectors (matrices, resp.), and lower-case letters, e.g., $x$, are used for scalars. The size of $N$-dimensional column vectors is $N \times 1$ while the size of $N$-dimensional row vectors is $1 \times N$. Analogously, the size of matrices with $N$ rows and $M$ columns is $N \times M$. $\mathbb{E}$ is the expectation operator. $|| \cdot ||_F$ is the Frobenius norm of matrices. $[X]_2$ is $XX^H$, and $\langle X, Y \rangle = \text{trace}(X^H Y)$. The notation $X \succeq 0$ ($X \succeq 0$, resp.) used for the Hermitian symmetric matrix $X$ indicates that it is positive definite (positive semi-definite, resp.). $I_n$ is the identity matrix of size $n \times n$. We also use $\langle A \rangle = \text{trace}(A)$, $x_S$ for the set $S$ used as a subscription represents the set $\{x_s : s \in S\}$, and $\mathbb{R}_+^N \triangleq \{(x_1, \ldots, x_n)^T : x_i > 0, i = 1, \ldots, n\}$.

### II. System Model for Improper Gaussian Signal Processing

Fig. 1 illustrates the downlink (DL) of a system consisting of $N$ cells under dense deployment, where the BS of each cell is equipped with $N_t$-transmit antennas (TAs) serving multiple single-antenna-aided users within its cell. In the $i$-th cell, there are $K$ energy-harvesting (EH) users (EU) indexed by $(i, e_1), \ldots, (i, e_K)$, who harvest energy transferred by the BS through the wireless channels and thus have to be located sufficiently near to their BS. There are $M$ information-receiving users (IUs) indexed by $(i, d_1), \ldots, (i, d_M)$, who receive and decode information transmitted by the BS through the wireless DL channels. Note that there is a potential overlap between the sets of EUs and IUs, whenever there are users, who receive both energy and information from the BS through the same wireless channels. Then

$$S_E \triangleq \{(s, e_\ell) : s = 1, \ldots, N; \ell = 1, \ldots, K\}$$

and

$$S_I \triangleq \{(s, d_\ell) : s = 1, \ldots, N; \ell = 1, \ldots, M\}$$

respectively represent the set of EUs and IUs. Under time-frequency-based information and energy transfer [7]–[9], the specific fraction of time $0 < 1/t_1 < 1$ is used for power transfer, while the remaining fraction of $0 < 1/t_2 < 1$ is used for information transfer. Let $h_{s,e_1} \in \mathbb{C}^{N_t \times N_t}$ be the channel spanning from the BS to the EU $(i, e_1)$, $x_{s,e_\ell} = v_{s,e_\ell}s_{s,e_\ell} \in \mathbb{C}^{N_t \times 1}$ be the beamformed energy signal intended for the EU $(s, e_\ell)$, where $v_{s,e_\ell} \in \mathbb{C}^{N_t \times 1}$ is the energy beamformer and $s_{s,e_\ell} \in \mathbb{C}^{N_t \times 1}$ is the energy symbol with $\mathbb{E}(|s_{s,e_\ell}|^2) = 1$. All
Gaussian source $s_{s,d} (\mathbb{E}(|s_{s,d}|^2) = 1$ and $\mathbb{E}((s_{s,d})^2) = 0)$ as [34]

$$x^s_{s,d} = w_{1,s,d}s_{s,d} + w_{2,s,d}s^*_{s,d}$$  (5)

with the aid of the beamformers $w_{1,s,d} \in \mathbb{C}^{N_i \times 1}$ and $w_{2,s,d} \in \mathbb{C}^{N_i \times 1}$. Then, the signal received in (4) at the IU $(i, d_j)$ is rewritten as

$$y_{i,d_j} = \sum_{(s,d) \in S_i} h_{s,i,d_j} (w_{1,s,d}s_{s,d} + w_{2,s,d}s^*_{s,d}) + n_{i,d_j}.$$  (6)

By writing

$$(x^s_{1,d_j})^* = \begin{bmatrix} w^*_{2,s,d} & w^*_{1,s,d} \\ s_{s,d} & s^*_{s,d} \end{bmatrix}$$

and defining $w_{s,d} = \{w_{j,s,d}, j = 1, 2\}$, we can express the augmented equation of (6) as

$$\tilde{y}_{i,d_j} \triangleq \begin{bmatrix} y_{i,d_j} \\ y_{i,d_j}^* \end{bmatrix} = \sum_{(s,d) \in S_i} \begin{bmatrix} h_{s,i,d_j} w_{1,s,d} \\ h^*_{s,i,d_j} w^*_{1,s,d} \end{bmatrix} \begin{bmatrix} s_{s,d} \\ s^*_{s,d} \end{bmatrix} + \begin{bmatrix} n_{i,d_j} \\ n^*_{i,d_j} \end{bmatrix} = \Lambda_{s,i,d_j} (w_{s,d}) \tilde{s}_{s,d} + \tilde{n}_{i,d_j}$$  (7)

for

$$\Lambda_{s,i,d_j} (w_{s,d}) \triangleq \begin{bmatrix} h_{s,i,d_j} w_{1,s,d} & h_{s,i,d_j} w_{2,s,d} \\ h^*_{s,i,d_j} w^*_{1,s,d} & h^*_{s,i,d_j} w^*_{2,s,d} \end{bmatrix},$$

which represents a linear mapping from $\mathbb{C}^{(2N_i) \times 1}$ to $\mathbb{C}^{2 \times 2}$, and

$$\tilde{s}_{s,d} \triangleq \begin{bmatrix} s_{s,d} \\ s^*_{s,d} \end{bmatrix} \in \mathbb{C}^2, \tilde{n}_{i,d_j} \triangleq \begin{bmatrix} n_{i,d_j} \\ n^*_{i,d_j} \end{bmatrix} \in \mathbb{C}^2.$$  (9)

It may be readily shown that

$$\mathbb{E}(|\tilde{s}_{i,d_j}|^2) = \mathbb{I}_2, \mathbb{E}(|\tilde{n}_{i,d_j}|^2) = \sigma^2 \mathbb{I}_2,$$  (10)

and

$$\mathbb{E}\left[|\Lambda_{s,i,d_j} (w_{s,d}) \tilde{s}_{s,d}|^2\right] = |\Lambda_{s,i,d_j} (w_{s,d})|^2.$$  (11)

The throughput at the IU $(i, d_j)$ expressed in nats/sec/Hz is given by the mutual information (MI) between $\tilde{y}_{i,d_j}$ and $\tilde{s}_{i,d_j}$ computed as [35]

$$\frac{1}{2t_2} r_{i,d_j} (w_{S_1})$$  (12)

for

$$r_{i,d_j} (w_{S_1}) = \ln \left| \mathbb{I}_2 + |\Lambda_{s,i,d_j} (w_{s,d})|^2 \left(\Psi_{i,d_j} (w_{S_1})\right)^{-1} \right|,$$  (13)

and

$$\Psi_{i,d_j} (w_{S_1}) \triangleq \sum_{(s,d) \in S_i \setminus \{(i,d_j)\}} |\Lambda_{s,i,d_j} (w_{s,d})|^2 + \sigma^2 \mathbb{I}_2.$$  (14)

The RF-to-harvested energy conversion function is in practice non-linear. However, there is no generally agreed accurate function at the time of writing. Hence, to avoid obfuscating the salient IGS-related trends, we have opted for this simple linear model.
Based on (2) and (12), we consider the following problem of max-min throughput optimization

\[
\begin{align*}
\max_{\mathbf{v}_{SE}, \mathbf{w}_{S_j}, \ell = (1, 2)^T \in \mathbb{R}^2_{+}, \gamma} & \quad r_{i,d_j} (\mathbf{w}_{S_j}) \\
\text{s.t.} (15a) & \quad (12) \\
\end{align*}
\]

where we have

\[
\begin{align*}
a^{(k)}_{i,d_j} & = r_{i,d_j} (\mathbf{w}^{(k)}_{S_j}) \\
& - [(\Lambda_{i,i,d_j} (\mathbf{w}^{(k)}_{S_j}))^2 (\Psi_{i,d_j} (\mathbf{w}^{(k)}_{S_j}))^{-1}] (17a) \\
B^{(k)}_{i,d_j} & = (\Lambda_{i,i,d_j} (\mathbf{w}^{(k)}_{S_j}))^H (\Psi_{i,d_j} (\mathbf{w}^{(k)}_{S_j}))^{-1}, (17b) \\
0 \leq C^{(k)}_{i,d_j} & = (\Psi_{i,d_j} (\mathbf{w}^{(k)}_{S_j}))^{-1} \\
& - [(\Lambda_{i,i,d_j} (\mathbf{w}^{(k)}_{S_j}))^2 + \Psi_{i,d_j} (\mathbf{w}^{(k)}_{S_j})] (17c)
\end{align*}
\]

Meanwhile, the RHS of (15b) is upper bounded as follows:

\[
2 \gamma t_2 \leq \gamma (\gamma t_2)^2 \left( \frac{\gamma}{\gamma (t_2)} + \frac{t_2}{\gamma (t_2)} \right)^2 .
\] (18)

Using (16) and (18), the nonconvex constraint (15b) is innerly approximated by the following convex constraint in the sense that any feasible point for the latter is also feasible for the former:

\[
r^{(k)}_{i,d_j} (\mathbf{w}_{S}) \geq \gamma (\gamma t_2)^2 \left( \frac{\gamma}{\gamma (t_2)} + \frac{t_2}{\gamma (t_2)} \right)^2 , (i, d_j) \in \mathcal{S}_f .
\] (19)

From (3), the LHS of (15c) is seen to be convex quadratic, hence the nonconvex constraint (15c) is said to be reverse convex and can be innerly approximated by a convex constraint by linearizing its LHS at \( \mathbf{v}^{(k)}_{S} \) [36]

\[
\sum_{\ell = 1}^{K} [2 \Re (\mathbf{h}_{i,e}^H \mathbf{w}^{(k)}_{S} \mathbf{h}_{i,e} + \mathbf{v}_{i,e}^H \mathbf{v}_{i,e})] - \mathbf{h}_{i,i,e}^H \mathbf{v}_{i,e} \geq \frac{\gamma}{\gamma (t_2)} + \frac{t_2}{\gamma (t_2)}, (i, d_j) \in \mathcal{S}_E ,
\] (20)

which was used in the previous treatises of [7]–[9] handling EH constraints.

At the \( k \)th iteration we solve the following convex problem, which provides a feasible value for (15), to generate the next feasible point \((\mathbf{v}^{(k+1)}_{SE}, \mathbf{w}^{(k+1)}_{S_I}, t^{(k+1)}, \gamma^{(k+1)})\) for (15):

\[
\max_{\mathbf{v}_{SE}, \mathbf{w}_{S_j}, \ell = (1, 2)^T \in \mathbb{R}^2_{+}, \gamma} \quad \text{s.t.} \quad (15d) - (15f), (19), (20).
\] (21)

This convex problem involves \( n_c = NN_f (K + 2M) + 3 \) decision variables and \( m_c = 1 + N (K + M + 1) \) quadratic constraints, hence its computational complexity is [37]

\[
O(m_c 2^{2n_c} (n^2_c + m_c)) .
\] (22)

Note that we have \( \gamma^{(k+1)} > \gamma^{(k)} \) as long as \((\mathbf{v}^{(k+1)}_{SE}, \mathbf{w}^{(k+1)}_{S_I}, t^{(k+1)}, \gamma^{(k+1)}) \neq (\mathbf{v}^{(k)}_{SE}, \mathbf{w}^{(k)}_{S_I}, t^{(k)}, \gamma^{(k)})\), because they respectively are the optimal solution and a feasible point for (21). This means that \((\mathbf{v}^{(k+1)}_{SE}, \mathbf{w}^{(k+1)}_{S_I}, t^{(k+1)}, \gamma^{(k+1)})\) is a better feasible point than \((\mathbf{v}^{(k)}_{SE}, \mathbf{w}^{(k)}_{S_I}, t^{(k)}, \gamma^{(k)})\) for (15). As such, the sequence \( \{(\mathbf{v}^{(k)}_{SE}, \mathbf{w}^{(k)}_{S_I}, t^{(k)}, \gamma^{(k)})\} \) of feasible points for (15) converges at least to a point satisfying the Karush-Kuh-Tucker (KKT) condition of optimality [38]. Our previous result (see e.g. [9]) shows that such a point often turns out to be the globally optimal solution of (15).
It is important to locate a feasible point \((\mathbf{v}_{S_E}^{(0)}, \mathbf{w}_{S_I}^{(0)}, \mathbf{t}^{(0)}, \gamma^{(0)})\) for (15) for initializing the path-following procedure. Let us fix \(\mathbf{t}^{(0)} = (\mathbf{t}_1^{(0)}, \mathbf{t}_2^{(0)})\) and good \(\gamma^{(0)}\) and randomly generate \((\mathbf{v}_{S_E}^{(0)}, \mathbf{w}_{S_I}^{(0)})\) feasible for (15e)-(15f). Then iterate as follows

\[
\max_{\mathbf{v}_{S_E}, \mathbf{w}_{S_I}, \eta} \eta \text{ s.t. } (15f) \quad (23a)
\]

\[
\begin{align*}
K \sum_{k=1}^{K} [2 \Re \{ (\mathbf{v}^{(k)}_{i,c_1})^H \mathbf{h}_{i,c_2} \mathbf{v}^{(k)}_{i,c_3} \} - |\mathbf{h}_{i,c_2}^H \mathbf{v}^{(k)}_{i,c_3}|^2] \\
\geq \frac{\gamma^{(0)}}{\zeta} \eta, \quad (i, \ell) \in S_T, \quad (23b)
\end{align*}
\]

\[
\frac{1}{t_1^{(0)}} \sum_{j=1}^{K} \eta \geq \frac{\gamma^{(0)}}{\zeta} \eta, \quad (i, \ell) \in S_E, \quad (23c)
\]

\[
\text{until reaching } \eta \geq 1 \text{ at } (\mathbf{v}_{S_E}^{(0)}, \mathbf{w}_{S_I}^{(0)}) \text{ in order to guarantee that } (\mathbf{t}^{(0)}, \gamma^{(0)}, \eta, \mathbf{v}_{S_E}^{(0)}, \mathbf{w}_{S_I}^{(0)}) \text{ is feasible for } (15). \]

Algorithm 1 represents the formal pseudo code of the above computational procedure.

Algorithm 1 IGS algorithm for (15)

1: **Initialization**: Set \(\kappa := 0\) and iterate (23) for finding a good initial feasible point \((\mathbf{v}_{S_E}^{(0)}, \mathbf{w}_{S_I}^{(0)}, \mathbf{t}^{(0)})\) for (15)

2: **Repeat until convergence of the objective in (15) is reached**: Solve the convex optimization problem (21) to generate the feasible point \((\mathbf{v}_{S_E}^{(k+1)}, \mathbf{w}_{S_I}^{(k+1)}, \mathbf{t}^{(k+1)})\) for (15); Reset \(\kappa := \kappa + 1\).

3: **Output** \(\mathbf{t}^{(k)}\), \(\mathbf{v}_{S_E}^{(k)}\), and \(\mathbf{w}_{S_I}^{(k)}\).

III. SIMPLIFIED IMPROPER GAUSSIAN SIGNALING

In (5), the improper Gaussian signal \(\mathbf{x}_{s,d}^{(1)}\) is generated as a widely linear transform of a proper Gaussian source \(s_{s,d}\). By contrast, in this section, the improper Gaussian signal \(\mathbf{x}_{s,d}^{(1)}\) in (4) is generated as a linear transform of an improper Gaussian source as follows

\[
\mathbf{x}_{s,d}^{(1)} = \mathbf{w}_{s,d} s_{s,d} + \mathbf{n}_{s,d},
\]

where \(s_{s,d}\) is a normalized improper Gaussian random variable \((\mathbb{E}[|s_{s,d}|^2] = 1)\), which is fully characterized by the augmented covariance defined in [13]:

\[
\mathbf{P}_{s,d} = \begin{bmatrix}
\mathbb{E}(|s_{s,d}^2|) & \mathbb{E}(s_{s,d}^2) \\
\mathbb{E}^*(s_{s,d}^2) & \mathbb{E}(|s_{s,d}^2|)
\end{bmatrix} =
\begin{bmatrix}
\mathbb{E}(s_{s,d} s_{s,d}^H) & \mathbb{E}^*(s_{s,d} s_{s,d}^H) \\
\mathbb{E}(s_{s,d}^H s_{s,d}) & \mathbb{E}(s_{s,d}^H s_{s,d}^H)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{1}{\sigma_{s,d}^2} q_{s,d} & 1 \\
1 & \frac{1}{\sigma_{s,d}^2}
\end{bmatrix}.
\]

(25)

\[\text{with } q_{s,d} \in \mathbb{C} \text{ satisfying the following convex quadratic constraint to make it qualified as a pseudo-covariance of } s_{s,d},\]

\[|q_{s,d}|^2 < 1, \quad (s, d) \in S_T,\]

which makes \(\mathbf{P}_{s,d}\) positive definite. Note that \(q_{s,d} = 0\) in (25) means \(\mathbb{E}(s_{s,d}^2) = 0\), i.e., \(s_{s,d}\) becomes proper.

By taking the square root according to

\[
\mathbf{p}_{s,d} = \sqrt{\mathbf{P}_{s,d}} = \begin{bmatrix} \alpha_{s,d} \beta_{s,d} \\ \beta_{s,d}^* \alpha_{s,d} \end{bmatrix} \geq 0,
\]

in conjunction with

\[
\alpha_{s,d} = \frac{1 + \sqrt{1 - |q_{s,d}|^2}}{\sqrt{2}}, \quad \beta_{s,d} = \frac{\sqrt{2}}{\sqrt{2}(1 + \sqrt{1 - |q_{s,d}|^2})^2},
\]

it can be readily shown that

\[
s_{s,d} = \alpha_{s,d} s_{s,d} + \beta_{s,d} \tilde{s}_{s,d},
\]

\[\text{for a normalized proper Gaussian } \tilde{s}_{s,d}. \]

Therefore (24) can be written in the widely linear form

\[
\mathbf{x}^{(1)}_{s,d} = \mathbf{w}_{s,d} \alpha_{s,d} \tilde{s}_{s,d} + \mathbf{w}_{s,d} \beta_{s,d} \tilde{s}_{s,d}^*,
\]

which a particular class of (5) associated with

\[
\mathbf{w}_{1,s,d} = \alpha_{s,d} \mathbf{w}_{s,d} \quad \text{and} \quad \mathbf{w}_{2,s,d} = \beta_{s,d} \mathbf{w}_{s,d}.
\]

The advantage of (24) over (5) is that for each \((s, \mathbf{d})\), the former involves only \(N_t + 1\) complex decision variables for information beamforming \((\mathbf{w}_{s,d})\) of dimension \(N_t\) plus the complex variable \(q_{s,d}\), while the latter involves \(2N_t\) complex decision variables \((\mathbf{w}_{s,d}, j = 1, 2, \text{each of dimension } N_t)\).

For the information transfer during the remaining \(1/t_2\) fractional time, the signal (4) received at the IU \((i, d)\) is now specified as

\[
y_{i,d} = \sum_{(s,d) \in S_T} \mathbf{h}_{s,i,d} \mathbf{w}_{s,d} s_{s,d} + n_{i,d}.
\]

(29)

By writing down its augmented form:

\[
\begin{bmatrix}
y_{i,d} \\
q_{i,d}
\end{bmatrix} = \sum_{(s,d) \in S_T} \mathbf{L}_{s,i,d} \begin{bmatrix}
\mathbf{w}_{s,d} \\
\mathbf{s}_{s,d}^*
\end{bmatrix} + \begin{bmatrix}
n_{i,d} \\
n_{i,d}^*
\end{bmatrix},
\]

for

\[
\mathbf{L}_{s,i,d} = \begin{bmatrix}
\mathbf{h}_{s,i,d} & 0 \\
0 & \mathbf{h}_{s,i,d}^* \mathbf{w}_{s,d}
\end{bmatrix} \in \mathbb{C}^{2 \times 2},
\]

which is a linear operator from \(\mathbb{C}^{N_t \times 1}\) to \(\mathbb{C}^{2 \times 2}\), we can readily determine the augmented covariance of the signal of interest in (29) as

\[
\Phi_{i,d}(\mathbf{w}_{s,d}, q_{i,d}) = \mathbf{L}_{s,i,d} \mathbf{P}_{s,d} \mathbf{L}_{s,i,d}^H + \sigma^2 \mathbf{I}_2.
\]

(31)

and the augmented covariance of the interference-plus-noise in (29) as

\[
\Gamma_{i,d}(\mathbf{w}_{s,d}, q_{i,d}) = \sum_{(s,d) \in S_T \setminus (i,d)} \mathbf{L}_{s,i,d} \mathbf{P}_{s,d} \mathbf{L}_{s,i,d}^H (\mathbf{w}_{s,d}) + \sigma^2 \mathbf{I}_2.
\]

(32)

The information throughput at user \((i, d)\) is then expressed as [12]

\[
\frac{1}{2t_2} r_{i,d}(\mathbf{w}_{s,d}, q_{s}),
\]

(33)
where

\[
\begin{align*}
r_{i,dj}(\mathbf{w}_{Si}, q_s) &= \ln \left| I_2 + \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_i, d_j) \left( \Gamma_{i,dj}(\mathbf{w}_{Si}, q_S) \right)^{-1} \right|. 
\end{align*}
\]

(34)

With \( \mathbf{x}^I_{s,d} \) defined by (24), the problem of max-min information throughput optimization subject to the EUs’ harvested energy and power constraints is formulated as follows instead of (15):

\[
\begin{align*}
\max_{t=(t_1, t_2) \in \mathbb{R}_+, \mathbf{v}_{SE}, \mathbf{w}_{Si}, q_s} \quad &\gamma \\
\text{s.t.} \quad & (15c), (15d), (26), (35a) \\
& \sum_{j=1}^{K} ||\mathbf{v}_{i,e_j}||^2 + \sum_{j=1}^{M} ||\mathbf{w}_{i,d_j}||^2 \leq P_i, \quad (35c) \\
& ||\mathbf{v}_{i,e_j}||^2 \leq P_{\max}(i, e_j) \in \mathcal{E}_i, \quad (35d) \\
& ||\mathbf{w}_{i,d_j}||^2 \leq P_{\max}(i, d_j) \in \mathcal{S}_i, \quad (35e)
\end{align*}
\]

where (35b) is the counterpart of (15b) for maximizing the IUs’ minimal throughput, while (35c) and (35d)-(35e) correspond to the power constraints (15c) and (15f), respectively. In (35), the constraint (26) is obviously convex, and the constraints (15c), (35c) and (35d)-(35e) are also convex just like their counterparts in (15).

The nonconvex constraint (35b) involves much fewer decision variables than its counterpart (15b) but the former also contains many crossed terms between beamformers and pseudo-covariances that require a different approximation technique.

Let \((t^{(\gamma)}, \gamma^{(\gamma)}, \mathbf{w}^{(\gamma)}_{Si}, \mathbf{v}^{(\gamma)}_{SE}, q^{(\gamma)}_S)\) be the feasible point for (35) found from the \((\gamma - 1)\)-th iteration. The nonconvex constraint (15d) in (35a) is innerly approximated by the convex constraint (20). However, we still need to develop an inner convex approximation of the nonconvex constraint (35b).

A. Path-following iteration

Use the equivalent representation

\[
\begin{align*}
r_{i,dj}(\mathbf{w}_{Si}, q_s) &= f_{i,dj}(\mathbf{w}_{i,dj}, P_{i,dj}, q_i, d_j) + g_{i,dj}(\mathbf{w}_{Si}, q_S), \quad (36)
\end{align*}
\]

in conjunction with

\[
\begin{align*}
f_{i,dj}(\mathbf{w}_{i,dj}, P_{i,dj}) &= \ln \left| I_2 + \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_i, d_j) \right| \\
&= 2 \ln |h_{i,i,dj}w_{i,dj}|^2 + \ln |P_{i,dj}|, \quad (37)
\end{align*}
\]

and

\[
\begin{align*}
g_{i,dj}(\mathbf{w}_{Si}, q_S) &= \ln \left| \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_i, d_j)^{-1} \right| \\
&+ \left| \Gamma_{i,dj}(\mathbf{w}_{Si}, q_S)^{-1} \right|, \quad (38)
\end{align*}
\]

Using the inequality (64) yields

\[
\begin{align*}
f_{i,dj}(\mathbf{w}_{i,dj}, P_{i,dj}) &\geq f_{i,dj}(\mathbf{w}_{i,dj}, P_{i,dj}) + 4 - \frac{2|h_{i,i,dj}w_{i,dj}|^2}{|h_{i,i,dj}w_{i,dj}|^2} - (P_{i,dj})^{-1} \\
&\geq -\frac{2|h_{i,i,dj}w_{i,dj}|^2}{2R\{h_{i,i,dj}P_{i,dj}\}} - |h_{i,i,dj}w_{i,dj}|^2 \\
&\geq \sum_{(s,d) \in \mathcal{S}_i} -\frac{\langle \chi_{i,dj,s,d}(\mathbf{w}_{s,d}) \rangle^2}{R_{s,d}} - \langle \mathbf{w}_{s,d} \rangle^2. \quad (39)
\end{align*}
\]

over the trust region

\[
\begin{align*}
2R\{h_{i,i,dj}P_{i,dj}\} &\geq |h_{i,i,dj}w_{i,dj}|^2 > 0. \quad (40)
\end{align*}
\]

Furthermore, using the inequality (62) in the Appendix yields

\[
\begin{align*}
&g_{i,dj}(\mathbf{w}_{Si}, q_S) + 2 - \langle \mathbf{B}_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) \rangle \\
&- \langle \mathbf{C}_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) \rangle = g_{i,dj}(\mathbf{w}_{Si}, q_S) + 2 - \sigma^2 \langle \mathbf{C}_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) \rangle \\
&\geq \sum_{(s,d) \in \mathcal{S}_i} -\frac{\langle \chi_{i,dj,s,d}(\mathbf{w}_{s,d}) \rangle^2}{R_{s,d}} - \langle \mathbf{w}_{s,d} \rangle^2. \quad (41)
\end{align*}
\]

where

\[
\begin{align*}
0 &< \mathbf{B}_{i,dj} \triangleq \left( \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj})^{-1} \\
&- \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) + \Gamma_{i,dj}(\mathbf{w}_{Si}, q_S)^{-1} \right) \\
0 &< \mathbf{C}_{i,dj} \triangleq \left( \Gamma_{i,dj}(\mathbf{w}_{Si}, q_S)^{-1} \\
&- \Phi_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) + \Gamma_{i,dj}(\mathbf{w}_{Si}, q_S)^{-1} \right)
\end{align*}
\]

and

\[
\begin{align*}
\chi_{i,dj,s,d}(\mathbf{w}_{s,d}) &\triangleq \mathcal{L}_{i,dj,s,d}(\mathbf{w}_{s,d}) \langle \mathbf{B}_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) \rangle^{1/2}, \\
\chi_{i,dj,s,d}(\mathbf{w}_{s,d}) &\triangleq \mathcal{L}_{i,dj,s,d}(\mathbf{w}_{s,d}) \langle \mathbf{C}_{i,dj}(\mathbf{w}_{i,dj}, q_{i,dj}) \rangle^{1/2}, \quad (42)
\end{align*}
\]

Let us introduce the positive definite matrix variables \( \mathbf{X}_{i,dj,s,d} \) of size \( 2 \times 2 \) satisfying the semi-definite constraints of

\[
\begin{align*}
[\chi_{i,dj,s,d}(\mathbf{w}_{s,d})]^2 &\leq \mathbf{X}_{i,dj,s,d}(s, d) \in \mathcal{S}_I \\
\mathbf{X}_{i,dj,s,d}(s, d) &\succeq 0. \quad (43)
\end{align*}
\]

Then, by using the inequality (65), we arrive at:

\[
\begin{align*}
\langle \chi_{i,dj,s,d}(\mathbf{w}_{s,d}) \rangle^2 &\leq \langle \mathbf{X}_{i,dj,s,d}(s, d) \rangle^2, \\
\frac{1}{4} \langle [\mathbf{X}_{i,dj,s,d}(s, d)]^2 \rangle^{1/2} &\leq \langle \mathbf{X}_{i,dj,s,d}(s, d) \rangle^2, \quad (44)
\end{align*}
\]

(46)

(47)
for
\[
X_{t,d_j,s,d}^{(k)} = \left[ x_{i,d_j,s,d} (w_{s,d})^{(k)} \right]^2, (s, d_t) \in S_I. \tag{48}
\]

Hence,
\[
g_{i,d_j}(w_{S_t}, q_{S_t}) \geq g_{i,d_j}^{(k)} \left( X_{S_t}, q_{S_t} \right) \tag{49}
\]
for
\[
g_{i,d_j}^{(k)} \left( X_{S_t}, q_{S_t} \right) \triangleq \frac{1}{4} \sum_{(s, d_t) \in S_I} \| (X_{i,d_j,s,d_t})^{-1/2} (X_{i,d_j,s,d_t} P_{s,d_t}) + X_{i,d_j,s,d_t} P_{s,d_t} \| F^{-1/2}.
\tag{50}
\]

At the \( k \)th iteration we solve the following convex optimization problem to generate the next iterative point \( (t^{(k+1)}, \gamma^{(k+1)}, w_{S_t}^{(k+1)}, v_{S_E}^{(k+1)}, q_{S}^{(k+1)}) \)
\[
\max_{t, w_{S_t}, q_{S_t}} \gamma \text{ s.t. (15d), (20), (26), (35c),}
\]
\[
(35d) - (35c), (40), (45), \tag{51a}
\]
\[
f_{i,d_j}^{(k)} (w_{i,d_j}, P_{i,d_j}) + g_{i,d_j}^{(k)} \left( X_{S_t}, q_{S_t} \right) \geq \frac{\gamma^{(k)}}{2} \left( \frac{\gamma^{(k)}}{\gamma^{(k)}} + \frac{t_2}{l_2} \right)^2, (i, d_j) \in S_I, \tag{51b}
\]

The computational complexity of this convex problem is (22) is determined by \( n_v = 3 + N [ M + N + 1 + \gamma M + K ] \) and \( m_c = 1 + N (2K + 4M + 1) \).

Note by observing (39) and (49) that the LHS of (51b) is a concave lower bounding approximation of the LHS of (35b), while by (18), the RHS of (51b) is a convex upper-bounding approximation of the RHS of (35b). Hence in fact the convex constraint (51b) is an inner approximation of the nonconvex constraint (35b). The convex problem (51) is then seen as an inner approximation of the nonconvex problem (35). Then \( \gamma^{(k+1)} > \gamma^{(k)} \) as far as \( (t^{(k+1)}, \gamma^{(k+1)}, w_{S_t}^{(k+1)}, v_{S_E}^{(k+1)}, q_{S}^{(k+1)}) \) and \( (t^{(k)}, \gamma^{(k)}, w_{S_t}^{(k)}, v_{S_E}^{(k)}, q_{S}^{(k)}) \) are the optimal solution and a feasible point for (51). As such, the sequence \( \{(t^{(k)}, \gamma^{(k)}, w_{S_t}^{(k)}, v_{S_E}^{(k)}, q_{S}^{(k)}) \} \) generated by (51) is of improved feasible points for the nonconvex problem (35) and it converges at least to a point satisfying the KKT condition of optimality [38].

B. Alternating descent iteration

One can see that the function \( r_{i,d_j}(w_{S_t}, q_{S_t}) \) defined by (34) is complex. We therefore develop an alternating procedure for its more efficient computation.

1) Alternating optimization in \( w_{S} \): By fixing \( q_{S_t} = q_{S_t}^{(k)} \), we address the problem
\[
\max_{t, w_{S_t}, q_{S_t}} \gamma \text{ s.t. (15c), (15d), (35c), (35d) - (35e),}
\]
\[
r_{i,d_j}(w_{S_t}, q_{S_t}^{(k)}) \geq 2\gamma t_2, (i, d_j) \in S_I. \tag{52a}
\]

Using the inequality (60) in the Appendix yields
\[
r_{i,d_j}(w_{S_t}, q_{S_t}^{(k)}) \geq 2\gamma t_2, (i, d_j) \in S_I.
\tag{52b}
\]

We solve the following convex optimization problem for generating the next feasible point \( (q^{(k+1)}_{S}, \gamma^{(k+1)}) \) for (35):
\[
\max_{q_{S_t}^{(k)}, \gamma^{(k+1)}} \gamma \text{ s.t. (15c), (20), (35c), (35d) - (35e),}
\]
\[
r^{(k+1)}_{i,d_j}(w_{S_t}^{(k)}, q_{S_t}^{(k)}) \geq 2\gamma t_2^{(k+1)}, (i, d_j) \in S_I. \tag{53}
\]

The computational complexity of this convex problem is (22) determined by \( n_v = 3 + N N t (M + K) \) and \( m_c = N (3K + 2M + 1) \).

Since the convex constraint (54b) is an inner approximation of the nonconvex constraint (52b), the convex problem (54) is seen as an inner approximation of the nonconvex problem (52). We then have
\[
\gamma^{(k+1)} \geq \gamma^{(k)}, \tag{55}
\]

because they are the optimal and a feasible value for (54).

2) Alternating optimization in \( q_{S} \): By fixing \( (t, w_{S_t}, v_{S_E}) = (t^{(k+1)}, w_{S_t}^{(k+1)}, v_{S_E}^{(k+1)}) \), we address the problem
\[
\max_{q_{S_t}} \gamma \text{ s.t. (26),}
\]
\[
r_{i,d_j}(w_{S_t}^{(k+1)}, q_{S_t}) \geq 2\gamma t_2^{(k+1)}, (i, d_j) \in S_I. \tag{56a}
\]

Using the inequality (61) in the Appendix yields
\[
r_{i,d_j}(w_{S_t}^{(k+1)}, q_{S_t}) \geq 2\gamma t_2^{(k+1)}, (i, d_j) \in S_I.
\tag{56b}
\]

We then solve the following convex optimization problem for generating the next feasible point \( (q_{S_t}^{(k+1)}, \gamma^{(k+1)}) \) for (35):
\[
\max_{q_{S_t}, \gamma^{(k+1)}} \gamma \text{ s.t. (26),}
\]
\[
r^{(k+1)}_{i,d_j}(q_{S_t}) \geq 2\gamma t_2^{(k+1)}, (i, d_j) \in S_I. \tag{57}
\]
We then iterate

\[ \gamma^{(k+1)} > \bar{\gamma}^{(k+1)} \geq \gamma^{(k)} \]

provided that \( \gamma^{(k+1)} > \bar{\gamma}^{(k+1)} \), hence the sequence \{\( (t^{(k)}, \gamma^{(k)}, w_{S_1}, v_{S_1}, q_{S_1}) \)\} is of feasible points for the nonconvex problem (35), which converges to a feasible point satisfying the KKT conditions for one of two variable sets \( (t, v_{S_E}, w_{S_1}) \) and \( q_{S_1} \), when the other is held fixed.

**C. Generating a good feasible point for (35)**

It is important to generate a good feasible point for (35). For this we fix \( t^{(0)} \) to satisfy (15d) and \( q_{S_1}^{(0)} \) (for instance \( q_{S_1}^{(0)} = 0.2 \) and reasonable \( \gamma^{(0)} \)). We then randomly generate \( w_{S_1}^{(0)} \) and \( v_{S_1}^{(0)} \) satisfying the convex constraints (35c)-(35e).

Let us set

\[
P_{i,d_j}^{(k)} = \begin{bmatrix}
1 & q_{i,d_j}^{(0)} \\
q_{i,d_j}^{(0)} & 1
\end{bmatrix}.
\]

We then iterate

\[
\max_{v_{S_E}, w_{S_1}, q_{S_1}} \eta 
\text{ s.t. } (23d), (35d) - (35e), (59a)
\]

\[
\tilde{r}_{i,d_j}^{(k)}(w_{S_1}) \geq 2\gamma^{(k)}(0)\eta_i, (i, d_j) \in S_1,
\]

\[
\sum_{j=1}^{K} ||v_{i,d_j}||^2 + \sum_{j=1}^{M} ||w_{i,d_j}||^2 \leq P, i = 1, \ldots, N
\]

(59b) until we have \( \eta > 1 \) for guaranteeing that \( (t^{(0)}, \gamma^{(0)}, \eta, w_{S_1}, v_{S_1}, q_{S_1}) \) is feasible for (35).

**D. Algorithm**

For optimizing a trade-off between the convergence speed and the solution optimality we propose Algorithm 2, which uses the alternating optimization until its convergence and then switches to the path-following optimization in order to converge at least to a locally optimal solution.

**Algorithm 2 s-IGS algorithm for (35)**

1. **Initialization**: Fix \( q_{S_1}^{(0)} \). Set \( \kappa := 0 \) and then iterate (59) for finding a good initial feasible point \( (v_{S_E}^{(0)}, w_{S_1}^{(0)}, t^{(0)}) \) for (35).

2. **Repeat until convergence of the objective in (35)**:
   - Solve the convex optimization problems (54) and (58) of alternating optimization to generate the feasible point \( (v_{S_E}^{(k+1)}, w_{S_1}^{(k+1)}, t^{(k+1)}) \) for (35); Reset \( \kappa := \kappa + 1 \).

3. **Repeat until convergence of the objective in (35)**:
   - Solve the convex optimization problem (51) of path-following optimization to generate the feasible point \( (v_{S_E}^{(k+1)}, w_{S_1}^{(k+1)}, t^{(k+1)}) \) for (35); Reset \( \kappa := \kappa + 1 \).

4. **Output** \( t = t^{(k)}, v_{S_E} = v_{S_1}^{(k)} \), and \( w_{S_1} = w_{S_1}^{(k)} \).

**IV. PERFORMANCE RESULTS**

In all our simulations we consider networks of three cells \((N = 3)\). The channel spanning from a BS to a user at a distance of \( d \) meters is expressed as \( \sqrt{10^{-3}\sigma I U} \), where \( \sigma_{PL} = 30 + 10\beta \log_{10}(d) \) is the path-loss in dB, and \( h \) is the Rician fading channel gain associated with a Rician factor of 10 dB for the EUs served by that BS only. Otherwise, \( h \) is the normalized Rayleigh fading channel gain. The path-loss exponent \( \beta \) is set to 3 for the Rician channels and to 2 for the Rayleigh channels. The power of the signal received by the UEs must exceed the threshold of \(-21 \text{dBm with } 13 \text{nm CMOS technology} [4] \) to facilitate EH. We set \( e_{min} = -20 \text{dBm, } \zeta = 0.5, P = 35 \text{dBm. The bandwidth is set to } B = 20 \text{MHz, the carrier frequency is set to } 2 \text{GHz, and the power spectral density of noise is } -174 \text{dBm/Hz.} \)

**A. NOMA favored scenario**

Fig. 2 illustrates a scenario, where \( K \) EUs \( (i, e_j), j = 1, \ldots, K \) also act as the first \( K \) IUs. The other \( K \) IUs \( (i, d_{K+j}), j = 1, \ldots, K \) in each cell are distributed near the cell boundary. Those IUs which are located near the cell-boundary, are not only in poorer channel conditions than the IUs \( (i, d_j), j = 1, \ldots, K \) but are then subject to intercell-interference. By bringing about the differentiated channel conditions between the near IUs \( (i, d_j), j = 1, \ldots, K \) and far IUs \( (i, d_{K+j}), j = 1, \ldots, K \), such scenario favours NOMA, helping it to perform better than the conventional OMA. Both NOMA and OMA use proper Gaussian signal for carrying information, i.e. \( q_{s,d_i} = 0 \) in (25) so the information signal \( x_{s,d_i}^1 \) defined by (24) or (5) for \( w_{s,d_i} = 0 \) is generated by linearly beamforming of a normalized proper Gaussian source \( s_{s,d_i} \). Under OMA, each IU \( (i, d_j), j = 1, \ldots, 2K \) decodes its own information \( s_{t,d_j} \) while under NOMA each pair of IUs \( (i, d_j) \) and \( (i, d_{K+j}) \), \( j = 1, \ldots, K \) decode the information.
for the IU \((i, d_{K+j})\) and then the IU \((i, d_j)\) subtracts \(s_{i, d_{K+j}}\) from its interference in decoding its own information \(s_{i, d_j}\).

Fig. 3 characterizes the convergence behaviour of the proposed Algorithms for \(N_t = 6\) and \(K = 3\), i.e. each BS is equipped with \(N_t = 6\) DL TAs and there are a total of 27 users served by the network. The NOMA PGS algorithm [11] converges rapidly as a benefit of the efficient approximation of the logarithmic functions. The convergence rate of the IGS and s-IGS algorithms is similar, but the computational complexity of the latter is significantly lower.

Fig. 4 plots the achievable minimum throughput under different numbers \(N_t\) of DL TAs for \(K = 2\) (18 users in total) and \(K = 3\) (27 users in total). Both the IGS and s-IGS outperform NOMA [11]. IGS outperforms s-IGS since the latter is a particular class of the former. All of them still benefit from the spatial diversity associated with the number \(N_t\) of BS TAs. This figure also shows the efficiency of the time fraction optimization as IGS, s-IGS and NOMA outperform their counter parts IGS \((t_1 = t_2 = 2)\), s-IGS \((t_1 = t_2 = 2)\) and NOMA \((t_1 = t_2 = 2)\), respectively, which use the half of the time-slot for power transfer and the remaining half for information transfer.

We now examine the achievable minimum throughput upon varying the BS transmit power budget \(P\) in Fig. 5 under \((N_t, K) = (6, 3)\). Both the IGS and s-IGA exploit the available transmit power much better than NOMA since the latter cannot use the total affordable power budget because its achievable minimum throughput is not sensitive to \(P \geq 33\) dBm. By contrast, by employing additional beamformers \(\mathbf{w}_{2, s, d_j}\) for the conjugate proper Gaussian information source \(s_{2, s, d_j}\) in (5) or optimizing the pseudo-covariance \(q_{s, d_j}\) in (25), IGS allows the total power budget be exploited for improving its throughput. Naturally, beyond a certain threshold, namely \(P = 41\) dBm in Fig. 5, its performance also becomes saturated. This should not be a surprise for interference-limited networks.

Fig. 6 portrays the users’ max-min throughput under \((N_t, K) = (6, 3)\) upon varying the EH threshold \(\epsilon_{\text{min}}\) to show the impact of the latter imposed on the former. As expected, the increase of the latter degrades the performance of the former.

Table I provides the rounded average number of iterations required required for the convergence of the three algorithms for \(K = 3\) under different number of BS TAs \(N_t\). For lower \(N_t\) the feasibility set becomes narrower, which forces all algorithms to converge slower.

<table>
<thead>
<tr>
<th>(N_t)</th>
<th>IGS</th>
<th>S-IGS</th>
<th>NOMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>30</td>
<td>31</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>28</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>22</td>
<td>15</td>
</tr>
</tbody>
</table>
1. General scenario

Fig. 7 illustrates a general scenario, where $M$ IUs are located outside the EH zone, hence they cannot act as EUs. The IUs’ channel conditions are not differentiated, therefore NOMA is inefficient. We thus compare IGS and s-IGS to the conventional PGS orthogonal multiple access (OMA), in which IU decodes its own message only.

Fig 8 shows the achievable minimum throughput for IGS, s-IGS and PGS OMA for different values of $N_t$. There are $K = 2$ EUs and $M = 4$ IUs for simulating Fig. 8(a), and $K = 3$ EUs and $M = 6$ IUs for simulating Fig. 8(b). As expected, IGS is the best performer, followed by s-IGS, while PGS OMA is the worst performer. Similarly to Fig. 4, this figure also includes the performance of IGS, s-IGS and PGS OMA at $t_1 = t_2 = 2$ to show the efficiency of the time fraction optimization.

Fig. 9 provides the achievable minimum throughput for varying values of the BS transmit power budget $P$. All three algorithms are capable of exploiting the affordable power budget to compensate for the increased distance from the BS to the IUs that makes the pathloss higher.

Finally, Table II provides the rounded average number of iterations for the convergence of IGS, s-IGS and PGS OMA for $(K, M) = (3, 6)$ and different values of $N_t$.

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>IGS</th>
<th>S-IGS</th>
<th>PGS OMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>17</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>19</td>
<td>19</td>
</tr>
</tbody>
</table>

V. Conclusions

We have involved improper Gaussian signaling (IGS), in both general format and a particular format (s-IGS), for improving the information throughput of a multi-cell energy-harvesting enabled network, which aims for transferring both information and energy over the same wireless channels within a time slot. In contrast to NOMA, which improves the network throughput by allowing the users of better channel condition to access and decode the information of the users of poorer channel condition, IGS is capable of improving the network throughput more substantially than NOMA, maintaining the users’ secrecy under OMA. Although the problem of max-min information user throughput subject to the EH thresholds and power budget is much more computationally challenging than its NOMA counterpart, the paper has developed path-following algorithms for its computation, which converge at least to a locally optimal solution. The numerical examples provided for networks serving 18 users and 27 users have confirmed the
advantages of IGS over NOMA and OMA proper Gaussian signaling. IGS is most beneficial in the most vital lower power regime or when we have a low number of transmit antennas (compared to the number of users).

The exploitation of IGS in massive multi-input multi-output energy-harvesting enabled networks is under current study.

**APPENDIX: FUNDAMENTAL INEQUALITIES**

The following inequalities hold true for matrices of dimension $2 \times 2$ [39] and [40]:

$$
\ln |I_2 + [V]^2(Y)^{-1}| \geq \ln |I_2 + [V]^2(Y)^{-1}| - \langle [V]^2(Y)^{-1} \rangle + 2\Re\{[V]^H(Y)^{-1}V\}
$$

$$
-((Y)^{-1} - (Y + [V]^2)^{-1}, [V]^2 + Y),
$$

\forall V, X \geq 0, Y > 0 \quad \& \quad \bar{V}, \bar{X} \geq 0, \bar{Y} > 0,

and

$$
\ln |I_2 + X(Y)^{-1}| \geq \ln |I_2 + X(Y)^{-1}| + 4
$$

$$
-(X + Y, (X + Y)^{-1}) - ((Y)^{-1}, Y)
$$

\forall V, X \geq 0, Y > 0 \quad \& \quad \bar{V}, \bar{X} \geq 0, \bar{Y} > 0,

and

$$
\log |X^{-1} + Y^{-1}| \geq \log |X^{-1} + Y^{-1}| + 2
$$

$$
-(\bar{X}^{-1} - (\bar{X} + \bar{Y})^{-1}, \bar{X})
$$

$$
-(\bar{Y}^{-1} - (\bar{X} + \bar{Y})^{-1}, \bar{Y})
$$

\forall X > 0, Y > 0 \quad \& \quad \bar{X} > 0, \bar{Y} > 0.

**Theorem 1:** The following inequalities hold true for all $X > 0, Y > 0$ and $\bar{X} > 0, \bar{Y} > 0$

$$
\langle X, Y \rangle \leq \frac{1}{4} ||X^{-1/2}XY^{1/2} + X^{1/2}YY^{1/2}||^2
$$

and

$$
\langle X, Y \rangle \leq \frac{1}{2} \left(||X^{-1/2}XY^{1/2}||^2 + ||X^{1/2}YY^{1/2}||^2\right)
$$

**Proof** It follows from the matrix inequality

$$
0 \leq (X^{-1/2}XY^{1/2} - X^{1/2}YY^{1/2})
$$

$$
\times (X^{-1/2}XY^{1/2} - X^{1/2}YY^{1/2})^H
$$

$$
= (X^{-1/2}XY^{1/2} + X^{1/2}YY^{1/2})
$$

$$
\times (X^{-1/2}XY^{1/2} + X^{1/2}YY^{1/2})^H
$$

$$
-2(X^{-1/2}XYX^{1/2} + X^{1/2}XXX^{1/2})
$$

and

$$
-2(X^{-1/2}XXY^{1/2} + X^{1/2}YY^{1/2})
$$

$$
\times (X^{-1/2}XXY^{1/2} + X^{1/2}YY^{1/2})^H
$$

$$
-2(X^{-1/2}XXY^{1/2} + X^{1/2}XXX^{1/2})
$$

and

$$
-2(X^{-1/2}XXY^{1/2} + X^{1/2}XXX^{1/2})
$$

$$
\times (X^{-1/2}XXY^{1/2} + X^{1/2}XXX^{1/2})^H
$$

$$
-2(X^{-1/2}XXY^{1/2} + X^{1/2}XXX^{1/2})
$$

and
that
\[
\frac{1}{2} \left( \frac{1}{2} \langle XY \rangle \right) = \left( \frac{1}{2} \langle XY \rangle \right) \leq \left( \frac{1}{2} \langle YY \rangle \right) = \left( \frac{1}{2} \langle YY \rangle \right).
\]

Therefore, we use
\[
\left( \frac{1}{2} \langle YY \rangle \right) = \left( \frac{1}{2} \langle YY \rangle \right) \geq 0
\]
that yields
\[
\left( \frac{1}{2} \langle YY \rangle \right) = \left( \frac{1}{2} \langle YY \rangle \right) \geq 0
\]
which is (66).

REFERENCES


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