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Signal Superposition in NOMA with Proper and Improper Gaussian Signaling

A. A. Nasir¹, H. D. Tuan², H. H. Nguyen³, T. Q. Duong⁴ and H. V. Poor⁵

Abstract—Recent findings in a single-cell two-user network have shown that a higher network throughput is achieved by using a common message to be decoded by both users and conveying partial information for both users, rather than the entire information for one of the two users. The latter is essentially the conventional non-orthogonal multiple access (NOMA), which performs better than orthogonal multiple access (OMA) only under users' highly dissimilar channel conditions. Unlike NOMA, the former performs consistently better than OMA. This paper generalizes such signalling strategy to a general multi-cell multiuser network, which leads to a new NOMA approach (called n-NOMA), in which each pair of users decodes a message that conveys partial information for one of them only. Unlike the conventional NOMA, whose performance is highly dependent on the users' pairing strategy, the proposed n-NOMA consistently outperforms both NOMA and OMA schemes. Both proper and improper Gaussian signaling is considered for all the concerned schemes and it is shown that the latter is clearly more advantageous than the former.

Index Terms—Signal superposition, orthogonal multiple access (OMA), non-orthogonal multiple access (NOMA), improper Gaussian signaling (IGS), max-min throughput optimization.

I. INTRODUCTION

The major challenge in providing wireless communications for the ever growing Internet-of-Things (IoT) applications is how to satisfy the demand for growing data throughput for a continuously-increasing number of connected devices under a limited available spectrum [1]. Inspired by Han-Kobayashi (HK) strategy [2], the signal superposition approach that uses common messages to be

decoded by the out-of-cell users, offers a quite effective way in mitigating inter-cell interference to improve the performance of cell-edge users in a multi-cell network [3]–[6]. On the other hand, non-orthogonal multiple access (NOMA) [7], [8], which has been emerged as an enabling technology for the fifth-generation-and-beyond wireless networks to meet the heterogeneous demands on massive connectivity and high throughput [9]–[14], is effective in mitigating intra-cell interference. This approach uses common messages to be decoded by different users of the same cells to convey the entire information for some of them. In practice, the performance gain (in terms of users' max-min throughput) by NOMA over conventional orthogonal multiple access (OMA) techniques, such as time-division multiple access and orthogonal frequency-division multiple access, strongly depends on how the users are paired. For a typical approach of pairing two users in NOMA, it is found that pairing two users in such a way that their channel conditions are most dissimilar, yields the highest performance gain over OMA [9].¹

Recently, it has been shown in [15], [16] that for a single-cell two-user network, by using a common message to convey partial information for both users, the network throughput can be enhanced. Compared to the conventional NOMA (which uses a common message to convey the entire information for one of the two users only) and OMA, such signal superposition results in the best throughput, no matter whether the two users are under similar or dissimilar channel conditions [16]. The main interest of this paper is to find out whether such signal superposition approach consistently outperforms the conventional NOMA and OMA in the more general and practical multi-cell multi-user networks. An another interesting question is whether splitting the common throughput among the paired users, as done in [16], is really necessary?

Furthermore, the information-bearing signals are often assumed proper Gaussian (PG), hence limiting their dimensionality. By relaxing the Gaussian properness, improper Gaussian signaling (IGS) [17] provides an opportunity to improve the network throughput [18]–[22]. Under dissimilar channel conditions of paired users, it has been clearly

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¹Generally, if the users' channels are ordered as $|h_1|^2 \leq |h_2|^2 \leq \dots \leq |h_N|^2$, where h_n denotes the channel gain between the base station and the ordered n th user, the ordered 1st and N th users have the most dissimilar channel conditions, while any two ordered users n and $n+1$ expect to experience similar channel conditions [7].

TABLE I: Overview of how the information is conveyed under a special case of two users.

Multiple access technique	Source through which information is conveyed to	
	User 1 (having a strong channel)	User 2 (having a poor channel)
MIS	private & common messages	private & common messages
NOMA	private message	common message
n-NOMA	private message	private & common messages
OMA	private message only	private message only

shown in [23] that OMA and (conventional) NOMA with IGS outperform their PG signaling (PGS) counterparts. However, it is still unknown whether such performance advantage of IGS remains intact under similar channel conditions of paired users. In addition, does the performance-superiority of HK-inspired signal superposition prevail under IGS?

It should be emphasized that this paper is not concerned with the rate region or capacity of multi-antenna broadcast channels under particular nonlinear encoding schemes [24]–[26]. Instead, the paper aims to address the problem of max-min users' achievable throughput optimization by designing transmit beamforming vectors at the base stations (BSs). The paper offers the following novel contributions:

- It shows that the signal superposition proposed in [15], [16], which uses the common messages to be decoded by users of the same cell to convey partial information for them, outperforms NOMA and OMA, in a general multi-cell multiuser network. For convenience of presentation, such signal superposition shall be generally referred to as the *mixed information superposition* (MIS) in this paper. For the particular case that the message intended for a user with a poorer channel condition is a superposition of two components: (i) a private message that is decoded by itself only, and (ii) a common message that is decoded by itself and its paired user, the MIS scheme leads to a new version of NOMA, called new NOMA (n-NOMA). Interestingly, performance of n-NOMA is very close to that of MIS. The n-NOMA is not only more efficiently implemented, but also more computationally efficient than MIS because it does not require to split the common throughput among the paired UEs. This simplifies signal processing before transmission and results in fewer decision variables. For easy reference, an overview of how the information is conveyed in MIS, NOMA, and n-NOMA is given in Table I.
- This paper is the first work that considers IGS for MIS (MIS-IGS) and demonstrates its advantage in terms of the users' max-min throughput. Furthermore, n-NOMA-PGS and n-NOMA-IGS are also considered for the first time. While existing works on MIS mainly consider two-user networks [16], MIS-IGS is studied for a general multi-cell multiuser network. Path-following algorithms are developed to obtain solutions of max-min throughput optimization problems under MIS-IGS, n-NOMA-PGS and n-NOMA-IGS.

- Most of previous works focus on two extreme scenarios: (i) users' channel conditions are highly dissimilar, under which NOMA performs the best, and (ii) users' channel conditions are very similar, under which NOMA is not useful since it performs even worse than OMA. The extensive simulations in this paper consider a more generalized scenario that has both sets of users having very different channel conditions, as well as very similar channel conditions. In addition, thanks to the computational efficiency of the developed algorithms, performance of the proposed algorithms is analyzed under both single-cell and multi-cell setups. The rigorously-conducted tests under several possible users' placements and different sets of simulation parameters, e.g., BS transmit power and the number of BS antennas, demonstrate that the proposed n-NOMA-IGS and MIS-IGS consistently outperform their PGS counterparts.

Organization: The rest of the paper is organized as follows. Section II is devoted to study MIS-PGS and n-NOMA-PGS. For ease of reference, the system model for NOMA-PGS is also described in Section II. Section III studies MIS-IGS and n-NOMA-IGS. Section IV provides comprehensive numerical results to corroborate the theoretical results established in Sections II and III. Finally, Section V concludes the paper.

Notation: Only design (decision) variables to be optimized are boldfaced. I_n is the identity matrix of size $n \times n$. The notations X^T , X^H , $\langle X \rangle$, and $|X|$ mean transpose, conjugate transpose, trace, and determinant of the matrix X , respectively, while $[X]^2 = XX^H$. The inner product $\langle X, Y \rangle$ of the matrices X and Y is defined as $\text{trace}(X^H Y)$. $\|\cdot\|$ stands for matrix's Frobenius norm or vector's Euclidean norm and $\Re\{x\}$ denotes the real part of the complex number x . $x \sim \mathcal{CN}(0, Z)$ means that x is a complex random vector following a circularly symmetric Gaussian distribution with zero mean and covariance matrix Z . $A \succ 0$ means that the matrix A is positive definite. $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

II. PROPER GAUSSIAN SIGNALING

Consider the downlink of a multi-cell multiuser wireless communication system which consists of K cells, where the BS of each cell is equipped with N_t antennas to serve N single-antenna users (also called user equipments or UEs) within its cell. Let $\mathcal{K} \triangleq \{1, 2, \dots, K\}$ and $\mathcal{N} \triangleq \{1, 2, \dots, N\}$. The n -th UE in the k -th cell is

referred to as UE $(k, n) \in \mathcal{M} \triangleq \mathcal{K} \times \mathcal{N}$. The case $K = 1$ corresponds to a single-cell setting, which will also be studied in Section IV together with the case of multi-cell setting ($K > 1$).

In every k -th cell, let UE (k, n) be paired with UE $(k, \pi(n))$ to form a virtual cluster, where for notational convenience, the pairing is chosen such that $\pi(n) = n + N/2$. It follows that the UEs in each cell can be divided into two sets $\mathcal{N}_1 \triangleq \{1, 2, \dots, N/2\}$ and $\mathcal{N}_2 \triangleq \{N/2 + 1, \dots, N\}$. As a generalization, UE $(k, n) \in \mathcal{M}_1 \triangleq \mathcal{K} \times \mathcal{N}_1$ is paired with UE $(k, \pi(n)) \in \mathcal{M}_2 \triangleq \mathcal{K} \times \mathcal{N}_2$. In this work, we adopt a simple user-pairing strategy which, instead of considering the complete channel information, is simply based on geographical locations of users. Due to small scale fading, the channel of a user changes more rapidly than its location. As such, user-pairing based on the complete channel information would require frequent update of the pairing strategy. In contrast, the adopted user-pairing strategy that is based on users' distances from the BSs requires less overhead for updating the users about their pairing strategy.

In the following, we first briefly describe the system model and problem formulation for NOMA, which is followed by the details of MIS and n-NOMA.

A. NOMA for PGS

The received signal at UE (k, n) , under PGS, is given by

$$y_{k,n} = \sum_{(k',n') \in \mathcal{M}} h_{k',k,n} x_{k',n'}^P + n_{k,n}, \quad (1)$$

where $h_{k',k,n} \in \mathbb{C}^{1 \times N_t}$ is the channel from BS $k' \in \mathcal{K}$ to UE (k, n) , $x_{k,n}^P \in \mathbb{C}^{N_t \times 1}$ is the signal intended for UE (k, n) , and $n_{k,n} \sim \mathcal{CN}(0, \sigma^2)$ is additive white Gaussian noise. In this paper, we assume that full channel state information is available, e.g., through coordination among the BSs [27].

Let $s_{k,n} \sim \mathcal{CN}(0, 1)$ be the message intended for UE (k, n) . In the conventional (proper Gaussian) signaling, each message $s_{k,n}$ is beamformed by a complex vector $\mathbf{w}_{k,n}^P \in \mathbb{C}^{N_t \times 1}$, such that the signal $x_{k,n}$ is given by

$$x_{k,n}^P = \mathbf{w}_{k,n}^P s_{k,n}. \quad (2)$$

Thus, the received signal in (1) at UE (k, n) is rewritten as

$$y_{k,n} = \sum_{(k',n') \in \mathcal{M}} h_{k',k,n} \mathbf{w}_{k',n'}^P s_{k',n'} + n_{k,n}. \quad (3)$$

Define $\mathbf{w}^P \triangleq \{\mathbf{w}_{k,n}^P, (k, n) \in \mathcal{M}\}$, which is the set of beamforming vectors under PGS.

Under NOMA, the information $s_{k,\pi(n)}$ intended for UE $(k, \pi(n))$ for $n \in \mathcal{N}_1$ is decoded by both UEs (k, n) and $(k, \pi(n))$, with the throughput $r_{k,\pi(n)}^{(n)}(\mathbf{w}^P)$ and $r_{k,\pi(n)}^{(\pi(n))}(\mathbf{w}^P)$ defined by

$$r_{k,\pi(n)}^{(\chi)}(\mathbf{w}^P) = \ln \left(1 + \frac{|h_{k,k,\chi} \mathbf{w}_{k,\pi(n)}^P|^2}{\hat{\Lambda}_{k,\chi}^N(\mathbf{w}^P)} \right), \quad (4)$$

for $\chi \in \{n, \pi(n)\}$, and

$$\hat{\Lambda}_{k,\chi}^N(\mathbf{w}^P) = \sum_{(k',n') \in \mathcal{M} \setminus \{(k,\pi(n))\}} |h_{k',k,\chi} \mathbf{w}_{k',n'}^P|^2 + \sigma^2.$$

Thus, the throughput for decoding message $s_{k,\pi(n)}$ is given by

$$r_{k,\pi(n)}^N(\mathbf{w}^P) = \min_{\chi \in \{n, \pi(n)\}} r_{k,\pi(n)}^{(\chi)}(\mathbf{w}^P). \quad (5)$$

Next, UE $(k, n) \in \mathcal{M}_1$ subtracts $s_{k,\pi(n)}$ from the superimposed received signal to decode its own message $s_{k,n}$ with the throughput

$$r_{k,n}^N(\mathbf{w}^P) = \ln \left(1 + \frac{|h_{k,k,n} \mathbf{w}_{k,n}^P|^2}{\Lambda_{k,n}^N(\mathbf{w}^P)} \right), \quad (6)$$

where

$$\Lambda_{k,n}^N(\mathbf{w}^P) = \sum_{(k',n') \in \mathcal{M} \setminus \{(k,n),(k,\pi(n))\}} |h_{k',k,n} \mathbf{w}_{k',n'}^P|^2 + \sigma^2.$$

The max-min throughput optimization problem is formulated as

$$\max_{\mathbf{w}^P} \min_{(k,n) \in \mathcal{M}_1} \{r_{k,n}^N(\mathbf{w}^P), r_{k,\pi(n)}^N(\mathbf{w}^P)\} \quad (7a)$$

$$\text{s.t.} \quad \sum_{n' \in \mathcal{N}} \|\mathbf{w}_{k,n'}^P\|^2 \leq P_k^{\max}, k \in \mathcal{K}. \quad (7b)$$

The above is a nonconvex problem due to the non-concave objective function (7a). The detailed computation to solve this problem is given in [28].

B. MIS for PGS

Under MIS, in addition to the signal $x_{k,n}^P \in \mathbb{C}^{N_t \times 1}$ that is intended exclusively for the individual UE (k, n) , BS k also transmits a common signal $x_{k,n,\pi(n)}^C \in \mathbb{C}^{N_t \times 1}$ to its serving paired UEs (k, n) and $(k, \pi(n))$. Thus, the transmitted signal of BS k is given by

$$\sum_{n=1}^N x_{k,n}^P + \sum_{n=1}^{N/2} x_{k,n,\pi(n)}^C. \quad (8)$$

It follows that the received signal at UE (k, n) is

$$\begin{aligned} \tilde{y}_{k,n} = & \sum_{(k',n') \in \mathcal{M}} h_{k',k,n} x_{k',n'}^P \\ & + \sum_{(k',n') \in \mathcal{M}_1} h_{k',k,n} x_{k',n',\pi(n')}^C + n_{k,n}. \end{aligned} \quad (9)$$

Let $s_{k,n} \sim \mathcal{CN}(0, 1)$ and $s_{k,n,\pi(n)}^C \sim \mathcal{CN}(0, 1)$ be the private message for UE (k, n) and the common message for UEs (k, n) and $(k, \pi(n))$, respectively. Similar to (2), where the message $s_{k,n}$ is precoded by a complex vector $\mathbf{w}_{k,n}^P$, here the message $s_{k,n,\pi(n)}^C$ is precoded by a complex vector $\mathbf{w}_{k,n,\pi(n)}^C \in \mathbb{C}^{N_t \times 1}$:

$$x_{k,n,\pi(n)}^C = \mathbf{w}_{k,n,\pi(n)}^C s_{k,n,\pi(n)}^C. \quad (10)$$

Therefore, the received signal in (9) at UE (k, n) can be rewritten as

$$\begin{aligned} \tilde{y}_{k,n} = & \sum_{(k',n') \in \mathcal{M}} h_{k',k,n} \mathbf{w}_{k',n'}^P s_{k',n'} \\ & + \sum_{(k',n') \in \mathcal{M}_1} h_{k',k,n} \mathbf{w}_{k',n'}^C s_{k',n'}^C \\ & + n_{k,n}. \end{aligned} \quad (11)$$

Define $\mathbf{w}^C \triangleq \{\mathbf{w}_{k,n,\pi(n)}^C, (k,n) \in \mathcal{M}_1\}$, which is the set of common beamforming vectors under PGS. With MIS, both UE (k, n) and UE $(k, \pi(n))$ first decode their common message $s_{k,n,\pi(n)}^C$ with the throughput

$$r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C) = \min_{\chi \in \{n, \pi(n)\}} r_{k,n,\pi(n)}^{(\chi)}(\mathbf{w}^P, \mathbf{w}^C) \quad (12)$$

where, for $\chi \in \{n, \pi(n)\}$,

$$r_{k,n,\pi(n)}^{(\chi)}(\mathbf{w}^P, \mathbf{w}^C) = \ln \left(1 + \frac{|h_{k,k,\chi} \mathbf{w}_{k,n,\pi(n)}^C|^2}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C)} \right), \quad (13)$$

with

$$\begin{aligned} \hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C) = & \sum_{(k',n') \in \mathcal{M}} |h_{k',k,\chi} \mathbf{w}_{k',n'}^P|^2 \\ & + \sum_{(k',n') \in \mathcal{M}_1 \setminus \{(k,n)\}} |h_{k',k,\chi} \mathbf{w}_{k',n'}^C|^2 + \sigma^2. \end{aligned}$$

After decoding $s_{k,n,\pi(n)}^C$, UEs (k, χ) , $\chi \in \{n, \pi(n)\}$, subtract it from their received signals to decode their private message $s_{k,\chi}$, with the throughput

$$r_{k,\chi}^P(\mathbf{w}^P, \mathbf{w}^C) = \ln \left(1 + \frac{|h_{k,k,\chi} \mathbf{w}_{k,\chi}^P|^2}{\Lambda_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C)} \right), \quad (14)$$

where

$$\begin{aligned} \Lambda_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C) = & \sum_{(k',n') \in \mathcal{M} \setminus \{(k,\chi)\}} |h_{k',k,\chi} \mathbf{w}_{k',n'}^P|^2 \\ & + \sum_{(k',n') \in \mathcal{M}_1 \setminus \{(k,n)\}} |h_{k',k,\chi} \mathbf{w}_{k',n'}^C|^2 + \sigma^2. \end{aligned}$$

Under MIS, the common throughput $r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C)$ shared by UE (k, n) and UE $(k, \pi(n))$ is further refined into individual throughputs $\mathbf{r}_{k,n}^C$ and $\mathbf{r}_{k,\pi(n)}^C$ for UE (k, n) and UE $(k, \pi(n))$, respectively, where $\mathbf{r}_{k,n}^C$ and $\mathbf{r}_{k,\pi(n)}^C$ satisfy

$$\mathbf{r}_{k,n}^C + \mathbf{r}_{k,\pi(n)}^C \leq r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C), \quad (k, n) \in \mathcal{M}_1. \quad (15)$$

For $\mathbf{r}^C \triangleq (\mathbf{r}_{k,n}^C, \mathbf{r}_{k,\pi(n)}^C)_{(k,n) \in \mathcal{M}_1}$, the max-min throughput optimization problem under MIS is formulated as the following nonconvex problem:

$$\max_{\mathbf{w}^P, \mathbf{w}^C, \mathbf{r}^C} \Gamma^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C, \mathbf{r}^C) \triangleq \min_{(k,n) \in \mathcal{M}} \{r_{k,n}^P(\mathbf{w}^P, \mathbf{w}^C) + \mathbf{r}_{k,n}^C\} \quad (16a)$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{(k,n) \in \mathcal{M}} \|\mathbf{w}_{k,n}^P\|^2 \\ & + \sum_{(k,n) \in \mathcal{M}_1} \|\mathbf{w}_{k,n,\pi(n)}^C\|^2 \leq P_k^{\max}, k \in \mathcal{K}, \end{aligned} \quad (16b)$$

$$r_{k,n,\pi(n)}^{(n)}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,n}^C + \mathbf{r}_{k,\pi(n)}^C, \quad (k, n) \in \mathcal{M}_1, \quad (16c)$$

$$r_{k,n,\pi(n)}^{(\pi(n))}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,n}^C + \mathbf{r}_{k,\pi(n)}^C, \quad (k, n) \in \mathcal{M}_1. \quad (16d)$$

C. New-NOMA (n-NOMA) for PGS

Note that the conventional NOMA, discussed in Section II-A, is seen as a particular case of the introduced MIS with no private message for UE $(k, \pi(n))$, i.e., $x_{k,\pi(n)}^P \equiv 0$, $(k, n) \in \mathcal{M}_1$ in (9), and with no common message share for UE (k, n) , i.e., $\mathbf{r}_{k,n}^C \equiv 0$, $(k, n) \in \mathcal{M}_1$ in (15), while both UEs (k, n) and $(k, \pi(n))$ still decode the message of UE $(k, \pi(n))$.

In general, the introduction of the common message $s_{k,n,\pi(n)}^C$ while retaining the private messages $s_{k,n}$ and $s_{k,\pi(n)}$ for both UEs (k, n) and $(k, \pi(n))$ in MIS offers a great flexibility and leads to consistent performance. In view of this, we propose a new and more flexible NOMA (n-NOMA) with the following features. Like NOMA, in n-NOMA there is no splitting of the common throughput among the paired UEs and the common message carries information only for UE $(k, \pi(n))$, i.e., $\mathbf{r}_{k,n}^C \equiv 0$, $(k, n) \in \mathcal{M}_1$ in (15). But unlike NOMA, in n-NOMA there is also a private message for UE $(k, \pi(n))$, i.e., $x_{k,\pi(n)}^P \neq 0$, $(k, n) \in \mathcal{M}_1$ in (9). In other words, the message for UE $(k, \pi(n))$ in n-NOMA is split into $s_{k,n,\pi(n)}^C$, which is decoded by both UEs (k, n) and $(k, \pi(n))$, and $s_{k,\pi(n)}$, which is decoded by itself only after subtracting $s_{k,n,\pi(n)}^C$ from its received signal.

The received signal model under n-NOMA is the same as expressed in (11) under MIS. Similar to MIS, both UE (k, n) and UE $(k, \pi(n))$ first decode their common message $s_{k,n,\pi(n)}^C$ with the throughput $r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C)$, defined in (12), and then they subtract it from their received signals to decode their private message $s_{k,\chi}$, $\chi \in \{n, \pi(n)\}$, with the throughput $r_{k,\chi}^P(\mathbf{w}^P, \mathbf{w}^C)$, defined in (14). However, under n-NOMA, the common message $s_{k,n,\pi(n)}^C$, which is decoded by both UEs (k, n) and $(k, \pi(n))$ with the throughput $r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C)$, carries information for the UE $(k, \pi(n))$ only, i.e.,

$$\mathbf{r}_{k,\pi(n)}^C \leq r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C), \quad (k, n) \in \mathcal{M}_1, \quad (17)$$

where $r_{k,n,\pi(n)}(\mathbf{w}^P, \mathbf{w}^C)$ is defined in (12).

For $\tilde{\mathbf{r}}^C \triangleq \left(\mathbf{r}_{k,\pi(n)}^C \right)_{(k,n) \in \mathcal{M}_1}$, the max-min throughput optimization problem under n-NOMA is formulated as the following nonconvex problem:

$$\begin{aligned} \max_{\mathbf{w}^P, \mathbf{w}^C, \tilde{\mathbf{r}}^C} \Gamma^{\text{NN}}(\mathbf{w}^P, \mathbf{w}^C, \tilde{\mathbf{r}}^C) \\ \triangleq \min_{(k,n) \in \mathcal{M}} r_{k,n}^{\text{NN}}(\mathbf{w}^P, \mathbf{w}^C, \tilde{\mathbf{r}}^C) \quad \text{s.t.} \quad (16b), \quad (18a) \end{aligned}$$

$$r_{k,n,\pi(n)}^{(n)}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1, \quad (18b)$$

$$r_{k,n,\pi(n)}^{(\pi(n))}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1, \quad (18c)$$

where

$$r_{k,n}^{\text{NN}}(\mathbf{w}^P, \mathbf{w}^C, \tilde{\mathbf{r}}^C) \triangleq \begin{cases} r_{k,n}^P(\mathbf{w}^P, \mathbf{w}^C), & (k,n) \in \mathcal{M}_1 \\ r_{k,n}^P(\mathbf{w}^P, \mathbf{w}^C) + \mathbf{r}_{k,n}^C, & (k,n) \in \mathcal{M}_2 \end{cases} \quad (19)$$

and $r_{k,n}^P(\mathbf{w}^P, \mathbf{w}^C)$ is given by (14).

D. Computation

We return to problem (16) for MIS max-min throughput optimization, where the objective function (16a) is non-concave, while the constraints (16c) and (16d) are nonconvex. To deal with the non-concave functions $r_{k,n}^P(\mathbf{w}^P, \mathbf{w}^C)$, $r_{k,n,\pi(n)}^{(n)}(\mathbf{w}^P, \mathbf{w}^C)$, and $r_{k,n,\pi(n)}^{(\pi(n))}(\mathbf{w}^P, \mathbf{w}^C)$ in (16), we shall find lower-bound concave approximations for functions $r_{k,\chi}^P(\mathbf{w}^P, \mathbf{w}^C)$, $k \in \mathcal{K}$, $\chi \in \{n, \pi(n)\}$, $n \in \mathcal{N}_1$, in (14) and $r_{k,n,\pi(n)}^{(n)}(\mathbf{w}^P, \mathbf{w}^C)$, $\chi \in \{n, \pi(n)\}$, $(k,n) \in \mathcal{M}_1$, in (13).

Let

$$\{w^{P,(\kappa)}, w^{C,(\kappa)}\} \triangleq \begin{cases} w_{k,n}^{P,(\kappa)}, & (k,n) \in \mathcal{M}; \\ w_{k,n,\pi(n)}^{C,(\kappa)}, & (k,n) \in \mathcal{M}_1 \end{cases}$$

be a feasible point for (16) that is found at the $(\kappa - 1)$ -th iteration. Applying the inequality (44) in the Appendix for $\mathbf{x} = h_{k,k,\chi} \mathbf{w}_{k,\chi}^P$, $\mathbf{y} = \hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C)$, $x^{(\kappa)} = h_{k,k,\chi} w_{k,\chi}^{P,(\kappa)}$, and $y^{(\kappa)} = \hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})$, a lower-bound concave approximation for $r_{k,\chi}^P(\mathbf{w}^P, \mathbf{w}^C)$, for $k \in \mathcal{K}$, $\chi \in \{n, \pi(n)\}$, $n \in \mathcal{N}_1$, is given by

$$\begin{aligned} r_{k,\chi}^{P,(\kappa)}(\mathbf{w}^P, \mathbf{w}^C) &\triangleq a_{k,\chi}^{\text{MIS},(\kappa)} \\ &+ 2b_{k,\chi}^{\text{MIS},(\kappa)} \Re \left\{ \left(h_{k,k,\chi} w_{k,\chi}^{P,(\kappa)} \right)^H h_{k,k,\chi} \mathbf{w}_{k,\chi}^P \right\} \\ &- c_{k,\chi}^{\text{MIS},(\kappa)} \left(|h_{k,k,\chi} \mathbf{w}_{k,\chi}^P|^2 + \hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C) \right) \leq \\ &r_{k,\chi}^P(\mathbf{w}^P, \mathbf{w}^C), \quad (20) \end{aligned}$$

with

$$a_{k,\chi}^{\text{MIS},(\kappa)} \triangleq r_{k,\chi}^P(w^{P,(\kappa)}, w^{C,(\kappa)}) - \frac{|h_{k,k,\chi} w_{k,\chi}^{P,(\kappa)}|^2}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})},$$

$$0 < b_{k,\chi}^{\text{MIS},(\kappa)} \triangleq \frac{1}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})},$$

$$\begin{aligned} 0 < c_{k,\chi}^{\text{MIS},(\kappa)} &\triangleq \frac{|h_{k,k,\chi} w_{k,\chi}^{P,(\kappa)}|^2}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})} \\ &\times \frac{1}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)}) + |h_{k,k,\chi} w_{k,\chi}^{P,(\kappa)}|^2}. \end{aligned}$$

Similarly, applying the inequality (44) in the Appendix for $\mathbf{x} = h_{k,k,\chi} \mathbf{w}_{k,n,\pi(n)}^C$, $\mathbf{y} = \hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C)$, $x^{(\kappa)} = h_{k,k,\chi} w_{k,n,\pi(n)}^{C,(\kappa)}$, and $y^{(\kappa)} = \hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})$, a lower-bound concave approximation for $r_{k,n,\pi(n)}^{(\chi)}(\mathbf{w}^P, \mathbf{w}^C)$, for $\chi \in \{n, \pi(n)\}$, $(k,n) \in \mathcal{M}_1$, is

$$\begin{aligned} r_{k,n,\pi(n)}^{(\chi,\kappa)}(\mathbf{w}^P, \mathbf{w}^C) &\triangleq a_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} \\ &+ 2b_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} \Re \{ (h_{k,k,\chi} w_{k,n,\pi(n)}^{C,(\kappa)})^H h_{k,k,\chi} \mathbf{w}_{k,n,\pi(n)}^C \} \\ &- c_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} \left(|h_{k,k,\chi} \mathbf{w}_{k,n,\pi(n)}^C|^2 + \hat{\Lambda}_{k,\chi}^{\text{MIS}}(\mathbf{w}^P, \mathbf{w}^C) \right) \leq \\ &r_{k,n,\pi(n)}^{(\chi)}(\mathbf{w}^P, \mathbf{w}^C), \quad (21) \end{aligned}$$

with

$$\begin{aligned} a_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} &\triangleq r_{k,n,\pi(n)}^{(\chi)}(w^{P,(\kappa)}, w^{C,(\kappa)}) \\ &- \frac{|h_{k,k,\chi} w_{k,n,\pi(n)}^{C,(\kappa)}|^2}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})}, \end{aligned}$$

$$0 < b_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} \triangleq \frac{1}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})}$$

$$\begin{aligned} 0 < c_{k,n,\pi(n)}^{\text{MIS},(\chi,\kappa)} &\triangleq \frac{|h_{k,k,\chi} w_{k,n,\pi(n)}^{C,(\kappa)}|^2}{\hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})} \\ &\times \frac{1}{|h_{k,k,\chi} w_{k,n,\pi(n)}^{C,(\kappa)}|^2 + \hat{\Lambda}_{k,\chi}^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)})}. \end{aligned}$$

Thus, at the κ -th iteration, we solve the following convex problem, to generate the next iterative feasible point $\{w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, r^{C,(\kappa+1)}\}$ for (16):

$$\begin{aligned} \max_{\mathbf{w}^P, \mathbf{w}^C, \mathbf{r}^C} \Gamma^{\text{MIS},(\kappa)}(\mathbf{w}^P, \mathbf{w}^C, \mathbf{r}^C) \\ \triangleq \min_{(k,n) \in \mathcal{M}} \{ r_{k,n}^{P,(\kappa)}(\mathbf{w}^P, \mathbf{w}^C) + \mathbf{r}_{k,n}^C \} \quad (22a) \end{aligned}$$

$$\text{s.t.} \quad (16b), \quad (22b)$$

$$r_{k,n,\pi(n)}^{(n,\kappa)}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,n}^C + \mathbf{r}_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1, \quad (22c)$$

$$r_{k,n,\pi(n)}^{(\pi(n),\kappa)}(\mathbf{w}^P, \mathbf{w}^C) \geq \mathbf{r}_{k,n}^C + \mathbf{r}_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1. \quad (22d)$$

This problem involves $1.5KN\mathcal{N}_t + KN$ decision variables and $K + 2.5KN$ constraints, so its computational complexity is $\mathcal{O}((1.5KN\mathcal{N}_t + KN)^3 (2.5KN + K))$ [29].

Algorithm 1 outlines the steps to solve the max-min throughput problem (16). Note that

$$\Gamma^{\text{MIS},(\kappa)}(w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, r^{C,(\kappa+1)}) > \Gamma^{\text{MIS},(\kappa)}(w^{P,(\kappa)}, w^{C,(\kappa)}, r^{C,(\kappa)})$$

Algorithm 1 MIS-PGS max-min throughput algorithm to solve (16)

- 1: **Initialization:** Set $\kappa := 0$ and initialize a feasible point $\{w^{P,(0)}, w^{C,(0)}\}$ satisfying the power constraint (16b)
- 2: **Repeat** until convergence of the objective in (16): Solve the convex problem (22) to generate the next feasible point $\{w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, r^{C,(\kappa+1)}\}$ for (16); Set $\kappa := \kappa + 1$.

because $\{w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, r^{C,(\kappa+1)}\}$ is the optimal solution of (22) while $\{w^{P,(\kappa)}, w^{C,(\kappa)}, r^{C,(\kappa)}\}$ is its feasible point. Therefore

$$\begin{aligned} \Gamma^{\text{MIS}}(w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, r^{C,(\kappa+1)}) &\geq \\ \Gamma^{\text{MIS},(\kappa)}(w^{P,(\kappa)}, w^{C,(\kappa)}, r^{C,(\kappa)}) &> \\ \Gamma^{\text{MIS},(\kappa)}(w^{P,(\kappa)}, w^{C,(\kappa)}, r^{C,(\kappa)}) &= \\ \Gamma^{\text{MIS}}(w^{P,(\kappa)}, w^{C,(\kappa)}, r^{C,(\kappa)}), \end{aligned} \quad (23)$$

where the last equality follows from the equality $r_{k,n}^{P,(\kappa)}(w^{P,(\kappa)}, w^{C,(\kappa)}) = r_{k,n}^P(w^{P,(\kappa)}, w^{C,(\kappa)})$, which is easily checked using the definition in (20). Algorithm 1 thus generates a sequence $\{w^{P,(\kappa)}, w_c^{P,(\kappa)}, r^{C,(\kappa)}\}$ of improved feasible points for (16). Following similar arguments as in [30], it can be easily shown that Algorithm 1 at least converges to a locally optimal solution of (16), which satisfies the Karush-Kuh-Tucker (KKT) optimality condition.

Analogously, in the n-NOMA max-min throughput optimization problem (18), the objective function (18a) is non-concave, while the constraints (18b) and (18c) are nonconvex.

Let

$$\{w^{P,(\kappa)}, w^{C,(\kappa)}\} \triangleq \left\{ w_{k,n}^{P,(\kappa)}, (k,n) \in \mathcal{M}; \right. \\ \left. w_{k,n,\pi(n)}^{C,(\kappa)}, (k,n) \in \mathcal{M}_1 \right\}$$

be a feasible point for (18) that is found at the $(\kappa - 1)$ -th iteration. At the κ -th iteration, the following convex problem is solved under n-NOMA to generate the next iterative feasible point $\{w^{P,(\kappa+1)}, w^{C,(\kappa+1)}, \tilde{r}^{C,(\kappa+1)}\}$ for (18):

$$\begin{aligned} \max_{w^P, w^C, \tilde{r}^C} \Gamma^{\text{nN},(\kappa)}(w^P, w^C, \tilde{r}^C) \\ \triangleq \min_{(k,n) \in \mathcal{M}} r_{k,n}^{\text{nN},(\kappa)}(w^P, w^C, \tilde{r}^C) \quad \text{s.t. (16b),} \end{aligned} \quad (24a)$$

$$r_{k,n,\pi(n)}^{(n,\kappa)}(w^P, w^C) \geq r_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1, \quad (24b)$$

$$r_{k,n,\pi(n)}^{(\pi(n),\kappa)}(w^P, w^C) \geq r_{k,\pi(n)}^C, (k,n) \in \mathcal{M}_1, \quad (24c)$$

with

$$\begin{aligned} r_{k,n}^{\text{nN},(\kappa)}(w^P, w^C, \tilde{r}^C) \\ \triangleq \begin{cases} r_{k,n}^{P,(\kappa)}(w^P, w^C), & (k,n) \in \mathcal{M}_1 \\ r_{k,n}^{P,(\kappa)}(w^P, w^C) + \mathbf{r}_{k,n}^C, & (k,n) \in \mathcal{M}_2 \end{cases}, \end{aligned} \quad (25)$$

Algorithm 2 n-NOMA-PGS max-min throughput algorithm to solve (18)

- 1: **Initialization:** Set $\kappa := 0$ and initialize a feasible point $\{w^{P,(0)}, w_c^{P,(0)}\}$ satisfying the power constraint (16b)
- 2: **Repeat** until convergence of the objective in (18): Solve the convex problem (24) to generate the next feasible point $\{w^{P,(\kappa+1)}, w_c^{P,(\kappa+1)}, \tilde{r}^{C,(\kappa+1)}\}$ for (18); Set $\kappa := \kappa + 1$.

where $r_{k,n}^{P,(\kappa)}(w^P, w^C)$ and $r_{k,n,\pi(n)}^{(\chi,\kappa)}(w^P, w^C)$ are the lower-bound concave approximations of $r_{k,n}^P(w^P, w^C)$ and $r_{k,n,\pi(n)}^{(\chi)}(w^P, w^C)$, that are obtained in (20) and (21), respectively.

Compared to the MIS-PGS iteration (22), the above n-NOMA-IGS iteration (24) involves the same number of $K + 2.5KN$ constraints but $KN/2$ fewer decision variables, so its computational complexity is $\mathcal{O}((1.5KNn_t + KN/2)^3(2.5KN + K))$ [29], which is lower than that of MIS-PGS.

Algorithm 2 outlines the steps to solve the max-min throughput problem (18).

III. IMPROPER GAUSSIAN SIGNALING

In contrast to PGS, IGS relaxes the Gaussian properness, allowing the real and imaginary parts of the complex Gaussian transmit signal to be correlated, which also results in non-zero pseudo-covariance matrix [31]. Thus, IGS possesses additional degrees of signaling freedom. IG signals can be generated from PG information bearing sources by widely linear precoding [20].

For interference-free channels, conventional PGS is the optimal method for communications. However, in the presence of multiuser and multi-cell interference (as considered in this paper), the use of IGS is expected to improve the achievable throughput. In the following, we describe the system models and present problem formulations for MIS and n-NOMA under IGS.

A. MIS for IGS

As Subsection II.B, let $x_{k,n}^P \in \mathbb{C}^{N_t \times 1}$ be the signal that is intended exclusively for the individual UE (k,n) , and $x_{k,n,\pi(n)}^C \in \mathbb{C}^{N_t \times 1}$ be the common signal to its serving paired UEs (k,n) and $(k,\pi(n))$. The transmitted signal of BS k is given by the equation (8) and the received signal at UE (k,n) is given by the equation (9).

Unlike PGS, IGS processes the private message $s_{k,n}$, $(k,n) \in \mathcal{M}$, for UE (k,n) and the common message $s_{k,n,\pi(n)}^C$, $(k,n) \in \mathcal{M}_1$, for the paired UEs (k,n) and $(k,\pi(n))$, by *widely linear* precoders to produce the fol-

lowing improper Gaussian signals for transmission [17]:

$$\mathbf{x}_{k,n}^P = \mathbf{w}_{1,k,n}^P s_{k,n} + \mathbf{w}_{2,k,n}^P s_{k,n}^*, \quad (26a)$$

$$\mathbf{x}_{k,n,\pi(n)}^C = \mathbf{w}_{1,k,n,\pi(n)}^C s_{k,n,\pi(n)}^C + \mathbf{w}_{2,k,n,\pi(n)}^C (s_{k,n,\pi(n)}^C)^*, \quad (26b)$$

where $\mathbf{w}_{i,k,n}^P \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{w}_{i,k,n,\pi(n)}^C \in \mathbb{C}^{N_t \times 1}$ for $i = 1, 2$, are the beamformers under IGS. Thus, instead of (11), the received signal at UE (k, n) is rewritten as

$$\begin{aligned} \tilde{y}_{k,n} = & \sum_{(k',n') \in \mathcal{M}} h_{k',k,n} (\mathbf{w}_{1,k',n'}^P s_{k',n'} + \mathbf{w}_{2,k',n'}^P s_{k',n'}^*) \\ & + \sum_{(k',n') \in \mathcal{M}_1} h_{k',k,n} (\mathbf{w}_{1,k',n',\pi(n')}^C s_{k',n',\pi(n')}^C \\ & + \mathbf{w}_{2,k',n',\pi(n')}^C (s_{k',n',\pi(n')}^C)^*) + n_{k,n}. \end{aligned} \quad (27)$$

Define $\tilde{\mathbf{w}}_{k,n}^P \triangleq \{\mathbf{w}_{i,k,n}^P, i = 1, 2\}$ and $\tilde{\mathbf{w}}_{k,n,\pi(n)}^C \triangleq \{\mathbf{w}_{i,k,n,\pi(n)}^C, i = 1, 2\}$. Also group the private beamforming vectors as $\tilde{\mathbf{w}}^P \triangleq \{\tilde{\mathbf{w}}_{k,n}^P, (k, n) \in \mathcal{M}\}$, and common beamformers as $\tilde{\mathbf{w}}^C \triangleq \{\tilde{\mathbf{w}}_{k,n,\pi(n)}^C, (k, n) \in \mathcal{M}_1\}$. Furthermore, in what follows, the following notations are used:

$$\begin{aligned} \bar{H}_{k',k,n} & \triangleq \begin{bmatrix} \Re\{h_{k',k,n}\} & -\Im\{h_{k',k,n}\} \\ \Im\{h_{k',k,n}\} & \Re\{h_{k',k,n}\} \end{bmatrix} \in \mathbb{R}^{2 \times (2N_t)}, \\ \bar{y}_{k,n} & \triangleq \begin{bmatrix} \Re\{\tilde{y}_{k,n}\} \\ \Im\{\tilde{y}_{k,n}\} \end{bmatrix} \in \mathbb{R}^2, \\ \bar{s}_{k,n} & \triangleq \begin{bmatrix} \Re\{s_{k,n}\} \\ \Im\{s_{k,n}\} \end{bmatrix} \in \mathbb{R}^2, \bar{n}_{k,n} \triangleq \begin{bmatrix} \Re\{n_{k,n}\} \\ \Im\{n_{k,n}\} \end{bmatrix} \in \mathbb{R}^2, \\ \bar{s}_{k,n,\pi(n)}^C & \triangleq \begin{bmatrix} \Re\{s_{k,n,\pi(n)}^C\} \\ \Im\{s_{k,n,\pi(n)}^C\} \end{bmatrix} \in \mathbb{R}^2. \end{aligned} \quad (28)$$

Also, define a linear mapping from $\mathbb{C}^{N_t} \times \mathbb{C}^{N_t}$ to $\mathbb{R}^{(2N_t) \times 2}$:

$$\begin{aligned} \mathcal{L}(\mathbf{u}) & = \mathcal{L}(\mathbf{u}_1, \mathbf{u}_2) \\ & = \begin{bmatrix} \Re\{\mathbf{u}_1\} + \Re\{\mathbf{u}_2\} & -\Im\{\mathbf{u}_1\} + \Im\{\mathbf{u}_2\} \\ \Im\{\mathbf{u}_1\} + \Im\{\mathbf{u}_2\} & \Re\{\mathbf{u}_1\} - \Re\{\mathbf{u}_2\} \end{bmatrix}. \end{aligned}$$

Thus, the received signal in (27) can be rewritten as

$$\begin{aligned} \tilde{y}_{k,n} = & \sum_{(k',n') \in \mathcal{M}} \bar{H}_{k',k,n} \mathcal{L}(\tilde{\mathbf{w}}_{k',n'}^P) \bar{s}_{k',n'} \\ & + \sum_{(k',n') \in \mathcal{M}_1} \bar{H}_{k',k,n} \mathcal{L}(\tilde{\mathbf{w}}_{k',n',\pi(n')}^C) \bar{s}_{k',n',\pi(n')}^C \\ & + \bar{n}_{k,n}. \end{aligned} \quad (29)$$

Note that $\mathbb{E}\{(\bar{s}_{k,n})^2\} = \mathbb{E}\{(\bar{s}_{k,n}^C)^2\} = \frac{1}{2} I_2$ and $\mathbb{E}\{[\bar{n}_{k,n}]^2\} = \frac{1}{2} \sigma^2 I_2$. As mentioned in Section II, both UE (k, n) and UE $(k, \pi(n))$ first need to decode their common message $s_{k,n,\pi(n)}^C$ with the throughput

$$\rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) = \min_{\chi \in \{n, \pi(n)\}} \rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \quad (30)$$

where for $\chi \in \{n, \pi(n)\}$,

$$\begin{aligned} \rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) = & \frac{1}{2} \ln \left| I_2 + [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^C)]^2 \right. \\ & \left. \times \tilde{\Psi}_{k,\chi}^{-1}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \right|, \end{aligned} \quad (31)$$

which is the mutual information between $\tilde{y}_{k,\chi}^C$ and $\bar{s}_{k,n,\pi(n)}^C$ [32], and

$$\begin{aligned} \tilde{\Psi}_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \triangleq & \sum_{(k',n') \in \mathcal{M}} [\bar{H}_{k',k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k',n'}^P)]^2 \\ & + \sum_{(k',n') \in \mathcal{M}_1 \setminus \{(k,n)\}} [\bar{H}_{k',k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k',n',\pi(n')}^C)]^2 + \sigma^2 I_2. \end{aligned}$$

After decoding $s_{k,n,\pi(n)}^C$, UEs (k, χ) , $\chi \in \{n, \pi(n)\}$ subtract it from their received signals to decode their private message $s_{k,\chi}$ with the throughput

$$\rho_{k,\chi}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) = \frac{1}{2} \ln \left| I_2 + [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^P)]^2 \Psi_{k,\chi}^{-1}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \right|, \quad (32)$$

where

$$\begin{aligned} \Psi_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \triangleq & \sum_{(k',n') \in \mathcal{M} \setminus \{(k,\chi)\}} [\bar{H}_{k',k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k',n'}^P)]^2 \\ & + \sum_{(k',n') \in \mathcal{M}_1 \setminus \{(k,n)\}} [\bar{H}_{k',k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k',n',\pi(n')}^C)]^2 + \sigma^2 I_2. \end{aligned}$$

Under MIS, the common throughput $\rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ is further refined into individual rates $\rho_{k,n}^C$ and $\rho_{k,\pi(n)}^C$ for UE (k, n) and UE $(k, \pi(n))$, respectively, where $\rho_{k,n}^C$ and $\rho_{k,\pi(n)}^C$ satisfy

$$\rho_{k,n}^C + \rho_{k,\pi(n)}^C \leq \rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C), \quad (k, n) \in \mathcal{M}_1. \quad (33)$$

For $\rho^C \triangleq (\rho_{k,n}^C, \rho_{k,\pi(n)}^C)_{(k,n) \in \mathcal{M}_1}$, the max-min throughput optimization problem under MIS is formulated as the following nonconvex problem

$$\begin{aligned} \max_{\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \rho^C} \quad & \hat{\Gamma}^{\text{MIS}}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \rho^C) \\ \triangleq \quad & \min_{(k,n) \in \mathcal{M}} \left\{ \rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) + \rho_{k,n}^C \right\} \end{aligned} \quad (34a)$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{(k,n) \in \mathcal{M}} \|\tilde{\mathbf{w}}_{k,n}^P\|^2 \\ & + \sum_{(k,n) \in \mathcal{M}_1} \|\tilde{\mathbf{w}}_{k,n,\pi(n)}^C\|^2 \leq P_k^{\max}, k \in \mathcal{K} \end{aligned} \quad (34b)$$

$$\rho_{k,n,\pi(n)}^{(n)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,n}^C + \rho_{k,\pi(n)}^C, \quad (k, n) \in \mathcal{M}_1, \quad (34c)$$

$$\rho_{k,n,\pi(n)}^{(\pi(n))}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,n}^C + \rho_{k,\pi(n)}^C, \quad (k, n) \in \mathcal{M}_1, \quad (34d)$$

where the objective function (34a) is non-concave, while the constraints (34c) and (34d) are nonconvex. To solve this problem, we need to deal with the non-concave functions $\rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, $\rho_{k,n,\pi(n)}^{(n)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, and $\rho_{k,n,\pi(n)}^{(\pi(n))}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ in (34). The foundation for lower-bound approximations of such functions to facilitate a path-following solution algorithm has been established in [23] and [33]. For simplicity, we use a result of [33].

Let

$$\begin{aligned} \{\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}\} \triangleq & \left\{ \tilde{\mathbf{w}}_{k,n}^{P,(\kappa)}, (k, n) \in \mathcal{M}; \right. \\ & \left. \tilde{\mathbf{w}}_{k,n,\pi(n)}^{C,(\kappa)}, (k, n) \in \mathcal{M}_1 \right\} \end{aligned}$$

be a feasible point for (34) that is found at the $(\kappa - 1)$ th iteration. Applying the inequality (43) in the Appendix for $\mathbf{X} = \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^P)$, $\mathbf{Y} = \Psi_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, $X^{(\kappa)} = \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^{P,(\kappa)})$, and $Y^{(\kappa)} = \Psi_{k,\chi}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)})$, the lower-bound concave approximation for $\rho_{k,\chi}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, for $k \in \mathcal{K}$, $\chi \in \{n, \pi(n)\}$, $n \in \mathcal{N}_1$, is given by

$$\begin{aligned} \rho_{k,\chi}^{P,(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) &\triangleq \frac{1}{2} \left(\tilde{a}_{k,\chi}^{(\kappa)} + 2 \left\langle \mathcal{A}_{k,\chi}^{(\kappa)}, \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^P) \right\rangle \right. \\ &\quad \left. - \left\langle \mathcal{B}_{k,\chi}^{(\kappa)}, [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^P)]^2 + \Psi_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \right\rangle \right) \\ &\leq \rho_{k,\chi}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \end{aligned} \quad (35)$$

with

$$\begin{aligned} 0 &> \tilde{a}_{k,\chi}^{(\kappa)} \triangleq 2\rho_{k,\chi}^P(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \\ &\quad - \langle [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^{P,(\kappa)})]^2 \Psi_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \rangle, \\ \mathcal{A}_{k,\chi}^{(\kappa)} &\triangleq \Psi_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^{P,(\kappa)}), \\ 0 &< \mathcal{B}_{k,\chi}^{(\kappa)} \triangleq \Psi_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \\ &\quad - \left([\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,\chi}^{P,(\kappa)})]^2 + \Psi_{k,\chi}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \right)^{-1}. \end{aligned}$$

Furthermore, applying the inequality (43) in the Appendix for $\mathbf{X} = \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^C)$, $\mathbf{Y} = \hat{\Psi}_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) + \sigma^2 I_2$, $X^{(\kappa)} = \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^{C,(\kappa)})$, and $Y^{(\kappa)} = \hat{\Psi}_{k,\chi}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) + \sigma^2 I_2$, the lower-bound concave approximation for $\rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, for $\chi \in \{n, \pi(n)\}$, $(k, n) \in \mathcal{M}_1$, is given by

$$\begin{aligned} \rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) &\triangleq \frac{1}{2} \left(\tilde{a}_{k,n,\pi(n)}^{(\chi)} + 2 \left\langle \mathcal{A}_{k,n,\pi(n)}^{(\chi)}, \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^C) \right\rangle \right. \\ &\quad \left. - \left\langle \mathcal{B}_{k,n,\pi(n)}^{(\chi)}, [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^C)]^2 + \hat{\Psi}_{k,\chi}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \right\rangle \right) \\ &\leq \rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \end{aligned} \quad (36)$$

with

$$\begin{aligned} 0 &> \tilde{a}_{k,n,\pi(n)}^{(\chi)} \triangleq 2\rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \\ &\quad - \langle [\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^C)]^2 \times \hat{\Psi}_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \rangle, \\ \mathcal{A}_{k,n,\pi(n)}^{(\chi)} &\triangleq \hat{\Psi}_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^{C,(\kappa)}), \\ 0 &< \mathcal{B}_{k,n,\pi(n)}^{(\chi)} \triangleq \hat{\Psi}_{k,\chi}^{-1}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \\ &\quad - \left([\bar{H}_{k,k,\chi} \mathcal{L}(\tilde{\mathbf{w}}_{k,n,\pi(n)}^{C,(\kappa)})]^2 + \hat{\Psi}_{k,\chi}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}) \right)^{-1}. \end{aligned}$$

Thus, at the κ -th iteration, we solve the following convex problem to generate the next iterative feasible point $\{\tilde{\mathbf{w}}^{P,(\kappa+1)}, \tilde{\mathbf{w}}^{C,(\kappa+1)}, \rho^{C,(\kappa+1)}\}$ for (34)

$$\begin{aligned} \max_{\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \rho^C} \quad &\hat{\Gamma}^{\text{MIS},(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \rho^C) \\ \triangleq \min_{(k,n) \in \mathcal{M}_1} \quad &\{\rho_{k,n}^{P,(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) + \rho_{k,n}^{C,(\kappa)}\} \text{ s.t. (34b),} \end{aligned} \quad (37a)$$

$$\rho_{k,n,\pi(n)}^{(n,\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,n}^{C,(\kappa)} + \rho_{k,\pi(n)}^{C,(\kappa)}, (k, n) \in \mathcal{M}_1, \quad (37b)$$

$$\rho_{k,n,\pi(n)}^{(\pi(n),\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,n}^{C,(\kappa)} + \rho_{k,\pi(n)}^{C,(\kappa)}, (k, n) \in \mathcal{M}_1. \quad (37c)$$

Algorithm 3 MIS-IGS max-min throughput algorithm to solve (34)

- 1: **Initialization:** Set $\kappa := 0$ and initialize a feasible point $\{\tilde{\mathbf{w}}^{P,(0)}, \tilde{\mathbf{w}}^{C,(0)}\}$ satisfying the power constraint (34b)
- 2: **Repeat** until convergence of the objective in (34): Solve the convex problem (37) to generate the feasible point $\{\tilde{\mathbf{w}}^{P,(\kappa+1)}, \tilde{\mathbf{w}}^{C,(\kappa+1)}, \rho^{C,(\kappa+1)}\}$ for (34); Set $\kappa := \kappa + 1$.

This problem involves $3KN N_t + KN$ decision variables and $K + KN$ constraints, so its computational complexity is $\mathcal{O}((3KN N_t + KN)^3 (KN + K))$ [29].

Algorithm 3 outlines the steps to solve the max-min throughput problem (34). Similar to (23), we have

$$\begin{aligned} \hat{\Gamma}^{\text{MIS},(\kappa)}(\tilde{\mathbf{w}}^{P,(\kappa+1)}, \tilde{\mathbf{w}}^{C,(\kappa+1)}, \rho^{C,(\kappa+1)}) &> \\ \hat{\Gamma}^{\text{MIS},(\kappa)}(\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}, \rho^{C,(\kappa)}) &, \end{aligned}$$

and like Algorithm 1, Algorithm 3 at least converges to a locally optimal solution of (34), which satisfies the KKT optimality condition.

B. n-NOMA for IGS

As explained in Section II-C, NOMA-IGS can be realized from MIS-IGS by forcing the private message for UE $(k, \pi(n))$ to zero, i.e., $x_{k,\pi(n)}^P \equiv 0$, $(k, n) \in \mathcal{M}_1$ in (26a) and by forcing no common message share for the UE (k, n) , i.e., $\rho_{k,n}^C \equiv 0$, $(k, n) \in \mathcal{M}_1$ in (33), while both UEs (k, n) and $(k, \pi(n))$ still decode the message of UE $(k, \pi(n))$. The detailed treatment of the max-min throughput optimization for NOMA-IGS as well as for OMA-IGS can be found in [23].

Regarding n-NOMA with IGS, the received signal model under n-NOMA-IGS is the same as expressed in (29) for MIS-IGS. Similar to MIS-IGS, both UE (k, n) and UE $(k, \pi(n))$ first decode their common message $s_{k,n,\pi(n)}$ with the throughput $\rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, defined in (30), and then they subtract it from their received signals to decode their private message $s_{k,\chi}$, $\chi \in \{n, \pi(n)\}$, with the throughput $\rho_{k,\chi}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, defined in (32).

However, under n-NOMA-IGS, the common message $s_{k,n,\pi(n)}$, which is decoded by both UEs (k, n) and $(k, \pi(n))$ with the throughput $\rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, carries information for UE $(k, \pi(n))$ only, i.e.,

$$\rho_{k,\pi(n)}^C \leq \rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C), (k, n) \in \mathcal{M}_1, \quad (38)$$

where $\rho_{k,n,\pi(n)}^C(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ is defined in (30). For $\tilde{\rho}^C \triangleq (\rho_{k,\pi(n)}^C)_{(k,n) \in \mathcal{M}_1}$, the max-min throughput optimization

problem under n-NOMA-IGS is formulated as the following nonconvex problem:

$$\begin{aligned} \max_{\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C} \quad & \hat{\Gamma}^{\text{NN}}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \\ \triangleq \quad & \min_{(k,n) \in \mathcal{M}} \rho_{k,n}^{\text{NN}}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \quad \text{s.t.} \quad (34\text{b}), \end{aligned} \quad (39\text{a})$$

$$\rho_{k,n,\pi(n)}^{(n)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,\pi(n)}^C, \quad (k,n) \in \mathcal{M}_1, \quad (39\text{b})$$

$$\rho_{k,n,\pi(n)}^{(\pi(n))}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,\pi(n)}^C, \quad (k,n) \in \mathcal{M}_1, \quad (39\text{c})$$

where

$$\rho_{k,n}^{\text{NN}}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \triangleq \begin{cases} \rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C), & (k,n) \in \mathcal{M}_1 \\ \rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) + \rho_{k,n}^C, & (k,n) \in \mathcal{M}_2 \end{cases} \quad (40)$$

and $\rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ is given by (32). In (39), the objective function (39a) is non-concave, while the constraints (39b) and (39c) are nonconvex.

Let

$$\{\tilde{\mathbf{w}}^{P,(\kappa)}, \tilde{\mathbf{w}}^{C,(\kappa)}\} \triangleq \begin{cases} \{w_{k,n}^{I,(\kappa)}, (k,n) \in \mathcal{M}; \\ w_{k,n,\pi(n)}^{I,(\kappa)}, (k,n) \in \mathcal{M}_1\} \end{cases}$$

be a feasible point for (39) that is found at the $(\kappa - 1)$ -th iteration. At the κ -th iteration, the following convex problem is solved under n-NOMA to generate the next iterative feasible point $\{w^{I,(\kappa+1)}, w_c^{I,(\kappa+1)}, \tilde{\rho}^{C,(\kappa+1)}\}$ for (18):

$$\max_{\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C} \quad \hat{\Gamma}^{\text{NN},(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \quad (41\text{a})$$

$$\triangleq \min_{(k,n) \in \mathcal{M}} \rho_{k,n}^{\text{NN},(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \quad \text{s.t.} \quad (34\text{b}), \quad (41\text{b})$$

$$\rho_{k,n,\pi(n)}^{(n),(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,\pi(n)}^C, \quad (k,n) \in \mathcal{M}_1, \quad (41\text{c})$$

$$\rho_{k,n,\pi(n)}^{(\pi(n)),(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) \geq \rho_{k,\pi(n)}^C, \quad (k,n) \in \mathcal{M}_1, \quad (41\text{d})$$

where

$$\begin{aligned} \rho_{k,n}^{\text{NN},(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C, \tilde{\rho}^C) \\ \triangleq \begin{cases} \rho_{k,n}^{P,(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C), & (k,n) \in \mathcal{M}_1 \\ \rho_{k,n}^{P,(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C) + \rho_{k,n}^C, & (k,n) \in \mathcal{M}_2, \end{cases} \end{aligned} \quad (42)$$

$\rho_{k,n}^{P,(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ and $\rho_{k,n,\pi(n)}^{(\chi),(\kappa)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ are the lower-bound concave approximations of $\rho_{k,n}^P(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$ and $\rho_{k,n,\pi(n)}^{(\chi)}(\tilde{\mathbf{w}}^P, \tilde{\mathbf{w}}^C)$, that are obtained in (35) and (36), respectively.

Compared to the MIS-IGS iteration (37), the above n-NOMA-IGS iteration (41) involves the same number of $K + KN$ constraints but $KN/2$ fewer decision variables, so its computational complexity is $\mathcal{O}((3KNN_t + KN/2)^3(KN + K))$ [29], which is lower than that of MIS-IGS.

Algorithm 4 outlines the steps to solve the max-min throughput problem (39).

Algorithm 4 n-NOMA-IGS max-min throughput algorithm to solve (39)

- 1: **Initialization:** Set $\kappa := 0$ and initialize a feasible point $\{w^{I,(0)}, w_c^{I,(0)}\}$ satisfying the power constraint (34b)
- 2: **Repeat** until convergence of the objective in (39): Solve the convex problem (41) to generate the next feasible point $\{w^{I,(\kappa+1)}, w_c^{I,(\kappa+1)}, \tilde{\rho}^{C,(\kappa+1)}\}$ for (39); Set $\kappa := \kappa + 1$.

Fig. 1: A network setup showing users' arrangements for the considered three scenarios in a single-cell network ($K = 1$).

IV. SIMULATION RESULTS

For simulation, we consider both single-cell network ($K = 1$) and a multi-cell network ($K = 3$) where the radius of each cell equals to 300 meters. The channel $h_{k',k,n}$ from BS $k' \in \mathcal{K}$ to UE (k,n) at a distance of d meters is generated as $h_{k',k,n} = \sqrt{10^{-\sigma_{\text{PL}}^2/10}} \tilde{h}_{k',k,n}$ with the path loss $\sigma_{\text{PL}}^2 = 38.46 + 10\beta \log_{10}(d)$ dB and the normalized channel gain $\tilde{h}_{k',k,n}$. Here, 38.46 dB is the free space path loss at a reference distance of 1 meter at a carrier frequency of 2 GHz and β is the path-loss exponent. Generally, the normalized channel $\tilde{h}_{k',k,n}$ is a Rayleigh fading channel gain with $\beta = 3.1$. The exception is for the channels between the BS and its own cell-centered (closest) UEs (in scenarios 2 and 3), which are modeled as Rician fading channel gains with a Rician factor of 0 dB and $\beta = 2$ [34]. The noise power spectral density is set to $\frac{\sigma_B^2}{B} = -174$ dBm/Hz with the bandwidth $B = 20$ MHz. For simplicity, the same power budget is set for each cell, i.e., $P_k^{\text{max}} \equiv P^{\text{max}}, \forall k \in \mathcal{K}$. Different numbers of users $N \geq 4$ are considered in the network. Unless specified otherwise, the transmit power budget is set to $P^{\text{max}} = 26$ dBm and the number of BS antennas is set to $N_t = N - 1$.

Fig. 2: Max-min throughput for Scenario 1 (in Fig. 1) versus $P^{\max}$ in a single-cell network $K = 1$.

Fig. 4: Max-min throughput for Scenario 3 (in Fig. 1) in a single-cell network versus transmit power budget $P^{\max}$.

Fig. 3: Max-min throughput for Scenario 2 (in Fig. 1) versus $P^{\max}$ in a single-cell network $K = 1$.

Three different scenarios are considered for simulations as illustrated in Fig. 1 for a single-cell network ($K = 1$) and Fig. 6 for a multi-cell network ($K = 3$). The details are as follows:

- *Scenario 1*: The first scenario assumes $N = 4$ UEs in

each cell, such that there are equal numbers of cell-centered UEs and cell-edge UEs. Due to the highly dissimilar channel conditions, each cell-centered UE is paired with the cell-edge UE.

- *Scenario 2*: In the second scenario, $N = 4$ UEs in each cell lie on the cell boundary. As a result, they all have similar channel conditions and thus are randomly paired with each other.
- *Scenario 3*: In the third scenario, we consider a generalized situation where there are unequal number of cell-centered UEs and cell-edge UEs. Assuming $N = 6$ UEs in each cell, there are 2 cell-centered UEs while the remaining 4 UEs are placed on the cell boundary. The cell-centered UEs are paired with the cell-edge UEs to allow pairing of users with the most dissimilar channel conditions, however, the remaining 2 cell-edge UEs have to be paired together. As a result, there is not only a set of users having dissimilar channel conditions, but also another set of users having similar channel conditions.

As mentioned in Section II, we adopt a simple user-pairing strategy, which is based on geographical locations of the users, and hence requires less overhead for updating the users about their pairing strategy. In the following, we present simulation results separately for a single-cell network and a multi-cell network, where performance of the proposed schemes is compared with performance of the conventional OMA and NOMA under PGS [28] and IGS [23].

Fig. 5: Max-min throughput for Scenario 3 (in Fig. 1) in a single-cell network versus the number of transmit antennas N_t .

A. Results for a Single-Cell Network ($K = 1$)

Under Scenario 1 of Fig. 1, Fig. 2 shows that the proposed MIS-IGS and n-NOMA-IGS yield similar performance as that of NOMA-IGS for $P^{\max} \leq 26$ dBm. However, they outperform NOMA-IGS at $P^{\max} \geq 30$ dBm. All common rates in MIS go to the cell-edge UEs, so indeed MIS behaves like n-NOMA, under which the message of the cell-edge users having poorer channel condition is split into the common message and private message. The usefulness of the private messages for the cell-edge users is seen for $P^{\max} \geq 30$ dBm. Fig. 2 also shows that when the channel conditions are highly dissimilar, MIS-PGS, n-NOMA-PGS, and NOMA-PGS have similar performance over the considered values of the transmit power budget.

Under Scenario 2 of Fig. 1, Fig. 3 shows that the proposed MIS-IGS and n-NOMA-IGS clearly outperform the OMA-IGS scheme. Here, the NOMA-IGS scheme delivers the worst performance among the IGS-based algorithms since all users are under similar channel conditions. Generally, Figs. 2 and 3 show that there is at least two-fold performance gain achieved by employing IGS instead of PGS.

Under Scenario 3 of randomly placed users in Fig. 1, Fig. 4 plots the max-min throughput versus transmit power budget $P^{\max}$, while Fig. 5 plots the max-min throughput versus the number of transmit antennas N_t . It can be seen again that the proposed MIS-IGS and n-NOMA-IGS clearly outperform the other counterpart schemes. Moreover, among the PGS-based schemes, the MIS-PGS

and n-NOMA-PGS clearly outperform the NOMA-PGS and OMA-PGS schemes. Fig. 5 shows that performance of NOMA-IGS can even be worse than that of OMA-PGS if $N_t \geq 7$ antennas are deployed at the BS under Scenario 3. This is because in addition to the set of users with dissimilar channel conditions, there is also another set of users with similar channel conditions and the performance of NOMA deteriorates if the users having similar channel conditions have to be paired together, as in Scenario 3.

B. Results for a Multi-Cell Network ($K = 3$)

Considering a three-cell network ($K = 3$), Figs. 7, 8, and 9 plot the optimized max-min throughput versus the transmit power budget $P^{\max}$ obtained under Scenarios 1, 2, and 3 depicted in Figs. 6a, 6b, and 6c, respectively. Under Scenario 3 of randomly placed users in Fig. 6c, Fig. 10 plots the sum-throughput versus the transmit power budget to illustrate the effect on the sum-throughput performance of difference schemes while aiming for the same max-min throughput objective. It can be observed from Figs. 9 and 10 that the relative performance of all schemes show the same trend, whether it is under the max-min throughput or the sum-throughput. Finally, under Scenario 3, Fig. 11 plots the max-min throughput versus the number of transmit antennas N_t . Similar to the single-cell network, Figs. 7, 8, 9, 10, and 11 clearly demonstrate the superiority of MIS-IGS and n-NOMA-IGS over their counterpart schemes and the benefit of employing IGS over PGS. Here, one can appreciate the computational efficiency of the developed algorithms, which allows us to easily analyze performance of the proposed algorithms under both single-cell and multi-cell setups. In this regard, the computational complexity comparison of different schemes is discussed next.

An important finding from Figs. 2-5 and Figs. 7-11 is that the proposed n-NOMA scheme yields performance very close to that of the MIS scheme. However, the former is computationally more efficient because there is no need to split the common throughput between the paired users. The computational complexity comparison of different schemes under IGS is shown in Table II. As expected, OMA-IGS and NOMA-IGS have the least computational complexity since they require the least number of variables and constraints to handle. Although MIS-IGS and n-NOMA-IGS have higher computational complexity than OMA-IGS and NOMA-IGS, the former schemes clearly outperform the latter ones for all different cases of user arrangements. Table II also shows that n-NOMA-IGS is more computationally efficient than MIS-IGS since the former requires fewer variables to handle. Table II shows the average number of iterations required for convergence under Scenario 3 with $N_t = 5$ BS antennas and $P^{\max} = 26$ dBm. It can be seen that that all schemes take similar numbers of iterations to converge.

Fig. 6: A network setup showing users' arrangements for the considered (a) Scenario 1, (b) Scenario 2, and (c) Scenario 3, in a three-cell network ($K = 3$).

TABLE II: Computational complexity comparison of IGS-based algorithms, where the average number of iterations (avg. # iterations) required for convergence are shown under Scenario 3 with $N_t = 5$ BS antennas and transmit power budget $P^{\max} = 26$ dBm.

Algorithms	avg. # iterations ($K = 1$)	avg. # iterations ($K = 3$)	# variables	# constraints
OMA-IGS	24.9	38.7	$2KN_t$	K
NOMA-IGS	30.7	36	$2KN_t$	K
MIS-IGS (Alg. 3)	30.4	43.5	$3KN_t + KN$	$K + KN$
n-NOMA-IGS (Alg. 4)	25.6	43	$3KN_t + 0.5KN$	$K + KN$

Fig. 7: Max-min throughput for Scenario 1 (in Fig. 6a) versus $P^{\max}$ in a three-cell network ($K = 3$).

Fig. 8: Max-min throughput for Scenario 2 (in Fig. 6b) versus $P^{\max}$ in a three-cell network ($K = 3$).

V. CONCLUSIONS

This paper has made several contributions in the design of transmit beamformers to maximize the users' minimum throughput in a multiuser multiple-input single-output net-

work. First, the advantage of employing MIS with IGS over the state-of-the-art NOMA and OMA schemes has been shown for various user geographical deployments. Second, a new NOMA scheme has been developed and

Fig. 9: Max-min throughput for Scenario 3 (in Fig. 6c) in a three-cell network ($K = 3$) versus the transmit power budget $P^{\max}$.

Fig. 10: Sum throughput for Scenario 3 (in Fig. 6c) in a three-cell network ($K = 3$) versus the transmit power budget $P^{\max}$.

Fig. 11: Max-min throughput for Scenario 3 (in Fig. 6c) in a three-cell network ($K = 3$) versus the number of transmit antennas N_t .

shown to consistently outperform both the NOMA and OMA schemes. It is also shown that the new NOMA is computationally more efficient than the MIS because the former involves $KN/2$ fewer decision variables compared to the latter, where K is the number of cells in the network

and N is the number of users per cell. On the other hand, the new NOMA yields similar performance as the MIS. For the sake of convenience in presenting the main idea of n-NOMA and for easy implementation, this paper has focused on the pairing of two users per group. An interesting future work could study the performance of n-NOMA and its comparison with other schemes under grouping multiple users per group.

APPENDIX: FUNDAMENTAL INEQUALITIES

For all matrices $\mathbf{X} \in \mathbb{C}^{n \times m}$ and $X^{(\kappa)} \in \mathbb{C}^{n \times m}$, and positive definite matrices $\mathbf{Y} \in \mathbb{C}^{m \times m}$ and $Y^{(\kappa)} \in \mathbb{C}^{m \times m}$, the following inequality holds true [33], [35, Theorem 1],

$$\ln |I_n + [\mathbf{X}]^2 \mathbf{Y}^{-1}| \geq a + 2\Re\{\langle \mathcal{A}, \mathbf{X} \rangle\} - \langle \mathcal{B}, [\mathbf{X}]^2 + \mathbf{Y} \rangle, \quad (43)$$

where

$$0 > a \triangleq \ln |I_n + [X^{(\kappa)}]^2 (Y^{(\kappa)})^{-1}| - \left\langle (X^{(\kappa)})^H (Y^{(\kappa)})^{-1} X^{(\kappa)} \right\rangle,$$

$$\mathcal{A} = (Y^{(\kappa)})^{-1} X^{(\kappa)},$$

$$0 \preceq \mathcal{B} = (Y^{(\kappa)})^{-1} - (Y^{(\kappa)} + [X^{(\kappa)}]^2)^{-1}.$$

A particular case of (43) is the following inequality for $\mathbf{x} \in \mathbb{C}$, $\mathbf{y} > 0$ and $x^{(\kappa)} \in \mathbb{C}$, $y^{(\kappa)} > 0$:

$$\begin{aligned} \ln \left(1 + \frac{|\mathbf{x}|^2}{\mathbf{y}} \right) &\geq \\ \ln \left(1 + \frac{|x^{(\kappa)}|^2}{y^{(\kappa)}} \right) - \frac{|x^{(\kappa)}|^2}{y^{(\kappa)}} + 2 \frac{\Re\{\langle x^{(\kappa)}, \mathbf{x} \rangle\}}{y^{(\kappa)}} \\ &\quad - \frac{|x^{(\kappa)}|^2}{y^{(\kappa)}(y^{(\kappa)} + |x^{(\kappa)}|^2)} (|\mathbf{x}|^2 + \mathbf{y}). \end{aligned} \quad (44)$$

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