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Directional Dependence of Extreme Metocean Conditions for Analysis and Design of Marine Structures

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Abstract

Marine structures are typically sensitive to the direction of wind and waves, especially in extreme metocean conditions. The extreme metocean conditions and their associated predicted directions are not easily reachable from traditional design methodologies. In this research, the most probable combinations of different extreme metocean conditions along with their associated direction are predicted for the HyWind Scotland wind farm, Scotland. To achieve this, the Hierarchical Bayesian Modelling approach is applied to define the Joint Probability Distribution Function (JPDF) of four combinations of metocean parameters, including wave direction, wind direction and wind-wave misalignment. The data is provided by the ERA-Interim dataset in 40 years (1979-2018). The JPDFs are composed of a marginal PDF of directional variables (a mixture of von-Mises Fischer distributions) and two conditional JPDFs which are defined to satisfy the periodicity and positivity of distribution parameters. Then, applying the Inverse First-Order Reliability Method (IFORM) to the JPDFs, the Environmental Contours (ECs) for four sets of metocean data are developed. The results show that extreme values obtained from ECs, including directional variables, are higher than the values of traditional linear ECs. The maximum 50-year extreme value of wind speed from the JPDF of wind direction, wind speed and wave height is 2 m/s higher than the same extreme extracted from the JPDF of wind speed, wave height and period. Another important observed point is that the direction at which the extreme of metocean parameters occurs is quite different from their dominant direction of wind rose or the most probable direction of their probability density function. According to the results, it seems for direction-dependent structures; the application of this method may lead to a more realistic presentation of joint occurrence of linear and directional metocean parameters.

Keywords: Extreme metocean analysis, Joint probability model, Linear and circular statistics, Directional dependence, First-Order Reliability Method (FORM), Rosenblatt transformation, Northwestern North Sea coastlines.

1. Introduction

The development of coastal and offshore structures towards areas with potentially more severe environmental conditions exposes the structures to extreme loading situations. Safety of such structures must, therefore, be ensured under the expected sea states during extreme metocean conditions. This necessitates a reliable estimation of specific important metocean parameters with very low probabilities of occurrence, which might lead to severe damages to the structure. Among all potential marine environmental factors, it is the wind and wave conditions to be considered as influential metocean parameters affecting significantly the design and integrity assessment of many marine engineering applications [1, 2]. Accordingly, current design standards and recommended practices, such as DNV GL's [3], NORSOK N-003 [4], API-RP-2A-WSD [5], ISO [6], IEC 64001-3 standard [7], and ABS [8], have issued guidelines prescribing modelling instructions on extreme wind and wave loadings through a series of load cases including either a single load pattern or a combination of load patterns. The required statistical measures of wind and waves in the specified extreme load cases are generally quantified through two types of variables: linear and circular, reflecting magnitude and direction of variables, respectively. The extreme values of linear variables in both single and combined loading patterns are typically assumed to be conditionally independent of their corresponding circular measures (i.e., linear circular-independent variables); and are associated with their frequently occurring values of their circular measures, such as mean or spectral peak directions [3-7, 9]. However, the probability of experiencing such direction-independent extreme loadings (i.e., single or combined) is very low and has often led to inconsistent estimation of environmental action effects associated with a particular recurrence period in other directions different than the typically considered direction (i.e., mean or spectral peak directions), which could result in inefficient and unreliable designs [10-12], especially in case of directional dependent structures such as non-axisymmetric support structures and mooring systems. This is due to the fact that these parameters are in reality correlated and direction-dependent so that due to the existence of usual multi-hour time lag between wind and wave fields during extreme events, a wind field with most likely less severe magnitude and different direction can be combined with a wave field during its extreme state [13-15].

Several recent studies have addressed this issue and provided some recommendations on probable appropriate treatments, see, e.g., [11, 12, 16-18]. On this basis, to deal with the directional dependence problem, a recommended approach is to apply the directional adjustment factors to the wind and wave extreme values obtained from omnidirectional marginal Probability Density Functions (PDFs) [11, 17, 18]. The directional dependent empirical formulations signifying wind and wave correlation can also be utilized

70 along with this method when time-series of data is not accessible for defining marginal PDFs of each
71 metocean variable [18]. A second approach to obtain directionality consistent extreme values is to develop
72 marginal PDFs within several directional bands, reflecting design requirements and constraints, for wind
73 and wave parameters. In this case, the directional characteristic values must be calculated such that the
74 overall non-exceedance probabilities obtained by the product of survival probabilities in all directions be
75 equal to the desired omnidirectional reliability level [12]. The main problem with these approaches is that
76 the dependence of linear and circular variables is either not taken into account or to some relatively large
77 extent ignored. A third and more realistic approach is to develop a joint probability model of occurrence for
78 metocean variables accounting different combinations of magnitudes and directions [16]. This approach is
79 a multivariate probabilistic method, in which the simultaneous occurrence of extreme values of linear
80 variables dependent on their circular measures could be predicted with an adequate degree of accuracy [3,
81 6, 7]. On this basis, consistent estimation of both single and combined wind and wave actions; and
82 consequently, extreme action effects can then be obtained [16]. The result, in this case, is an envelope also
83 known as “environmental contour”, which is, in fact, a simple, economic but yet sophisticated method
84 from computational and accuracy point of views to define an appropriate combination of metocean variables
85 with a specified MRP [1, 2]. The quality of environmental contour generally depends on several crucial
86 factors including derivation methodology, probabilistic description of variables, fitting process and
87 goodness-of-fit criteria, joint probability models (JPMs) (i.e., marginality or conditionality of each
88 parameter or in another word structural dependence of variables). In dealing with such difficulties, several
89 computational techniques to establish such multivariate probabilistic description including Inverse First
90 Order Reliability Method (IFORM) [19], Inverse Second-Order Reliability Method (ISORM) [20]), Highest
91 Density Contour (HDC) method [21], and hyper plans-/Monte Carlo-based approaches [22] have been
92 developed. The newly established direct sampling method has been also used for the development of ECs
93 in higher dimensions and directional variables [23, 24]. Moreover, different JPMs [10, 25-27], as well as
94 various parametric [28] and non-parametric [29] statistical approaches have been presented and examined
95 to define marginal/conditional PDF of variables.

96

97 **2. The current stage of directional metocean modelling and challenges**

98 The omni-directional and directional design conditions in combination of individual extreme values
99 have been the subject of many studies (directional adjustment factors and direction-dependent marginal
100 PDFs) [11, 12, 16-18]. The directionally effects in metocean joint modelling has also been addressed [10,

25-27]. However, the problem of making consistent design criteria using multivariate JPMs considering both intensities and directions of wind and waves is has not been appropriately studied. With this in mind, the main objective of this study is to quantify the influence of direction on the combined extreme values. To accomplish our goal, a series of direction-independent and -dependent multivariate joint metocean models using the Conditional Modelling Approach (CMA) are developed, and the extreme metocean conditions are compared by omnidirectional and directional contours. The joint probability distribution of random variables to capture their inherent interdependence can be estimated by different methods. There is no preferred or recommended method for producing environmental contours from site-specific data sets; however, the differences in the methods mainly result from how dependencies or correlations are defined. Therefore, choosing the best method, which neither under- nor over-estimates the required quantities, is not always straightforward. The CMA can be implemented in the joint probability modelling when the sufficient amount of data in the form of simultaneous multi-variate time series (hindcast or recorded) is available. In the CMA, the first random variable is divided into the bins and a conditional probability function for the second random variable associated with each bin is estimated by analyzing the collected data in each bin. When the environmental data is not present in the form of recorded or hindcast data and just the marginal distribution of data besides their correlations is in hand, an approximate joint probability distribution approach, such as copula [30] and Nataf-based [31] models are used, avoiding the need to search for conditional independence. Nonetheless, the problem with copula and Nataf distribution approaches is that since dependence structure is predefined based on certain correlations, the use of correlation coefficients together with the marginal distribution may not capture the effect of dependence between the variables as it is applicable by described conditional distributions. So, it might not be possible to reflect the underlying shape of the data distribution well. In this case, the shapes of contours are generally rigid concerning the given sample data [32, 33].

This study proposes four different sets of conditional joint models to evaluate the influence of directionality on the extreme metocean condition. The paper is organized as follows: In Section 3, the study area along with utilized metocean data, are presented. In Section 4, the basic methodology for the long-term description of metocean and considered conditional models for metocean linear and circular variables along with multivariate distribution for each representative model is described. The analysis framework and fitting procedure implemented in this study are presented in Section 5. The application of developed joint models in extreme value analysis is discussed in details in Section 6. At last, the discussion on the directional dependency of extreme metocean conditions and remarks are provided in Sections 8 and 9.

3. Study area and metocean data description

The area considered in the present study is located at the north-western portion of North Sea approximately 25 km off the northeast coast of Scotland, Peterhead, where a multi-MW floating offshore wind farm known as “Hywind Scotland Pilot Park” over an area of 15 km² with a water depth around 95-120 m is currently operating. An overview of the study area and study point location (57°29'00"N and 1°21'00"W) is presented in Figure 1. To develop the joint distribution of wind and wave parameters, simultaneous wind and wave time histories at the location for a regular time interval of hours over several years (e.g., typically 3-6 hours and 30-50 years, respectively [1]) are needed. In this study, the required long-term data to describe wind and wave environmental conditions are provided by ERA-Interim dataset, a dataset from Numerical Weather Prediction Model (NWPM) of European Center for Medium-range Weather Forecast (ECMWF). [34]. This database is produced by much more accurate atmospheric modelling and assimilation techniques (i.e., better resolution and description of the wind and wave fields) compared to previous ECMWF datasets such as ERA-15 and ERA-40. The accuracies of the ERA-Interim reanalysis wave and wind field data against measurements have also been comprehensively evaluated and verified in various locations, see e.g., [35]. The data contains information regarding wind parameters, including speed and direction along with wave characteristics consisting of wave height, wave period, and direction of the wave. It covers the span of 1979-2018 (40 years) with a temporal resolution of six-hours which means there are about 58440 short-term environmental condition data points in the gathered database.

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Figure 1. Location for study point at Hywind Scotland pilot wind park (yellow point) concerning the coast of Peterhead (red point). (For colour interpretation of this figure, the reader is referred to the web version of this article).

4. Methodology

Development of environmental contours (i.e., line in 2D cases and on a surface in 3D cases) is a process composed of generally two main steps: (1) Defining the dependence structure between metocean variables and (2) Establishing the environmental contours either directly in the original variable space (e.g., Monte Carlo-based approaches) or by transforming the variables between physical and standard normal spaces (e.g. IFORM). In the present study, the dependence between variables is described using the Conditional Modelling Approach (CMA) [36], while the joint models of the environmental variables are developed based on inverse first-order reliability method (IFORM) utilizing the Rosenblatt transformation [19, 37]. The theoretical elements of each step are described in details in the following subsections.

4.1. Dependence modelling of variables

One of the critical issues to obtain practical and reliable environmental contours is to describe the structure of dependence between intensities and associated directions of wind and wave characteristic parameters. To cope with this aspect, recent studies, see, e.g., [10, 25-27], have mainly adopted two

techniques. In the first approach, the directional distributions are conditioned on their most correlated characteristic parameters (i.e., one dependent parameter for conditional variables) as is expected to be compatible with the physics of the phenomenon [26]. For instance, wind direction depends on wind speed, but not vice versa. This model is sophisticated and realistic but can be computationally expensive as it turns out that there is no guaranty to have a unique conditional PDF for each circular variable in all directions. Therefore, alternative simplified but yet sufficiently accurate approaches with up to two dependent parameters for conditional variables have been suggested in order to reduce computational efforts. In these models, as the most widely used ones, the directional distributions are incorporated into at most tri-variate joint probability functions through marginal distributions [10, 25, 27]. This is generally a reasonable assumption mainly for the sake of convenience as it is easier to specify one unique PDF to each circular variable or their misalignment in all directions. In this case, the statistical dependencies in generated JPMs have also been defined using IFORM based Nataf transformation approach and CMA [25].

On this premise, the dependence between linear and circular metocean variables in the present study is modelled based on the Conditional Modelling Approach, (CMA). This method assumes that the probability density function of the combination of parameters can be described by the product of marginal distribution of a primary variable and some conditional PDFs of other ones. Therefore, the Joint Probability Distribution Function (JPDF) of random variables $(x_1, x_2, \dots, x_n)$ is written as:

$$f_{X_1, X_2, \dots, X_n}(x_1, x_2, \dots, x_n) = f_{X_1}(x_1) f_{X_2|X_1}(x_2|x_1) \dots f_{X_n|X_2, X_1, \dots, X_{n-1}}(x_n|x_1, x_2, \dots, x_{n-1}) \quad (1)$$

where $f_{X_1}(x_1)$ is the marginal PDF of the primary variable and $f_{X_n|X_2, X_1, \dots, X_{n-1}}(x_n|x_1, x_2, \dots, x_{n-1})$ is the conditional PDF of the x_n given x_1 to x_{n-1} .

Considering the exchangeability of parameters priority in the CMA, one can assume different forms of priority in the Joint probability model. Here, due to the high number of mixture components in the distributions fitted to directional parameters, and to fit less sophisticated forms of interpolation functions, the JPM is set so that the primary variable is the directional one (when one of the variables is directional). On this basis, in the present study, four sets of environmental contours from a combination of six main metocean parameters including both linear and circular variables (i.e., wind speed, U_w , significant wave height, H_s , peak wave period, T_p , wind direction, θ_{wind} , wave direction, θ_{wave} , and wind and wave misalignment, $\Delta\theta$) have been developed. The JPDF of these sets is first described in Table 1. In all the distributions, it is assumed that the dependence between the second and third variable is negligible as

described by [38]. Each distribution, hereafter termed as D1-D4, is a product of three distributions as defined by Equation 1. These underlying distributions are categorized into two main classes of marginal and conditional distributions. The base distributions of D1-D4 JPMs are a set of marginal distributions, as discussed in Subsection 4.1.1 and three forms of conditional distribution functions as displayed in Table 1.

Table 1.

The underlying distributions for Environmental Contours developed in this research.

JPDF	X_1	X_2	X_3	$f_{X_1}(x_1)$	$f_{X_2 X_1}(x_2 x_1)$	$f_{X_3 X_2}(x_3 x_2)$
D1	U_w	H_s	T_p	Weibull	LLCW	LLCL
D2	θ_{wave}	H_s	T_p	movMF	CLCW	LLCL
D3	θ_{wind}	U_w	H_s	movMF	CLCW	LLCW
D4	$\Delta\theta$	U_w	H_s	movMF	CLCW	LLCW

4.1.1. Marginal PDF of circular variables

The most naturally observed PDFs for circular parameters are shown to be the wrapped Cauchy, wrapped normal, and circular normal or von-Mises Fischer distribution functions [39, 40]. Among these forms, the von-Mises Fischer distribution function, which reduces to von-Mises distribution in the 2D case and is the normal distribution function projected over the perimeter of the circle, has shown to be the most suitable statistical distribution to model the directionality of metocean parameters, e.g., wind and wave directions [27, 41-43]. This distribution as a symmetric unimodal distribution is defined by its mean value, $\mu \in (0, 2\pi)$, and a concentration parameter, $\kappa > 0$, as:

$$f(\theta) = \frac{1}{2\pi I_0(\kappa)} \exp[\kappa \cos(\theta - \mu)] \quad (2)$$

where $I_0(\kappa)$ is the modified Bessel function of the first kind and zero-order. A mixture of von-Mises Fisher distributions can also be written as:

$$f_{\Theta_w}(\theta) = \sum_{i=1}^n \omega_i f_i(\theta)$$

$$0 < \omega_i \leq 1, \quad (3)$$

$$\sum_{i=1}^n \omega_i = 1$$

were ω_i are weight coefficients. In the present study, the utilized Expectation-Maximization (EM) algorithm for the log-likelihood of the mixture in the package movMF [44] is applied to find the mixture parameters.

4.1.2. Conditional PDFs of linear and circular variables

The conditional dependence of variables in D1-D4 models is modelled by the three distributions. These distributions appear in different JPMs (i.e., D1-D4) and their parameters are estimated from the underlying data for each distribution.

4.1.2.1. Linear-Linear Conditional Weibull Distribution (LLCW)

The conditional Weibull distribution of variable x_i given the linear variable x_j is defined by Equation 7. The shape and scale parameters of the Weibull distribution fitted to X_j are defined as power functions of the primary variable (X_i):

$$f_{X_i|X_j}(x_i|x_j) = \beta_{ll} \frac{x_j^{\beta_{ll}-1}}{\alpha_{ll}^{\beta_{ll}}} \exp\left(-\left(\frac{x_j}{\alpha_{ll}}\right)^{\beta_{ll}}\right) \quad (4)$$

where

$$\alpha_{ll} = c_1 + c_2 x_i^{c_3} \quad (5)$$

$$\beta_{ll} = c_4 + c_5 x_i^{c_6} \quad (6)$$

This model has been applied to demonstrate the dependence between wind speed and wave height in the D1, D3, and D4 models (see also Table 1).

4.1.2.2. Linear-linear Conditional Lognormal Distribution (LLCL)

The conditional lognormal distribution of a linear x_i given the linear variable x_j can be written as:

$$f_{X_i|X_j}(x_i|x_j) = \frac{1}{\sqrt{2\pi}\sigma_{\ln x} x_i} \exp\left(-\frac{(\ln(x_i) - \mu_{\ln x})^2}{2\sigma_{\ln x}^2}\right) \quad (7)$$

in which

$$\mu_{\ln x} = d_1 + d_2 x_j^{d_3} \quad (8)$$

$$\sigma_{\ln x}^2 = d_4 + d_5 \exp(-d_6 x_j) \quad (9)$$

The distribution of wave period given the wave height is assumed to follow this model for D1 and D2 models (see also Table 1).

241 4.1.2.3. Circular-Linear Conditional Weibull Distribution (CLCW)

242 The conditional Weibull distribution of a linear variable x_i , e.g., wave height, dependent on a directional
 243 variable, θ_w is defined by:

$$244 f_{X_i|\theta_j}(x_i|\theta_j) = \beta_{cl} \frac{x_j^{\beta_{cl}-1}}{\alpha_{cl}^{\beta_{cl}}} \exp\left(-\left(\frac{x_j}{\alpha_{cl}}\right)^{\beta_{cl}}\right) \quad (10)$$

245 where:

$$246 \alpha_{cl} = a_0 + \sum_{i=1}^n a_n \cos(n\theta_j) + b_n \sin(n\theta_j) \quad (11)$$

247 and

$$248 \beta_{cl} = a_0 + \sum_{i=1}^n a_n \cos(n\theta_j) + b_n \sin(n\theta_j) \quad (12)$$

249 The functions defined by Equation 5 and Equation 6 are Fourier series with non-negative coefficients since
 250 the shape and scale parameters of Weibull distribution cannot take negative values, the function should also
 251 be periodic in the period of $0^\circ \sim 360^\circ$ and be able to resemble the shape of data points with reasonable
 252 accuracy. This model has been applied to demonstrate the dependence of wind speed and wave height with
 253 their corresponding individual directions as well as misalignments in the D2-D4 models (see also Table 1).

254 4.2. Construction of Environmental Surface (ES) for D1-D4

255 A well-known way of tackling problems associated with events of small exceedance probabilities is
 256 using the structural reliability methods. If we can assume the performance function of a structural system
 257 with $g(X)$, where X is the vector of random variables, the $g(X) = 0$ is the failure surface or boundary
 258 of the system. The failure probability is defined then by:

$$259 p_f = \Pr(g(X) < 0) = \int_{g(X) < 0} f_X(X) dX \quad (13)$$

260 where $f_X(x)$ is the joint PDF of random variables. The integral can be solved by direct numerical integration
 261 methods if the $f_X(X)$ and $g(X)$ are known functions. The last section described the definition of $f_X(X)$ by
 262 CMA. The second unknown function is a failure surface ($g(X)$). Most of the structural reliability methods
 263 are defined to find a solution for the failure surface at the design point and the IFORM is set so that the
 264 failure surface $g(X)$ can be assumed to have a tangent hyperplane in the design point.

265 This method transforms the Correlated X variables from the physical space to the uncorrelated
 266 Gaussian U variables in the standard normal space. The distance of the design point from the origin in the
 267 standard normal space is called the reliability index β . For the case of standard normal space the reliability
 268 index is written as:

$$269 \quad \beta = \Phi^{-1}(1 - p_f) \quad (13)$$

270 where Φ is the CDF of the standard normal distribution function.

271 The standard normal space which is defined by a hypersphere which can be related to the probability of
 272 occurrence of variables and their associated return period as follows:

$$273 \quad \|U\|^2 = \beta^2 \quad (14)$$

274 where β is the target reliability level associated with return period R by the following relation:

$$275 \quad \beta = \Phi^{-1}\left(1 - \frac{T_s}{365.25 * 24 * R}\right) \quad (15)$$

276 Here T_s denotes the duration of each short term observation (here 6-hours) and R is assumed to be in
 277 years.

278 Through the definition of variables by CMA, the ECs are defined for each value of β . The Rosenblatt
 279 transformation can be written as:

$$\begin{aligned} 279 \quad U_1 &= \Phi^{-1}(F_{X_1}(x_1)) \\ 280 \quad U_2 &= \Phi^{-1}(F_{X_2|X_1}(x_2|x_1)) \\ U_3 &= \Phi^{-1}(F_{X_3|X_2}(x_3|x_2)) \end{aligned} \quad (16)$$

281 where $U_1 \dots U_n$ are the base vectors of the hypercube and defined by:

$$\begin{aligned} 281 \quad U_1 &= \beta_r \sin \varphi \cos \theta \quad 0 \leq \theta \leq 2\pi \\ 282 \quad U_2 &= \beta_r \sin \varphi \sin \theta \quad 0 \leq \varphi \leq \pi \\ U_3 &= \beta_r \cos \varphi \end{aligned} \quad (17)$$

283 Utilizing this method, any combination of metocean variables can produce their environmental contours
 284 when their joint probability distribution function is appropriately described.

285

286 **5. Joint models fitting procedure and specifications**

287 The joint models described in Section 4 has been fitted to the metocean data using MATLAB and the
 288 movMF package in CRAN-R [44]. To this aim, first, the marginal PDF of the primary variable is estimated

289 and then according to the order of variables in each model, the data has been sorted in bins, and the functions
 290 have been fitted to each related conditioned parameter. For example, the D1 JPM as described in Table 1 is
 291 a product of marginal PDF of wind speed ($f_{U_w}(u_w)$) as will be mentioned in Table 2 and two conditional
 292 JPPDFs. The first one is the LLCW of wave height given wind speed ($f_{H_s|U_w}(h_s|u_w)$) and the second one is
 293 the LLCL of wave period given the wave height $f_{T_p|H_s}(t_p|h_s)$. Accordingly, for the $f_{H_s|U_w}(h_s|u_w)$, the
 294 values of wave height H_s are sorted and binned according to their corresponding values of wind speed U_w
 295 , and then the conditional Weibull distribution of Equation 4 is fitted to the sorted and binned data by
 296 nonlinear-least squares method to define the parameters of Equations 5 and 6. For the $f_{T_p|H_s}(t_p|h_s)$ after
 297 binning and sorting the data, the LLCL of Equation 7 with parameters defined in Equations 8 and 9 is fitted
 298 in the same way. After defining the parameters of all the distributions, the ECs are produced applying the
 299 methodology described in Section 4.2. The overall process of the methodology applied in this research to
 300 conclude the directional interference of metocean parameters is illustrated in Figure 2.

Figure 2. A detailed flowchart of the analysis framework steps.

6. Results

In this section, two methods used to calculate the extreme values of linear and circular variables with a particular recurrence period are described. The first method commonly utilized in practice is a univariate distribution modelling of the data, in which the dependency of variables is ignored, and a second method proposed in this paper consisting a series of multivariate/joint distribution of linear and circular variables.

6.1. Raw data marginal distributions and correlation

In this section, the extreme values of linear variables from their marginal distributions are calculated from the marginal distributions fitted to them. To this aim, the marginal distributions of linear metocean variables (i.e., significant wave height (H_s), mean wind speed (U_w), mean wave period (T_p)) are displayed in Figure 2, while the probability density function of directional variables (i.e., mean wave and wind directions (θ_{wave} and θ_{wind}) as well as wind and wave misalignment ($\Delta\theta$) are shown in Figure 3. Table 1 also lists the variables and the distributions fitted in Figure 2 and Figure 3d-f. Moreover, the number of components estimated for each circular variable is presented in

Table 2. The details of the components are listed in Table A1 of the Appendix. The rose diagram for wind direction, wave direction, and wind-wave misalignment are plotted in Figure 3a-c. The correlation coefficients between the parameters as an informative measure of the concurrency of variables were also calculated using the Spearman's Rho [45] formulation and listed in

Table 3. These correlations between circular and linear variables (l-c), (l-l) for linear and (c-c) for circular variables will be discussed later. The Spearman's Rho is a measure of association which evaluates the monotonicity in the values of two variables, i.e. their tendency to change together in the same way, but not necessarily with a constant rate. If two variables are in a robust relationship but their relation is not monotonic, the Spearman's Rho may be significantly low. As it is expected, (l-l) coefficients showing a correlation between (H_s, U_w) and then (H_s, T_p) hold the highest values in

Table 3. The dependency between (U_w, T_p) is considerably low in contrast to the other (l-l) correlations, which was the base for the assumption of dependency between (U_w, T_p) in the D1 distribution. The definition of (l-c) and (c-c) correlation coefficients, based on the Spearman's Rho, are different than (l-l) and thus quite different values should be expected. When it comes to calculation of this measure for circular and linear variables, the concept becomes more complicated; i.e. the tendency of variables to decrease or

decrease in the same way for the circular and linear variables is not physically easy to interpret. While in the case of (l-l) and (c-c) the concurrency of variables can be observed and understood more easily.

Figure 3. Marginal distribution of linear (left panel) and directional (right panel) variables for the metocean data (from 1979 to 2018): (a) Mean wind speed, (b) Significant wave height, (c) Mean wave period, (d) Mean wave direction, (e) Mean wind direction, (f) wind-wave misalignment.

As shown in

Table 3., the (l-c) correlation coefficient of (H_s, θ_{wind}) is higher than that of (H_s, θ_{wave}) . Here a question may arise that why the wave height is more dependent on the wind direction than on the wave direction. Referring to the origin of Spearman's Rho, which is how the H_s and θ_{wave} can affect interchangeably, may help to answer the question. Since the waves are known to be mainly wind-generated, it is acceptable that the waves are affected by the wind direction and also because the wave direction is dependent on the

topology of the site and the changes in the wave direction toward the shoreline and changes in the depth. The correlation between $(U_w, \Delta\theta)$ and that of $(H_s, \Delta\theta)$, on the other hand, are close to each other, while the misalignment has a bit more tendency to be affected by U_w . The (c-c) correlation between $(\theta_{wave}, \Delta\theta)$ is the only negative value in Table 3. While the (c-c) correlation coefficient of $(\theta_{wind}, \Delta\theta)$ takes the reasonably high value of 0.161. This means the increase in the misalignment is much more dependent on the wind direction, while it is inversely dependent on the wave direction. Another issue about the slightly low values of (c-c) and (l-c) correlation coefficients can be explained by the fact that although winds generate waves, there are usually time lags between highest values of winds and waves. Therefore, the extreme values of wind speeds and wave heights do not coincide, and this leads to lower values of correlation coefficients.

Table 2.
Metocean variables used in this research and their marginal distributions.

Variable	Distribution
Mean wind speed (U_w)	Weibull ($\alpha = 8.742$ $\beta = 2.206$)
Significant wave height(H_s)	Weibull ($\alpha = 1.766$ $\beta = 1.916$)
Peak wave period (T_p)	Log-normal ($\mu = 1.847$ $\sigma = 0.206$)
Mean wind direction (θ_{wind})	A mixture of von-Mises (n=8)
Mean wave direction (θ_{wave})	A mixture of von-Mises (n=6)
The absolute value of $\theta_{wave} - \theta_{wind}$, ($\Delta\theta$)	A Mixture of von-Mises (n=12)

Table 3.
The Spearman's Rho correlation coefficient between variables.

	U_w	H_s	T_p	θ_{wind}	θ_{wave}	$\Delta\theta$
U_w	1	0.773	0.149	0.141	0.348	0.101
H_s		1	0.644	0.108	0.069	0.094
T_p			1	0.185	0.346	0.149
θ_{wind}				1	0.369	0.161
θ_{wave}					1	-0.131
$\Delta\theta$						1

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Figure 4. Scatter plot of wind direction vs. wave direction.

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Figure 4 is the scatter plot of wave and wind directions. The area between $120^{\circ} \sim 300^{\circ}$ is lighter in density and less scattered which is in accordance with the density plots as shown in Figure 3d and Figure 3e. The rose diagram for wind speed and direction, wave height and direction and wind-wave misalignment and wind speed are plotted in Figure 5a-c. The rose plot of wind and wave misalignment in Figure 5c shows the gradual decrease in the density of wind speeds with the increase of misalignment from 0° to 360° . It can also be seen that the misalignment between wave and wind direction is less than 90° in a majority of wind and wave combinations. It is to be noted that the wind-wave misalignment is defined as the absolute value of the difference between wind and wave directions, as mentioned in Table 1.

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Figure 5. Density of directional parameters in different directions (a) Wind direction vs. wind speed (m/s) (b) Wave direction vs. wave height (m) (c) Absolute value of wind-wave misalignment vs. wind direction (m/s).

6.2. Multivariate extreme metocean conditions calculation

The environmental contours for distributions D1 to D4 models have been developed using the above-described methodology. The results for a mean return period of 50 years are presented in Figure 6 to Figure 9, respectively. Each figure is composed of four parts (a-d), where the overall view of the surface is displayed in part (a) and the contours of surface elevation in three planes are shown in parts (b) to (d).

6.2.1. D1 model

The developed EC for D1 model (i.e., U_w , H_s and T_p) is displayed in Figure 6. In this case, the related parameters of wind speed marginal distribution as the primary variable are reported in

Table 2, and the parameters of the conditional distribution of wave height and wave period are listed in Table A4 and Table A5. According to Figure 6a., the entire environment surface is expanded in $T_p \sim (2.5 \text{ s}-12.2 \text{ s})$, $H_s \sim (0-7.4 \text{ m})$, and $U_w \sim (0-26 \text{ m/s})$, such that the contours of U_w shrink to smaller areas and get concentrated on more specified values of H_s and T_p as U_w increases. It can also be seen that the contour line corresponding to $U_w=26 \text{ m/s}$ is located where H_s and T_p experience fewer values than the corresponding values at the contour line associated with $U_w=25 \text{ m/s}$ meaning that the maxima of T_p , H_s and U_w are not located at the same representative point. This is more evident in Figure 6(b-d) where the values of H_s and T_p for the highest range of U_w between 6.17 m and 6.94 m and 10.27 s and 11 s, respectively, while the extreme value of wave height and wave period (i.e., 7.35 m, 12.2 s respectively) occur in lower values of wind speed, (i.e., 24.74 m/s, 8.5 m/s respectively).

(a)

(b)

(c)

(d)

Figure 6. ES (environmental surface) envelope developed for D1 distribution: a) The overall view of the surface, b) contour lines of U_w in (m/s), c) contour lines of T_p in (s), and d) contour lines of H_s in (m).

6.2.2. D2 model

The ES (environmental surface) developed based on D2 model for wave parameters, including significant wave height (H_s), mean wind direction, and (θ_{wave}) mean wave period (T_p) is shown in Figure 7. The parameters of the model for wave direction as the primary variable are reported in Table A1. This ES (environmental surface) is an extension of the LonoWe model (which is a JPDF of wave height and period) for all the wave directions. Table A6 and Table A4 also report the related parameters of wave height and wave period distributions, respectively.

Figure 7a displays the surface and the underlying data, and it shows that there are multiple peaks in the H_s - T_p values in specific wave directions showing irregular and non-monotonic behaviour. Figure 7b shows the contours of H_s - T_p for different wave directions. As the contours take more extreme sea states, they tend to get deformed and have a sharp edge on the most extreme point. The least value of T_p in the contours is more than 2.3 s which complies with the underlying distribution of T_p as shown in Figure 3c that shows the minimum value of T_p at the 2.2 s. The highest value of T_p in almost all the contours is 12.2 s. The most extreme contour exists at the 106° where $H_s=8.4$ m and $T_p=12.2$ s. The other peaks in the surface also appear close to the 0° . This also supports what can be inferred from Figure 3d, where the density of directions near 0° is higher than in the other regions. If the design approach suggests selecting the design direction based on the density plot of wave directions, the 0° can be the right choice according to Figure 3d. However, the present method reveals that the most extreme sea states occur in the 106° , while this direction does not take a high-density value in the density plot (Figure 3b). This kind of understanding cannot be comprehended just from the rose plot of wave direction (Figure 4a) or the density plot (Figure 3b). Figure 7c also demonstrates the regions of high H_s values with contours of $H_s=7$ m and 8 m around 106° . Figure 7d demonstrates the levels of T_p for different values of H_s and θ_{wave} . The contour of $T_p=12$ s also shows a peak around the most severe direction ($\theta_{wave} = 106^\circ$).

(a)

(b)

(c)

(d)

Figure 7. ES developed for D2 distribution: a) The overall view of surface b) Contour lines of θ_{wave} in (Degrees), c) Contour lines of H_s in (m), and d) Contour lines of T_p in (s)

6.2.3. D3 model

The ES developed based on D3 model for the combination of wind direction, speed, and wave height is presented in Figure 8. The related distribution parameters of wind direction, wind speed, and wave height are tabulated in Table A2, Table A7, and Table A5, respectively. Figure 8a shows the surface with the peaks on both sides of 0° (i.e., 5° and 354°) and in the 159° . In the direction of 159° the surface shows the extreme sea state of $H_s=8.19$ m and $U_w=28.28$ m/s. Figure 8b shows the contours of θ_{wind} in the U_w-H_s plane. A pronounced feature of these contours is their tendency to have a virtual tangent line in their right side, none of the contours crosses this line. The line can be physically related to the wave heights which are possible to be developed in the associated wind speed. For example, in the $U_w=15$ m/s, the H_s in none of the contours exceeds the value of 5.6 m. The interpretation of the reason for such a phenomenon needs a more precise analysis of the physical interaction of wind and waves. The wave data used in this research is the significant height of combined wind waves and swell as provided by ECMWF in the ERA-Interim database and there is no information available on the percentage of each component. So it can not be concluded directly from the shape of contours that the wave heights are related to the wind speeds (due to the existence of swells). But since the contours are generated from the JPMs derived from the raw data without applying any other filter. To explain this phenomenon, as an observed feature the physical interaction between wind speeds, fetch length, duration of blowing, water depth and bathymetry shape should be investigated which is out of the scope of this research. The contours of U_w-H_s are plotted in Figure 8c in which the region around the 159° direction is highlighted with contours of increasing values of the Tform 25 to 28 m/s. U_w same region is also observable with high values of H_s in Figure 8d where the contours of U_w in the $H_s - \theta_{wind}$ plane are plotted.

(a)

(b)

(c)

(d)

Figure 8. ES developed for D3 distribution: a) The overall view of the surface, b) Contour lines of θ_{wind} in (Degrees), c) Contour lines of U_w in (m/s), and d) Contour lines of H_s in (m).

6.2.4. D4 model

The ES for the wind-wave misalignment, wind speed, and wave height, developed based on the D4 model is shown in Figure 9. The related distribution parameters for misalignment, wind speed, and wave height can be found in Table A3, Table A8, and Table A5, respectively. It can be seen from Figure 9a that the value of misalignment increases as its probability of occurrence decreases, which complies with Figure 3f. This means wind and waves frequently happen from close directions. The contour lines of H_s and U_w shown in Figure 9(b-d) also show that the most extreme combination of $\Delta\theta$, U_w and H_s occurs in the 68° where U_w and H_s are 27.41 m/s and 7.1 m, respectively. Additionally, it is seen that high values of H_s and U_w are also likely to occur in 142° and 191° misalignments. Moreover, from Figure 9c and Figure 9d, it can be seen that the contour lines corresponding to the highest values of H_s and U_w are mostly located in the misalignment values less than 180° signifying lower probability of occurrences of opposite directions for wind and waves in the severe sea states.

(a)

(b)

(c)

(d)

Figure 9. The ES developed for D4 distribution: a) The overall view of the surface, b) contours of $\Delta\theta$ in (Degrees), c) contours of U_w in (m/s) d) contours of H_s in (m).

494 **7. Model diagnostics and validation**

495 The purpose of applying the probability models in this research is to describe the dependence of

496 variables and generate samples similar to underlying data. The accuracy of the fits of the proposed model

497 was investigated by nonlinear regression. Afterwards, a series of samples were generated by Monte Carlo

498 (MC) method and the RMSE (Root mean square Error) of the model was studied through comparison of

499 the underlying hindcast data and generated samples. The methodology of developing the JPDFs and the

500 ECs was described in Sections 4 and 5. To test the models, the R-squared values of the fits were observed

501 calculated. As an example, the fits of the LLCL and CLCW distributions fitted to raw data for the D2 model

502 besides their R-squared values are reported in Figure 10.

Figure 10. The fits and raw data of D2 model a) The mean parameter of LLCL, R-squared: 0.959, b) The square of the standard deviation parameter of LLCL, R-squared: 0.997, c) The scale parameter of CLCW, R-squared: 0.979, and d) The shape parameter of CLCW, R-squared: 0.828

The main scope of this research as mentioned before is to provide the designers with the joint probability of metocean parameters. The models are expected to generate a series of data which are likely to occur realistically. Monte-Carlo methods are usually used to generate pseudo-random samples from complex distributions [26, 46]. Latin Hypercube Sampling (LHS) as a more efficient way can also be applied to generate samples [47, 48]. Both of these methods were used herein to validate the JPDFs. The samples generated by the MC are compared versus the hindcast data to test the accuracy of the models. To better illustrate the correspondence of model versus hindcast variables, the histograms of generated samples are compared in Figure 11. The samples of (U_w, H_s, T_p) predicted by D1 and θ_{wind} , θ_{wave} and $\Delta\theta$ generated respectively by D2, D3 and D4 as shown are in good accordance. The RMSE, as a popular goodness of fit test, was also utilized to check the results. The 3-variate samples of data were generated from D1-D4 distributions by MC method. Then the accuracy of the fits was evaluated by the RMSE (Root Mean Square Error) statistic [46]. Two sets of this statistic were computed for the models. The RMSE of the whole JPDFs

are listed in Table 1. Then the RMSE of pairs of variables are reported in Table 2. As can be seen, the low values (near zero) show adequate accuracy of the model.

Table 4.
The RMSE of the JPDPFs

	D1	D2	D3	D4
RMSE	0.0151	0.0082	0.0065	0.0062

Table 5.
The RMSE of pairs of variables

		U_w	H_s	T_p
D1 (U_w, H_s, T_p)	U_w		0.0216	0.0207
	H_s			0.0131
D2 (θ_{wave}, H_s, T_p)	θ_{wave}		0.0104	0.0129
	H_s			0.0217
D3 (θ_{wind}, U_w, H_s)	θ_{wind}	0.0053	0.0119	
	U_w		0.0217	
D4 ($\Delta\theta, U_w, H_s$)	$\Delta\theta$	0.0058	0.0101	
	U_w		0.0217

(a)

(b)

(c)

(d)

Figure 11. The generated vs. hindcast data: a) U_w from D1, b) H_s from D1, c) T_p from D1, d) θ_{wave} from D2, e) θ_{wind} from D3, f) $\Delta\theta$ from D4.

8. Discussion and investigation of extreme values

To discuss the results of implemented IFORM on the combinations of circular and linear metocean conditions, two sets of comparisons have been made. First, the maximum values of linear variables extracted from extreme contours of four JPMs (i.e., D1-D4 models) are compared. Then the extreme contours for two sets of variables from different JPMs are chosen to be contrasted.

8.1. Discussion on the extreme values in contrast to marginal distributions

According to the results of applying the environmental contours method on the JPMs D1-D4 models (Table 2), the contours where the extreme value of each linear variable occurs were selected to compare the extremes extracted from different JPMs. The environmental contours associated with the highest value of parameters as defined and explained in Section 6, were compared to the extreme values associated with a 50-year MRP from marginal distributions in Table 6. It can be seen that the values of H_s and U_w are in close ranges for the D1 model, (U_w, H_s, T_p) , and marginal distributions. However, the extreme contours for the D2 model, $(\theta_{wave}, H_s, T_p)$, and D3 model, $(\theta_{wind}, U_w, H_s)$, show higher values for H_s and U_w than those

of marginal distributions. While in the D4 model, $(\Delta\theta, U_w, H_s)$, the extreme values are near the marginal ones.

Table 6.

The extreme values for the 50-year return period from marginal distribution and D1-D4 JPMs for linear variables.

Parameter	U_w	H_s	T_p
Marginal distribution	26.56 m/s	6.35 m	15.33 s
D1 (U_w, H_s, T_p)	26 m/s	7.02 m	11.04 s
D2 (θ_{wave}, H_s, T_p)	-	8.4 m	12.2 s
D3 (θ_{wind}, U_w, H_s)	28.8 m/s	8.18 m	-
D4 ($\Delta\theta, U_w, H_s$)	27.4 m/s	7.1 m	-

8.2. Comparison of extreme contours

The extreme contours of specified variables from different JPMs are compared and evaluated. The extreme values from different JPMs show considerably different values. Reviewing the process of building this series of JPDPs may help to explain such differences. To find the parameters of JPDPs, the underlying data of each JPDP is at first sorted in the form of tables with specified bin numbers; then the prescribed function forms are fitted to the distribution parameters. Researchers have applied the D1 model to find the extreme sea states associated with specified MRPs [38]. In the case of the D1 model, which is the JPDP of wind speed, wave height, and wave period, there is an increasing monotonic relation between wind speed, wave height, and period.

Moreover, the definition of JPDP is somehow based on the physical relation of U_w , H_s , and T_p , while in the case of D2-D4 models, where the primary variable which has the marginal distribution is a circular variable, there is not inherently a monotonic relation between the increases of parameters. Since the winds and waves coming from different directions usually have various sources of generation, the data in these distributions in each bin of sorted data may come from different sources and show different behaviours; and thus, different parameter distributions can be fitted to them. The data in a specified bin may maintain a higher mean value; and thus, a Weibull distribution with much higher parameters than that of the aggregate data is fitted to this subpopulation. This may lead to a higher probability prediction for these bins. Vanem has recently investigated the effect of data sub-sampling on the resulted contours to account for seasonality [49]. The research shows that subsampling data for different seasons can result in the contours which are

located in higher sea states or lower sea states, i.e. the contours for harsher seasons are much higher than contours of seasons with calmer weather. However, the contours can be moderated by some methods to be applied for practical applications which are out of the scope of the current research.

The definition of distributions and the order of marginal-conditional distributions affect the way the data is sorted and binned and the distributions fitted to each bin. This definition influences the shape of the ES, and thus the final extreme contour is highly dependent on the definition of distributions. To further investigate the issue, the most extreme contours from different distributions are compared in Figure 12. As can be seen, the contours which are frequent in different distributions (D1-D4 models) are chosen. First, the extreme values of contours of U_w-H_s from the D1 model, D3 model, and D4 model are plotted in Figure 12a. The contour of $\theta_{wind} = 159^\circ$ derived from the ES of the D3 model (θ_{wind}, U_w, H_s) is located in higher values, as mentioned in Table 6. The second highest value is associated with $\Delta\theta = 68^\circ$ from the ES of D4 model ($\Delta\theta, U_w, H_s$). The lowest value locates on the contour associated with D1 model (U_w, H_s, T_p).

Figure 10b also shows the results of comparison of contours of H_s-T_p from the D1 and D2. The extreme value of the combination of H_s-T_p derived from the D1 model (U_w, H_s, T_p) which is located at $U_w = 26m/s$ is a limited area, while the contour of H_s-T_p derived from the D2 model (JPDF of θ_{wave}, H_s and T_p) is quietly extended in a big area of wave height and periods, where the $\theta_{wave} = 106^\circ$. This can be explained by a look at the difference of the form of ES of D1 and D2 models. The ES created by D1 model is physically interpreted by the inherent nature of wind and waves and has usually shown the fastest winds to be accompanied by higher waves. The generation of waves is dependent on the existence of wind. However, the problem of concurrency and the definition of wind seas and swells can be taken into account. It is clear that the sea states become more severe as higher winds blow; and hence, it can be accepted that high amounts of wave are concurrent with limited conditions of wind speed. The ES created by the JPM of D1 model (U_w, H_s, T_p) expands in a big area of (H_s, T_p) in the low values of U_w , but the cross-sectional area reduces with the increase of U_w , as can be seen in Figure 6a the number of data points also decreases as U_w increases. In the case of directional variables, on the other hand, there is no physical meaning between the direction and wind and wave linear characteristics. Wind and waves generally may come from any direction without any limitations on their speeds and heights, respectively. The distributions which create

the contours rely on data in bins associated with each direction. The spreading of data in each bin of direction as in (D2-D4 models) is more varied than its diversity in the bins of wave period as in D1 model. This can also be explained by the physical limitations on the wave and wind characteristics like wave steepness. Accordingly, the contours of wave and wind created by directional considerations cover a wide span of wind and wave linear characteristics. As can be seen in Figure 7-Figure 12, the cross-section of the ESs associated with D2-D4 models which contain directional variables is then extended along the direction axis with more or less continuous densities spread all over the axis.

Figure 12. The highest contours created by different distributions. a) contours of $(U_w - H_s)$ b) contours of $(H_s - T_p)$

9. Conclusion

The present research was conducted to reach to an understanding of the concurrence of metocean conditions in the HyWind Scotland site, Scotland. To achieve this, four combinations of metocean parameters were selected, and their JPDFs were created. Then, the ESs (environmental surfaces) were developed for these combinations of parameters utilizing the IFORM and the CMA. Analysing the surfaces and their underlying contours, the subsequent outcomes can be comprehended:

- The traditional methods of design environment choosing suggest the use of rose diagrams and directional density plots to find the direction of wind or wave in which the structure will be damaged more. This research proposes the use of ESs created considering the directionality of metocean parameters to find the design direction.
- Comparison of the results shows that using just the rose plot or marginal PDF of directional data for prediction of the design wind or wave direction may lead to choosing the design direction which is not associated with the most extreme sea states, since the extreme waves or winds may come from directions which are not frequent in the site. Thus, it is necessary to utilize approaches which consider the concurrency of metocean parameters to forecast the combinations with higher precision.
- The shape and size of 3D ESs which account for directionality, are almost the same as the shape of their underlying ECs in 2D. e.g., The contours developed by D2, which is the JPDF of wave parameters $(\theta_{wave}, H_s, T_p)$ are similar to contours of the omnidirectional form of (H_s, T_p) .
- The misalignment of wind and waves in the HyWind Scotland site is remarkable so that the most extreme winds and waves may act in quite the opposite directions. That may happen due to the nature of waves and winds affected by different metocean processes in the North Sea.

In conclusion, the results show that the most extreme environmental conditions usually are concurrent with the directions which are not frequent. The analysis of concurrency of extreme metocean conditions through the ESs of directional variables reveals that the extreme values are dependent on the distributions on which ESs are based. Since ESs are dependent on the order of parameters in each JPDF, these distributions should be chosen based on the relative importance of metocean variables in the design process.

648 **Appendix**

649 **Table A1.**
650 Wave direction distribution.
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No. of component	Parameters of Mixture of von-Mises Fischer		
	α	μ	κ
1	0.166	-1.464	1.975
2	0.181	0.111	11.114
3	0.193	1.887	2.852
4	0.203	-0.321	2.537
5	0.136	2.915	2.302
6	0.120	2.686	1.711

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653 **Table A2.**
654 Wind direction distribution.

No. of component	Parameters of Mixture of von-Mises Fischer		
	α	μ	κ
1	0.081	0.343	1.595
2	0.150	1.181	4.232
3	0.107	3.031	1.435
4	0.0961	0.721	1.631
5	0.0721	0.238	1.343
6	0.0768	0.215	1.272
7	0.261	1.809	3.207
8	0.156	0.682	2.957

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667 **Table A3.**
668 Wind–wave misalignment distribution coefficients.

No. of component	Parameters of Mixture of von-Mises Fischer		
	α	μ	κ
1	0.117	0.963	9.018
2	0.115	0.464	27.316
3	0.053	1.903	2.495
4	0.104	1.012	7.775
5	0.067	2.047	2.518
6	0.069	0.135	131.492
7	0.068	1.742	3.152
8	0.053	1.737	2.8
9	0.108	2.346	3.316
10	0.108	-2.991	3.68
11	0.068	1.537	3.657
12	0.069	1.614	3.192

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670 **Table A4.**
671 Parameters of the conditional distribution of wave period on wave height.

Parameter	Value
c_1	0.35
c_2	0.522
c_3	1.43
d_1	0.04326
d_2	0.3926
d_3	-0.001

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673 **Table A5.**
674 Parameters of the conditional distribution of wave height on wind speed.

Parameter	Value
c_1	0.0411
c_2	1.536
c_3	0.7539
d_1	0.02
d_2	1.755
d_3	2.099

680 **Table A6.**
 681 Parameters of the conditional distribution of wave height on wave direction.

Parameter	Value	
	α	β
a ₀	1.749	2.18
a ₁	-0.117	0.236
b ₁	-0.0.	-0.093
a ₂	-0.063	-0.126
b ₂	-0.124	0.167
a ₃	0.106	-0.021
b ₃	-0.101	0.037
a ₄	0.118	0.00
b ₄	0.045	0.023
a ₅	0.008	-0.004
b ₅	0.016	0.001
a ₆	0.005	0.003
b ₆	-0.052	0.026
a ₇	0.03	-0.022
b ₇	-0.019	0.02
a ₈	0.0001	0.02
b ₈	0.006	-0.022
a ₉	0.007	0.003
b ₉	-0.003	0.013
a ₁₀	0.011	0.014
b ₁₀	-0.006	0.004

695 **Table A7.**
696 Parameters of the conditional distribution of wind speed on wind direction.

Parameter	Value	
	α	β
a ₀	8.376	2.18
a ₁	1.09	0.236
b ₁	0.476	0.069
a ₂	-0.28	-0.126
b ₂	-0.547	-0.014
a ₃	-0.13	-0.021
b ₃	0.227	0.021
a ₄	0.095	-0.006
b ₄	0.098	0.033
a ₅	0.772	0.004
b ₅	0.004	0.012
a ₆	0.029	0.032
b ₆	-0.11	1.62e-05
a ₇	-0.007	-0.011
b ₇	-0.066	-0.009
a ₈	-0.047	-0.002
b ₈	0.063	0.007

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698 **Table A8.**
699 Parameters of the conditional distribution of wind speed on wind-wave misalignment.

Parameter	Value	
	α	β
a ₀	8.107	2.249
a ₁	-0.717	0.134
b ₁	0.834	-0.029
a ₂	-0.396	0.016
b ₂	1.077	0.058
a ₃	-0.121	0.014
b ₃	0.708	0.023
a ₄	0.025	0.038
b ₄	0.245	-0.016
a ₅	0.057	0.045
b ₅	0.133	-0.008
a ₆	-0.043	0.016
b ₆	0.248	-0.033
a ₇	-0.031	0.006
b ₇	0.245	-0.004
a ₈	0.029	0.027
b ₈	0.182	-0.008

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