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On Realized Volatility of Crude Oil Futures Markets: Forecasting with Exogenous Predictors under Structural Breaks

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Abstract

We introduce Infinite Hidden Markov (IHM) models to forecasting realized volatilities of crude oil futures markets with exogenous factors. With these IHM models, we lift the restriction of a pre-defined number of regimes and allow for an unknown number of different parameter regimes and breakpoints. We employ two types of infinite hidden Markov models to accommodate structural breaks incurred by policy changes, exogenous shocks, and other factors. We find that IHM-HAR models outperform all other non-switching variants. In regard to forecasting performance, IHM-HAR models with exogenous factors such as realized volatilities of competing futures markets and the S&P500 are superior model choices for short-term forecasts. For longer-term forecasts, the equity channel shows only little positive impact. Evidence of economic gains in portfolio construction based on IHM-HAR forecasts is provided.

Keywords: Crude Oil, Forecasting, HAR Models, Markov Switching, Realized Volatility

JEL classification: C24; E32; Q43

1. Introduction

Understanding oil price uncertainty is of crucial importance to oil exporting and importing nations as well as to institutional and private investors in the energy markets and is subject to extensive academic research (see, for example, Klein & Walther, 2016, Ji et al., 2018, 2019, Alqahtani et al., 2019, Ma et al., 2019b, and references therein). Hence,

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both regulators and investors inevitably need accurate forecasts of oil market volatility for portfolio optimization, hedging, derivatives pricing and value-at-risk estimation. This study extends extant knowledge in this domain by analysing the incremental benefit of exogenous variables and infinite hidden Markov models for oil volatility forecasting.

The increasing availability of high-frequency financial market data has facilitated a growing body of literature in the research domain of oil volatility forecasting. Sévi (2014), Prokopczuk et al. (2016), Gong & Lin (2017), Liu et al. (2018) and Ma et al. (2018b) investigate the forecasting performance of various specifications of heterogeneous autoregressive- (HAR-)type models. Chatziantoniou et al. (2019) study the usefulness of harnessing futures data for forecasting the realised volatility of the spot prices of Brent. Haugom et al. (2014), Gong & Lin (2018) and Lv (2018) analyse the contribution of the Crude Oil Volatility Index (OVX) in HAR models for forecasting the realized volatility of crude oil. Lyócsa & Molnár (2018) study the potential of harnessing information from a related energy commodities (oil and natural gas) in a HAR setting. Ma et al. (2018c) focus on the impact of jump intensity on the volatility of futures in the oil futures market using HAR models for realized range-based volatility. Ma et al. (2018a) investigate the predictive power of uncertainty and market sentiment indexes, macroeconomic fundamentals and technical indicators with respect to future monthly oil price volatility. Yang et al. (2019) analyze the role of investor sentiment and leverage effects for volatility forecasting of crude oil futures in a HAR model setting while Zhang et al. (2019b) compare the predictive ability of forecast combination and shrinkage methods for oil price volatility.

We contribute to this literature by investigating whether the incorporation of exogenous predictors under structural breaks can improve the forecasting accuracy of HAR models for the realized volatility of futures on the Western Texas Intermediate (WTI) and Brent crude oil. More specifically, the statistical and economical significance of adding equity market information and another international oil price benchmark are studied by means of infinite hidden Markov models.

The idea to harness information from equity markets is motivated by the survey of Degiannakis et al. (2018) which identifies only three studies examining the informational con-

tent of stock markets when forecasting oil prices. Efimova & Serletis (2014) use S&P500 daily returns (along with energy commodity returns) for forecasting oil price volatility and document that equity prices are not superior compared to other information channels. In contrast, Phan et al. (2016) investigate the volatility interaction between US crude oil and equity markets using intraday data and conclude that using volatility from other markets improves predictability in both in-sample and out-of-sample analyses. Last, Degiannakis & Filis (2017) provide evidence that different information channels—among which stock index volatilities—enhance the predictive accuracy of realized volatility of Brent at all considered forecasting horizons. Based on these promising results and the scarcity of relevant research, Degiannakis et al. (2018) call for further analysis in this domain, in particular using high frequency data which are likely to contain rich information for forecasting purposes. Further evidence for the potential of equity markets to contribute to enhancing the predictability of oil markers stems from the volatility spillover literature (e.g., Souček & Todorova, 2013; Li et al., 2016) and recent forecasting studies (e.g., Wen et al., 2019; Ma et al., 2019a).

Additionally, when modelling the volatility of WTI and Brent individually, we include the realized volatility of the other oil market as an exogenous factor in order to capture potential spillover effects. The extensive literature on volatility spillovers and co-integration between WTI and Brent (Maslyuk & Smyth, 2009, Kaufmann & Banerjee, 2014, Ji & Fan, 2015, Klein, 2018, Zhang et al., 2019a) suggests that harnessing information from the other market may be useful for forecasting purposes and portfolio construction.

The notion of structural breaks and application of regime switching approaches have received attention in studies on oil volatility during the last decade (e.g., Chen et al., 2014; Ewing & Malik, 2016; Chai et al., 2018; Herrera et al., 2018; Lux et al., 2016). The potential for volatility forecasting improvements of the oil price is subject of an increasing attention in the literature. Arouri et al. (2012) investigate the performance of the conditional volatility forecasts of oil spot and futures prices with structural breaks and GARCH models. Salisu & Fasanya (2013) use daily data for Brent and WTI to model oil price volatility with structural breaks. Ma et al. (2017) employ Markov regime

switching approaches to forecast the realized volatility of crude oil futures. Wen et al. (2016) forecast the volatility of crude oil futures using HAR-type models with structural breaks. Lux et al. (2016) applies a Markov-switching multifractal model and a number of GARCH-type models to model and forecast oil price volatility. Wang et al. (2016) document the benefits of a Markov switching multifractal volatility approach compared to both popular GARCH-class and historical volatility models for crude oil. Pan et al. (2017) introduce a regime switching GARCH-MIDAS model to investigate the relationships between oil price volatility and its macroeconomic fundamentals. Zhang et al. (2019c) estimate and forecast crude oil price volatility using single-regime GARCH and regime-switching GARCH models. The majority of these studies provide evidence that models allowing regime switching increase the forecasting ability significantly than those without regime switching approaches.

Inspired by this notable potential for improving the accuracy of volatility forecasts, the study at hand employs two types of infinite hidden Markov models to accommodate structural breaks. While existing models usually require an ex ante definition of the number of possible regimes, the approach we choose does not impose such a strict limitation but rather allows for an arbitrary number of regimes. Our aim is to establish whether the substantial flexibility of letting the data to speak for itself during the estimation process enhances the accuracy of volatility forecasts and if potential statistical improvements can be translated into economic gains.

Our main findings and contributions to literature on crude oil volatility are threefold. First, IHM-HAR specifications outperform their non-switching counterparts in terms of estimation quality and fit. This observation is in line with the notion that models that are more flexible in terms of parameter evolution are likely to yield better results than simpler models. Second, we provide strong evidence that the short-, medium-, and long-term realized volatility forecasting benefit to a great extent from introducing IHM structures for both WTI and Brent futures series. For WTI, exogenous factors such as the equity market volatility improve short-term forecasts while for longer forecasting horizons, the benefit from harnessing additional volatility series diminishes. Modelling different regimes

of realized volatility is found to make a higher contribution to forecasting accuracy than including exogenous factors, especially in the long-run. For Brent, IHM-HAR models outperform non-switching variants while including exogenous factors does not add notable value for forecasting purposes. Lastly, we find portfolios based on volatility forecasts of IHM models to produce higher returns and higher utility. We hence confirm that IHM-HAR models outperform non-switching HAR specifications in terms of data fit, realized volatility forecasting, and portfolio performance.

The paper is structured as follows. Section 2 introduces the methodology of the study. Section 3 describes the data set. Section 4 reports and discusses the results of the empirical study and Section 5 concludes.

2. Empirical Methodology

2.1. Heterogeneous Autoregressive Realized Volatility Models

From the intraday prices for each crude type or index, we calculate intraday returns as the logarithmic difference of two consecutive 5-min prices

$$r_{t,j} = 100 * (\log p_{t,j} - \log p_{t,j-1}), \text{ for } j = 2, \dots, M, \quad (1)$$

where M refers to the total number of 5-min prices per day t . This yields $M - 1$ intraday returns. We calculate the realized volatility, RV_t , of day t as sum of squared intraday returns,

$$RV_t = \sum_{j=1}^M r_{t,j}^2. \quad (2)$$

The weekly ($h = 5$ trading days) and monthly ($h = 22$ trading days) averages of the daily realized volatility, denoted by $RV_{t:t-4}$ and $RV_{t:t-21}$, respectively, are defined as

$$RV_{t:t-h+1} = h^{-1} \sum_{i=0}^{h-1} RV_{t-i}.$$

With the daily, weekly, and monthly realized volatility, we build on the *Heterogeneous Autoregressive* model (HAR) of Corsi (2009) which reads

$$RV_t = \beta_0 + \beta_1 RV_{t-1:t-1} + \beta_2 RV_{t-1:t-5} + \beta_3 RV_{t-1:t-22} + \varepsilon_t, \quad (3)$$

where ε is an i.i.d. error term with zero mean.

We extend the baseline HAR model of Eq. (3) with exogenous factors such as the other markets realized volatility (RV^{Eoil}) and realized volatility of a stock market index (RV^{SPX}).¹ By extending the base HAR model, we measure the impact of exogenous factors under structural breaks in what follows.

Firstly, we augment the lattice structure of the base HAR model in Eq. (3) with RV measures of the competing crude oil market. For example, modeling the realized volatility of WTI futures prices, we include realized volatility measures of Brent futures as exogenous factors. This extensions reads

$$\begin{aligned} RV_t = & \beta_0 + \beta_1 RV_{t-1:t-1} + \beta_2 RV_{t-1:t-5} + \beta_3 RV_{t-1:t-22} \\ & + \beta_4 RV_{t-1:t-1}^{Eoil} + \beta_5 RV_{t-1:t-5}^{Eoil} + \beta_6 RV_{t-1:t-22}^{Eoil} + \varepsilon_t. \end{aligned} \quad (4)$$

In order to measure the impact and volatility spillover of equity markets on crude oil futures, we augment Eq. (4) with the realized volatility of the S&P500² and extend the model to

$$\begin{aligned} RV_t = & \beta_0 + \beta_1 RV_{t-1:t-1} + \beta_2 RV_{t-1:t-5} + \beta_3 RV_{t-1:t-22} \\ & + \beta_4 RV_{t-1:t-1}^{Eoil} + \beta_5 RV_{t-1:t-5}^{Eoil} + \beta_6 RV_{t-1:t-22}^{Eoil} \\ & + \beta_7 RV_{t-1:t-1}^{SPX} + \beta_8 RV_{t-1:t-5}^{SPX} + \beta_9 RV_{t-1:t-22}^{SPX} + \varepsilon_t. \end{aligned} \quad (5)$$

¹See Section Data for more details on data selection and periods.

²Obtained from high frequency data as well, see Section Data for more details.

As a last base model variant, we isolate the effect of equity markets and reduce Eq. (5) to

$$\begin{aligned}
RV_t = & \beta_0 + \beta_1 RV_{t-1:t-1} + \beta_2 RV_{t-1:t-5} + \beta_3 RV_{t-1:t-22} \\
& + \beta_4 RV_{t-1:t-1}^{SPX} + \beta_5 RV_{t-1:t-5}^{SPX} + \beta_6 RV_{t-1:t-22}^{SPX} + \varepsilon_t.
\end{aligned}
\tag{6}$$

It has been shown that the equity channel contributes to forecasting quality in crude oil markets (Phan et al., 2016, Degiannakis & Filis, 2017) for high frequency data but its effect on daily resolution might be negligible (Efimova & Serletis, 2014). We revisit the equity channel by simultaneously controlling for an unknown number of shocks. Additionally, we include the competing oil market as exogenous factor to model directional spillover effects. The literature on volatility spillovers and co-integration between WTI and Brent markets is extensive and well documented (Maslyuk & Smyth, 2009, Kaufmann & Banerjee, 2014, Ji & Fan, 2015, Klein, 2018) and Infinite Hidden Markov (IHM-)HAR models offer to revisit this relationship under the presence of breakpoints as described below. As shown in literature, oil price fluctuations can be greatly affected by many factors such as announcement of macroeconomic news, unexpected Black Swan events, war and conflicts, disaster and extreme weather, and risk contagion from financial crises. These events may result in unexpected structural breaks in the oil price volatility (Choi & Hammoudeh, 2010, Pan et al., 2017, Ma et al., 2017). Especially after the 2008 global financial crisis, the crude oil market experienced a sharp decline. The market sentiment was more sensitive and the market became more vulnerable which may result in several regime switching events of high-frequency volatility. Moreover, there is evidence that regime switching structures improve the forecasting performance of high-frequency volatility models (Bauwens et al., 2017, Gong & Lin, 2018). Therefore, we introduce regime switching structures to accommodate unknown structural breaks in realized volatility of crude oil prices.

2.2. Infinite Hidden Markov (IHM) models and their estimation

As pointed out by Song (2014), structural break models with a pre-specified number of regimes may have estimation bias because pre-break data is not directly used in the parameters estimation of the new born regimes. Modelling regime switching and structural breaks with an infinite hidden Markov (IHM) model circumvent these issues (Song, 2014).

To this end, we introduce the IHM structure to model unknown structural breaks of oil price volatilities.

All base HAR models defined previously can be rewritten as a general linear regression

$$y_t = \mathbf{X}_t \boldsymbol{\alpha} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma^2), \quad (7)$$

where $\mathcal{N}$ refers to a Normal distribution and σ^2 is the variance of the disturbances. These HAR models are implemented with constant coefficients and variances. In what follows, we remove this restriction of time-invariant specifications. Recent research motivates to consider structural breaks as their modeling improves the forecasting accuracy with HAR models. The most important reason might be that high-frequency data based realized volatilities of financial assets are subject to unknown structures (Bollerslev et al., 2015, Tian et al., 2017, Luo & Ji, 2018, Luo et al., 2019). Having seen an unparalleled financialization of the oil sector, leading to a massive increase in volumes of crude oil futures markets we extend the baseline HAR models in Eq. (3)-(6) with time-varying coefficients and variances. Regime switches and structural breaks are governed by infinite Hidden Markov switching processes.

Assuming the presence of structural breaks in realized volatility of crude oil futures markets, Eq. (7) can be redefined with state-dependent coefficients and variances as follows:

$$y_t = \mathbf{X}_t \boldsymbol{\alpha}_{s_t} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_{s_t}^2), \quad (8)$$

where the HAR coefficients $\boldsymbol{\alpha}_{s_t}$ and variance $\sigma_{s_t}^2$ are governed by a single state variable s_t .

We now follow a simplified version of the IHM-HAR class outlined in Luo et al. (2019). For this IHM-HAR model, the variation coefficients and variances are governed by the same Markov state variable s_t , which follows an infinite Hidden Markov switching process

with

$$\begin{aligned}
\boldsymbol{\alpha}_{s_t} &\sim \mathcal{N}(\boldsymbol{\alpha}_0, \Sigma_0), \quad \sigma_{s_t}^2 \sim \mathcal{IG}(\phi_0, \nu_0), \\
s_t | s_{t-1}, \{\mathbf{p}_k^s\}_{k=1}^\infty &\sim \mathbf{p}_{s_{t-1}}^s, \\
\mathbf{p}_k^s | c^s, \rho^s, \boldsymbol{\pi}^s &\sim \text{DP}(c^s, (1 - \rho^s)\boldsymbol{\pi}^s + \rho^s \delta_k), \\
\boldsymbol{\pi}^s | \gamma^s &\sim \mathbf{SBP}(\gamma^s).
\end{aligned} \tag{9}$$

Here, the coefficients $\boldsymbol{\alpha}_{s_t}$, and variances $\sigma_{s_t}^2$ are sampled from two base distributions. We apply a Gaussian distribution (with parameters $\boldsymbol{\alpha}_0$ and Σ_0) for α_i and an Inverse Gamma distribution (with parameters ϕ_0 and ν_0) for σ_i , according to recent literature (see Luo et al., 2019, and references therein). Hence, the regime indicator s_t is driven by transition probabilities with infinite dimension, $\mathbf{p}_k^s$.

In particular, this infinite states parameter $\mathbf{p}_k^s$ follows a Dirichlet process (DP). Applying this process for transition probabilities is outlined in Ferguson (1973) while we use the version of Fox et al. (2011). The DP process is characterized by a positive concentration parameter c and the base distribution G_0 of form $G_0 = (1 - \rho)\pi + \rho\delta_i$, with $0 < \rho < 1$ as sticky parameter which enhances the state self-transition probability and δ_i is the degenerate probability measure sampled from G_0 .

The parameter π in the last line of Eq. (9) is calculated from a *stick-breaking process* (**SBP**) following Sethuraman (1994). The **SBP** can be described as breaking a stick of length 1 for infinite times. ‘‘Breaking the stick’’ the i -th time is determined by $V_i \sim \mathcal{B}(1, \gamma)$ where $\mathcal{B}$ is a Beta distribution. Then, $\pi_i = V_i \prod_{j=1}^{i-1} (1 - V_j)$ and we define $\pi | \gamma \sim \mathbf{SBP}(\gamma)$.

As last switching framework, we only allow the variances to be state dependent and assume the coefficients of the respective HAR model to be time-invariant. The main difference between the IHM-HAR model and IHMC-HAR model is the coefficients of the IHM-HAR model are time-varying and regime dependence while those of IHMC-HAR model are specified to be time-constant. Then, only the variances are driven by a infinite

Hidden Markov switching process and the model framework for the IHMC-HAR reads

$$\begin{aligned}
\boldsymbol{\alpha} &\sim \mathcal{N}(\boldsymbol{\alpha}_c, \Sigma_c), \quad \sigma_{s_t}^2 \sim \mathcal{IG}(\phi_0, \nu_0), \\
s_t | s_{t-1}, \{\mathbf{p}_k^s\}_{k=1}^\infty &\sim \mathbf{p}_{s_{t-1}}^s, \\
\mathbf{p}_k^s | c^s, \rho^s, \boldsymbol{\pi}^s &\sim \text{DP}(c^s, (1 - \rho^s)\boldsymbol{\pi}^s + \rho^s \delta_k), \\
\boldsymbol{\pi}^s | \gamma^s &\sim \mathbf{SBP}(\gamma^s),
\end{aligned} \tag{10}$$

with distributions and parameter definitions analogously to the IHM-HAR.

Specifically, the infinite hidden Markov HAR models allows an infinite number of regimes and recurring states. IHM models can be considered as a Markov switching model with a countably infinite number of states. Given a finite dataset, the number of states is estimated along with all the other parameters. When new data arrives, a new state is automatically introduced and incorporated into forecasts to capture new features of the conditional density, which cannot be captured by fixed state MS models. Moreover, the transition matrix is unbounded so that both recurring states from the past and new states can be introduced to capture structural breaks (Maheu & Yang, 2016). To this end, the constructed models are able to capture dynamic instability of oil prices and unknown structural breaks in oil volatility, thus improving estimation and forecasting precision. Figure 1 depicts the parameter interaction with respect to Eq. 9 in the mathematical framework of the IHM model.

We employ the Markov chain Monte Carlo (MCMC) posterior sampler to estimate the IHM process. The main procedures are introduced as follows.³ Referring to Song (2014) and Hou (2017), we employ the beam sampler approach to obtain samples from the exact posterior distribution proposed by Van Gael et al. (2008) to solve the problem of the infinite number of regimes assumption in the IHM framework.

In the empirical estimations, considering both the issues of slow convergence of MCMC and computational feasibility of estimation time, we set MCMC replications to 10 000 after a burn-in period of 5 000. The final 5 000 samples retained are used to compute all

³For a more detailed description on the estimation procedure we refer the interested reader to Song (2014), Hou (2017), and Luo et al. (2019).

Figure 1: Description of the IHM model functioning. Note: ψ is the parameter set including the coefficients and variances and H is the hyper-parameter set determining the evolution of coefficients and variances.

posterior estimates.

2.3. Out-of-sample analysis and forecasts

In order to compare the models based on their forecasting performance, we define a relatively long out-of-sample window such that the Model Confidence Set (MCS) of Hansen et al. (2011) yields robust results. Approximately one third of the observations are used as out-of-sample period while the first two thirds serve as a training window. Following recent literature in forecasting (Corsi, 2009, Corsi & Renò, 2012, Sévi, 2014, Degiannakis et al., 2020, Luo et al., 2019), forecasting realized volatilities over different horizons is carried out iteratively or on aggregated level. We obtain one-day ahead or short term ($h = 1$), one-week ahead or medium term ($h = 5$), and one-month ahead or long-term ($h = 22$) predictions for the predefined out-of-sample period.

Adjusting for different trading times and bank holidays, we double-match the time series depending on the forecasting objective. Forecasting the volatility of WTI futures, we match volatility series of Brent and the S&P500 according to the trading dates of the WTI futures series and use a zero order hold for synchronization; while for forecasting Brent futures, we match all volatility series to the respective Brent trading days analogously.

One-step ahead forecasts are obtained by estimating the HAR variant with a heteroskedasticity consistent estimation with an expanding training window. Multi-step ahead forecasts can be obtained in different ways. At least two widely used approaches are established in literature. One approach aims to forecast the accumulated or average realized volatility over h days as carried out in Corsi (2009), Degiannakis & Filis (2017), Degiannakis et al. (2020), and Todorova (2015) for example. The other approach is to

obtain point forecasts for the h -day ahead realized volatility by iterating one-day ahead forecasts and utilizing these forecasts as pseudo-observations to obtain the next forecast (Marcellino et al., 2006). The latter approach is very sensitive to misspecification and the forecasting precision would rapidly decrease if these effects are present as demonstrated by Anderson & Meisch (2003) and Sizova (2011).

Circumventing the systematic deterioration of forecasting precision, we follow Corsi (2009) and apply the direct forecasts for the h -day ahead cumulative realized volatility

$$RV_{t+1:t+h|t} = \frac{1}{h} \sum_{i=1}^h RV_{t+i|t}, \text{ for } h = 1, 5, 22,$$

translating to forecasting the realized volatility over an h -day period (daily, weekly, and monthly) rather than obtaining a point forecast. We choose this approach as it proves to be the only feasible approach for the economic evaluation based on forecasts which follows in Subsection 2.3.2.

Combining the IHM and IHMC variant of Markov-type HAR models, we obtain 12 models in total as we include the baseline models as benchmark. We evaluate these models by a density forecast based on the log marginal likelihood statistic. For the direct forecasts, we evaluate the unbiasedness of the forecasting model based on loss functions such as the Mean Square Error (MSE) and Quasi-likelihood (QLIKE) loss functions. These results are tested with the Model Confidence Set (MCS) of Hansen et al. (2011) to identify the possible outperforming models. In the text which follows, the employed performance evaluation criteria are outlined.

2.3.1. Statistical evaluation

We apply the nested loss functional of Patton (2011), where for $b = 0$ and $b = -2$, the loss functional degenerates to two conventional loss functions; the MSE and QLIKE,

respectively. The loss functional is given as

$$L(RV_{true,t}, RV_{for,t}) = \begin{cases} \frac{1}{T_2} \sum_{t=T_1+1}^T \left(\frac{1}{(b+1)(b+2)} (RV_{true,t}^{b+2} - RV_{for,t}^{b+2}) - \frac{1}{b+1} RV_{for,t}^{b+1} (RV_{true,t} - RV_{for,t}) \right), & \text{for } b \notin \{-1, -2\} \\ \frac{1}{T_2} \sum_{t=T_1+1}^T RV_{for,t} - RV_{true,t} + RV_{true,t} \log \left(\frac{RV_{true,t}}{RV_{for,t}} \right), & \text{for } b = -1 \\ \frac{1}{T_2} \sum_{t=T_1+1}^T \frac{RV_{true,t}}{RV_{for,t}} - \log \frac{RV_{true,t}}{RV_{for,t}} - 1, & \text{for } b = -2, \end{cases}$$

where T_2 refers to the total number out-of-sample forecasts, T_1 to the first observation in the out-of-sample window, $RV_{true,t}$ denotes the actual realized volatility on day t , and $RV_{for,t}$ refers to the volatility forecast, where both point and aggregated measures and forecasts are feasible choices.

Additionally, we compute the P -statistic of Blair et al. (2001) to overcome the scale-dependent drawbacks of the loss functions. The P -statistic is computed as

$$P - \text{stat} = 1 - \frac{\sum_{t=T_1}^T RV_{true,t} - RV_{for,t}}{\sum_{t=T_1}^T RV_{true,t} - \overline{RV}_{true}}.$$

2.3.2. Economic evaluation

The statistical evaluation outlined above aims at assessing the forecast accuracy of the employed forecasting models. For practical purposes, this statistical evaluation of a model set relative to each other might not be of priority. More importantly, market participants are concerned with economic gains or value-added generated by employing more sophisticated and calculation-intensive models. In order to measure any gain in terms of an investor's utility—usually directly related with higher returns while holding the risk constant or similar returns with lower risk, or any favorable combination of the two—we follow the standard mean-variance utility which reads

$$U(R_p) = \mathbb{E}(R_p) - \frac{1}{2}\gamma\mathbb{V}(R_p),$$

where γ denotes a *risk aversion rate*, $\mathbb{E}(R_p)$ denotes the expected portfolio return R_p , and $\mathbb{V}(R_p)$ is the portfolio variance.

Following Campbell & Thompson (2008) and Neely et al. (2014), an investor now allocates capital in a risky and risk-less asset. The risky assets are our crude oil futures, based on volatility forecasts of time $t + 1$, a measure for its riskiness. The optimal weight of the risky asset is

$$\hat{w}_t = \frac{1}{\gamma} \frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2},$$

where $\hat{r}_{t+1}$ is the predicted excess return and $\hat{\sigma}_{t+1}^2$ is the forecasted variance. Based on the strategy outlined in Campbell & Thompson (2008) as well as in Neely et al. (2014), we model the excess returns as a moving average process on a rolling basis of one trading year, yielding 256 trading days and utilize one step-ahead predictions.

We use 5-year U.S. treasury bills as a proxy of the risk-free asset and assume that there is virtually no sovereign risk associated. Hence, the portfolio variance equals the weighted variance of the crude oil futures. For the realized portfolio return at time $t + 1$, $R_{p,t+1}$, we obtain

$$R_{p,t+1} = \hat{w}_t r_{t+1} + (1 - \hat{w}_t) r_{t+1}^f,$$

where r_{t+1} and r_{t+1}^f are the realized excess returns of our WTI or Brent futures and the risk-free return at time $t + 1$, respectively.

The economic evaluation is carried out for all forecasting horizons. For the multi-step ahead forecasts, the portfolio calculations are repeated for all possible horizons on a weekly basis for $h = 5$ (e.g., Monday to Monday, Tuesday to Tuesday) and on a monthly basis for $h = 22$ (e.g. the 1st day of a month to the first of the following month (or the closest trading day equaling 22 trading days, the 2nd day of the month and so forth). Hence, for weekly and monthly portfolio optimizations, we obtain moving averages over weekly and monthly windows such that

$$r_{t+1:t+h} = \frac{1}{h} \sum_{i=1}^h r_{t+i}. \quad (11)$$

While calculating the portfolio return based on forecasted volatilities provides first evidence on the usefulness of the employed model, we seek to quantify a possible outperformance in terms of gained utility to compare models relative to each other. In order to

quantify this gain, we calculate the economic value Δ for which it holds that

$$\sum_{t=T_1+1}^T U(R_{p,t}^k) = \sum_{t=T_1+1}^T U(R_{p,t}^l - \Delta),$$

for two portfolios R_p^k and R_p^l . The economic value describes how much return an investor with a specific risk aversion rate forgoes by changing the portfolio allocation from portfolio k to portfolio l . For the utility function U , we rely on Fleming & Hernández-Hernández (2003), Bollerslev et al. (2016), and Callot et al. (2017), and employ the quadratic utility with a risk aversion γ , which reads for an arbitrary portfolio $R_{p,t}^k$

$$U(R_{p,t}^k, \gamma) = (1 + R_{p,t}^k) - \frac{\gamma}{2(1 + \gamma)}(1 + R_{p,t}^k)^2, \quad (12)$$

to calculate Δ according to Eq. (11) with the baseline model as benchmark portfolio. For the risk aversion rate, choices vary in literature. We use a mild risk aversion rate of $\gamma = 1$ and a strong risk aversion rate with $\gamma = 10$ to cover different investor attitudes. Note that gamma directly affects the utility in Eq. (12) and hence, the economic value Δ .

3. Data set

In this paper, we acquire high-frequency data of the most liquid WTI and Brent futures as well as the S&P500 index in 5-min intervals from Thomson Reuters Tick History and Tick Data Database, respectively. For WTI, we make use of data coming from the NYMEX/CME with ticker symbol CL. For Brent, data originates from the ICE with ticker symbol LCO. With the commonly chosen frequency of 5-min transaction data, we retain most of the information needed to reliably calculate realized volatilities but also to cancel out undesirable consequences of microstructure effects present in high-frequency data (Liu et al., 2015). From the available futures contracts, we sample the contract with the highest trading volume ensuring that we make use of present liquidity and its informational content. For a vast majority of days, the front month futures contracts of WTI and Brent are the most liquid contracts chosen for constructing our realized volatility

measures.⁴

The observation period spans January 3, 2013 to May 11, 2018. During synchronization of the data sets, we remove daily observations of days that were not traded in all markets. This could be due to differing bank holidays in North American and European markets. By data pre-processing and synchronization, we finally obtain 1360 daily observations of realized volatilities for WTI, Brent, and the S&P500 index.

For the out-of-sample analysis, the training window spans from January 3, 2013 to June 30, 2016 which yields $n_{\text{is}} = 900$ observations in the in-sample and $n_{\text{os}} = 460$ observations in the out-of-sample period running from July 1, 2016 to May 11, 2018. Descriptive statistics and preliminary analyses of realized volatilities defined in Eq. (2), based on scaled log-returns given in Eq. (1), are given in Table 1.

	Mean	StD	Skewness	Kurtosis	JB	LB $Q(5)$	LB $Q(10)$	LB $Q(22)$	ADF
WTI	1.751	2.368	5.68	67.17	240670***	991.45***	1657.89***	2973.02***	-15.36***
Brent	3.094	4.045	2.99	14.57	11561***	2410.94***	4608.15***	8400.84***	-13.80***
SPX	0.944	1.783	20.43	582.68	19291224***	220.23***	307.46***	387.97***	-20.12***

*** denotes significance at the 1% level.

Table 1: Descriptive statistics and preliminary analysis of the realized volatilities of WTI and Brent futures as well as of the S&P500 index.

Being defined to only take positive real values as variances are always positive, all realized volatilities show pronounced right-skew and a very high kurtosis. Subsequently, the Jarque-Bera test for normality is rejected at the 1% level. The Ljung-Box tests on 5, 10, and 22 lags ($Q(5)$, $Q(10)$, and $Q(22)$) show highly significant autocorrelations, underlying the necessity to have an autoregressive structure in modeling realized volatilities. Hence, the application of HAR models with their lagged structure on daily, weekly, and monthly average realized volatilities is justified by the data in particular view of the exogenous factors. The augmented Dickey-Fuller test rejects the presence of a unit root, which comes as no surprise for a time series being constructed from sums of squared log-returns. In order to account for possible heteroskedasticity in residuals, we employ White’s heteroskedasticity-adjusted robust standard errors (White, 1980) for model inference. Figure 2 visualizes the synchronized daily realized volatilities for prices of the most

⁴As we construct our time series as outlined above, the list of contracts used for WTI and Brent is available in the appendix.

liquid futures of WTI and Brent as well as index futures on the S&P500. As first observation on these realized volatilities, we find clustering as already picked up by the Ljung-Box tests. Additionally, we notice that the evolution of realized volatility is different across the WTI and Brent.

Figure 2: Realized volatilities of the most liquid futures prices of WTI (*top*), Brent (*middle*), and the S&P500 (*bottom*), between January 2, 2013 and May 11, 2018.

4. Results

4.1. In-sample analysis

Firstly, we estimate the IHM and IHMC variants of the base HAR, HAR-Eoil, HAR-Eoil-SPX, and HAR-SPX, which totals 12 models for each WTI and Brent, respectively, on our full in-sample data set. In order to explore the impact of exogenous factors on the realized volatility of oil futures and for the sake of brevity, we report the results for the

Markov-switching IHM-HAR, IHM-HAR-Eoil, IHM-HAR-Eoil-SPX, and IHM-HAR-SPX only.⁵

Table 2 and Table 3 present the mean of the estimates and their variances across the IHM model variants for WTI and Brent, respectively. The mean is calculated as the average of the parameter estimates across the estimated regimes. For WTI and Brent, we find that short-, medium-, and long-term volatilities have a significant impact across all regimes.

For WTI futures, realized volatilities of Brent in the IHM-HAR-Eoil and IHM-HAR-Eoil-SPX model have a significant, yet very small impact on daily realized volatilities. We find the long-term average realized volatility ($RV_{t-1:t-22}^{Eoil}$) to have the highest impact at 0.0471 (0.0007) and 0.081 (0.0016) for each model. For the other averages, the mean of the estimated parameters is close to zero suggesting little to no impact.⁶ The impact of the equity channel, measured by the realized volatility of the S&P500 shows some surprising effects in the mean of the parameter estimates across regimes. We find the impact to be significant for the short-, medium-, and long-term measures of the S&P500 for the IHM-HAR-Eoil-SPX and the IHM-HAR-SPX, while the effects are more pronounced in the latter model. We observe a positive impact for the short- and the medium term. The average realized volatility of the S&P500 over one and five days positively impacts the realized volatility of the WTI. If markets become more volatile—for example in periods of market turmoil—we identify a direct positive spillover to energy markets measured by the parameter averages 0.0211 (0.0010) and 0.0914 (0.0038) for RV_{t-1}^{SPX} and $RV_{t-1:t-5}^{SPX}$. The impact of the long-run realized volatility of the S&P500, $RV_{t-1:t-22}^{SPX}$, on WTI markets is negative, however. An increasing long-term volatility has a decreasing impact on the daily realized volatility of WTI futures. These effects are statistically significant across all regimes and across the IHM-HAR-Eoil-SPX and the IHM-HAR-SPX models. This might be an indication that the equity channel in the long-term loses impact and coincide with

⁵All other in-sample estimation results for the base models and the IHMC variants, where the parameters are time-invariant, are available upon request.

⁶We note that the exogenous oil factor RV^{Eoil} has the same scaling as the endogenous realized volatility RV .

a levelling off of commodity futures volatility.

For Brent, the WTI as exogenous factor has a much higher impact than vice versa, which underlines a information spillover from WTI to Brent markets (Klein, 2018). The equity market proxied by S&P500, on the other hand, has little to no impact on Brent futures, which comes as no surprise as the S&P500 indexes the North-American market. Parameter estimates for RV^{SPX} are very low but significantly different from zero.

	IHM-HAR		IHM-HAR-Eoil		IHM-HAR-Eoil-SPX		IHM-HAR-SPX	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
Intercept	0.4472	0.0377	0.2962	0.0140	0.3686	0.0247	0.4533	0.0427
RV_{t-1}	0.2358	0.0125	0.2618	0.0150	0.2179	0.0103	0.2081	0.0093
$RV_{t-1:t-5}$	0.2609	0.0176	0.2900	0.0193	0.3396	0.0256	0.2291	0.0137
$RV_{t-1:t-22}$	0.2466	0.0152	0.2346	0.0134	0.1925	0.0100	0.2916	0.0180
RV_{t-1}^{Eoil}			0.0038	0.0007	-0.0131	0.0001		
$RV_{t-1:t-5}^{Eoil}$			0.0336	0.0008	0.0078	0.0003		
$RV_{t-1:t-22}^{Eoil}$			0.0471	0.0007	0.0831	0.0016		
RV_{t-1}^{SPX}					0.0319	0.0007	0.0211	0.0010
$RV_{t-1:t-5}^{SPX}$					0.0064	0.0004	0.0914	0.0038
$RV_{t-1:t-22}^{SPX}$					-0.0371	0.0012	-0.0562	0.0017

Table 2: Mean and variance of the estimates of the time-varying, switching parameters for the Infinite Hidden Markov switching HAR models for WTI.

	IHM-HAR		IHM-HAR-Eoil		IHM-HAR-Eoil-SPX		IHM-HAR-SPX	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
Intercept	0.3877	0.0271	0.4567	0.0379	0.4627	0.0439	0.3561	0.0223
RV_{t-1}	0.2553	0.0149	0.2169	0.0092	0.2276	0.0109	0.2622	0.0146
$RV_{t-1:t-5}$	0.2406	0.0224	0.2438	0.0133	0.2132	0.0117	0.2698	0.0183
$RV_{t-1:t-22}$	0.2768	0.0159	0.2304	0.0121	0.1994	0.0111	0.2677	0.0163
RV_{t-1}^{Eoil}			0.0025	0.0003	0.0038	0.0002		
$RV_{t-1:t-5}^{Eoil}$			0.0279	0.0005	0.0547	0.0007		
$RV_{t-1:t-22}^{Eoil}$			0.0635	0.0015	0.1244	0.0033		
RV_{t-1}^{SPX}					0.0374	0.0009	0.0081	0.0015
$RV_{t-1:t-5}^{SPX}$					-0.0075	0.0002	0.0006	0.0002
$RV_{t-1:t-22}^{SPX}$					0.0101	0.0006	-0.0225	0.0008

Table 3: Mean and variance of the estimates of the time-varying, switching parameters for the Infinite Hidden Markov switching HAR models for Brent.

With respect to the Hidden Markov structure, Figure 3 and Figure 4 visualize the posterior mean of the number of regimes and heat maps that mark regime changes. For these heat maps, lighter colors describe a higher likelihood of observations being in the same regime. Darker colors describe a lower likelihood of identical regimes (Hou, 2017, Luo et al., 2019). We note that WTI appears to have a higher number of possible regimes, and subsequently, a higher likelihood of different regimes than Brent. This is of interest as

it indicates that realized volatilities of Brent and WTI appear to be structurally different, if we allow for an unknown number of Markov switches. When keeping the parameters fixed and only allowing for the variance to switch in the IHMC-HAR variant, we observe a drastically lower number of regimes in the lower panels of Figures 3 and 4. Furthermore, the heat maps then reveal that similarities in regimes exist. The likelihood of one regime only from January 2013 to October 2014 is relatively high for both Brent and WTI. From November 2014 to September 2017, we observe a different set of regimes that—relative to each other—seems similar. This effect is more pronounced for Brent than for WTI. From September 2017 regimes change again and the likelihood to stay in this new regime which is different to previous ones is high, indicated by a light area in the plots.

(a) IHM-HAR-Eoil-SPX

(b) IHMC-HAR-Eoil-SPX

Figure 3: Posterior mean of number of regimes and heat map of regimes for the WTI in-sample.

Structural breaks of oil volatility are identified by the IHM-HAR model and the IHMC-HAR model. Particularly, the results imply that changing situations—including exogenous shocks, macro environments, and energy markets—result in many unknown structural breaks in a relatively short-term oil futures time series. As shown in Figure 3 and

(a) IHM-HAR-Eoil-SPX

(b) IHMC-HAR-Eoil-SPX

Figure 4: Posterior mean of number of regimes and heat map of regimes for the Brent in-sample.

Figure 4, there are significantly more breakpoints in the period of 2014-2017 than before 2014, which is just in response to the sudden fall in oil prices in the second half year of 2014 induced by the serious imbalance between supply and demand in oil market caused by the rapid increase of the US shale oil production and OPEC's decision not to cut production. In the meantime, during the period from 2014 to 2017, the global political situation is more turbulent in which geopolitical risks are rising and black swan incidents occur frequently such as the UK exiting from the EU, OPEC's first agreement to production cuts within eight years, and Syrian refugee crisis and Yemen conflict which further increased the vulnerability and uncertainty of the global crude oil market. These findings are consistent with the existing literature (Pan et al., 2017, Ma et al., 2017, Gong & Lin, 2018).

In conclusion, we find that WTI and Brent futures markets are different in terms of their realized volatility structure. WTI futures feature more hidden Markov-regime

switches, if we allow for an ex-ante unspecified number of regimes modelled by Infinite Hidden Markov models. We find an elevated likelihood for stable regimes from January 2013 to October 2014, November 2014 to September 2017, and November to December 2017, if we only allow the variance to switch. Naturally, we find the S&P500 to have a higher impact on WTI futures than on Brent futures, which underlines the locality of the equity channel which supports and extends the findings of Efimova & Serletis (2014) and Degiannakis & Filis (2017). The realized volatility of WTI as exogenous factor has a significant impact on Brent futures volatility. We can hence confirm this directional spillover identified in literature (Elder & Jin, 2007, Elder et al., 2014, Klein, 2018)

4.2. Out-of-sample evaluation

4.2.1. Statistical evaluation

Turning towards the out-of-sample forecasts, we firstly evaluate the density forecast performance of the 12 individual volatility models according to the log marginal likelihood statistic across different forecasting horizons (Table 4).

We find that IHM-HAR specifications are performing superior in forecasting the realized probability density. In particular for longer forecasting horizons, the superiority of the Infinite Markov models becomes apparent in comparison to the non-switching base variants. This is consistent with (Luo et al., 2019) who also find HAR models with infinite Hidden Markov regime-switching structures have better precision than the benchmark HAR models. The implications of these findings are further discussed below.

The forecast accuracy is evaluated based on the MSE and QLIKE loss functions according to Patton (2011) as well as the P -statistic of Blair et al. (2001) to measure the proportion of variance explained by the forecasts. Table 5 and Table 6 report the error functions in detail for each model at daily, weekly, and monthly forecasting horizons. We apply the model confidence set of Hansen et al. (2011) in order to identify outperforming models. It becomes evident that for both WTI and Brent, all IHM-HAR variants outperform the base HAR models. In particular for the medium- and long-term forecast of five- and 22-days ahead, the Hidden-Markov switching models dominate non-switching variants, which is additionally confirmed by P -stats close to one in the respective model

	WTI			Brent		
	$h = 1$	$h = 5$	$h = 22$	$h = 1$	$h = 5$	$h = 22$
IHM-HAR	-431.255	35.412	461.932	-676.700	-185.909	284.486
IHMC-HAR	-487.302	-261.166	-219.608	-986.766	-556.778	-381.635
HAR	-850.287	-616.814	-478.424	-1195.337	-885.474	-833.932
IHM-HAR-Eoil	-406.288	37.425	450.152	-696.611	-186.797	275.322
IHMC-HAR-Eoil	-465.066	-268.038	-237.736	-992.567	-555.823	-396.827
HAR-Eoil	-840.089	-609.891	-446.129	-1197.153	-886.025	-837.891
IHM-HAR-Eoil-SPX	-403.301	6.071	417.439	-683.172	-193.453	257.084
IHMC-HAR-Eoil-SPX	-466.947	-262.656	-171.973	-990.668	-549.303	-391.531
HAR-Eoil-SPX	-840.578	-607.958	-444.369	-1196.510	-883.195	-837.272
IHM-HAR-SPX	-426.035	31.287	420.804	-701.894	-194.776	276.548
IHMC-HAR-SPX	-489.007	-262.460	-206.813	-980.956	-550.215	-380.879
HAR-SPX	-848.516	-607.494	-480.016	-1194.684	-881.981	-832.150

Note: Greater values of the log marginal likelihood imply better density forecast performance. Values of $h = 1$, $h = 5$ and $h = 22$ correspond to the 1-step, 5-step, and 22-step forecasts. We combine two types of infinite hidden Markov models with HAR models; the Infinite hidden Markov model (IHM) and the Infinite hidden Markov model with time-constant coefficients (IHMC), while Eoil refers to Exogenous oil predictors and SPX to the S&P500 index.

Table 4: Results for density forecasts based on log marginal likelihood

grouping. This finding is consistent with the results of an increased number of regimes during most of our sample period shown in Figure 3 and Figure 4. When the market is in a period of instability and high risk, ignoring potential structural breakpoints affects the prediction performance of volatilities. In particular, the longer the prediction time is, the more likely there is a structural break point. Our finding that the medium- and long-term forecast of regime switching HAR variants significantly dominate non-switching variants provide strong evidence of this result.

For WTI, the IHM-HAR, IHM-HAR-Eoil-SPX, and IHM-HAR-SPX are the outperforming models when compared to their corresponding IHMC and non-switching modifications for all forecasting horizons. If we compare models with different exogenous variables, we find some differences in their performance in terms of forecasting horizons. For short-term forecasting, exogenous factors improve the forecasting performance quite substantially with the IHM-HAR-Eoil-SPX model yielding lowest MSE and QLIKE. For medium- to long-term forecasts, the IHM-HAR without any exogenous factors exhibits a performance similar to those specifications with exogenous factors. Furthermore, IHM-HAR models with exogenous factors do not appear to outperform IHM-HAR models.

For Brent, the results indicate an even clearer outperformance of the IHM variants of the HAR model. Only the Hidden Markov models are in the MCS for all forecasting

horizons. Neither the IHMC nor the non-switching model variants are considered to be of sufficient forecasting performance. In some contrast to WTI as well as to the in-sample analysis, the endogenous IHM-HAR seems to be sufficient for forecasting when comparing loss functions with IHM-HAR models augmented by exogenous factors. The IHM-HAR has predominantly the lowest MSE and QLIKE values across all models and horizons, which makes the realized volatility of Brent less responsive to exogenous impacts, at least from a forecasting perspective.

In conclusion, HAR specifications that allow for infinite Hidden Markov switching in the parameters as well as the variance clearly outperform non-switching models as well as models that only feature differing variance regimes for both WTI and Brent futures markets. However, the impact of selected exogenous factors differ across WTI and Brent volatility forecasting. For WTI, exogenous channels and factors such as the S&P500 improve the forecasting performance for short-term horizons while for the month-ahead forecasts, the benefit of including exogenous factors diminishes. In contrast, including exogenous oil factors for Brent does not improve forecasting accuracy. There are two possible reasons which can explain these observations. First, we note that the S&P500 is a regionally different equity channel for Brent and with this, is less likely to contribute to improvement of forecasting performance. With increasingly regional price characteristics of WTI markets, reflecting the supply and demand situation of the US after the financial crisis, the link between WTI markets and the S&P500 has strengthened in the short term. Findings of Zhang & Ma (2019) are in support of this argument as they find that the simultaneous risk spillover between WTI and the S&P500 stock market is fairly pronounced. The second reason is that structural breakpoints are usually caused by emergencies and exogenous shocks including risk spillover from other financial markets. Thus, our HAR specifications allowing for infinite Hidden Markov switching can partly offset these effects of exogenous factors on volatility forecasting.

4.2.2. Economic evaluation

For the economic evaluation, we apply two different risk aversion levels in the utility function in Eq. (12), the mild risk aversion rate $\gamma = 1$ and the strong risk aversion rate

	$h = 1$			$h = 5$			$h = 22$		
	MSE	QLIKE	P -Stat	MSE	QLIKE	P -Stat	MSE	QLIKE	P -Stat
IHM-HAR	1.077**	0.248*	0.575	0.0880**	0.0256**	0.9174	0.0064**	0.0034**	0.9884
IHMC-HAR	1.141**	0.302	0.550	0.4351	0.1555	0.5920	0.1963	0.0902	0.6446
HAR	1.084**	0.241*	0.573	0.3982	0.1210	0.6266	0.1909	0.0762	0.6543
IHM-HAR-Eoil	1.059**	0.227*	0.583	0.0980**	0.0276**	0.9081	0.0070**	0.0037**	0.9873
IHMC-HAR-Eoil	1.115**	0.254*	0.561	0.3880	0.1180	0.6362	0.1586	0.0634	0.7128
HAR-Eoil	1.067**	0.242*	0.579	0.4093	0.1440	0.6162	0.2023	0.0960	0.6337
IHM-HAR-Eoil-SPX	1.050**	0.221**	0.586	0.1116*	0.0280**	0.8953	0.0091*	0.0043*	0.9836
IHMC-HAR-Eoil-SPX	1.110**	0.244*	0.563	0.4127	0.1306	0.6130	0.1470	0.0610	0.7337
HAR-Eoil-SPX	1.070**	0.241*	0.578	0.4039	0.1402	0.6212	0.2299	0.0878	0.5837
IHM-HAR-SPX	1.083**	0.248*	0.573	0.1047*	0.0282**	0.9018	0.0160*	0.0046**	0.9711
IHMC-HAR- SPX	1.151**	0.313	0.547	0.4487	0.1707	0.5793	0.1948	0.0942	0.6472
HAR- SPX	1.075**	0.238*	0.577	0.3821	0.1150	0.6417	0.2803	0.0617	0.4924

Note: Lower values of MSE and QLIKE loss functions imply a higher forecast precision. We also compute the P -stat according to Blair et al. (2001) to measure the proportion of variance explained by the forecasts: P -stat = $1 - \frac{\sum_{t=T_1}^T RV_{true,t} - RV_{for,t}}{\sum_{t=T_1}^T RV_{true,t} - \overline{RV_{true,t}}}$. Higher values of the P -stat suggest a better forecast precision. The values $h = 1$, $h = 5$, and $h = 22$ correspond to the 1-step, 5-step, and 22-step forecasts. Asterisks * and ** refer to corresponding MCS p -values of $> 10\%$ and $> 25\%$, respectively.

Table 5: Forecast precision evaluations of realized volatilities of WTI futures for different forecast horizons.

	$h = 1$			$h = 5$			$h = 22$		
	MSE	QLIKE	P -Stat	MSE	QLIKE	P -Stat	MSE	QLIKE	P -Stat
IHM-HAR	1.4686**	0.3930**	0.6441	0.113**	0.014	0.922	0.012**	0.002**	0.986
IHMC-HAR	1.8496	0.5631	0.5517	0.550	0.086	0.620	0.304	0.061	0.652
HAR	1.8704	0.5597	0.5467	0.546	0.067	0.623	0.335	0.045	0.616
IHM-HAR-Eoil	1.5028**	0.3996**	0.6358	0.117**	0.014	0.920	0.013**	0.002**	0.986
IHMC-HAR-Eoil	1.8436	0.5622	0.5532	0.548	0.086	0.622	0.322	0.064	0.631
HAR-Eoil	1.9116	0.5720	0.5367	0.561	0.070	0.612	0.367	0.052	0.579
IHM-HAR-Eoil-SPX	1.4985**	0.3978**	0.6368	0.123**	0.015	0.915	0.015	0.002**	0.983
IHMC-HAR-Eoil-SPX	1.8442	0.5602	0.5530	0.531	0.081	0.633	0.309	0.058	0.646
HAR-Eoil-SPX	1.9008	0.5725	0.5393	0.552	0.069	0.619	0.369	0.053	0.577
IHM-HAR-SPX	1.4828**	0.3954**	0.6406	0.123*	0.015	0.915	0.012**	0.002**	0.986
IHMC-HAR- SPX	1.8502	0.5599	0.5516	0.532	0.081	0.633	0.309	0.058	0.646
HAR- SPX	1.8601	0.5591	0.5492	0.536	0.067	0.630	0.328	0.045	0.624

Note: The lower values of MSE and QLIKE loss functions imply a higher forecast precision. We also compute the P -stat according to Blair et al. (2001) to measure the proportion of variance explained by the forecasts: P -stat = $1 - \frac{\sum_{t=T_1}^T RV_{true,t} - RV_{for,t}}{\sum_{t=T_1}^T RV_{true,t} - \overline{RV_{true,t}}}$. Higher values of the P -stat suggest a better forecast precision. The values $h = 1$, $h = 5$, and $h = 22$ correspond to the 1-step, 5-step, and 22-step forecasts. Asterisks * and ** refer to corresponding MCS p -values of $> 10\%$ and $> 25\%$, respectively.

Table 6: Forecast precision evaluations of realized volatilities of Brent futures for different forecast horizons.

$\gamma = 10$. For both risk preferences, corresponding to two different types of investors, we now compare the economic value of the forecasts over the three forecasting horizons (daily, weekly and monthly).

Table 7 shows the economic values for a portfolio comprising of WTI futures for different forecasting models. Integrating infinite hidden Markov models is proven to benefit the portfolio performance. All IMH and IHMC models show positive economic values relative to the base model across all forecasting horizons. Several interesting observations can be

made in this regard. Firstly, the economic value of the IHMC variants is higher than that of the IHM variant for portfolios containing WTI futures. Secondly, the economic benefits of applying IHM(C) models are more pronounced for cases of mild risk aversion with $\gamma = 1$. Thirdly, we cannot identify a single IHM(C) model that yields highest economic gains across all forecasting horizons relative to their base model. For example, for the one day-ahead forecast and $\gamma = 1$, the IHMC-HAR yields the highest economic value with $\Delta = 2.913$ while for five days-ahead, the IHM-HAR-Eoil produces the highest portfolio gain measured as $\Delta = 2.417$, relative to the non-switching variant. Lastly and most importantly, IHM(C) models show higher economic values for longer forecasting horizons. This result underlines the superior performance of these models with respect to the base models and provides evidence that these more sophisticated HAR extensions do not only offer statistical improvement but also add economic value, in particular in the long run.

For Brent, the analysis of the economic gain is given in Table 8. The results are similar to those obtained for WTI. IHM(C) models show consistently positive economic values relative to the base HAR models. Again, the economic gain is higher for the mildly risk averse investor than for the strongly risk averse investor. For Brent, however, the economic gain does not increase with the forecasting horizon as we observe for WTI and the economic gain varies over the forecasting horizons. Similar to the results of WTI, we cannot identify a single model dominating all other model variants. The results for Brent are insofar different to those for the WTI as the economic gain is in general of smaller magnitude. This indicates that the economic gain, derived from the forecasting performance of these IHM(C) model variants, is different between the two markets and can be explained by differences in forecasting performances of these models.

Ultimately, we interpret our findings as evidence that realized volatilities of the S&P500 and the corresponding oil market as exogenous factors have differing explanatory power across the North American and European oil futures markets. As in the statistical evaluation, we detect differences in the impact of exogenous factors on portfolio returns based on forecasts of realized volatilities of WTI and Brent futures.

For WTI, we find that including realized volatilities of Brent offers almost no economic

gains in terms of improvement of portfolio returns, e.g. for $h = 1$ and the IHM-HAR we obtain a portfolio return of 7.202% while for the IHM-HAR-Eoil including realized volatilities of Brent we obtain 7.266%. For longer forecasting horizons, the return of the portfolio obtained by IHM-HAR-Eoil model shows even worse performance than the IHM-HAR model. A significant improvement of portfolio returns, translating to a significant economic gain, is found when including realized volatility of the S&P500. All HAR-SPX variants (HAR-SPX, IHMC-HAR-SPX, and IHM-HAR-SPX) yield positive economic gains with respect to models not including the SPX as well as for all other models for WTI. For Brent, we find that the SPX as exogenous factor yields no significant economic gain. This, however, should not come as a surprise as the S&P500 covers the North American stock market and is less relevant for the European oil benchmark. On the other hand, including realized volatilities of the WTI as exogenous factors produces significantly higher portfolio returns in the short ($h = 1$) and medium ($h = 5$) run. For the long run with $h = 22$, we find no evidence that exogenous factors have an impact for Brent.

There are two possible contributing factors to why HAR models with IHM structures provide surpluses in portfolio returns compared with benchmark HAR models. First, models with IHM structure have time-varying coefficients and variances, which can accommodate unknown structural changes in oil price volatility. The improvement of oil volatility forecast performances using Markov regime switching has been proven in literature (Ma et al., 2017, Di Sanzo, 2018). Second, specifying HAR models with heteroskedastic and flexible error structures might increase portfolio performances as shown in Luo et al. (2019). In addition, from an economic perspective, due to the occasionally high volatility and jumps of crude oil prices, investors need to adjust their portfolio strategies in time according to the changes of risk spillovers and exogenous shocks across the markets.

In conclusion, we find differing results for economic gains and absolute portfolio returns for WTI and Brent. Including the S&P500 positively impacts portfolio results for WTI while it has no effect on Brent. Including realized volatility of oil futures from the other market only has an impact on Brent where including information from WTI futures markets improves portfolio results but not vice versa. We again confirm the leading role

of WTI over Brent as discussed in Elder et al. (2014) and Klein (2018), which is not free of controversy (Ji & Fan, 2015). Overall, we find that IHM and IHMC variants of the HAR base model always improve portfolio performance and yield positive economic values relative to non-switching models.

	$h = 1$				$h = 5$				$h = 22$			
	$\gamma = 1$		$\gamma = 10$		$\gamma = 1$		$\gamma = 10$		$\gamma = 1$		$\gamma = 10$	
	r	Δ	r	Δ	r	Δ	r	Δ	r	Δ	r	Δ
IHM-HAR	7.202	0.494	2.621	0.052	8.170	1.278	2.716	0.135	6.215	1.507	2.506	0.159
IHMC-HAR	9.576	2.913	2.859	0.308	8.906	2.033	2.789	0.215	5.915	1.202	2.476	0.127
HAR	6.715	—	2.572	—	6.918	—	2.590	—	4.732	—	2.357	—
IHM-HAR-Eoil	7.266	1.375	2.628	0.145	8.147	2.417	2.713	0.255	6.105	1.792	2.495	0.189
IHMC-HAR-Eoil	8.211	2.335	2.722	0.246	7.912	2.176	2.690	0.230	5.348	1.020	2.419	0.108
HAR-Eoil	5.914	—	2.492	—	5.780	—	2.477	—	4.342	—	2.318	—
IHM-HAR-Eoil-SPX	7.257	1.338	2.627	0.141	8.164	2.276	2.715	0.240	6.174	1.691	2.502	0.178
IHMC-HAR-Eoil-SPX	8.366	2.467	2.738	0.260	7.998	2.105	2.698	0.222	5.585	1.091	2.443	0.115
HAR-Eoil-SPX	5.940	—	2.495	—	5.935	—	2.492	—	4.510	—	2.335	—
IHM-HAR-SPX	7.607	0.205	2.662	0.022	8.297	0.581	2.728	0.061	6.258	1.047	2.510	0.110
IHMC-HAR-SPX	9.961	2.604	2.897	0.275	9.197	1.505	2.818	0.159	5.950	0.734	2.479	0.077
HAR-SPX	7.405	—	2.641	—	7.727	—	2.671	—	5.228	—	2.407	0.159

Note: r is the portfolio return and Δ is the economic value of different HAR models with infinite hidden Markov switching structures against the corresponding benchmark HAR models. We consider two types of risk aversion rates: $\gamma = 1$ and $\gamma = 10$.

Table 7: Economic evaluations for WTI over different forecast horizons.

	$h = 1$				$h = 5$				$h = 22$			
	$\gamma = 1$		$\gamma = 10$		$\gamma = 1$		$\gamma = 10$		$\gamma = 1$		$\gamma = 10$	
	r	Δ	r	Δ	r	Δ	r	Δ	r	Δ	r	Δ
IHM-HAR	4.062	0.617	2.193	0.065	3.657	0.081	2.151	0.009	3.404	0.272	2.108	0.029
IHMC-HAR	3.602	0.155	2.147	0.016	3.903	0.330	2.175	0.035	3.565	0.434	2.124	0.046
HAR	3.448	—	2.132	—	3.576	—	2.143	—	3.135	—	2.081	—
IHM-HAR-Eoil	5.132	1.824	2.300	0.192	3.708	0.220	2.156	0.023	3.397	0.324	2.108	0.034
IHMC-HAR-Eoil	3.579	0.258	2.145	0.027	3.914	0.427	2.176	0.045	3.552	0.481	2.123	0.050
HAR-Eoil	3.322	—	2.119	—	3.491	—	2.134	—	3.076	—	2.075	—
IHM-HAR-Eoil-SPX	5.284	1.951	2.316	0.205	3.667	0.125	2.152	0.013	3.401	0.305	2.108	0.032
IHMC-HAR-Eoil-SPX	3.527	0.179	2.140	0.019	3.867	0.327	2.172	0.034	3.482	0.386	2.116	0.041
HAR-Eoil-SPX	3.349	—	2.122	—	3.543	—	2.139	—	3.100	—	2.078	—
IHM-HAR-SPX	4.570	1.112	2.244	0.117	3.664	0.015	2.151	0.002	3.407	0.245	2.109	0.026
IHMC-HAR- SPX	3.552	0.090	2.142	0.009	3.847	0.200	2.170	0.021	3.523	0.362	2.120	0.038
HAR- SPX	3.463	—	2.134	—	3.649	—	2.150	—	3.164	—	2.084	—

Note: r denotes the portfolio return and Δ is the economic value of different HAR models with infinite hidden Markov switching structures against the corresponding benchmark HAR models. We consider two types of risk aversion rates: $\gamma = 1$ and $\gamma = 10$.

Table 8: Economic evaluations for Brent over different forecast horizons.

5. Conclusions

While existing literature suggests that Markov switching volatility models outperform non-switching specifications for crude oil price series, these models usually require an ex

ante definition of the number of possible states. In this paper, we lift this strict limitation with introducing Infinite Hidden Markov models, which allow for an arbitrary number of regimes, solely determined by the time series during the estimation process.

We contribute to the literature on crude oil volatility firstly by applying these IHM models to models of realized volatility (HAR) and secondly, by augmenting these novel IHM-HAR models with exogenous factors in order to capture spillover and equity channel effects. In this vein, we revisit the established links between equity and commodity markets (Degiannakis & Filis, 2018) under the existence of an unknown number of breakpoints and different regimes, which extends the findings of, for example, Zhang & Zhang (2015), Ewing & Malik (2016), and Herrera et al. (2018).

Our results are manifold and of high relevance as they offer an encompassing answer to the academic discussion on regime switching in crude oil markets and their volatility. Firstly, we note that IHM-HAR variants are outperforming their non-switching variants in terms of estimation quality and fit. This observation is purely of statistical nature and does not come as a surprise since more sophisticated models—in particular those that are more flexible in terms of parameter evolution—yield better results than simpler models. With comparison to literature, we find the number of regimes produced by the IHM part to be notably higher than what has previously been applied. Secondly, we provide strong evidence that forecasting realized volatility over the short-, medium-, and long-term benefits tremendously from introducing IHM structures for both WTI and Brent futures series. For WTI, exogenous factors such as the equity channel improve short-term forecasts—which is in line with literature (Phan et al., 2016, Degiannakis & Filis, 2017) and in contrast to findings on daily frequency of Efimova & Serletis (2014)—while for longer forecasting horizons, their impact is reduced. It appears that modeling different regimes of realized volatility benefits forecasting more than including exogenous factors, at least for the long-run. For Brent, it holds that IHM-HAR models outperform non-switching variants. While we observe a higher fit when including exogenous factors, forecasting does not necessarily benefit from those factors for Brent unanimously for all factors. The endogenous IHM-HAR yields sufficient forecasts in a sense that exogenous

IHM-HAR variants do not outperform endogenous specifications at all horizons for all factors. One possible explanation for the present but low impact of the S&P500 might be the transition of the U.S. from an oil importer to an exporter. With this transition, the S&P500 might have lost some of its global indicator function, at least with regard to the oil demand of the U.S. This is relevant to policy makers as it shows that volatility in European oil markets becomes less sensitive to U.S.-specific exogenous factors. However, we still find WTI to be in a leading role in terms of volatility spillover. Lastly, we find IHM models to produce positive economic gains by their superior forecasting ability. Portfolios that are constructed based on volatility forecasts of IHM models produce higher returns and higher utility. We hence show that IHM-HAR models outperform non-switching variants in data fit, in prediction of future realized volatility for all forecasting horizons, and in portfolio performance. These findings improve the understanding of the volatility behavior of these markets and are useful for portfolio managers and policy makers. Our findings indicate that realized volatilities of crude oil markets are prone to structural changes and that single-specification models underperform in mapping these features. This requires risk management or prediction applications to account for rapid changes in volatility behavior of these futures markets.

Future research with IHM-HAR models could extend the number and nature of exogenous factors included, in order to determine their usefulness for prediction of realized variance. The IHM component offers an additional tool to distinguish between breaks and exogenous impacts and refines the understanding thereof. An additional question which has not been addressed in much detail in this area of research concerns the number of regimes and their similarity over time. We detect phases of similar regimes for both WTI and Brent. Future research could aim at explaining what makes these regimes similar and how the changes within regimes emerge across different markets.

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Appendix A. Additional tables

1	CLG3	17	CLM4	33	CLV5	49	CLH7
2	CLH3	18	CLN4	34	CLX5	50	CLJ7
3	CLJ3	19	CLQ4	35	CLF6	51	CLK7
4	CLK3	20	CLU4	36	CLG6	52	CLM7
5	CLM3	21	CLV4	37	CLH6	53	CLN7
6	CLN3	22	CLX4	38	CLJ6	54	CLQ7
7	CLQ3	23	CLZ4	39	CLK6	55	CLU7
8	CLU3	24	CLF5	40	CLM6	56	CLV7
9	CLV3	25	CLG5	41	CLN6	57	CLX7
10	CLX3	26	CLH5	42	CLQ6	58	CLZ7
11	CLZ3	27	CLJ5	43	CLU6	59	CLF8
12	CLF4	28	CLK5	44	CLV6	60	CLG8
13	CLG4	29	CLM5	45	CLX6	61	CLH8
14	CLH4	30	CLN5	46	CLZ6	62	CLJ8
15	CLJ4	31	CLQ5	47	CLF7	63	CLK8
16	CLK4	32	CLU5	48	CLG7	64	CLM8

Table A.9: WTI futures contracts used to construct the data set.

1	LCOG3	17	LCOM4	33	LCOV5	49	LCOG7
2	LCOH3	18	LCON4	34	LCOX5	50	LCOH7
3	LCOJ3	19	LCOQ4	35	LCOZ5	51	LCOJ7
4	LCOK3	20	LCOU4	36	LCOF6	52	LCOK7
5	LCOM3	21	LCOM5	37	LCOG6	53	LCOM7
6	LCON3	22	LCOV4	38	LCOH6	54	LCON7
7	LCOQ3	23	LCOX4	39	LCOJ6	55	LCOQ7
8	LCOU3	24	LCOZ4	40	LCOK6	56	LCOU7
9	LCOV3	25	LCOF5	41	LCOM6	57	LCOV7
10	LCOX3	26	LCOG5	42	LCON6	58	LCOZ7
11	LCOZ3	27	LCOH5	43	LCOQ6	59	LCOX7
12	LCOF4	28	LCOJ5	44	LCOU6	60	LCOF8
13	LCOG4	29	LCOK5	45	LCOV6	61	LCOG8
14	LCOH4	30	LCON5	46	LCOX6	62	LCOH8
15	LCOJ4	31	LCOQ5	47	LCOZ6	63	LCOJ8
16	LCOK4	32	LCOU5	48	LCOF7	64	LCOK8

Table A.10: Brent futures contracts used to construct the data set.