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Electron trajectories associated with laser-driven coherent synchrotron emission at the front surface of overdense plasmas

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We present an in-depth analysis of an ultrafast electron trajectory type that produces attosecond electromagnetic pulses in both the reflected and forward directions during normal incidence, relativistic laser-plasma interactions. Our particle-in-cell simulation results show that for a target which is opaque to the frequency of the driving laser pulse the emission trajectory is synchrotronlike but differs significantly from the previously identified figure-eight type which produces bright attosecond bursts exclusively in the reflected direction. The origin and characteristics of this trajectory type are explained in terms of the driving electromagnetic fields, the opacity of the plasma, and the conservation of canonical momentum.

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I. INTRODUCTION

When an initially solid target is irradiated by the leading edge of an intense laser pulse, a dense plasma rapidly forms at its surface. Typically this plasma is opaque and reflective to the frequency of the incoming laser pulse ω_L and acts macroscopically as a plasma mirror [1]. At a microscopic level this reflection is facilitated by collective electron motion, which is driven by the electromagnetic fields at the interface. If the peak field strength E_{L0} of the laser pulse is sufficiently high such that the dimensionless parameter $a_0 = |eE_{L0}/m_e c \omega_L| \geq 1$, then the motion of these electrons becomes relativistic and a process known as surface high-harmonic generation (SHHG) can occur [2]. As a consequence of SHHG, the spectrum of the reflected beam will consist of integer multiples, or harmonics, of the driving laser frequency, which can extend well into the extreme ultraviolet (XUV) region and beyond [3]. For thin foil targets which are approximately 100–1000 nm thick, high harmonics are also emitted in the forward direction and can be collected at the rear side of the plasma [4]. By filtering out the low-order harmonics from the beam, a series of bright subfemtosecond pulses is unveiled. Research into the production of these attosecond pulses is motivated principally by their potential application in performing time-resolved measurements of electronic and atomic phenomena which evolve on similar timescales [5,6].

A variety of theoretical models have been developed to describe the reflected emission in specified regimes such as the oscillating mirror model [7,8], the theory of relativistic spikes [9,10], and the relativistic electronic spring [11,12]. Numerical particle-in-cell (PIC) simulations [13,14] are also extensively used, since their comparatively *ab initio* approach enables the study of complex dynamics that are beyond the

scope of such models. These simulations have shown that in optimized interaction conditions, dense electron bunches can form at the plasma surface and produce particularly bright attosecond bursts via coherent synchrotron emission (CSE) as they are accelerated into curved trajectories [15]. Recent research has placed emphasis on understanding and controlling the shape of these trajectories to increase the strength of the reflected emission, including using tailored waveforms [16] or two-color laser pulses [17] to drive the process or by employing a multiple reflection geometry [18]. The two-color technique has since been verified experimentally, with significant enhancements to the intensity of the harmonics being observed [19].

Forward CSE, though not as extensively studied as the reflected case, has recently shown great promise as a coherent XUV source, particularly when the target is irradiated by the laser pulse at normal incidence [4,20–22]. Experiments which employ this normal incidence in transmission (NIIT) geometry have shown CSE spectra with a shallow intensity scaling with frequency [4] and a strong ellipticity dependence [21] which can be exploited via polarization gating techniques [21,23] to isolate an attosecond pulse. In this geometry, the forward coherent wake emission process [24–27] is also extinguished, which could otherwise degrade the effective temporal duration of the pulses. Furthermore, the attosecond pulse structure of forward CSE is immediately apparent since the frequencies below the plasma frequency are automatically filtered out as the radiation passes through the bulk plasma [27,28]. These observations highlight the suitability of the NIIT configuration in producing intense isolated attosecond pulses that are ideal for attosecond pump-probe experiments.

To fully optimize the relativistic electron dynamics associated with CSE, their synchrotronlike emission trajectories must be thoroughly understood. However, at present there have been few dedicated and detailed studies relating to these trajectories and none relating to forward emission or the role

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played by plasma opacity. In the case of an ultrathin (~ 4 nm) foil target irradiated by a linearly polarized laser pulse at normal incidence, simulations show that under idealized conditions the emitting electrons undergo figure-eight-type trajectories [29]. At the top and bottom of the 8, the electrons experience a large acceleration that is perpendicular to their relativistic velocity and emit synchrotron radiation exclusively in the reflected direction. However, the absence of forward emission suggests that an alternative trajectory type must occur for thicker targets (~ 100 – 1000 nm) in order to explain the experimental observations of forward CSE. The case of a two-color driving laser pulse irradiating a thicker plasma target to produce reflected harmonics has also been investigated [17]. However, there has not yet been a detailed analysis of the emitting electron trajectory type that occurs when a single color laser pulse interacts with an opaque target, nor a description of how forward CSE relates to these trajectories.

Here we present an in-depth numerical analysis of a laser-driven CSE trajectory type that produces attosecond electromagnetic pulses in both the reflected and forward directions using a normal incidence geometry. We describe the roles played by the electromagnetic fields, the opacity of the plasma, and the conservation of canonical momentum in driving this process. We emphasize that we do not observe any figure-eight emitting electron trajectories in a plasma that is opaque to the frequency driving laser pulse, but do so in a plasma that turns relativistically transparent [30] during the interaction.

The remainder of this article is structured as follows. In Sec. II we give the initial conditions for the simulated interaction and discuss the one-dimensional (1D) geometry it employs. Our main results are given in Sec. III and are divided into four parts. In Sec. III A we give a high-level overview of the simulated interaction and identify a relativistic electron trajectory type which occurs at the front surface of the opaque plasma. In Sec. III B we show that when an electron bunch is accelerated into the identified trajectory shape, attosecond pulses are produced at two distinct points, one of which propagates in the reflected direction and the other in the forward direction. We show that the instantaneous motion of the bunch at each emission point is synchrotronlike by analyzing the velocity and acceleration components of one of its constituent particles and directly calculate the electromagnetic fields it emits. In Sec. III C the motion of this representative particle is explained by analyzing the fields, forces, and accelerations it experiences. In Sec. III D we compare this trajectory type to a figure-eight type which we observe in a second simulation using a target that becomes relativistically transparent during the interaction. We conclude in Sec. IV by summarizing our main results and highlighting some of their implications for future research.

II. PIC SIMULATION TECHNIQUE AND INITIAL CONDITIONS IN THE 1D GEOMETRY

A. 1D PIC simulations

Numerical simulation using the particle-in-cell approach is an established and trusted tool for studying laser-plasma interactions at a microscopic level [13,14]. In PIC codes the plasma is represented by a collection of numerical macroparticles which move in continuous phase space according to the

Newton-Lorentz equations. Each individual macroelectron or macroion represents a large number of physical particles of their respective species and since they have the same charge-to-mass ratio they follow the same trajectory when under the influence of an electromagnetic field. The macroparticles contribute to charge and current densities that are defined on a fixed spatial grid, which consists of numerous cells. Maxwell's equations are used to calculate the electromagnetic fields on the grid points which are interpolated back to the position of the macroparticles. The results in this article are obtained using the 1D PIC code PICWIG, which is described in Ref. [31]. In 1D codes, each physical quantity depends only on one spatial longitudinal coordinate and there are no variations in the other transverse directions. Each macroparticle in a 1D simulation may therefore be conceptualized as an infinite sheet of charge in three spatial dimensions.

B. Simulation parameters

The simulation for the opaque target models a linearly polarized, Gaussian laser pulse, with a pulse duration (measured as the FWHM of intensity envelope), central wavelength, and normalized vector potential of $\tau_L = 5$ fs, $\lambda_L = 800$ nm, and $a_0 = 20$, respectively. Mathematically, the electric field of the input laser pulse may be expressed, up to an offset in time t , as $(a_0 m_e c \omega_L / e) \exp(-t^2 / \sigma^2) \cos(\omega_L t)$, where $\sigma = \tau_L / \sqrt{2 \ln 2}$ and $\omega_L = 2\pi c / \lambda_L$ is the laser frequency. The laser pulse propagates along the x axis and its electric and magnetic fields are polarized along the y and z axes, respectively. The shape of the input laser pulse is illustrated in Fig. 1(a).

This pulse is normally incident onto a fully ionized, initially cold, carbon plasma with an initial peak density of $n_{e0} = 100n_c$, where $n_c = \epsilon_0 m_e \omega_L^2 / e^2$ is the critical density. The density profile consists of an 800-nm-thick top-hat region, positioned with its front surface at $x = 0$, along with a front-side exponential density ramp, which extends to $x/\lambda_L = -1$ and takes the form $n_e(x) = n_{e0} \exp(x/L)$, where the scale length $L = \lambda_L / 10$. The ions are treated as an immobile neutralizing background. The initial electron density profile and the effective ion density profile are illustrated in Fig. 1(b). The simulation box, which spans the region $x = [-10\lambda_L, 10\lambda_L]$, consists of 1000 cells per λ_L and initially 300 macroelectrons per populated cell. For the target that turns relativistically transparent during the interaction, discussed in Sec. III D, the input parameters are the same as those listed above, except the plasma is much thinner, consisting solely of a 30-nm top-hat region without a front-side density ramp.

C. 1D interaction equations

Throughout this article, we refer to three main equations to help interpret our results. These are significantly simplified due to the 1D interaction geometry and the linear polarization of the applied fields [13]. In the simulated interaction we have only three nonzero-field components (E_x , E_y , and B_z) and two current density components (j_x and j_y) which are defined on the numerical grid. Similarly, each particle in the simulation only has two momentum components (p_x and p_y) and their associated velocity components (v_x and v_y).

The first main equation we use is the Newton-Lorentz equation which may be written, for this simplified

FIG. 1. Initial (a) laser and (b) plasma parameters used in the principal PIC simulation of this article.

geometry, as

$$\begin{aligned} \mathbf{F} &= \frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ &= q(E_x + v_y B_z)\hat{\mathbf{x}} + q(E_y - v_x B_z)\hat{\mathbf{y}}, \end{aligned} \quad (1)$$

where q is the charge of the particle and the fields are given at the position of the particle.

The second equation is the conservation of canonical momentum in this geometry [32]. This is obtained by converting to the potential formulation via $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$. In this geometry the transverse fields are $E_y = -\partial A_y/\partial t$ and $B_z = \partial A_y/\partial x$, so for each particle we have

$$\begin{aligned} \frac{dp_y}{dt} &= q(E_y - v_x B_z) \\ &= -q\left(\frac{\partial A_y}{\partial t} + v_x \frac{\partial A_y}{\partial x}\right) = -q \frac{dA_y}{dt}, \end{aligned}$$

where the fields and potential are again defined at the particle position. For a particle with initial transverse momentum $p_{y0} = 0$, integration of the above gives

$$p_y = -qA_y, \quad (2)$$

which is the equation expressing the conservation of canonical momentum.

The third equation we use is the solution to the wave equation. By substituting the potential relations for E_y and B_z into Ampère's law and using the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, the wave equation is obtained

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] A_y(x, t) = -\mu_0 j_y(x, t).$$

In solving this equation (e.g., using the method in Ref. [33]), the transverse electric field $E_y(x, t)$ observed at position x and time t due to the current density $j_y(x', t_r)$ located at position x' at the retarded time $t_r = t - |x - x'|/c$ may be written as

$$E_y(x, t) = -\frac{1}{2\epsilon_0 c} \int j_y(x', t_r) dx'. \quad (3)$$

III. SIMULATION RESULTS

A. Overview of simulated interaction and relativistic electron trajectories at the plasma surface

1. Electromagnetic field components

Figures 2(a) and 2(b) show the transverse electromagnetic field components $E_y(x, t)$ and $B_z(x, t)$ (blue to red) overlaid on the electron density $n_e(x, t)$ (gray to black) during the simulated interaction. Although all frequencies of electromagnetic radiation (including the high-frequency CSE) are shown on these plots, it is the fundamental frequency of the laser pulse ω_L which is the strongest and most visible. These plots show that the simulated laser pulse initially propagates from left to right, before interacting with and being reflected from the front surface of the plasma. This reflection is explained by fundamental plasma physics. Since the plasma is overdense, it is opaque to frequencies less than the plasma frequency ω_{p0} , which includes that of the laser pulse and low-order harmonics (here $\omega_{p0} = \sqrt{n_{e0}/n_c} \omega_L \approx 10\omega_L$).

Since this is a normal incidence interaction, the reflected fields superpose with the incoming fields, forming nodes and antinodes in the overlap region. The fields which drive the SHHG process are those directly at the front surface of the plasma and are discussed in more detail in Sec. III C. From this high-level perspective however, the transverse field distributions appear quite simple and are analogous to the textbook example of a plane wave normally incident onto a conductor. Here the simulation shows that E_y (B_z) has nodes (antinodes) at $x = -(k/2)\lambda_L + x_{rp}$ for $k = \{0, 1, 2, \dots\}$, where x_{rp} is the reflection point. In between these positions, E_y (B_z) has antinodes (nodes) at $x = -[(2k+1)/4]\lambda_L + x_{rp}$. The third and final field component in this simulation is the longitudinal $E_x(x, t)$ (Coulomb) field shown in Fig. 2(c), which is caused by deviations from local charge neutrality arising from longitudinal motion of the particles.

2. Relativistic electron dynamics at the plasma surface

These fields directly drive electron motion within the skin depth at the front surface of the plasma. Figures 3(a) and 3(b) show the electron density overlaid with the longitudinal $j_x(x, t)$ and transverse $j_y(x, t)$ current density components, respectively. In full detail these currents are quite complex; however, if attention is restricted to the most prominent motion of the plasma surface, it appears regular and periodic, undergoing transverse oscillations once every laser period

FIG. 2. Transverse electromagnetic fields (a) $E_y(x, t)$ and (b) $B_z(x, t)$ (blue to red) overlaid on the electron density $n_e(x, t)$ (gray to black) during the simulated interaction. Since the target is overdense, the transverse fields in front of its surface are a superposition of incoming and reflected waves and the long-wavelength fields tend towards zero inside the plasma. (c) Longitudinal $E_x(x, t)$ field, which is caused by deviations from local charge neutrality arising from the longitudinal motion of the particles.

and longitudinal oscillations twice every laser period. This apparent regularity emerges from electron motion that is aperiodic at the single-particle level. Figure 3(c) highlights six individual macroelectron trajectories in the xt plane. Those in red are selected from three different regions throughout the width of the oscillation that occurs at $t/T_L \approx 9.2$, while those in blue are from the next oscillation at $t/T_L \approx 9.7$. Figure 3(d) shows these trajectories in the xy plane. Here the y coordinates

FIG. 3. Electron density overlaid with the (a) longitudinal $j_x(x, t)$ and (b) transverse $j_y(x, t)$ current density components. The overall motion of the front surface consists of transverse oscillations once every laser period and longitudinal oscillations twice every laser period. Six individual macroelectron trajectories are selected from different regions across the width of two successive electron oscillations. Also shown is their motion in the (c) xt and (d) xy planes.

of each particle were obtained by integrating its transverse momentum (starting at $t/T_L = 8$) and setting the constant of integration such that $y(t/T_L = 10.5) = 0$, for visibility. These particles are observed being pulled towards the plasma surface and being accelerated into a loop before being injected into the plasma where they continue their motion in a straight line towards the rear side of the target.

In Sec. III B we describe how this trajectory type facilitates attosecond pulse production in both the reflected and forward directions. To aid this ensuing description, we highlight here three fundamental characteristics of this relativistic motion.

(a) Electrons throughout the width of the oscillation undergo the same trajectory shape, with the proviso that those electrons towards the front of the oscillation, i.e., the left side, in this case, undergo larger loops than those towards the rear.

(b) As the electrons enter the plasma during the final stage of their trajectory, they have a near-zero transverse momentum, i.e., their motion is almost parallel to the x axis. This is a simple consequence of the conservation of canonical momentum. Since the plasma is opaque to frequencies less than ω_p , the region beyond its surface has $eA_y/m_e c \approx 0$. Therefore, from Eq. (2), the normalized transverse momentum of these electrons $p_y/m_e c$ must also approach zero.

(c) Each half-period, a new group of electrons is pulled towards the surface and undergoes the same trajectory shape, except the orientation alternates between clockwise and counterclockwise in the xy plane.

B. Attosecond pulse production

1. CSE from electron bunches

During the simulated interaction, bunches of electrons are accelerated into the identified trajectory shape and produce attosecond bursts of CSE in both the reflected and forward directions. To illustrate the origin of the high-frequency radiation, Fig. 4 shows the electron density (gray to black) at the front surface of the plasma, for one half-period during the peak of the interaction. Here the color scale is the same as that in Fig. 2. Overlaid in magenta is $|eE_{y,\text{filt}}/m_e c \omega_L|^2$, where $E_{y,\text{filt}}$ is the transverse electric field filtered between $\omega/\omega_L = 100$ and 200. This figure shows that a bunch of electrons is pulled from the plasma and produces a burst of high-frequency radiation in the reflected direction at $t/T_L \approx 9.635$, before it is accelerated towards the plasma and emits a forward burst at $t/T_L \approx 9.97$. The density profile of the bunch at each emission time is inset into the plots. Here, since both CSE bursts originate from the same bunch (corresponding to the primary bunch described in Ref. [22]), the forward emission mechanism is fundamentally different from that associated with wave breaking as discussed in Refs. [27,34].

The pulse duration of both bursts of CSE are approximately Fourier limited, for this choice of frequency filtering, at ~ 27 as measured by the FWHM of the intensity profile. However, the peak intensity of the reflected emission is ~ 30 times greater than the forward. There are a number of important factors which influence the relative intensity of forward and reflected CSE. Here the disparity can largely be attributed to differences in the peak density of the emitting electron bunch. The line-out insets in Fig. 4 show that the bunch has a peak

FIG. 4. Collective electron dynamics and attosecond emission surrounding the (a) reflected and (b) forward emission points. The electron density $n_e(x, t)$ (gray to black) is shown at the front surface of the plasma during the peak of the simulated interaction. Overlaid (magenta) is the square of the transverse electric field $|eE_{y,\text{filt}}/m_e c \omega_L|^2$, where $E_{y,\text{filt}}$ is the transverse electric field filtered in the range $\omega/\omega_L = 100$ –200.

density of $\sim 250n_c$ at the reflected emission point but becomes more dispersed and is only $\sim 50n_c$ at the forward emission point. A simple argument is that, all else being equal, the intensity of CSE produced by a perfectly coherent electron bunch should scale with the square of its density. Therefore, one might expect this ~ 5 times difference in peak density to result in reflected emission that is ~ 25 times stronger than the forward, which is roughly what is seen here. However, there are other influencing factors such as the exact density profile of the bunch, since it is not perfectly localized, and the relative phase of the trajectories along this profile. Another important factor is the characteristics of the trajectory shape surrounding each emission point, which is now described.

2. Synchrotronlike trajectory of the emitting electron bunch

Figure 5(a) shows the motion, in the xy plane, of a selected macroelectron that belongs to the bunch and contributes to both reflected and forward emissions. Figures 5(b) and 5(c) show, respectively, the velocity $v_{x,y}(t)$ and acceleration $a_{x,y}(t)$ components of this particle. In this geometry, the emission of

FIG. 5. (a) Trajectory (shown as dots) of the representative macroelectron at equally spaced time intervals from $t/T_L = 8$ to $t/T_L = 10.5$. Also shown are the (b) velocity $v_{x,y}(t)$ and (c) acceleration $a_{x,y}(t)$ components of the highlighted particle. At $t/T_L \approx 9.64$ and 10, the electron bunch has both a relativistic longitudinal velocity and a large transverse acceleration, which results in the emission of synchrotron radiation.

synchrotron radiation requires the particle to simultaneously possess a large longitudinal velocity v_x and a large transverse acceleration a_y . The plots show that these conditions are satisfied at both reflected ($t_{\text{refl}}/T_L \approx 9.64$) and forward ($t_{\text{forw}}/T_L \approx 10$) emission times. The longitudinal velocity of the particle is $v_x/c \approx -0.96$ at t_{refl} and $v_x/c \approx 0.98$ at t_{forw} , with the transverse acceleration being at a peak for both. The conditions for synchrotron radiation are therefore satisfied at both emission times.

However, we note that there are differences in how this transverse acceleration affects the behavior of the v_y component surrounding these points; though at both times $v_y/c \approx 0$, at the reflected emission point the transverse velocity of the particle changes sign, whereas at the forward emission point

FIG. 6. Comparison between the attosecond pulses obtained directly from the PIC simulation (orange dotted line) and those calculated from the single-particle trajectory (blue solid line): (a) reflected pulse at position $x/\lambda_L = -8$ and (b) forward pulse collected at $x/\lambda_L = 9.1$. In both cases, the pulses are filtered to only include frequencies in the range $\omega/\omega_L = 100$ –200.

the transverse velocity tends to, but does not pass through, zero. At both emission times, the longitudinal acceleration component $a_x(t) \approx 0$.

To verify that this electron trajectory produces attosecond pulses in both the forward and reflected directions we directly calculate the transverse electric field it radiates via Eq. (3). For an individual particle, the current density may be written as $j_y(x', t_r) = -en_e l_0 \delta(x - x_e(t_r)) v_y(t_r)$, where $x_e(t_r)$ is its position, $-en_e l_0$ is its areal charge density, and δ is the Dirac delta function. Under this condition, Eq. (3) simplifies to

$$E_y(x, t) = \frac{en_e l_0}{2\epsilon_0 c} \frac{1}{1 - s v_x(t_r)/c} v_y(t_r).$$

Here $s = 1$ ($s = -1$) when the observation point is to the right (left) of the particle. In dimensionless units this becomes

$$\frac{e E_y(x, t)}{m_e c \omega_L} = \pi \left(\frac{l_0}{\lambda_L} \right) \left(\frac{n_e}{n_c} \right) \frac{1}{1 - s v_x(t_r)/c} \frac{v_y(t_r)}{c}. \quad (4)$$

For the selected particle, we have $l_0/\lambda_L = 1 \times 10^{-3}$ (the simulation grid size) and $n_e/n_c \approx 0.28$. Using this particle, Fig. 6 shows the results of this calculation for frequencies in the range $\omega/\omega_L = 100$ –200. Figures 6(a) and 6(b) show, respectively, the reflected and forward emission from the single particle (blue), overlaid on the total emission taken from the simulation results (orange dotted line). The intensity of the pulses calculated from the single-particle trajectory is much lower than that observed from the simulation, because

FIG. 7. (a)–(c) Particle trajectory in the xy plane overlaid with arrows that represent the magnitude and direction of the six acceleration terms defined by Eq. (5) at each point. To help interpret these results, (d) $E_x(x, t)$, (e) $E_y(x, t)$, and (f) $B_z(x, t)$ (blue to red) are shown in the vicinity of the emitting particle over the same time period (yellow line).

the latter has many particles contributing coherently to the emission. However, these plots verify that the identified trajectory emits the high-frequency emission at the expected times.

C. Fields and forces that drive the electron bunch trajectory

Here we describe the contribution of each electromagnetic field component in driving the identified trajectory. This is achieved numerically by using the field strengths experienced by the particle during the simulation to calculate its acceleration components directly from the Lorentz force.

From fundamental relativistic dynamics, the ordinary acceleration vector $\mathbf{a}(t)$ due to a force $\mathbf{F}(t)$ acting on a particle of mass m traveling with a velocity vector $\mathbf{v}(t)$ and Lorentz factor $\gamma(v^2) = 1/\sqrt{1 - v^2/c^2}$ may be written as

$$\mathbf{a} = \frac{1}{\gamma m} \left(\mathbf{F} - \frac{(\mathbf{v} \cdot \mathbf{F})\mathbf{v}}{c^2} \right).$$

When combined with Eq. (1), this acceleration may be written in component form as $\mathbf{a} = a_x \hat{\mathbf{x}} + a_y \hat{\mathbf{y}}$, where

$$\begin{aligned} a_x &= \frac{q}{\gamma m} \left(E_x + v_y B_z - \frac{(E_x v_x + E_y v_y) v_x}{c^2} \right), \\ a_y &= \frac{q}{\gamma m} \left(E_y - v_x B_z - \frac{(E_x v_x + E_y v_y) v_y}{c^2} \right), \end{aligned} \quad (5)$$

and the fields are defined at the position of the particle and are functions of time.

The role played by each electromagnetic field component in driving this motion is illustrated in Fig. 7. Figures 7(a)–7(c) show the particle trajectory in the xy plane overlaid with arrows that represent the magnitude and direction of the six acceleration terms defined by Eq. (5) at each point. To help interpret these results, Figs. 7(d), 7(e), and 7(f) show, respectively, $E_x(x, t)$, $E_y(x, t)$, and $B_z(x, t)$ (blue to red) in the vicinity of the emitting particle over the same time period (yellow line).

Despite the apparent complexity of Eq. (5), the motion of the particle can be explained qualitatively using Fig. 7. This figure shows that in this moderately relativistic regime, contributions to the acceleration principally come from the $\mathbf{F}/\gamma m$ components that are displayed in Figs. 7(a) and 7(b). We divide the trajectory into four main phases, denoted by I–IV, which begins when the plasma surface is compressed and the particle starts to experience the strong electromagnetic fields at $t/T_L \approx 9.25$.

(I) The restoring qE_x (Coulomb) force accelerates the particle in the reflected direction. As $|v_x|$ becomes relativistic, the $-qv_x B_z$ force accelerates the particle transversely. For this particular laser half-period, this transverse acceleration is in the downward direction.

FIG. 8. PIC code simulation results showing emitting electron dynamics in a relativistically transparent target. The electron density (gray to black) is overlaid with the (a) $E_x(x, t)$, (b) $E_y(x, t)$, and (c) $B_z(x, t)$ field components (blue to red). The position of one macroelectron which typifies the collective motion is plotted with the yellow line. The black dots in (d) show the trajectory of this selected particle in the xy plane and the yellow arrows represent the magnitude and direction of the acceleration vector at each point. In this case, the particle exhibits a figure-eight motion. Also shown are the (e) velocity and (f) acceleration components of this particle.

(II) The qE_y force provides a strong transverse acceleration that opposes and overcomes the earlier acceleration by the $-qv_x B_z$ force. In this case, this net acceleration is in the upward direction. This causes the transverse momentum of the particle $p_y/m_e c$ to pass through zero and change sign, which coincides with the reflected emission point.

During phases III and IV, the particle passes through a node of E_y and an antinode of B_z . The relativistic particle now undergoes a rotation by the strong B_z field.

(III) The $qv_y B_z$ force accelerates the particle in the forward direction, attaining a relativistic longitudinal velocity.

(IV) The $-qv_x B_z$ force accelerates the particle transversely, bringing $p_y/m_e c$ towards zero. In this case the acceleration is in the downward direction. This reduction of the transverse momentum to zero coincides with the forward emission point.

Following phase IV, the particle crosses the surface of the plasma, is screened from the driving fields, and continues at a constant velocity towards the rear side of the target. At this point, the plasma surface is again compressed and the process repeats with a different group of electrons. However, since the sign of the transverse fields is switched, the orientation of the particle trajectory is inverted from clockwise to counterclockwise, as previously discussed with reference to Fig. 3.

D. Comparison to the figure-eight trajectory type

We now compare the identified CSE trajectory type to that found in a relativistically transparent plasma. For this comparison, results from a second PIC simulation are used. The input parameters are the same as those listed in Sec. II, except the plasma is much thinner, consisting solely of a 30-nm top-hat region without a front-side density ramp.

Figure 8 shows results from this second simulation. Figures 8(a), 8(b), and 8(c) show the electron density (gray to black) overlaid with the $E_x(x, t)$, $E_y(x, t)$, and $B_z(x, t)$ field components (blue to red), respectively. Here the time axis has been restricted to one laser period at the peak of the simulated interaction. From these plots, the transparency of the plasma is apparent since the transverse fields propagate through the target and remain in phase with each other. This is in contrast to the formation of nodes and antinodes observed in front of the opaque plasma surface as previously discussed with reference to Fig. 2.

In this optimized case, the electrons collectively oscillate about the immobile ions in a regime which we call coherent whole foil motion. The position of one macroelectron which typifies the collective motion is plotted with the yellow line. The black dots in Fig. 8(d) show the trajectory of this selected particle in the xy plane and the yellow arrows represent the

magnitude and direction of the acceleration vector at each point. In this case, the particle exhibits a figure-eight motion similar to that identified in Ref. [29]. The orientation of this trajectory, which comes fundamentally from the driving electromagnetic fields, is denoted by the black curved arrow. The velocity and acceleration components of this particle are shown in Figs. 8(e) and 8(f), respectively.

At the bottom and top of the 8, which occur at $t/T_L \approx 10$ and 10.4, respectively, the particle has a relativistic longitudinal velocity in the reflected direction while simultaneously experiencing a strong transverse acceleration. Therefore, a burst of synchrotron emission is produced in the reflected direction at both of these points. However, negligible high-frequency emission is produced in the forward direction from this trajectory shape. This is because when the particle has a relativistic velocity in the forward direction, e.g., at $t/T_L \approx 10.25$, it does not experience any net acceleration. From Figs. 8(a)–8(c) it is clear that this is simply because the fields are near zero at this point in the trajectory.

IV. CONCLUSION

We conclude by summarizing our main results and highlighting some of their implications for future research.

In this article we identify a relativistic trajectory type whereby electrons are first pulled into the vacuum in front of the overdense plasma surface where they undergo roughly three-quarters of a loop before being injected into the plasma with a relativistic longitudinal velocity and a near-zero transverse velocity. This process repeats with a new group of electrons every half laser period, with the orientation of the loop alternating between clockwise and counterclockwise in the plane defined by the electric field polarization and propagation direction of the driving laser pulse.

When an electron bunch is accelerated into the identified trajectory shape, attosecond pulses are produced at two distinct points, one of which propagates in the reflected direction and the other in the forward direction. At both emission points the instantaneous motion of the bunch is synchrotronlike, with its relativistic longitudinal velocity being accompanied by a strong transverse acceleration. However, the character of this motion and the role of the electromagnetic fields immediately surrounding the two emission points are fundamentally different. For the reflected emission, the electron bunch is first pulled away from the target by the restoring Coulomb field which is set up following the compression of the plasma surface. As its longitudinal velocity becomes relativistic, the magnetic field of the laser pulse accelerates the bunch transversely. However, this transverse acceleration is then opposed and overcome by the electric field of the laser pulse. As the

transverse momentum of the bunch passes through zero a burst of synchrotron emission is produced which propagates in the reflected direction. For the forward emission, the bunch is accelerated first longitudinally and then transversely by the magnetic field of the laser pulse. Here the emission occurs immediately prior to the injection of the bunch into the plasma when its transverse momentum tends towards, but does not pass through, zero. This reduction towards zero may be seen as a consequence of the conservation of canonical momentum, since when the bunch enters the opaque target it is shielded from the strong low-frequency electromagnetic fields.

In a plasma that turns relativistically transparent during the interaction, a figure-eight trajectory is observed under optimized conditions which only emits strong attosecond radiation in the reflected direction. These contrasting electron dynamics are a consequence of the plasma opacity via its effects on the electromagnetic fields that drive the motion.

The simulation results in this article contribute towards the understanding of surface high-harmonic generation at a fundamental level. The trajectory type we observe provides a microscopic explanation for the recent experimental observations of forward harmonics [4,20,21], for which the figure-eight picture is insufficient. We identify the central role played by the conservation of canonical momentum which may provide an alternative starting point for theoretical modeling of forward attosecond emission. Our analysis of the fields and forces which drive CSE in this geometry provides a conceptual framework which can help facilitate optimization of the emission process towards its potential applications in attosecond pump-probe experiments. The observation that forward emission is strong for a laser pulse interacting with an opaque plasma, but weak with a transparent plasma under optimized conditions, may suggest an alternative route towards isolating an attosecond pulse. Carefully choosing the thickness of the target such that it becomes relativistically transparent during the peak interaction period may limit the time window during which forward emission occurs, via opacity gating, which could be used in conjunction with established techniques such as polarization gating [10,21,23,35]. In a broader context, these results also contribute to the understanding of plasma heating mechanisms (specifically relating to Brunel-type absorption [36]) and are therefore applicable to other research areas which employ this normal incidence geometry such as ion acceleration [37–39] and plasma shuttering [40].

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