Frequency-Diverse Computational Automotive Radar Technique for Debris Detection


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Abstract—Frequency-diversity is a computational imaging technique that can offer all-electronic imaging systems by leveraging spatio-temporally incoherent radiation patterns as an enabling technology. This approach exhibits a significant contrast to conventional imaging modalities, such as synthetic aperture radars (SAR) and phased arrays in that the raster scanning requirement of a scene to be imaged (mechanical or electronic) can be broken and replaced by a quasi-random interrogation of the scene. This aspect of frequency-diverse computational imaging systems significantly simplifies the physical hardware requirements of conventional radars. Despite this advantage, the application of the frequency-diversity technique has been mostly limited to static imaging scenarios, where the position of the scene to be imaged remains fixed over the data acquisition cycle. This limitation hinders the frequency-diverse computational radars from being deployed for applications where the scene dynamics may vary over the data acquisition cycle, such as in automotive radars. In this paper, we demonstrate that by modifying the sensing matrix to account for the movement of the radar platform, frequency-diverse computational imaging radars can be successfully used in debris detection on roads. We show that operating within the frequency band of 77-81 GHz, the presented dynamic frequency-diverse radar technique can produce high fidelity point spread function (PSF) patterns eliminating the distortions caused by the motion of the radar. We also prove that the PSF patterns of the radar are in excellent agreement with theoretical diffraction limited resolution limits.

Index Terms—Computational imaging, frequency-diversity, debris detection, millimeter-wave, radar, Doppler.

I. Introduction

Computational imaging has recently received significant traction as a new means of enabling millimeter-wave radar technology [1-11]. A distinct type of computational imaging technique, known as frequency-diversity, has been studied in several applications, such as millimeter-wave radars for security-screening [12-18] and ultrasound transducers for nondestructive testing [19-21] to name a few. The frequency-diverse computational radar technique relies on frequency controlled quasi-random modes to probe the imaged scene and encode the backscattered measurements. An estimate of the scene information can then be reconstructed from these encoded measurements using various computational techniques. In comparison to conventional imaging modalities, such as synthetic aperture radars (SAR) [22-26] and phased arrays [27-30], computational frequency-diverse millimeter-wave radars bring several advantages. First, frequency-diverse radar systems exhibit all-electronic operation. This suggests that sampling of the scene is achieved without the need for any moving apparatus. Second, frequency-diverse radars remove the requirement of raster scanning the imaged scene. Instead, they use spatio-temporally varying quasi-random radiation patterns to encode and compress the backscattered radar measurements into a substantially reduced number of channels. This advantage of the frequency-diverse radars significantly reduces the hardware constraints. Third, despite offering all-electronic operation, frequency-diverse radars do not rely on complex phase shifting circuits to achieve beam synthesis. This is a significant advantage of frequency-diverse radars over their phased array counterparts. These advantages on the system hardware layer come at the cost of a more complicated signal processing layer for the frequency-diverse computational imaging. However, with the increased computing power of today’s computing systems, as will be shown later in this article, this challenge can be overcome.

In the context of frequency-diverse imaging, research has been mostly limited to applications where the scene position relative to the radar remains static. In other words, as the radar performs a frequency sweep, there is no variation in the position of the imaged object. Whereas such an assumption can hold for certain applications, such as security-screening [12], where the imaged object operates in a cooperative mode and remains static while the measurements are taken, in many other applications, this approach is not feasible. One particular example to this problem can be thought in the context of...
automotive radars. For example, for the frequency-diverse imaging technique shown in [12], it is indicated that the system can perform 70 frequency sweeps per second, resulting in a sweep time, \( t_s \), on the order of ms. Considering the same system for a vehicle moving at a constant speed \( v \), the vehicle moves by a distance \( v \times t_s \) by the end of the frequency sweep with respect to its original position at the beginning of the sweep \( t=0 \). In this application, because the radar platform is mounted on a moving vehicle, it is vital that the radar system can operate in real-time and compensate for the dynamic nature of the imaging problem at hand. In this paper, we demonstrate the first proof-of-concept study of the frequency-diverse computational imaging technique applied to dynamically varying scenes. Different from the quasi-static approach where multi-looks of a single object can be taken in a static manner and stitched together afterwards [12, 31], the presented study demonstrates, the application of a millimeter-wave computational, frequency-diverse radar technique to dynamically varying scenes while the data is acquired. The novelty of the presented work can be highlighted under three aspects: First, this is the first time that the mathematical framework behind the frequency-diverse computational imaging technique is redeveloped to handle dynamically varying scenes. Second, this is the first time that the application of the frequency-diverse computational imaging technique is studied and demonstrated for an automotive radar. Third, and finally, to the best of our knowledge, this is the first time that the application of a compressive system that can reconstruct 3D images using a single antenna (with a single channel) is presented for automotive radar applications. Therefore, this work addresses a major limitation of the frequency-diversity technique, which has historically been limited to static and quasi-static scenes. As a particular application example, we present this technique in the context of automotive radars operating within the frequency-band of 77-81 GHz. The outline of this paper is as follows: In Section II, we explain the frequency-diverse computational imaging technique and its application to static and dynamic scenes. We show that a compensation technique needs to be applied to the sensing matrix to successfully recover the scene information when the scene dynamics change during data acquisition. In Section III, we present an example imaging scenario involving a frequency-diverse radar integrated with an automotive platform moving at a speed of 30 miles per hour (mph), or 48 kilometers per hour (km/h), towards the imaged scene. We present the point-spread-function (PSF) patterns of the radar with and without the modified sensing matrix and prove that without compensation, the reconstructed images are heavily distorted. Using the developed technique, we later study a debris detection scenario. We also present a systematic study of different imaging parameters, such as positioning errors of the vehicle, system noise, positioning and size of the debris object, and their effect on the reconstructed images. Finally, in Section IV, we provide our concluding remarks and further applications of the proposed technique.

II. FREQUENCY-DIVERSE COMPUTATIONAL IMAGING

Frequency-diversity is a computational imaging technique that relies on encoding the scene information onto a set of spatially varying radiation patterns controlled by a simple frequency sweep. A computational radar leveraging this principle can be synthesized using frequency-diverse antennas [3, 12]. Using the concept of frequency-diversity, an operating frequency-band is sampled at certain intervals governed by the Q-factor of the antennas to illuminate the scene with quasi-random field patterns and collect the backscattered signals. Mathematically, using the first Born approximation, the imaging equation can be given as follows:

\[
g_{\text{Meta}} = H_{\text{Meta}}f_{\text{Meta}} + n_{\text{Meta}} \tag{1}
\]

Eq. (1) constitutes a forward-model in which \( g \) is the measured signal collected from the imaged scene, \( H \) is the sensing matrix, \( f \) is the scene susceptibility distribution and \( n \) is the measurement noise. It should be noted that, in this paper, we consider scalar fields and the bold font in (1) is to denote the vector-matrix notation.

For a frequency-diverse radar producing \( M \) measurement modes to image a scene consisting of \( N \) voxels, the size of the sensing matrix is \( M \times N \). This suggests that, in the most general case, the sensing matrix is a fat matrix and, hence, does not have an exact inverse. To solve the inverse problem and retrieve an estimate of the scene information, \( f_{\text{est}} \), several approaches can be used, such as single-shot matched-filtering or iterative algorithms based on least-square techniques [32]. In this work, we make use of the matched-filter approach, which consists of an adjoint operation on the sensing matrix applied to the backscattered measurements as follows:

\[
f_{\text{est}} = H'g \tag{2}
\]

Eq. (2) suggests that retrieving an estimate of the scene information can be achieved by doing a phase compensation of the sensing matrix applied on the measured radar data.

A significant challenge in the data acquisition step is the fact that the scene information, or the objective function, for the inverse problem defined in (1) needs not be changed as the data acquisition cycle is in progress. To put this statement into context, let us consider the data acquisition cycle for a frequency-diverse computational radar. From our previous work [1, 12], we consider a linearly-swept stepped frequency continuous wave (SFCW) as the waveform of choice for the presented work. Using the SFCW technique, Fig. 1 depicts a complete frequency sweep cycle.

Fig. 1 shows that given a frequency band of operation (bandwidth), the imaging bandwidth is sampled at \( K \) frequencies. At each frequency interval (from \( f_1 \) to \( f_K \)), the radar transmits and receives data, with the width of this interval is known as the dwell time (\( t_d \)). This process is repeated for \( K \) frequency points in a single chirp, resulting in a total sweep time, \( t_s = K \times t_d \). For an imaging scenario where the scene remains static with respect to the radar platform, this approach does not pose a challenge from a data acquisition perspective. However, for an application where the scene
changes rapidly, variations in the objective function (scene information) can easily fall within the single SFCW cycle. In order to address this challenge, the sensing matrix of the computational radar must be compensated.

![Fig. 1. Depiction of a single chirp for a SFCW radar.](image)

For the imaging problem at hand, we consider an automotive radar application for close proximity debris detection. The radar consists of a single frequency-diverse antenna, transmit and receive, similar to the frequency-diverse antenna presented in [33]. The operating frequency-band for the radar is selected to be 77 – 81 GHz, which is known as the automotive short range radar (SRR) band [34]. The Q-factor of the antenna is selected to be $Q = 330$ and the 77-81 GHz band is sampled at 101 frequency points. From our previous work on frequency-diverse radars [1, 12], we assume that the total sweep time for a single SFCW chirp is $t_c = 10$ ms, suggesting that the dwell time for each data acquisition point across the 77-81 GHz band is $t_d = 10$ ms / 101 = 99 $\mu$s. Whereas this sweep time can seem to be excessive for automotive radars, our purpose in this work is to show that the proposed compensation algorithm can work with even such large sweep times, and hence displacements. It should also be mentioned that the time domain impulse response width of a frequency-diverse antenna is directly proportional to the Q-factor [1, 3]. With a Q-factor 330, the time domain impulse response of the frequency diverse antenna is on the order of 20 $\mu$s [3], significantly smaller than the assumed dwell time, $t_d = 99$ $\mu$s. A detailed discussion on the design of frequency-diverse computational antennas is given in [35]. For a vehicle traveling at a speed of 30 mph, the maximum Doppler frequency can be calculated to be 3.6 kHz at 80 GHz, resulting in a coherence time of 9.28 ms, which is substantially larger than the dwell time $t_d = 99$ $\mu$s at each frequency sampling point. Therefore, in this proof-of-concept study, we assume that the caused Doppler shift is negligible across the dwell time at each frequency sampling point $t_d$.

Assuming that the vehicle travels at a constant speed of $v = 30$ mph (speed limit in the UK for urban areas), it is evident that at each new measurement forming the rows of the sensing matrix, the car moves by 1.3 mm, corresponding to 0.35$\lambda$, where $\lambda$ denotes the wavelength at 80 GHz. This suggests that at each frequency sampling point, $f_i, i = 1\rightarrow M$, the radar platform advances towards the imaged scene by a distance of 0.35$\lambda$. In other words, by the end of a single chirp, the car travels by a total distance of 0.1341 m, which is 35.7$\lambda$ at 80 GHz. With no compensation applied in the sensing matrix, this creates a substantial discrepancy in the forward-model of (1), effectively destroying the reconstructed images as will be shown in the next section. To illustrate the difference between the static and dynamic cases, in Fig. 2, we compare the sensing matrix in the range plane ($xy$-plane) for three discrete frequencies within the SFCW sweep ($f_1, f_2, f_3$) and we choose an arbitrary pixel as the imaged object. As depicted in Fig. 2(a), for the static case, the positions of the vehicle and the imaged object remain fixed during the frequency sweep. In contrast, as shown in Fig. 2(b), for the dynamic case, this position information changes as the frequency is swept from $f_1$ to $f_3$. It should be noted that in Fig. 2(b), for the sake of clarity, we depict the dynamic case by moving the object with respect to a stationary vehicle. This is reciprocal to moving the vehicle with respect to a stationary object.

To compensate for the effect of the movement on the frequency-diverse radar, we can leverage several techniques. The first technique could be thought of a hardware-based solution. In this approach, we can leverage several frequency-diverse antennas placed at different depths and sequentially activate them as the vehicle moves to reduce the imaging problem at hand to a static problem. However, such a solution would increase the hardware complexity of the radar system. Alternatively, in this paper, we demonstrate that by appropriately modifying each row of the sensing matrix by taking into account the new position of the radar as the frequency is swept, we can eliminate this distortion. This modification of the sensing matrix can be achieved by
multiplying each row of the sensing matrix by the corresponding Green’s function to propagate the panel fields to the correct position that matches with the physical position of the radar with respect to the scene as the vehicle moves. This simple modification is equivalent to updating the location coordinates of the antenna in the calculation of the Green’s function when the transmit and receive antenna fields are propagated to the imaged scene to build the sensing matrix, $H$.

The sensing matrix is proportional to the radiated fields by the frequency-diverse antenna as follows [11]:

$$\mathbf{H} \propto \mathbf{E}_{Tx} \mathbf{E}_{Rx} \quad (3)$$

In (3), $\mathbf{E}_{Tx}$ and $\mathbf{E}_{Rx}$ are the radiated fields by the frequency-diverse antenna propagated to the imaged scene:

$$E_{Tx}(\mathbf{r}^t, f(t)) = \left[ J_{r}(\mathbf{r}(t)) \right] e^{-i \frac{2 \pi f(t)}{c} |\mathbf{r}(t) - \mathbf{r}|} \partial^2_{\mathbf{r}} \mathbf{r}(t) \quad (4)$$

$$E_{Rx}(\mathbf{r}^t, f(t)) = \left[ J_{r}(\mathbf{r}(t)) \right] e^{-i \frac{2 \pi f(t)}{c} |\mathbf{r}(t) - \mathbf{r}|} \partial^2_{\mathbf{r}} \mathbf{r}(t) \quad (5)$$

In (4) and (5), $J(\mathbf{r})$ denotes the surface current distribution of the frequency-diverse antenna while $\mathbf{r}$ and $\mathbf{r}^t$ denote the radar and scene coordinates, respectively. The frequency-diverse radar that we study in this paper consists of a single antenna sharing the same aperture to transmit and receive, and hence $J_{Tx}=J_{Rx}$ and $E_{Tx}=E_{Rx}$. Because the radar location with respect to the scene changes over time, $t$, the antenna coordinates are time dependent, $\mathbf{r}(t)$, and linking the antenna location with the measurement time we have:

$$\mathbf{r}(t) = \mathbf{r}_0 + \int_0^t \mathbf{v}(t) \partial t \quad (6)$$

In (6), $\mathbf{r}_0$ denotes the initial position of the frequency-diverse radar before the sweep is initiated ($t=0$). Alternatively, for the sake of simplicity, we can also consider a reciprocal situation with the coordinate system defined with respect to the vehicle, considering that the target is moving toward the vehicle instead. In this case, (6) can be represented in terms of the scene coordinates as follows:

$$\mathbf{r}^t(t) = \mathbf{r}_0^t + \int_0^t \mathbf{v}^t(t) \partial t \quad (7)$$

In (7), $\mathbf{r}_0^t$ is the initial scene position at the time the frequency sweep is initiated ($t=0$) and $\mathbf{v}^t$ is the speed at which the scene is moving towards the radar. In this case, the transmit and receive fields in the scene domain can be represented as:

$$E_{Tx}(\mathbf{r}^t(t), f(t)) = \left[ J_{r}(\mathbf{r}(t)) \right] e^{-i \frac{2 \pi f(t)}{c} |\mathbf{r}(t) - \mathbf{r}|} \partial^2_{\mathbf{r}} \mathbf{r}(t) \quad (8)$$

$$E_{Rx}(\mathbf{r}^t(t), f(t)) = \left[ J_{r}(\mathbf{r}(t)) \right] e^{-i \frac{2 \pi f(t)}{c} |\mathbf{r}(t) - \mathbf{r}|} \partial^2_{\mathbf{r}} \mathbf{r}(t) \quad (9)$$

Although in this paper we focus on the on the movement along the range direction (x-axis), $v=v_x=30\text{mph}$, for the sake of completeness, the coordinates projected along the principal axes of the Cartesian coordinate system can be given as follows:

$$\dot{x}(t) = x_0 - v_x t \quad (10)$$

$$\dot{y}(t) = y_0 + v_y t \quad (11)$$

$$\dot{z}(t) = z_0 + v_z t \quad (12)$$

In (10-12), $\mathbf{v} = \{v_x, v_y, v_z\}$ and the negative sign on the car speed along the x-axis accounts for the definition of the reciprocal coordinates (target is moving towards the vehicle). The linear approximation in (10-12) is justified given the sweep time considered, $t_s=10$ ms. Following the definition of SFCW parameters in Fig. 1, using frequency step indices as $q \in \{1, ..., K\}$ corresponding to frequencies $\{f_1, ..., f_K\}$:

$$t_q = (q + \frac{1}{2}) t_s \quad (13)$$

$$\dot{x}_q(t) = \dot{x}_0 - v_x t_q \quad (14)$$

$$\dot{y}_q(t) = \dot{y}_0 + v_y t_q \quad (15)$$

$$\dot{z}_q(t) = \dot{z}_0 + v_z t_q \quad (16)$$

Using the updated coordinates in (14-16), we can interpolate the sensing matrix on these new locations for each index $q$ in the entire sweep time. In the case of a nearest neighbor approach, we find the closest samples to our initial grid sampled by $\delta_x$, $\delta_y$ and $\delta_z$. A spatial offset of integer number of samples is defined for each dimension, denoted $m_x$, $m_y$ and $m_z$:

$$m_x = \lfloor v_x t_q / \delta_x \rfloor \quad (17)$$

$$m_y = \lfloor v_y t_q / \delta_y \rfloor \quad (18)$$

$$m_z = \lfloor v_z t_q / \delta_z \rfloor \quad (19)$$

Finally, the updated fields can be given as follows:
\[ E_{\text{R}}(x', y', z', f(q)) \rightarrow E_{\text{R}}(x'+m_1\delta_x, y'+m_1\delta_y, z'+m_1\delta_z, f(q)) \]  
\[ (20) \]

\[ E_{\text{R}}(x', y', z', f(q)) \rightarrow E_{\text{R}}(x'+m_2\delta_x, y'+m_2\delta_y, z'+m_2\delta_z, f(q)) \]  
\[ (21) \]

As will be shown in the next section, updating the sensing matrix to account for the movement during the frequency sweep is sufficient to successfully recover the scene information using a computational frequency-diverse radar for dynamic scenes.

III. RESULTS AND DISCUSSION

A. Validation of the Technique and Resolution Analysis

To validate the proposed technique, we first characterize the PSF patterns of the frequency diverse radar on the move. The size of the frequency-diverse aperture is assumed to be 30 cm x 30 cm, imaging a point scatter placed at a distance of \( d = 1 \) m along the x-axis. The frequency-diverse antenna is modelled as an array of cavity-backed metamaterial elements radiating across the 77-81 GHz operating frequency band. The radiation mechanism for each metamaterial element can be derived in the form of a magnetic dipole and the antenna radiated fields propagated to the imaged scene are then calculated using dyadic Green’s functions at each frequency to form the sensing matrix \( \mathbf{H} \) [35]. For this PSF analysis, we study three cases. Case 1 represents a static scenario where the position of the scene with respect to the frequency diverse radar is fixed (no movement is involved). For case 2, we study a scenario where the radar moves at \( v = 30 \) mph, but for the adjoint operation of (2), we make use of the static sensing matrix (no compensation for the radar movement is applied). Finally, for case 3, we study a scenario where the radar moves at a speed of \( v = 30 \) mph and the sensing matrix is compensated for this movement.

For these studies, using standard radar resolution equations below [32], the diffraction limited resolution limit of the frequency diverse aperture is calculated as \( \delta_{cr} = 1.23 \) cm in cross-range (yz-plane) and \( \delta_{cr} = 3.75 \) cm in range (x-axis), respectively.

\[ \delta_{cr} = \frac{\lambda d}{D} \]  
\[ (22) \]

\[ \delta_r = \frac{c}{2BW} \]  
\[ (23) \]

In (22) and (23), \( \lambda \) is the wavelength, \( d \) is the imaging distance, \( D \) is the frequency-diverse aperture size, and \( BW \) is the frequency bandwidth of operation. The size of the imaged scene is 10 cm x 10 cm x 10 cm, and the scene is discretised into cubic voxels of \( \delta_x = \delta_y = \delta_z = 1 \) cm. Hence the total number of voxels can be calculated as \( N = 1000 \). Discretization of the scene is done below the resolution limit to interpolate and smooth the reconstructed images for plotting. Hence, for the imaging problem at hand, the size of the sensing matrix is \( M = 101 \times N = 1000 \), corresponding to 0.8 Mb with single precision. For the considered problem in this paper, solving the inverse problem of (2) can be achieved in real-time, especially by using parallel-processing solutions, such as field programmable gate arrays (FPGAs) and general purpose graphical processing units (GPGPUs) [37, 38]. In this work, we make use of a NVIDIA GeForce GTX Titan X GPU and solving (2) for \( f_{\text{ct}} \) using a GPU acceleration takes around 0.015 s. The reconstructed PSF pattern of the frequency-diverse radar for case 1 (static) is shown in Fig. 3.

![Fig. 3. Reconstructed PSF pattern for the static scenario (case 1) (a) 3D view (b) range (top) view. Colorbar is in dB scale.](image-url)
Fig. 4. Reconstructed PSF pattern for the dynamic case without movement compensation in the sensing matrix (case 2) (a) 3D view (b) range (top) view. Colorbar is in dB scale.

Analysing Fig. 4, it is evident that the reconstructed image is heavily distorted by the mismatch introduced between $g$ and $H$ due to the movement of the radar platform towards the imaged scene. Because, for this scenario, the sensing matrix is not compensated for the movement of the radar, the reconstructed PSF pattern in Fig. 4 does not contain any useful information.

Finally, the reconstructed PSF pattern for case 3 (dynamic, with movement compensation) is shown in Fig. 5.

Fig. 5. Reconstructed PSF pattern for the dynamic case with movement compensation applied to the sensing matrix (case 3) (a) 3D view (b) range (top) view. Colorbar is in dB scale.

Analysing Fig. 5, it can be seen that the reconstructed PSF pattern is free from the distortions present in Fig. 4 caused by the motion of the radar during the SFCW acquisition cycle. The range plot along the x-axis shown in Fig. 5(b) clearly shows the correct distance for the imaged point scatter at $d=1$ m. Another important feature observed in this reconstruction is that the resolution is slightly improved due to the reduced distance of the frequency-diverse radar as it moves closer to the imaged object during the frequency sweep. For case 3, the cross-range and range resolution limits were calculated to be 1.3 cm and 4.1 cm, respectively.

B. Application as an Automotive Radar and Debris Detection

Following the analysis of the PSF patterns, we study a potential application of the presented frequency-diverse automotive radar for the detection of possible debris on roads. For this study, a fully reflective debris considered to be metal is placed at a distance of $d=2$ m away from the frequency-diverse antenna. The size of the debris is 9.5 cm (y-axis) x 3.5 cm (z-axis). Due to the limited integration space available on vehicles for radar antennas, we make use of a single frequency-diverse antenna with a size of 10 cm x 10 cm. A depiction of the debris detection scenario is given in Fig. 6(a) whereas the frequency-diverse antenna as the enabling medium radiating spatio-temporarily varying quasi-random bases is shown in Figs. 6(b)-6(d).
Fig. 6. Depiction of the studied close-range debris detection scenario. (a) The vehicle with the frequency-diverse antenna moves towards the object at \( v = 30 \text{ mph} \) speed (b) frequency-diverse antenna: front-view with the aperture radiated field at 77 GHz (left) and back-view with the single compressed channel (right) (c) frequency-diverse antenna: front-view with the aperture radiated field at 79 GHz (left) and back-view with the single compressed channel (right) (d) frequency-diverse antenna: front-view with the aperture radiated field at 81 GHz (left) and back-view with the single compressed channel (right). Dimensions: \( D = 10 \text{ cm} \) and \( h = 0.5 \text{ cm} \).

Similar to the PSF analyses carried out, it is assumed that the car moves at a constant speed of \( v = 30 \text{ mph} \) in the direction of the debris. Processing the backscattered measurements using the modified sensing matrix, we reconstruct the image shown in Fig. 7. For reconstruction, the sensing matrix is modified to take into account the movement of the radar platform (a) 3D view (b) range (top) view. Colorbar is in dB scale.

Analysing the reconstructed images in Fig. 7, the presence of the imaged debris is evident. The range plot shown in Fig. 7(b) correctly predicts the initial distance of the debris from the radar, \( d = 2 \text{ m} \).

In order to put the presented frequency-diverse technique into context, in Table I, we provide a comparison between different types of automotive radar sensor modalities studied in the literature and the presented work in this paper. Analysing Table I, it is evident that most sensor topologies studied in the literature are various forms of array antenna architectures. Majority of these works rely on a linear array architecture [39-44, 47, 48], synthesizing a fixed, broadside radiation pattern. Using a planar array topology [45, 46], beam scanning in 3D space can be achieved within a certain frequency band of operation. However, this is accomplished at the expense of increased hardware complexity, requiring a substantially increased number of antennas (and channels) in comparison to the presented computational imaging concept in this paper, which can capture and reconstruct 3D images using a single antenna with a single channel. This advantage of the proposed technique can substantially simplify the hardware requirements for automotive radar applications.
Table I. Comparison of existing sensor techniques for automotive radar to the presented frequency-diverse technique.

<table>
<thead>
<tr>
<th>Work</th>
<th>Frequency Band (GHz)</th>
<th>Sensor Type</th>
<th>All-Electronic 3D Imaging</th>
</tr>
</thead>
<tbody>
<tr>
<td>[40]</td>
<td>76-77</td>
<td>Patch Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>[41]</td>
<td>77</td>
<td>Patch Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>[42]</td>
<td>76.2 – 77.8</td>
<td>Slot Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>[43]</td>
<td>77 – 81</td>
<td>Grid Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>[45]</td>
<td>77</td>
<td>Orthogonal Linear Array</td>
<td>Possible. Uses 4 transmit and 4 receive channels to produce multiple fixed beams to cover both azimuth and elevation.</td>
</tr>
<tr>
<td>[46]</td>
<td>76 – 81</td>
<td>Rectangular carved horn array</td>
<td>Possible. Uses an array of 64 (8x8) carved horn antennas.</td>
</tr>
<tr>
<td>[47]</td>
<td>77 – 81</td>
<td>Grid Antenna Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>[48]</td>
<td>75 – 82</td>
<td>Slotted Waveguide Array</td>
<td>No. Linear array with fixed broadside radiation.</td>
</tr>
<tr>
<td>This Work</td>
<td>77-81 GHz</td>
<td>Frequency-diverse aperture</td>
<td>Yes. Backscattered signal is compressed into a single channel using a single sensor. 3D images from compressed data are reconstructed.</td>
</tr>
</tbody>
</table>

C. Uncertainty Analysis for the Forward-Model

For the studied imaging problem, two types of uncertainties can be present in the forward-model of (1). First is the measurement noise, which can be present in any radar system. In this work, we can analyse the effect of noise uncertainty by adding a distortion to the measured signal, $g$, as a function of different signal-to-noise (SNR) levels. To this end, we use the measured signal, $g$, for the original reconstruction in Fig. 7 (noiseless) and add to it a Gaussian white noise modelled distortion at SNR rates of 0 dB, 5 dB, 10 dB and 20 dB, respectively. The reconstructed images as a function of noise uncertainty are shown in Fig. 8.

Analysing Fig. 8, it is evident that the presented technique is robust to measurement noise. Whereas we observe a slight distortion in the reconstructed image below 5 dB SNR level, increasing the SNR above this level, the fidelity of the reconstructed image remains almost the same.

The second uncertainty can be observed in the presence of non-precise calculation of the position of the vehicle with respect to the imaged object as the vehicle moves. We emphasize that the goal of this paper is to show that the frequency-diverse computational imaging technique, which has been historically limited to static scenes, can be adopted to dynamically varying scenes in the context of automotive radars. Hence, the analysis of inaccurate calibration of vehicle’s trajectory is not the main scope of this paper, nevertheless, we present a systematic study to demonstrate the effect of positioning uncertainty on the reconstructed images. To this end, in Fig. 9, we study four scenarios. Scenario 1: Ideal case – the vehicle moves at a speed of $v=30$ mph for the forward-model and for the adjoint operation this speed is registered correctly ($v=30$ mph). Scenario 2: The vehicle moves at a speed of $v=30$ mph for the forward-model but for the adjoint operation, the vehicle speed is registered as $v=30.5$ mph in error. Scenario 3: The vehicle moves at a speed of $v=30$ mph for the forward-model but the adjoint operation, the vehicle speed is registered as $v=32.5$ mph in error. Scenario 4: The vehicle moves at a speed of $v=30$ mph for the forward-model but for the adjoint operation, the vehicle speed is registered as $v=35$ mph in error. The reconstructed images for these scenarios are shown in Fig. 9. For this analysis the SNR level was chosen to be 20 dB.
Fig. 9. Effect of position uncertainty in image reconstruction
(a) Scenario 1: original case – no positioning error is present
(b) Scenario 2: $\Delta d=0.1$ mm positioning error for each frequency step (c) Scenario 3: $\Delta d=0.2$ mm positioning error for each frequency step (d) Scenario 4: $\Delta d=0.3$ mm positioning error for each frequency step. Colorbar is in dB scale.

Analysing Fig. 9, it is evident that positioning uncertainties, when not correctly calculated, can substantially affect the reconstructed images. For the studied scenario 2 shown in Fig. 9(b), the vehicle moves at the intended speed of $v=30$ mph for the forward-model, suggesting that at each frequency sampling point interval, the vehicle moves by 1.3 mm. For the adjoint operation, however, miscalculating this speed as $v=30.5$ mph, the vehicle moves by 1.4 mm at each frequency sampling point interval. As a result, by the end of the frequency sweep of 101 frequency points, we have a total positioning error of 1 cm, or $2.67\lambda$ at 80 GHz, degrading the reconstructed image as can be seen in Fig. 9(b). Increasing the positioning error, we can see that the reconstructed images in Figs. 9(c) and 9(d) are heavily distorted. For the studied scenario 3 shown in Fig. 9(c), the vehicle moves at a speed of $v=30$ mph for the forward-model and is assumed to move at $v=32.5$ mph for the adjoint operation in error, resulting in a total positioning error of 2 cm, or $5.3\lambda$ at 80 GHz. Finally, for the studied scenario 4 in Fig. 9(d), the vehicle moves at a speed of $v=30$ mph for the forward-model and is assumed to move at $v=35$ mph for the adjoint operation in error, resulting in a total positioning error of 3 cm, $8\lambda$ at 80 GHz.

D. Systematic Analysis of the Debris Position and Size for the Debris Detection Scenario

Finally, to further understand the imaging characteristics of the proposed technique as an automotive radar application, we study the size and position of the imaged debris, and investigate the reconstructed images as a function of these parameters. First, in Fig. 10, we vary the position of the object from $d=2$ m to $d=5$ m. For each studied scenario, we use the original imaging parameters, i.e. the vehicle moves at a constant speed of $v=30$ mph and the sensing matrix is compensated with respect to the movement of the frequency-diverse antenna as detailed in Section II. The selected SNR level is 20 dB. The reconstructed images of the debris as a function of varying position are shown in Fig. 10.

As shown in Fig. 10, as the debris is positioned further away, the cross-range profile of the reconstructed image widens and becomes less informative due to the worsening cross-range resolution. This is expected, because, from (22), the cross-range resolution of the radar is directly impacted by the imaging distance. For each case in Fig. 10, we can clearly identify the position of the debris in range. As can be seen from (23), for the presented scenario, the range-resolution is governed by the imaging bandwidth, which remains constant at 4 GHz (77-81 GHz).

Next, we study the same imaging scenario, but as a function of varying debris size. For this study, the debris size is varied from half of the original size, 4.75 cm x 1.75 cm to 1.5 times the original size, 14.25 cm x 5.2 cm. The reconstructed images are shown in Fig. 11. As can be seen in Fig. 11, the outline of the reconstructions is sensitive to the size of the debris and increasing in size as the debris size is increased. An important point can be seen in Fig. 11(c), where the physical size of the debris exceeds the size of the frequency diverse antenna in the y-axis, limiting the reconstruction fidelity. This is expected and can be overcome by using a larger aperture.
to retrieve high fidelity PSF patterns from the radar. Although shown for W-band, the proposed technique can be scaled to other frequencies and find applications in biomedical imaging, non-destructive evaluation and spectroscopy where the objective function has dynamic characteristics and may vary during the data acquisition cycle.

Fig. 11. Reconstructed images of the debris for varying size (a) half of the original size \((W=4.75 \text{ cm}, L=1.75 \text{ cm})\) (b) original size \((W=9.5 \text{ cm}, L=3.5 \text{ cm})\) (c) 1.5 times the original size \((W=14.25 \text{ cm}, L=5.2 \text{ cm})\). The size of the frequency-diverse antenna is \(D=10 \text{ cm}\). The debris is shown in purple color and the antenna is in orange color. Colorbar is in dB scale.

IV. CONCLUSIONS

We have presented a proof-of-concept study for a computational frequency-diverse automotive radar scheme for detection of debris at W-band frequencies. It has been presented that for the imaging problem at hand, the scene characteristics change during the SFCW data acquisition cycle and hence the sensing matrix of the computational radar needs to be compensated for this variation to achieve imaging. We have successfully demonstrated this compensation technique in the context of a computational frequency-diverse automotive radar for debris detection and validated that the proposed radar technique can reconstruct images free of movement related artifacts. We have further studied the imaging characteristics of the frequency-diverse radar by analyzing the PSF patterns and shown that the proposed movement compensation applied to the sensing matrix is vital

REFERENCES


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