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Published in:
Energy Economics

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
Link to publication record in Queen's University Belfast Research Portal

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Reviewing the Oil Price–GDP Growth Relationship: 
A Replication Study

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**Abstract**

This paper presents a thorough replication of \textsuperscript{Hamilton} (2003) which in turn replicates and extends the findings of four seminal papers regarding the oil price–GDP growth relationship. Firstly, we replicate the empirical results obtained with the oil price measures of \textsuperscript{Hamilton} (1983), \textsuperscript{Mork} (1989), \textsuperscript{Lee et al.} (1995), \textsuperscript{Hamilton} (1996), and \textsuperscript{Hamilton} (2003) by using an identical data set of real and nominal oil prices. Secondly, we extend the data sets to 2019Q4 and apply the same methodology. We find that for more recent data the explanatory power of the proposed oil price measures on GDP growth rates is still present, albeit on a slightly weaker magnitude. Extending the ARX(4), we only find little evidence that oil price decreases impact GDP growth rates on the full data set. Parameter stability tests suggest that the impact of oil price decreases might be limited to certain periods.

**Keywords:** Oil prices, GDP growth, Asymmetry, Nonlinearity  
**JEL classification:** C24; E32; F43; Q43

1. Introduction


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This increase of interest in this particular topic is motivated by the importance of this commodity as strategic intermediary input during the production process of almost all goods, and as a final good for all economic sectors (for example transport, agriculture, and service). However, despite the large number of studies examining the impact of oil price changes on the macroeconomy, this nexus continues to spark intense debate. For instance, while Hamilton’s studies (Hamilton, 2003, 2011b) show that oil price shocks still have a significant impact on the U.S. real GDP growth, other research suggests that with recent data the negative relationship between oil price changes and the U.S. economy is still unclear or not present at all (Kilian & Vigfusson, 2011a).

This paper replicates and extends the findings of Hamilton (2003) who, in turn, reviews and replicates the findings of seminal studies focusing on the oil price–GDP relationship (Hamilton, 1983, Mork, 1989, Lee et al., 1995, Hamilton, 1996). We replicate these empirical results on their respective data range. Then, we update the data to 2019Q4 and re-estimate all models for nominal and real oil prices.

The five papers replicated in this study are among the most cited in the academic energy economics literature (cited more 10,000 times in total) and are widely used in public policy papers (Anderson & Meisch, 2003, Jackson et al., 2019). These five selected papers investigate the oil price–GDP link and each of them proposes a new measure of oil price changes to understand their impact on the macroeconomy. Table 1 gives an overview of recession and factors related to crude oil as a contributing factor. Fig. 1 visualizes recessions and the preceding oil price movements.

The first paper we focus on is Hamilton (1983) who examines the oil–GDP link motivated by the observation that seven of the eight postwar recessions in the United States have been preceded by a dramatic increase in oil prices. The results of Hamilton (1983) demonstrate that oil prices have a negative and significant impact on gross national product (GNP) for the periods of 1949Q2 to 1972Q4 and 1973Q1 to 1980Q3. Hamilton’s results show that the coefficients associated to the first period ($T = 96$ observations) are
greater (in absolute terms) than those of the second period ($T = 31$ observations).

The second paper which we take into consideration regarding the derivation of a new oil price measure is Mork (1989) who proposes measures that account for increases and decreases of oil prices separately. Mork (1989) builds his approach on the observation that mostly oil price increases are present in data of previous studies. Mork (1989) examines whether the findings of Hamilton (1983) continue to hold if more recent data are used, including a period of an oil price collapse and other significant decreases. Extending the findings of Hamilton (1983), Mork (1989) finds evidence that oil price increases have a negative and significant impact on the U.S real GNP. However, Mork’s results show that oil prices decreases are found to have little, if not zero, correlation to GNP growth. Overall, Mork (1989) provides strong evidence for the asymmetric impact of oil prices on the macroeconomy and finds that accounting for this asymmetric impact might solve problems of parameter instability (Mork, 1989, Tab. 1 and 2, pp. 742-743). Comparing the results of Hamilton (1983) to Mork (1989), we observe that the impact of oil prices on the U.S. real GNP is decreasing for more recent data and larger sample size.

The third oil price measure which is applied in Hamilton (2003) and replicated in this paper is derived in Lee et al. (1995) who introduce a normalized positive and negative oil price measure. The idea behind these measures is that sudden oil price changes in less volatile times have a greater impact than in times of erratic oil prices. Using similar model frameworks as in Hamilton (1983) and Mork (1989), Lee et al. (1995) find that only normalized positive oil price changes are a contributing factor to economic growth in the U.S.

The last two measures which play an important role in this replication are those of Hamilton (1996) and Hamilton (2003). The former paper introduces the one-year net oil prices increases measure while the latter updates this measure to span three years of oil price changes. Hamilton (1996) bases theses measures on the phenomenon that almost all

--- Fig. [approximately here] ---

---

\footnote{As an example, the third and fourth lag coefficients, associated with symmetric oil prices, are $-0.170$ and $-0.177$ for the period 1949Q2 - 1972Q4 (Hamilton, 1983, p. 244, Table 5), compared to a lower values of $-0.017$ and $-0.029$ for the period 1949Q1 - 1986Q1.}
oil prices decreases after 1986 are followed immediately by oil prices increases indicating the former are corrections of the latter or inversely. Empirically, this one-year net oil price increases measure is negatively and significantly related to the U.S. real GDP growth for the full sample of 1948Q1-1994Q2 and the sub-sample before the first oil prices shock of 1948Q1-1973Q3. For the sub-sample 1973Q4-1994Q2, only the third coefficient associated to the net oil price increases is negative and significant at 5% level. The three-years net oil prices measure of Hamilton (2003) is proposed after the observation that the oil price increase of 1999, which qualifies as a one-year net oil price increase according to Hamilton (1996) measure, is not followed by any decreases in the real economic growth in the year 2000. The economic explanation of Hamilton (2003) is that if the net oil price increases of 1999 does not cause consumers and firms to adjust their spending plans within one year, then the measure should be extended to more quarters which yields the three-years net oil price measure. Hamilton (2003) finds that this measure is more appropriate to capture the effect of oil prices increases on the U.S. real GDP as it now also explains the recession of 2001. Using a sample covering the period from 1949Q2 to 2001Q3, for a total of $T = 210$ observations, the empirical findings of Hamilton (2003) show that the three-years net oil price has a negative and significant impact on the economic activity proxied by the real economic growth.

We review the findings of Hamilton (2003) and replicate the estimations carried out with the oil price measures defined in Hamilton (1983), Mork (1986), Lee et al. (1995), Hamilton (1996), and Hamilton (2003). On an identical observation period, we are able to confirm parameter estimates and obtain values close to those reported in Hamilton (2003). Minor differences might be due different data sets and a more recent deflator. After the successful replication, we extend the data set to 2019Q4 and repeat the estimations. We find that within the proposed model framework of Hamilton (2003), oil price changes—measured with five different transformations—retain their predictive power in explaining GDP growth rates. However, as already noted in Hamilton (2003), the explanatory power and factor loads decrease for larger data sets and more recent data. Identical to the original data set of Hamilton (2003), we find a significant reaction of GDP growth rates
to oil price changes with a lag of four quarter when recent data up to 2019Q4 is used. We extend the ARX(4) to additionally include oil price decreases. Our results show that oil price decreases have little to no impact on GDP growth. This is underlined by exclusion tests. Further analysis shows, however, that parameters in this extended ARX(4) are unstable over the whole period which can be attributed to changes in parameters of oil price decreases.

The remainder of this article is structured as follows. The different oil price measures and the ARX($p$) framework are introduced in Section 2. Section 3 describes our data sets and gives an overview of preliminary tests thereof. In Section 4, we present the results of the replication study as well as of an extension of these findings with data up to 2019Q4. Section 5 concludes this replication article.

2. Empirical Methodology

2.1. Oil Price Measures

In order to replicate the above mentioned studies, we define the utilized oil price measures in what follows. Let $O_t$ denote oil price changes defined by the following log-difference

$$O_t = 100 \times (\log p_t - \log p_{t-1}),$$

where $p_t$ denotes an oil price at time $t$ in quarterly resolution. This linear, symmetric measure $O_t$ is applied in Hamilton (1983). Based on the assumption that oil price changes have an asymmetric effect on GDP growth rates, nonlinear—or asymmetric—oil price measures are proposed in subsequent research. We replicate four of these studies and use the oil price measures introduced in Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003). We note that Hamilton (1983, 1996, 2003) makes use of nominal oil price changes (denoted by $O_{t}^{\text{nom}}$), whereas Mork (1989) and Lee et al. (1995) utilize real oil price changes (denoted by $O_{t}^{\text{real}}$).

2.1.1. Mork’s asymmetric approach

Mork (1989) is a direct response to Hamilton (1983). This response applies a refined modelling framework, motivates using refiners acquisition cost (RAC) after 1974Q2 in-
stead of the producer price index (PPI), and tests oil price increases and decreases, whereas no significant effects of the latter are identified. Mork (1989) justifies testing decreases in addition to increases is by highlighting that the data set of Hamilton (2003) is mainly characterized by price increases, leaving decreases untestable. Mork (1989, p. 740) notes that “[Hamilton’s] study pertained to a period in which all the large oil price movements were upward, and thus it left unanswered the question whether the correlation persists in periods of price decline. Moreover, the price variable he used was somewhat distorted by price controls in the 1970s.” Hence, Mork (1989) updates the data set to 1988Q2 and corrects the oil price measure for several controls. The asymmetric measure proposed by Mork (1989) for price increases is given by

\[ O_{\text{Mork},t}^+ = \max \left( O_{t}^{\text{real}}, 0 \right) , \]

and analogously for oil price decreases, defined as

\[ O_{\text{Mork},t}^- = \min \left( O_{t}^{\text{real}}, 0 \right) , \]

where \( O_{t}^{\text{real}} \) denotes the real oil price changes of the PPI and RAC\(^2\) and \( O_{\text{Mork},t}^+ \) and \( O_{\text{Mork},t}^- \) are the positive and negative censored parts of the real oil price changes, respectively.

2.1.2. Volatility-scaled oil prices of Lee et al.

Lee et al. (1995) revisits Hamilton (1983) from a modeling approach but propose a scaling of oil prices which takes into consideration sudden shocks measured by price volatility. Their motivation stems from the assumption that oil price shocks have a varying impact on GNP growth rates, depending on how surprising the timing of the shock is. It is assumed that shocks in stable price periods have larger impact on the macroeconomy than during periods of volatile prices where this specific shock could be considered a reaction to a previous shock. Lee et al. (1995) then argue that adjusting oil price changes

\(^2\)Mork (1989) constructs the price set as follows. From 1949Q1 to 1971Q2, the rate of change of PPI is used. From 1971Q3 to 1974Q1, the rate of PPI change is multiplied by the annual log RAC change in order to counter the price control measures in the 70s, and from 1974Q2, the change rate of the RAC is used. See Mork (1989, p. 741) for more detail.
by its volatility accounts for these effects as well as the idea of an outbreak of its prior pattern.

Formally, the measure of Lee et al. (1995) builds on the estimation of an AR\(p\)-GARCH(1,1) model of oil price changes. Then, both oil price increases and decreases are divided by their estimated GARCH volatility to obtain the measures. The AR\(p\)-GARCH(1,1) reads

\[
O_t^{\text{real}} = \mu + \sum_{i=1}^{p} \alpha_i O_{t-i}^{\text{real}} + e_t, \tag{1}
\]

where \(\mu\) is an unconditional mean, the AR order lag is set to \(p = 4\) following Mork (1989), and \(\alpha_i\) are the parameters of the autoregressive part. The disturbance term \(e_t\) is modeled as GARCH(1,1) process with variance \(h_t\) which is defined as

\[
e_t = \sqrt{h_t} \zeta_t, \tag{2}
\]

\[
h_t = \gamma_0 + \gamma_1 e_{t-1}^2 + \gamma_2 h_{t-1},
\]

where \(\gamma_0, \gamma_1,\) and \(\gamma_2\) refer to GARCH parameters and \(\zeta_t \sim \mathcal{N}(0, 1)\) i.i.d. for all \(t = 1, \ldots, n\). The standardized residual \(e_t^* = e_t/\sqrt{h_t}\) is then censored and we obtain the positive and negative oil price measures of Lee et al. (1995) by

\[
O_{LNR,t}^{+} = \max(e_t^*, 0), \quad \text{and}
\]

\[
O_{LNR,t}^{-} = \min(e_t^*, 0).
\]

2.1.3. Hamilton’s adjusted measures

Hamilton (1996) proposes a new measure that contrasts oil price increases to decreases in the following four quarters. Hamilton (1996, p. 216) notes “if one wants a measure of how unsettling an increase in the price of oil is likely to be for the spending decisions of consumers and firms, it seems more appropriate to compare the current price of oil with where it has been over the previous year.” As a consequence, Hamilton (1996) defines the new measure as difference of the current increase of nominal oil prices and the maximum of increases during the previous four quarters. This measure is formally defined as
In this study, we additionally define a measure for price decreases which has not been applied in the original article. The decrease measure reads

\[ O_{\text{Ham1},t}^- = 100 \cdot \min (\log p_t - \log (\min (p_{t-1}, p_{t-2}, p_{t-3}, p_{t-4})), 0). \]

This negative measure is motivated from later studies such as Kilian & Vigfusson (2011b).

While the measure outlined in Hamilton (1996) spans four quarters and hence, assumes that price corrections are happening within a year, the Asian Crisis 1997-1998 had only little immediate impact on economic growth in the U.S. although oil prices drop to roughly $12 per barrel. The recession, identified by NBER (see also Table III), caused by the wake of this crisis hit the U.S. in 2001, around three years later. Hamilton (2003) suggests to extend the look-back period of the Hamilton (1996) measure, \( O_{\text{Ham1},t}^+ \), to three years or 12 quarters to account for longer reaction times within a three year window. This adjusted measure is denoted \( O_{\text{Ham3},t}^+ \) and defined analogously as

\[ O_{\text{Ham3},t}^+ = 100 \cdot \max (\log p_t - \log (\max (p_{t-1}, p_{t-2}, \ldots, p_{t-12})), 0), \]

which now spans twelve quarters. We calculate the net decrease as

\[ O_{\text{Ham3},t}^- = 100 \cdot \min (\log p_t - \log (\min (p_{t-1}, p_{t-2}, \ldots, p_{t-12})), 0), \]

which is not tested in the original article.

2.2. ARX(4) models and an extended version

We follow the model framework outlined in Hamilton (2003) for the replication exercise in this paper. Hence, we formulate a univariate autoregressive model structure that seeks to explain the impact of oil price shocks as exogenous factor on U.S. GDP growth. Let \( p \) denote the autoregressive lags of GDP growth rates and \( q \) denotes the lags of quarterly
oil price measures defined in Section 2.1. The ARX($p$) model reads

$$y_t = \mu_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \delta_i O^+_{(M),t-i} + u_t,$$

(3)

where $y_t$ refers to the rate of real GDP growth and $O^+_{(M),t}$ proxies the positively censored oil price changes with $\{M\}$ as a placeholder for the measures defined in the previous subsections, namely $O^+_{\text{Mork},t}$, $O^+_{\text{LNR},t}$, $O^+_{\text{Ham1},t}$, and $O^+_{\text{Ham3},t}$. The parameters $\phi_i$ are restricted to contain no unit root while $\delta_i \in \mathbb{R}$. Loosening these restrictions has negligible impact on parameter estimates as demonstrated in Charteddine et al. (2019). The error term $u_t$ is Gaussian i.i.d.

In addition to the replication of Hamilton (2003), we extend Eq. (3) to account for both positive and negative oil price changes. Kilian & Vigfusson (2011a) find that utilizing only positively censored data to explain GDP growth rates leads to an overestimation of their impact on real GDP growth in VAR models. Hence, we extend Eq. (3) with negative measures and obtain a joint model:

$$y_t = \mu_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \delta_i O^+_{(M),t-i} + \sum_{i=1}^{q} \gamma_i O^-_{(M),t-i} + u_t,$$

(4)

where $p$, $q$, and $O^+/-$ are defined as above with $\gamma_i \in \mathbb{R}$.

3. Data

For reason of comparison, we use both nominal oil price changes as in Hamilton (1983, 1996, 2003), and the real oil price changes as in Mork (1989), Lee et al. (1995), and Kilian & Vigfusson (2011b) for example. However, for the replication of results in Hamilton (2003) we follow the data notation therein. Overall, the data sample covers a period of more than 70 years of quarterly observations ranging from 1947Q2 to 2019Q4 and provides a total number of $T = 291$ quarters.

As mentioned in Section 2, we use quarterly data for the ARX($p$) model. Quarterly growth rates of chain-weighted real GDP are collected from the U.S. Bureau of Economic Analysis (BEA). We process logarithmic returns multiplied by 100. The calculation basis
is the USD value of 2012. The monthly nominal crude oil producer price index (PPI) is collected from U.S. Bureau of Labor Statistics (BLS) and converted to quarterly observations by using the end-of-quarter monthly observation. Real oil prices are calculated by multiplication with the CPI deflator with 2012 USD basis.3

3.1. Data sets of the original papers

As each of the five articles proposes a new measure of oil price changes applied on more recent data, end dates of the observation periods differ. Across these papers, oil prices are either obtained from the producer price index (PPI) or refiners acquisition costs (RAC). An overview on the original data sets of Hamilton (1983), Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003) is given in Table 2.

Our data set diverges from the data used in these papers in the following ways. Firstly, for the replication results presented in Table 3, we use identical end points as in the replications of Hamilton (2003), 2001Q3, whereas the original papers use data up to 1980Q3, 1988Q2, 1992Q2, and 1994Q2, respectively. Secondly, while Hamilton (1983), Mork (1989), and Lee et al. (1995) use the Gross National Product, Hamilton (1996, 2003) and this paper use the Gross Domestic Product. Additionally, since the data provided by the Bureau of Economic Analysis is seasonally adjusted, our real GDP time series is somewhat different to the one employed by Hamilton (2003). Our sampling of GDP rates is based on the 2012 dollar value, whereas the dollar value of 1996 is used for the data set in Hamilton (2003). Thirdly, these five articles also use different starting dates, i.e. 1948Q2, 1949Q1, 1949Q1, 1948Q1, and 1948Q2. In this replication, we decide to use the longest possible period, i.e. to use 1948Q2 as Hamilton (1983). Thus, we have four lags, starting 1947Q2 for the first period of the ARX(4) model. Lastly, the oil prices used in the five papers are the Producer Price Index (PPI, Hamilton, 1983, 1996, 2003) and the

3The GDP data is retrieved from www.bea.gov/data/gdp/gross-domestic-product on 02/04/2020 and fred.stlouisfed.org/series/CPIAUCSL (CPI) on 02/14/2020, respectively. We selected the seasonally unadjusted oil price time series WPU0561 via data.bls.gov/cgi-bin/dsrv?wp on 02/04/2020. The original data, the constructed data, the code, and outputs are available in the supplementary material to this article.
Refiners’ Acquisitions Costs (Mork, 1989, Lee et al., 1995). Here, we use the PPI. In addition, Hamilton (2003) calculates the real oil price changes by subtracting the GDP deflator. We use the monthly CPI to deflate monthly nominal oil prices before converting them to quarterly, real oil price changes.

3.2. Updating the data sets

In addition, we extend the data set to 2019Q4 and re-estimate all models on more recent data to compare findings and implications with the original papers.

Given these data sets, the calculation of the oil price measures is straightforward. The difference between these measures, in particular in scaling and filtering, becomes apparent from Fig. 2, which plots the oil price measures from 1947Q2 to 2019Q4. Comparing Hamilton’s measures (Hamilton, 1996, 2003) to those of Mork (1989) and Lee et al. (1995) reveals the filtering character of Hamilton’s measures which are becoming zero for price movements that are corrections to previous movements as intended.

——— Fig. 2 approximately here. ———

4. Results

Firstly, we repeat the estimations and replicate the results reported in Hamilton (2003). Secondly, for reasons of comparability with earlier studies and replication of their findings, we subsequently repeat all estimations on the full sample from 1947 to 2019. Results are reported in Subsection 4.2. Thirdly, in Subsection 4.3 we extend the framework and include positive and negative oil price changes in order to quantify their joint effects. Lastly, we assess parameter stability in the linear ARX(4) model in Subsection 4.4.

4.1. Replication of previous studies with identical sample period

We apply the linear ARX(4) outlined in Eq. (3) of this paper. We replicate the estimations of Eq. (1.5), Eq. (1.6), Eq. (1.8), Eq. (3.2), and Eq. (3.8) of Hamilton (2003). In this subsection only, all references to equations refer to the original article of Hamilton (2003). Parameter estimates of the original paper carry an H as superscript while replicated parameters are indicated with r.
The results of the replication are reported in Table 3. Columns (a) represent the results from Hamilton (2003), while columns labelled with (b) present replication results obtained by applying an identical model framework and data periods as in Hamilton (2003) with our data set.

The results of estimating Eq. (1.5), the framework of Hamilton (1989) with data from 1948Q2 to 1980Q4, show that only the fourth lag of nominal oil changes has a significant negative impact (coefficients of $-0.0595$). With the fully estimated model, this means that a 10% increase of nominal oil prices leads to an approx. 1.4% decrease of real GDP growth four quarters later (Hamilton, 2003, p. 369). Comparing columns (a) and (b) shows that we are able to replicate those estimates. For the only significant parameter regarding oil price changes, $\delta_4$, we estimate $\delta_4^r = -0.0595$, while Hamilton (2003) reports $\delta_4^H = -0.064$.

This estimation is repeated for data from 1948Q2 to 2001Q3 in Eq. (1.6). Again the only statistically significant lag of oil price changes is $\delta_4$ with an estimated coefficient of $\delta_4^r = -0.0153$ which is close to the reported $\delta_4^H = -0.0160$ in Hamilton (2003). Both coefficients are significant at the 5% level. Those results indicate that oil price changes have a lowered impact on GDP growth if the data set is extended to 2001Q3.

We continue our discussion and analysis of the results of the estimation of Eq. (1.8) in column (a) and (b) of Table 3 with the measure of Mork (1989), $O_{Mork,t}^+$. This measure only considers positive oil price changes. Again, the only significant positive oil price change is $\delta_4$ estimated at $\delta_4^r = -0.0211$ which is similar to $\delta_4^H = -0.023$ estimated in Hamilton (2003). Both standard errors are identical at 0.009.

The results of using Hamilton’s 1y net oil price increases, $O_{Haml,t}^+$ are presented in column Eq. (3.2), column (a) and (b) in Table 3. Our estimation results in column (b) mirror those reported in Hamilton (2003) in column (a). Yet again, the only significant coefficient is at fourth lag, estimated at $\delta_4^r = -0.0314$ while $\delta_4^H = -0.031$. Finally, by using the 3y net oil price increases, $O_{Ham3,t}^+$ only $\delta_4$ is found to be significant at the 5% level and replicated estimates align with those reported in Hamilton (2003).
adjusted oil price measure by Lee et al. (1995) are statistically significant at 1%. We confirm that $\delta_3$ and $\delta_4$ are highly significant and have only small differences in the parameters using our data set. However, the GARCH process (Eq. 1 and 2) to obtain the oil price measure is different. We choose to re-estimate the AR-GARCH coefficients, given the fact that our data set is somewhat different. Hamilton (2003) reports the following parameter estimates:

\[
O_t^{\text{real}} = -0.4965 + 0.436O_{t-1}^{\text{real}} - 0.401O_{t-2}^{\text{real}} + 0.244O_{t-3}^{\text{real}} - 0.238O_{t-4}^{\text{real}} + e_t \\
h_t = 1.49 + 2.208e_{t-1}^2 + 0.197h_{t-1}. \tag{5}
\]

Our estimations, however, yield the following estimates with $p$-values in parenthesis:

\[
O_t^{\text{real}} = 0.0410 + 0.4826O_{t-1}^{\text{real}} - 0.2173O_{t-2}^{\text{real}} + 0.2199O_{t-3}^{\text{real}} + 0.1408O_{t-4}^{\text{real}} + e_t \\
\begin{array}{c}
\text{(0.8066)} \\
\text{(0.0001)} \\
\text{(0.0001)} \\
\text{(0.0001)} \\
\text{(0.0001)}
\end{array}
\]

\[
h_t = 0.5275 + 8.2177e_{t-1}^2 - 0.0003h_{t-1}. \\
\begin{array}{c}
\text{(0.0529)} \\
\text{(0.0004)} \\
\text{(0.6493)}
\end{array}
\]

The difference in the GARCH process might also explain the slightly different estimated coefficients of our sample (b) and the reported coefficients in Hamilton (2003) (a). The parameters reported in Hamilton (2003) are the same as in Lee et al. (1995). The latter note that even for other GARCH specifications, the main results remain unchanged. When we use the originally reported GARCH equation, that is with estimates given in Eq. (5), we find similar results for the later analysis on an extended data set.

In conclusion, we are able to replicate the estimations carried out in Hamilton (2003), applying the measures of Hamilton (1983), Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003). In addition, we note that a slightly different data set (using PPI and CPI, and a different basis for GDP calculation) causes only minor deviations in terms of difference of the replicated and reported parameter estimates. In the next subsection, we repeat our estimations on an updated data set.
4.2. Results on an updated data set

This section discusses whether the empirical findings of the five highlighted studies are still valid when newer data up to 2019Q4 are appended.

The results of estimating the oil price–GDP growth relationship with recent data based on the different measures, \( O_t \), \( O_{Mork, t}^+ \), \( O_{LNR, t}^+ \), \( O_{Ham1, t}^+ \), and \( O_{Ham3, t}^+ \), are reported in Table 4. Turning to the impact of oil prices measures on the economic growth, columns 2 and 3 refer to results when estimating the ARX(4) model using a linear functional form for the exogenous oil price changes variable \( O_t \) without any censoring. The results show that, in line with previous studies, only the fourth lag of oil price changes is significant at the 10% level. Moreover, our results confirm the findings of Hamilton (2003) who postulates that the impact of positive oil price changes decreases as the covered time span increases. For instance, the fourth lag of the oil price changes is estimated at \(-0.064\) in Hamilton (2003, p. 369, Eq. (1.6)) for the period 1948Q2 to 1980Q4. Extending the period to 2001Q4, it decreases to \(-0.016\). For the period 1948Q2 to 2019Q4, we obtain a statistically significant estimate of \(-0.0067\) reported in Table 4. This can be explained by the fact the recent periods are characterized by higher volatility of oil prices. As mentioned in previous studies, the shortcoming of the linear ARX(4) is the assumption that negative oil price changes have the same impact on GDP growth than positive changes.

The results of applying the oil price measures of Mork (1989), \( O_{Mork, t}^+ \), with the extended model defined in Eq. (4) are reported in Table 4, columns 4 and 5. Our results show that the measure of Mork (1989) is significant for the first and fourth lag at 10% and 5%, respectively, indicating that this measure depicts a short term reaction in the first quarter.

The results on the extended time frame obtained with the measures introduced in Lee et al. (1995) are very close to those reported for the replicated period until 2001Q3. We
estimated the following GARCH model based on Eq. 1 and 2 with p-values in parentheses:

\[
O_t^\text{real} = 0.1196 - 0.3972O_{t-1}^\text{real} - 0.1120O_{t-2}^\text{real} + 0.2047O_{t-3}^\text{real} + 0.1990O_{t-4}^\text{real} + \varepsilon_t \\
h_t = 0.4806 + 6.7799\varepsilon_{t-1}^2 + 0.0437h_{t-1}.
\]

For this extended data set, the model follows a different, but also explosive GARCH process for real oil prices as reported in Lee et al. (1995) and Hamilton (2003). However, the third and fourth lag of the heteroskedasticity-adjusted, real oil price increases remain statistically significant for explaining real GDP growth.

The results of applying the net oil price measure of Hamilton (1996) and Hamilton (2003), \(O_{t}^\text{Ham1,4}\) and \(O_{t}^\text{Ham3,1}\), are in line with the findings of other oil price proxies; only the fourth lag is highly significant with an estimated coefficient of \(\delta_4 = -0.0256\) and \(-0.0330\), respectively. Allowing for a 10% level of significance, we also find the coefficient for the first lag of the 3y measure to be statistically significant.

Our empirical results show that the 1- and 3-year net oil price increases have an even lower impact for the extended sample. For instance, using data up to 2019Q4, the estimated \(\delta_4\) are lower in absolute terms than the reported coefficients by Hamilton (2003) in Table 3. With a longer time period covered with the models, the behavior of consumers and their reactions to price, demand, and supply changes might have changed. By taking into consideration the volatility of oil price changes, they become less sensitive to fluctuation of oil prices. This could hold true for the period after 2001, in particular, as these years are characterized by extremely erratic prices compared to earlier subsets. Hamilton (2003) already suggests that the 1y net oil price measure is not consistent with recent oil price fluctuations relative to 2003.

An important observation that arises from Table 4 is the decrease in the intercept of the real GDP equation when comparing the 1948-2001 regression with the results of the 1948-2019 regression. This result may reflect the fact that GDP growth slowed down in the 21st century compared to the 20th century.

In the next subsection, we augment the original framework with negative oil price
measures, following an analogous definition as in the original papers and test their impact on the extended data set.

4.3. Extending the framework: combining positive and negative oil price changes

Kilian & Vigfusson (2011a, p. 427) suggest that if “both energy price increases and decreases matter for the real GDP but at a different extend, then the censored regressor model is likely to overestimate the effect of an energy price increase.” Following this argument, one may assume that all results reported in Table 3 and Table 4 are biased by overestimating the impact of price changes on real GDP since all of them are obtained with positively censored price measures only. To assess the impact of omitting the negative (price decreases) part, we re-estimate all censored models of the previous subsection including both positive and negative measures, $O_{\{M\},t}^+$ and $O_{\{M\},t}^-$, as defined in Eq. (4) on the extended data set.

The estimation results are reported in Panel A of Table 8. As in Table 3 and Table 4, parameters $\delta_i$ refer to oil price increases while we now extend the framework for also measuring oil price decreases estimated with $\gamma_i$. Most importantly, we observe that the estimated coefficients associated with negative (decreasing) oil price measures are all insignificant at the 10% level except for the model specification of Lee et al. (1995). For the latter, we observe that the first and the third lag for heteroskedasticity-adjusted oil price decreases have a positive association with the GDP and are significant at 5% and 10%, respectively. While the size of the parameters is smaller in absolute terms compared to oil price increases, we find that oil price decreases also decrease the GDP, if adjusted for heteroskedasticity of the oil price.

Moreover, the results show that the inclusion of the negative oil price measures impacts neither the magnitude of positive oil price measures nor their significance. Changes in parameter estimates are negligible. For example, the estimated coefficient at the fourth lag of $O_{\{Ham\},t}^+$ is $\delta_4 = -0.0256 (0.003)$ if no decreases are included. If we include decreases, the estimation yields $\delta_4 = -0.0265 (0.003)$. In terms of significance there are only minor differences between including or omitting the negative measures as all lag coefficients of the oil price proxies keep their level of significance for this data set.
In order to statistically assess whether the oil price changes have a significant impact on real GDP growth, we conduct exclusion tests for each oil price measure. Results are given in Panel B of Table 5. We test the exclusion of oil price increases, oil price decreases, and their joint exclusion, where the latter assumes that oil prices do not impact GDP growth.

Unsurprisingly, the hypothesis that oil price increases do not have an impact on GDP, hypothesized by $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$, is unanimously rejected for all oil price measures, indicating that price increases play a significant role in the outlined ARX(4) for GDP growth rates. For oil price decreases, the joint hypothesis $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ cannot be rejected translating to no significant impact of oil price decreases on GDP growth at 5%. For the Lee et al. (1993) oil price measure, we can reject at 10% as expected from the parameter tests. Testing both restrictions jointly, we cannot reject the hypothesis that increases and decreases are not relevant to the ARX(4) GDP growth framework which is attributed to the significance of price increases.

Motivated by the theoretical and empirical findings of Kilian & Vigfusson (2011b), we include both positive and negative oil price measures. For this particular data and model set, omitting the negatively censored data has no impact on the results. Factor loads of the negative proxies are statistically insignificant for three out of four models and proxies. The results from the ARX(4) model suggest that especially oil price increases are important for the oil price-GDP relation. We find slightly significant impact of oil price decreases on GDP growth for the heteroskedasticity-adjusted oil price measure. We also conclude that excluding negative oil price changes in this ARX(4) model does not overestimate the impact of oil price increases on GDP growth.

——— Table 5 approximately here. ———

4.4. Stability tests

An important issue that might arise when using long periods of data is the possibility of structural changes. This instability of the relationship between oil prices and GDP growth is tested in literature using the Chow test and the tests of Andrews (1993) and Andrews & Ploberger (1994). The choice of these tests in this study is mainly due to
comparing results with previous studies that we are replicating.\footnote{For additional robustness checks, we employ structural breaks tests as introduced in Bai \& Perron (2003). We also allow the error distributions to differ across sub-samples. However, results obtained with this test only differ slightly from those described in detail in this study. Results of the structural breaks tests of Bai \& Perron (2003) are available upon request.}

In what follows, we test the framework of Hamilton (2003), Eq. (3), that only includes oil price increases and the extended framework, Eq. (4), including increases and decreases separately to analyze parameter stability over the extended data set from from 1948Q2 to 2019Q4. The results of the Sup $F$, Avg $F$, and Exp $F$ (Andrews \& Ploberger, 1994) tests are reported in Table 6 with their $p$-values in parentheses.

Firstly, we focus on the original framework without extension for oil price decreases. The results are presented in the first row for each Panel A-E. For the uncensored measure and the measures of Mork (1989) and Lee et al. (1995), the tests indicate a rejection of the null hypothesis of stability over the complete sample. Interestingly, for the net oil price measures (Hamilton, 1996, 2003), we cannot reject parameter stability. Secondly, when testing the extended framework that includes increases and decreases, we find that the hypothesis of parameter stability is not rejected for the heteroskedasticity-adjusted oil price measure by Lee et al. (1995). For the measures by Mork (1989) and Hamilton (1996), we can reject the hypothesis of stable parameters of the whole sample period at 5% and 1%, respectively. Due to the many zero observations for oil price decreases, we were not able to conduct the test for the 3-year net oil prices and thus, do not report findings on this measure. In fact, the first non-zero observation for oil price decreases appears in 1984Q2 (see the bottom right plot of Figure 2).

Combining these findings with the results of Subsection 4.3, we deduce the following. For oil price measures where we do not find statistical significant parameters for oil price decreases the parameters seem to be unstable in their estimates in the extended ARX(4); including them yields a rejection of stability tests. This suggests that their relevance might change over time or is limited to only a subset of our data set. While the estimation results for price decreases clearly rule out any contribution to GDP growth for the full sample, we might find a reaction of GDP to oil price decreases in recent years where demand...
shocks are linked with recessions. However, from the above tests, we cannot infer on any significance of this effect. One contributing factor to the detected instability could be the existence of breakpoints in the relationship of oil prices and GDP growth. These breakpoints might not be breaks in the classical sense but rather induced by the behaviour of oil prices as such. Prior to 1973, oil prices behave like step functions and thereafter, like a price driven by supply and demand as well as policy changes. Other major events and periods, such as the Great Moderation, might also be a contributing factor to the above detected instability. Significant price movements and sudden bursts might play a role as well (Pavlidis et al., 2018, Kruse & Wegener, 2020). In addition, the stable parameters of the heteroskedasticity-adjusted oil price measures may also provide an indication that more volatile periods cause the instability of the other measures.

Table 6 approximately here.

5. Conclusions

In this replication paper, we review the findings of Hamilton (2003) who estimates the oil price–GDP link with different measures for oil price changes. These measures are defined in the papers of Hamilton (1983), Mork (1989), Lee et al. (1995), Hamilton (1996), and Hamilton (2003), which represent some of the most influential papers with focus on the oil-macroeconomy relationship.

We are able to confirm all estimation results. Minor differences are caused by slightly varying data sets of this paper. After this successful replication of the findings of Hamilton (2003), we augment the data set with recent data up to 2019Q4 and repeat the estimations of the ARX(4) framework. We find that the relationship between oil price increases and GDP growth rates remains intact. We still detect a significant reaction of GDP growth rates to oil price increases with a lag of four quarters. However, this impact is reduced in its magnitude when compared to findings on earlier data sets, as suggested by Hamilton (2003) already. We then extend the ARX(4) framework to model the impact of oil price increases and decreases jointly. Estimation results and exclusion tests underline that oil

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5We thank Lutz Kilian for an extensive discussion on this topic.
price decreases have almost no significant impact on GDP growth rates when applying the extended ARX(4) to the full data set. Parameter stability tests reveal that parameters for the net oil price measures remain stable across the data set while for other oil price measures, we detect instabilities. Interestingly, for models where oil price decreases seem to have no significant impact, parameter stability is rejected in the extended ARX(4). This might be an indication that oil price decreases have a time varying impact or their impact is limited to a certain subset of our data set. A possible breakpoint in this behavior could be the Great Moderation of 1984. We note that this instability is no indication for significance of the possibly varying impact of oil price decreases.

The observations and findings of this paper might motivate further analyses of oil price decreases and their impact on GDP growth rates, in particular in recent years.

Acknowledgements

We are thankful to the editor Richard Tol, James Hamilton, with whom we had a fruitful discussion on a preliminary set of results, and an additional anonymous reviewer. Their comments significantly improved this manuscript. Part of the work has been conducted during Thomas Walther’s research time as Assistant Professor at the University of St. Gallen, Institute for Operations Research and Computational Finance. This publication is part of SCCER CREST, which is supported by Innosuisse. Thomas Walther is grateful for the funding received for conducting this research.

References


Table 1: Oil price episodes, principal factors, and their corresponding U.S. economic recessions following Hamilton (2011a). A more detailed history can also be found in Barsky & Kilian (2004).

<table>
<thead>
<tr>
<th>Oil Price Episode</th>
<th>Principal factors</th>
<th>NBER recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947–1948</td>
<td>Post-War demand boom for energy and decreases of oil production due to shorter work week create shortages in some U.S. states.</td>
<td>11/1948–10/1949</td>
</tr>
<tr>
<td>1952–1953</td>
<td>Disruption of oil supply due to Iranian nationalization of the oil industry in summer '51; decrease of world supply by 19mb/month due to boycott of Iranian oil; disruption in US supply due to oil refinery workers strikes.</td>
<td>07/1953–05/1954</td>
</tr>
<tr>
<td>1956–1957</td>
<td>Suez Canal is nationalized by Egyptian leaders in July '56 pulling 10.1% of world's oil production from the markets.</td>
<td>08/1957–04/1958</td>
</tr>
<tr>
<td></td>
<td>No oil price increase that corresponds to the U.S. economic recession of 1960–1961.</td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>Long-lasting decline in U.S. reserves due to strike by east coast fuel oil deliverers in December '68; nationwide strike in the oil, chemical, and atomic workers union January '69; oil price increase by 7%</td>
<td>12/1969–11/1970</td>
</tr>
<tr>
<td>1970</td>
<td>Trans-Arabian pipeline rupture in May '70 in Syria leads to 8% increase in nominal oil price.</td>
<td></td>
</tr>
<tr>
<td>1973–1974</td>
<td>The Arab members of the OPEC announce an embargo on oil export to some countries supporting Israel leading to a 7.5% shortfall of total world production.</td>
<td>11/1973–05/1975</td>
</tr>
<tr>
<td>1978–1979</td>
<td>Iranian revolution; due to the large public protests in '78 and the strikes in the oil sector in the fall of '78 induce a decrease in world's oil production of 7%.</td>
<td>01/1980–07/1980</td>
</tr>
<tr>
<td>1990–1991</td>
<td>First Gulf War; Iraq invades Kuwait in August '90; as a consequence, oil production plummets by 6.9% during this period</td>
<td>07/1990–03/1991</td>
</tr>
<tr>
<td>1997–1998</td>
<td>The east Asian crisis decreases the oil demand and consequently oil prices diminish to $12 per barrel.</td>
<td>03/2001–11/2001</td>
</tr>
<tr>
<td>1999–2000</td>
<td>Asian demand for oil recovers rapidly after the '97 crisis; price increases by 38% between November '99 and November '00.</td>
<td></td>
</tr>
<tr>
<td>2007–2008</td>
<td>Oil demand increases by 5mb/d between '03 and '05; oil price increases from $55 per barrel in 2005 to more than $140.</td>
<td>12/2007–06/2009</td>
</tr>
<tr>
<td>2014-2016</td>
<td>Global oil demand decrease and U.S. shale oil boom leading to an oil price decrease of 70%</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Overview of the different observation periods, oil price proxies, and macroeconomic measure for Hamilton (2003) and the replicated papers therein.

<table>
<thead>
<tr>
<th>Publication</th>
<th>Start Date</th>
<th>End Date</th>
<th>Oil Price</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamilton (1983)</td>
<td>1948Q2</td>
<td>1980Q3</td>
<td>PPI</td>
<td>GNP</td>
</tr>
<tr>
<td>Mork (2003)</td>
<td>1949Q1</td>
<td>1988Q2</td>
<td>PPI &amp; RAC</td>
<td>GNP</td>
</tr>
<tr>
<td>Lee et al. (1996)</td>
<td>1949Q1</td>
<td>1992Q2</td>
<td>RAC</td>
<td>GNP</td>
</tr>
<tr>
<td>Hamilton (1996)</td>
<td>1948Q1</td>
<td>1994Q2</td>
<td>PPI</td>
<td>GDP</td>
</tr>
<tr>
<td>Hamilton (2003)</td>
<td>2001Q3</td>
<td>2001Q3</td>
<td>PPI</td>
<td>GDP</td>
</tr>
</tbody>
</table>

Note: *Different to what is stated in Hamilton (2003) p. 369), the start date of the respective study is in fact 1948Q2, as confirmed by the accompanied data files and the author himself.

Table 3: Parameter estimates for the replication of the results in Hamilton (2003) using identical periods from 1948Q2 to 1980Q4 for Eq. (1.5), and 1948Q2 to 2001Q3 for the rest of equations.

<table>
<thead>
<tr>
<th></th>
<th>Eq. (1.5)</th>
<th>Eq. (1.6)</th>
<th>Eq. (1.8)</th>
<th>Eq. (3.2)</th>
<th>Eq. (3.6)</th>
<th>Eq. (3.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>p-value</td>
<td>Coef.</td>
<td>p-value</td>
<td>Coef.</td>
<td>p-value</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.19</td>
<td>0.0988</td>
<td>0.72</td>
<td>0.6911</td>
<td>0.88</td>
<td>0.8027</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.20</td>
<td>0.2065</td>
<td>0.28</td>
<td>0.2825</td>
<td>0.26</td>
<td>0.2796</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.06</td>
<td>0.1148</td>
<td>0.13</td>
<td>0.1656</td>
<td>0.12</td>
<td>0.1620</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.09</td>
<td>0.1543</td>
<td>0.06</td>
<td>0.1159</td>
<td>0.07</td>
<td>0.1253</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>0.04</td>
<td>0.0890</td>
<td>0.07</td>
<td>0.0687</td>
<td>0.07</td>
<td>0.0687</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.003</td>
<td>-0.0092</td>
<td>-0.003</td>
<td>-0.0048</td>
<td>-0.011</td>
<td>-0.0122</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.036</td>
<td>-0.0166</td>
<td>-0.003</td>
<td>-0.0055</td>
<td>-0.005</td>
<td>-0.0048</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-0.036</td>
<td>-0.0090</td>
<td>-0.004</td>
<td>-0.0122</td>
<td>-0.005</td>
<td>-0.0048</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>-0.064</td>
<td>-0.0595</td>
<td>-0.016</td>
<td>-0.1513</td>
<td>-0.023</td>
<td>-0.0211</td>
</tr>
</tbody>
</table>

Note: Column (a) refers to the results presented in Hamilton (2003), while column (b) presents our replicated parameter estimates. We report standard errors in parentheses below the estimated parameters.

Table 4: Regression results from the linear Hamilton (1983) model using different oil price measures for the period 1948Q2 to 2019Q4.

<table>
<thead>
<tr>
<th></th>
<th>Coef.</th>
<th>p-value</th>
<th>Coef.</th>
<th>p-value</th>
<th>Coef.</th>
<th>p-value</th>
<th>Coef.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.5299</td>
<td>(0.000)</td>
<td>0.7483</td>
<td>(0.000)</td>
<td>0.7522</td>
<td>(0.000)</td>
<td>0.7307</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.246</td>
<td>(0.000)</td>
<td>0.2815</td>
<td>(0.000)</td>
<td>0.2825</td>
<td>(0.000)</td>
<td>0.2721</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.1708</td>
<td>(0.006)</td>
<td>0.1538</td>
<td>(0.012)</td>
<td>0.1874</td>
<td>(0.020)</td>
<td>0.1944</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.0850</td>
<td>(0.171)</td>
<td>-0.1049</td>
<td>(0.087)</td>
<td>-0.1245</td>
<td>(0.038)</td>
<td>-0.0926</td>
<td>(0.126)</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>0.0695</td>
<td>(0.242)</td>
<td>-0.0976</td>
<td>(0.096)</td>
<td>-0.0283</td>
<td>(0.619)</td>
<td>-0.0880</td>
<td>(0.128)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.0025</td>
<td>(0.499)</td>
<td>-0.0112</td>
<td>(0.076)</td>
<td>-0.1844</td>
<td>(0.021)</td>
<td>-0.0102</td>
<td>(0.246)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-0.0032</td>
<td>(0.383)</td>
<td>-0.0059</td>
<td>(0.347)</td>
<td>0.0549</td>
<td>(0.495)</td>
<td>-0.0076</td>
<td>(0.276)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-0.0027</td>
<td>(0.451)</td>
<td>-0.0099</td>
<td>(0.113)</td>
<td>-0.2493</td>
<td>(0.002)</td>
<td>-0.0105</td>
<td>(0.235)</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>-0.0067</td>
<td>(0.065)</td>
<td>-0.0138</td>
<td>(0.029)</td>
<td>-0.3859</td>
<td>(0.000)</td>
<td>-0.0256</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

Log-likelihood and information criteria

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>-358.588</th>
<th>-353.879</th>
<th>-342.697</th>
<th>-351.263</th>
<th>-346.713</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>2.5616</td>
<td>2.5288</td>
<td>2.4509</td>
<td>2.5105</td>
<td>2.4788</td>
<td></td>
</tr>
<tr>
<td>SIC</td>
<td>2.6763</td>
<td>2.6435</td>
<td>2.5656</td>
<td>2.6253</td>
<td>2.5936</td>
<td></td>
</tr>
<tr>
<td>HQC</td>
<td>2.6076</td>
<td>2.5748</td>
<td>2.4968</td>
<td>2.5565</td>
<td>2.5248</td>
<td></td>
</tr>
</tbody>
</table>

Restrictions of oil price increases $H_0$: $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$

$F$-stat: 1.2249 (0.300) 3.5843 (0.007) 9.5068 (0.000) 4.9287 (0.000) 7.3264 (0.000)

Note: The oil price measure refer to the censored $O_t$ of Hamilton (1983), the asymmetric measure $O_{\text{Mark}_t}^+$ of Mork (2003), the volatility scaled measure $O_{\text{LNR}_t}^+$ of Lee et al. (2003), and Hamilton’s measure $O_{\text{Ham}_t}^+$ (Hamilton 2003) as well as $O_{\text{Ham}_3}^+$ (Hamilton 2003).
Table 5: Regression results and restriction tests for the linear extended Hamilton (1983) model using different oil price measures and by including the negative counterparts for the period 1948Q2 to 2019Q4.

### Panel A: Estimation results

<table>
<thead>
<tr>
<th></th>
<th>$O_{\text{Mork},t}$</th>
<th>$O_{\text{LNR},t}$</th>
<th>$O_{\text{Ham1},t}$</th>
<th>$O_{\text{Ham3},t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p-value</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.7696 (0.000)</td>
<td>0.9207 (0.000)</td>
<td>0.7848 (0.000)</td>
<td>0.7921 (0.000)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.2747 (0.000)</td>
<td>0.2663 (0.000)</td>
<td>0.2634 (0.000)</td>
<td>0.2429 (0.000)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.1529 (0.014)</td>
<td>0.1789 (0.003)</td>
<td>0.1442 (0.019)</td>
<td>0.1426 (0.019)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>-0.1005 (0.106)</td>
<td>-0.1170 (0.060)</td>
<td>-0.0976 (0.113)</td>
<td>-0.0996 (0.100)</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>-0.0993 (0.096)</td>
<td>-0.0247 (0.663)</td>
<td>-0.0946 (0.108)</td>
<td>-0.0979 (0.089)</td>
</tr>
</tbody>
</table>

| $\delta_1$     | -0.0129 (0.066)      | -0.2334 (0.004)     | -0.0108 (0.224)      | -0.0199 (0.054)      |
| $\delta_2$     | -0.0056 (0.418)      | 0.0232 (0.778)      | -0.0105 (0.238)      | -0.0145 (0.156)      |
| $\delta_3$     | -0.0105 (0.123)      | -0.3006 (0.000)     | -0.0113 (0.210)      | -0.0164 (0.111)      |
| $\delta_4$     | -0.0127 (0.059)      | -0.4072 (0.000)     | -0.0265 (0.003)      | -0.0330 (0.001)      |
| $\gamma_1$     | 0.0068 (0.223)       | 0.1542 (0.034)      | 0.0035 (0.588)       | 0.0059 (0.507)       |
| $\gamma_2$     | -0.0025 (0.659)      | 0.0549 (0.452)      | 0.0018 (0.783)       | -0.0035 (0.697)      |
| $\gamma_3$     | 0.0032 (0.575)       | 0.1332 (0.067)      | 0.0063 (0.317)       | 0.0079 (0.379)       |
| $\gamma_4$     | -0.0045 (0.436)      | 0.0547 (0.452)      | 0.0029 (0.712)       | 0.0023 (0.792)       |

Log-likelihood and information criteria

<table>
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<tr>
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<th>LL</th>
<th>AIC</th>
<th>SIC</th>
<th>HQC</th>
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### Panel B: Restriction tests

Restrictions of oil price increases $H_0$: $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$

| F-Stat | 3.5780 (0.007) | 10.9726 (0.000) | 5.2236 (0.000) | 7.4656 (0.000) |

Restrictions of oil price decreases $H_0$: $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$

| F-Stat | 0.5676 (0.686) | 1.9786 (0.098) | 0.4219 (0.793) | 0.3666 (0.832) |

Restrictions for both oil price increases and decreases $H_0$: $\delta_1 = \ldots = \delta_4 = \gamma_1 = \ldots = \gamma_4 = 0$

| F-Stat | 2.0648 (0.039) | 5.8096 (0.000) | 2.6548 (0.008) | 3.8131 (0.000) |

Note: The oil price measures refer to the uncensored $O_t$ of Hamilton (1983), the asymmetric measures $O_{\text{Mork},t}$ of Mork (1982), the volatility scaled measures $O_{\text{LNR},t}$ of Lee et al. (2003), and Hamilton’s measures $O_{\text{Ham1},t}$ (Hamilton, 1983) as well as $O_{\text{Ham3},t}$ (Hamilton, 2002).
Table 6: Andrews & Ploberger (1994) test for linear models with data from 1948Q2 to 2019Q4. The test has the null hypothesis that the parameters of the oil price measures are stable over the sample.

<table>
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<tr>
<th>Panel</th>
<th>Test</th>
<th>Stat</th>
<th>p-value</th>
<th>Date</th>
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<td>18.0400</td>
<td>(0.025)</td>
<td>1985Q4</td>
<td>6.6540</td>
<td>(0.012)</td>
<td>9.2910</td>
<td>(0.015)</td>
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<td>B</td>
<td>2</td>
<td>18.5441</td>
<td>(0.021)</td>
<td>1987Q1</td>
<td>7.1999</td>
<td>(0.007)</td>
<td>9.3935</td>
<td>(0.014)</td>
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<tr>
<td></td>
<td>3</td>
<td>24.3187</td>
<td>(0.039)</td>
<td>1993Q3</td>
<td>10.2206</td>
<td>(0.014)</td>
<td>14.8850</td>
<td>(0.018)</td>
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<tr>
<td>C</td>
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<td>7.6807</td>
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<td>4.1818</td>
<td>(0.377)</td>
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<tr>
<td></td>
<td>5</td>
<td>15.8489</td>
<td>(0.408)</td>
<td>1984Q2</td>
<td>6.1593</td>
<td>(0.249)</td>
<td>10.2444</td>
<td>(0.184)</td>
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<tr>
<td>D</td>
<td>6</td>
<td>10.5024</td>
<td>(0.329)</td>
<td>1970Q3</td>
<td>3.7681</td>
<td>(0.166)</td>
<td>6.3859</td>
<td>(0.105)</td>
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<td>7</td>
<td>29.9834</td>
<td>(0.005)</td>
<td>1960Q1</td>
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<td>(0.004)</td>
<td>14.0279</td>
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<tr>
<td>E</td>
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<td>(0.529)</td>
<td>1971Q1</td>
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<td>(0.384)</td>
<td>4.3406</td>
<td>(0.347)</td>
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</table>

Note: The results reported in this table correspond to Table 2 of Hamilton (2003) where only the coefficients of the lagged oil price measures are allowed to change. We report the Wald $F$-statistics of the Andrews (1993) and Andrews & Ploberger (1994), which follow an asymptotic $\chi^2(K)$ distribution with $K$ numbers of tested parameters for instability. The $F$-statistics are calculated for each possible break point $\tau$ between $t_1 = [0.15T]$ and $t_2 = [0.85T]$, where $T$ is the total number of observations. Then, the statistics are $\text{Sup } F = \max_{t_1 < \tau < t_2} F_\tau$, $\text{Avg } F = \ln \left( \frac{1}{t_2-t_1-1} \sum_{\tau=t_1}^{t_2} F_\tau \right)$, and $\text{Exp } F = \ln \left( \frac{1}{t_2-t_1-1} \sum_{\tau=t_1}^{t_2} \exp \left( \frac{1}{2} F_\tau \right) \right)$. The $p$-values are calculated following the procedure of Hansen (1997) as implemented in E-Views 9.5.
Figure 1: Crude oil price increases and decreases (solid lines) and U.S. recessions (shaded areas). Source: Authors calculations and NBER data.
Figure 2: Nonlinear oil price measures from 1947Q2 to 2019Q4 of Mork (1989) (top left), Lee et al. (1995) (top right), Hamilton (1996) (bottom left), and Hamilton (2003) (bottom right) applied throughout the paper. Shaded areas indicate the US NBER recession periods.