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A new hierarchical approach for the optimal design of a 5-dof hybrid serial-parallel kinematic machine

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Abstract

Parallel kinematic machines are highly nonlinear and strongly coupled systems, which makes the driving-structural-dimensional parameters integrated design extremely difficult. To solve this problem, this paper proposes a new hierarchical optimal approach for a 5 degree-of-freedom hybrid serial-parallel kinematic machine. Considering application requirements in high speed machining, a global kinematic index, a global dynamic index, a global elastodynamic index as well as a global stiffness index are introduced and considered as objective functions. Taking the extension ratio and rotation angles as constraint conditions, optimal design is developed through a hierarchical optimization procedure. Results demonstrated that the proposed approach enables the robot to achieve good kinematic, dynamic and stiffness performance simultaneously and can be applied to other PKMs for machining.

Keywords: Parallel kinematic machine, optimal design, dimensional synthesis, stiffness, natural frequency

Notation

a	radius of the platform's circumscribed circle	M_k	the k th row of the mass matrix M^*
a_k	vector pointing from P to A_k in B_4 -xyz	$\tilde{M}$	mass matrix expressed in the joint space
b_k	distance between B_4 and B_k	m_p	mass matrix of the platform measured in B_4 -xyz
b_k	vector pointing from B_4 to B_k	n_j	direction of the spring axis
b_x	distance from B_4 to B_2 (B_3)	p	pitch of the lead-screw
b_y	distance from B_4 to B_1	Q^*	Coriolis force
C^*	coefficient matrix of $\dot{U}$	q_i	length of leg_i
C_p	compliance matrix of the PKM	$\dot{q}_i$	velocity of the i th prismatic joint
D_1	external diameter of stretchable limb	q_{4min}	minimum length of leg_4
D_2	external diameter of oscillating limb	q_{4max}	maximum length of leg_4
D_3	external diameter of thick limb	q_{max}	maximum length of the leg whose extension ratio is the largest among three UPS legs
D_4	external diameter of thin limb	q_{min}	minimum length of the leg whose extension ratio is the largest among three UPS legs
D^{ai}	transformation matrix of U_p with respect to ρ_{Ai}	R	radius of the task workspace
D^{li}	transformation matrix of U_p with respect to δ_{Ai}	R	rotation transformation matrix of P -uvw with respect to B_4 -xyz
f^*	driving force of the system	R_i	transformation matrix of B_i -xyz with respect to B_4 -xyz
F	synthesis index with weight factors	r_{Ai}	vector pointing from B_4 to point A_i
F	global external force vector	r_j	direction and distance to the line of action
F^*	external force applied at the platform in the rigid dynamic model	r_p	position of the point P
F_1	global kinematic index	s_j	unit eigenscrew to eigenstiffness λ_j
$\bar{f}_1$	average of the first order natural frequency	t_1	thickness of stretchable limb

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F_2	global dynamic index	t_2	thickness of oscillating limb
F_3	global elastodynamic index	t_3	thickness of thick limb
F_4	global stiffness index	t_4	thickness of thin limb
$\bar{F}_1$	average value of F_1	T_{Ai}	reaction moment of joint A_i
$\bar{F}_2$	average value of F_2	T_i	nodal transformation matrix of $B_i\text{-}x_iy_iz_i$ with respect to $B_4\text{-}xyz$
$\bar{F}_3$	average value of F_3	T_{Mi}	reaction force applied at joint M_i ($M=A, B$) in $B_4\text{-}xyz$
F_{Ai}	reaction force of joint A_i	$\mathbf{u}$	unit vector of u axis
$\mathbf{f}_i$	external force vector of the leg_i	$\mathbf{U}$	global coordinate vector
$\mathbf{f}_{Mi}$	reaction force applied at joint M_i ($M=A, B$) in $B_i\text{-}x_iy_iz_i$	$\dot{\mathbf{U}}^*$	velocity of the platform
$\mathbf{F}_{Mi}$	reaction force applied at joint M_i ($M=A, B$) in $B_4\text{-}xyz$	$\ddot{\mathbf{U}}^*$	acceleration of the platform
$\mathbf{f}_p$	external force including the platform's gravity	$\mathbf{u}_i$	coordinate vector of the leg_i
$\mathbf{G}^*$	gravity of the system	$\mathbf{v}$	unit vector of v axis
h	height of the task workspace	V	volume of the task workspace
H	distance between the right plane of the task workspace and the global coordinate system	$\mathbf{v}_p$	linear velocity of point P
h_j	pitch of s_j	w_{4x}	coordinate in the x axis of $\mathbf{w}_4$
I_k	equivalent load moment of inertia applied to the servomotor	w_{4y}	coordinate in the y axis of $\mathbf{w}_4$
$I_{k\text{mean}}$	average value throughout the whole task workspace	w_{4z}	coordinate in the z axis of $\mathbf{w}_4$
I_M	rotor inertia of the servomotor	$\mathbf{w}_i$	unit vector of leg_i
I_P	inertia matrix of the platform measured in $B_4\text{-}xyz$	$\mathbf{w}_P$	angular velocity of the platform
$\mathbf{J}$	generalized Jacobian matrix	ψ	Euler angle about the x axis
$\mathbf{J}_a$	active Jacobian matrix	ψ_{i0}	allowable angle between the y_i axis and y axis
$\mathbf{J}_c$	constraint Jacobian matrix	θ	Euler angle about the v axis
$\mathbf{J}_{pa}$	homogeneous Jacobian matrix	θ_{i0}	allowable angle between the z_i axis and z axis
$\mathbf{K}$	stiffness matrix of the PKM	μ	extension ratio
k^*	average stiffness of six spring constants at any configuration	μ_0	allowable extension ratio
$\bar{k}^*$	average of k^* throughout the whole task workspace	η	condition number of the homogeneous Jacobian matrix
$\mathbf{k}_i$	stiffness matrix of the leg_i	λ	eigenstiffness of the stiffness matrix $\mathbf{K}_p$
k_j	spring constant	ζ	eigenscrew of the stiffness matrix $\mathbf{K}_p$
k_{Mix}	linear stiffness of joint M_i ($M=A, B$) in x_i axis	δ_{Ai}	linear displacement of the interface point A_{iP}
k_{Miy}	linear stiffness of joint M_i ($M=A, B$) in y_i axis	ρ_{Ai}	angular displacement of the interface point A_{iP}
k_{Miz}	linear stiffness of joint M_i ($M=A, B$) in z_i axis	ε_{Ai}	linear displacement of the interface point A_{iL}
k_{Miu}	angular stiffness of joint M_i ($M=A, B$) about x_i axis ($i=1\sim 4$)	$\mathbf{e}_p$	linear displacement of the platform
k_{Miv}	angular stiffness of joint M_i ($M=A, B$) about y_i axis	ζ_{Ai}	angular displacement of the interface point A_{iL}

	axis ($i=1\sim4$)		
k_{Miw}	angular stiffness of joint M_i ($M=A, B$) about z_i axis ($i=1\sim4$)	ξ_p	angular displacement of the platform
k_{Mi}	linear stiffness matrix of joint M_i ($M=A, B$)	$\tau_{a,k}$	driving torque of leg_k
k_{M2i}	angular stiffness matrix of joint M_i ($M=A, B$)	$\tau_{a,kmax}$	maximum driving force required to realize the unit acceleration of the platform
K_p	stiffness of the platform	τ_p	external moment at the platform
l_1	length of stretchable limb	τ_{Mi}	reaction moment applied at joint M_i ($M=A, B$) in $B_i-x_iy_iz_i$
l_2	length of oscillating limb	λ_j	eigenstiffness
l_3	length of thick limb	$\lambda_{a/b}$	platform/base ratio
l_4	length of thin limb	$\lambda_{R/b}$	workspace/footprint ratio
M	mass matrix of the PKM in the elastodynamic model	$\lambda_{h/R}$	height/radius ratio of the task workspace
M^*	mass matrix in the rigid dynamic model	Δ	conversion between two types of coordinates
m_i	mass matrix of the leg_i	ω_{Lk}	angular velocity of the leg_k

1. Introduction

Parallel kinematic machines (PKMs) have been applied in the locomotive and aeronautical industries due to high stiffness, high speed and high accuracy. However, the obvious disadvantage of PKMs is their limited workspace due to distributions of legs. In order to increase workspace and dexterity of PKMs, hybrid serial-parallel kinematic machines (HSPKMs) are proposed by connecting serial kinematic machines (SKMs) serially to PKMs. Therefore, HSPKMs will present good performance of PKMs and large workspace of SKMs theoretically, which makes them an alternative solution to high speed machining (HSM). This has been fully exemplified by the commercial success of Exechon [1,2] and the Tricept robots [3,4]. However, most existing HSPKMs are still in the laboratory stage. Inspired by the Tricept and Exechon, a novel HSPKM named Trimule [5] was proposed by Tianjin University as shown in Fig. 1. The HSPKM consists of a 3 degree-of-freedom (DOF) PKM for positioning and a 2-DOF SKM for machining, which can be used in milling and drilling for large scale components.

Fig. 1. A CAD model of the Trimule robot.

Optimal design of this kind of robots in the early design stage is very essential, which will influence the machining accuracy, machining efficiency and control performance of PKMs for HSM. Constructing an optimization model consists of defining *design variables*, *objective functions* and *constraint conditions*. In general, design variables can be classified into *driving parameters* (mainly referring to parameters of the driving system, such as speed, torque and rotor moment of servomotors), *dimensional parameters* (mainly including size of the platform and base, length of legs) and *structural*

parameters (parameters of legs' cross-sections, for example, radius, thickness, width and height). Due to the complicated structural features of this kind of PKMs, most work related to the early optimal design is single objective optimization based on a simple and intuitive kinematic performance. Kinematic indices are mainly related to dexterity, workspace, and motion/force transmission measures [6-10].

However, optimal design based on kinematic indices cannot guarantee good dynamic and stiffness performance especially when PKMs are working at a high speed and high acceleration, which will affect machining efficiency and accuracy in practice. Therefore, combining kinematic indices and dynamic/stiffness indices, two-objective optimization is conducted. Dynamic indices are usually based on the system's inertial matrix. In order to minimize the required driving force and maximize the reachable workspace, optimal design of both symmetric and unsymmetrical hexapod PKMs was conducted [11]. Based on the torque and power indices, dynamic optimization of the Delta robot was conducted [12]. Different from pick-and-place parallel manipulators, PKMs usually work at a relatively low speed. Therefore, dynamic optimization of PKMs is very scarce. In order to achieve good position accuracy, stiffness needs to be considered. In order to maximum system stiffness and workspace, optimization for a 3-DOF parallel manipulator was conducted [13]. It is noted that only actuator's stiffness is considered in most literatures, while joints and limbs are assumed to be rigid [14].

Even stiffness indices are considered, elastodynamic indices should be considered for PKMs may own large inertia, which is also conducive to vibration reduction and vibration suppression. Considering the conditional number of the Jacobian matrix and the first order natural frequency simultaneously, Wu et al [15] presented optimal design of a 2-DOF PKM with redundancy. Taking workspace and the first order resonance frequency as objective functions, optimization of the Gantry-Tau PKM was conducted [16]. Actually, due to the complex structures, there is no too much work about optimization based on elastodynamic indices of PKMs for HSM.

It is noted that optimization with two indices are not enough according to application requirements of PKMs. Thus, more and more work is focused on multi-objective optimization to achieve a good and comprehensive performance system. Taking workspace volume, dexterity, static efficiency and stiffness as objective functions, Russo et al proposed a multi-objective optimization method for a 3-UPR PKM for a robotic leg [17]. Considering motion transmission, velocity transmission and acceleration transmission, the multi-objective optimization of a 3-PUU PKM was carried out by the improved non-dominated sorting genetic algorithm [18]. Combining transmission, stiffness and dynamic indices, architecture optimization of a parallel manipulator for pick-and-place applications was conducted [19].

Although there are many work focused on optimization design of PKMs, most design variables in the published literatures are mainly about dimensional parameters. Optimization of structural parameters and matching design of driving parameters to the whole system are seldom discussed. Actually, structural and driving parameters have an important influence on the system's stiffness and dynamic performance, which should be considered in the optimal design [20-23]. Since a PKM is a highly nonlinear and strongly coupled system with multi-inputs and multi-outputs, it will be a challenging but meaningful work to deal with the integrated design of dimensional, structural and driving parameters.

The Trimule robot is a new type of HPKMs and has potential to be applied in HSM. Thus, this paper will deal with the integrated design of driving-dimensional-structural parameters of this robot. Considering application requirements, a kinematic index, two dynamic indices as well as a stiffness index are introduced and taken as objective functions. Since a lot of design variables are involved, it would be a huge workload to optimize all with one-time optimization. Therefore, a hierarchical approach is proposed. Besides, the proposed approach can make each design stage concise. Considering the coupling relationships between indices and design variables, dimensional synthesis, structural parameters optimization and matching design of driving parameters are conducted step by step. The reminder of this paper is organized as follows. Inverse kinematics of the Trimule robot is described in Section 2, followed by the definition of design variables and constraint conditions in Section 3. A performance evaluation system is built in Section 4, based on which a hierarchical optimal design method is introduced in Section 5. Finally, main conclusions are drawn in Section 6.

2. Inverse kinematics of the robot

As shown in Fig. 1, the Trimule mainly consists of one base, one platform, four legs, one A axis and one C axis. The four legs are denoted as $leg_1 \sim leg_4$, respectively. $leg_1 \sim leg_3$ connect to the base by a universal joint (U joint) while connecting to the platform through a spherical joint (S joint). Therefore, they have an identical UPS architecture, where P stands for a linear actuator. leg_4 is slightly different from the above three legs in that it is connected to the platform fixedly. Kinematically, leg_4 can be regarded as a UP architecture, where P represents a prismatic joint. Structure of the A axis and C axis is relatively simple, which will not be analyzed in the followings.

Then, the inverse kinematic solutions, namely the length and unit vector of the leg_k can be derived

$$q_k = |\mathbf{r}_p + \mathbf{a}_k - \mathbf{b}_k|, \mathbf{w}_k = (\mathbf{r}_p + \mathbf{a}_k - \mathbf{b}_k) / q_k. \quad (8)$$

Taking derivation of Eqs. (2) and (3) yields

$$\mathbf{v}_p = \dot{q}_k \mathbf{w}_k + q_k (\boldsymbol{\omega}_{Lk} \times \mathbf{w}_k) - \boldsymbol{\omega}_p \times \mathbf{a}_k, \quad (9)$$

$$\mathbf{v}_p = \dot{q}_4 \mathbf{w}_4 + q_4 (\boldsymbol{\omega}_p \times \mathbf{w}_4), \quad (10)$$

where $\mathbf{v}_p$ and $\mathbf{w}_p$ are the linear velocity of the point P and the angular velocity of the platform, respectively; $\dot{q}_i$ and $\boldsymbol{\omega}_{Lk}$ denote the velocity of the i th prismatic joint and the angular velocity of the leg_k , respectively.

Taking inner dot of Eq. (9) by $\mathbf{w}_k$ leads to

$$\dot{q}_k = \mathbf{w}_k^T \mathbf{v}_p + (\mathbf{a}_k \times \mathbf{w}_k)^T \boldsymbol{\omega}_p. \quad (11)$$

Taking inner dot of Eq. (10) by $\mathbf{u}$ and $\mathbf{v}$ ($\mathbf{u}$ and $\mathbf{v}$ are unit vectors of u and v axes) respectively yields

$$\mathbf{u}^T \mathbf{v}_p + q_4 (\mathbf{u} \times \mathbf{w}_4)^T \boldsymbol{\omega}_p = 0, \quad \mathbf{v}^T \mathbf{v}_p + q_4 (\mathbf{v} \times \mathbf{w}_4)^T \boldsymbol{\omega}_p = 0. \quad (12)$$

According to the principle of linear superposition, the angular velocity of the platform can be expressed as

$$\boldsymbol{\omega}_p = \dot{\psi} \mathbf{x} + \dot{\theta} \mathbf{v}. \quad (13)$$

Taking inner dot of Eq. (13) by $\mathbf{n} = \mathbf{v} \times \mathbf{x}$ yields

$$\mathbf{n}^T \boldsymbol{\omega}_p = 0. \quad (14)$$

Eqs. (11)~(12) and (14) can be written in a matrix form as follows

$$\mathbf{J} \begin{bmatrix} \mathbf{v}_p \\ \boldsymbol{\omega}_p \end{bmatrix} = \dot{\mathbf{q}}, \quad (15)$$

where $\dot{\mathbf{q}} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad 0 \quad 0 \quad 0]^T$; $\mathbf{J} = \begin{bmatrix} \mathbf{J}_a \\ \mathbf{J}_c \end{bmatrix}$ is the generalized Jacobian matrix of the PKM module, $\mathbf{J}_a$ and $\mathbf{J}_c$ are defined as the active Jacobian matrix and the constraint Jacobian matrix respectively, which can be expressed as

$$\mathbf{J}_a = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 \\ \mathbf{a}_1 \times \mathbf{w}_1 & \mathbf{a}_2 \times \mathbf{w}_2 & \mathbf{a}_3 \times \mathbf{w}_3 \end{bmatrix}^T, \quad \mathbf{J}_c = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{0} \\ q_4 (\mathbf{u} \times \mathbf{w}_4) & q_4 (\mathbf{v} \times \mathbf{w}_4) & \mathbf{n} \end{bmatrix}^T. \quad (16)$$

3. Question description and constraint conditions

3.1. Design variables

Design variables involved in this paper are driving parameters (parameters of selected servomotors), structural parameters (external diameters and thickness of $legs$ 1~4) and dimensional parameters (size of the platform and base, length of $legs$ 1~4 and the distance between the task workspace and the origin of the global coordinate system). To make it clear, relationship between the task workspace and dimensional parameters is depicted in Fig. 3. The task workspace is referred to a cylinder whose radius is R and height is h . Herein, H represents the distance between the right plane of the task workspace and the global coordinate system. b_x and b_y are distance from B_4 to B_2 (B_3) and B_1 , respectively. Since the base is an equilateral triangle, there exists $b_y = \sqrt{3}b_x$.

As can be seen from Fig. 3, the minimum length and maximum length of leg_4 can be expressed as

$$q_{4\min} = H, \quad q_{4\max} = \sqrt{R^2 + (H + h)^2}. \quad (17)$$

Similarly, the minimum length and maximum length of $leg_1 \sim leg_3$ can be expressed in certain equations related to a , b_x , b_y , R , h and H . For the convenience of serialization design, take normalization with b_x as

$$\lambda_{a/b} = a / b_x, \quad \lambda_{R/b} = R / b_x, \quad \lambda_{H/b} = H / b_x, \quad (18)$$

where $\lambda_{a/b}$ is defined as the platform/base ratio; $\lambda_{R/b}$ is defined as the workspace/footprint ratio.

Define $\lambda_{h/R} = h / R$ as the height/radius ratio of the task workspace, which has an allowable range of 0.4~0.5 when referring to design of Tricept. Therefore, optimization of this PKM can be described as *how to determine design parameters to make the PKM perform best with the given $\lambda_{h/R} = h / R$ and corresponding constraint conditions.*

Fig. 3. Relationship between the task workspace and dimensional parameters.

3.2. Constraint conditions

In this paper, the following two constraint conditions are considered.

In order to assure the lateral stiffness of each UPS leg, the extension ratio is defined as

$$\mu = \frac{q_{\max} - q_{\min}}{q_{\min}} \leq \mu_0, \quad (19)$$

where μ_0 is the allowable extension ratio, which is usually set to be 0.7~0.8. $q_{\max}$ and $q_{\min}$ are the maximum and minimum length of the leg whose extension ratio is the largest among the three UPS legs.

Rotation angle of the universal joint should be limited as

$$|\theta_i|_{\max} \leq \theta_{i0}, |\psi_i|_{\max} \leq \psi_{i0}, \quad (20)$$

where θ_{i0} and ψ_{i0} represent the two allowable rotation angles of the universal joint in leg_i , respectively. To be specific, ψ_{i0} denotes the allowable angle between the y_i axis and y axis while θ_{i0} represents the allowable angle of leg_i about the y_i axis.

4. Performance evaluation system

In order to evaluate the performance of the system comprehensively, a performance evaluation system is proposed, which includes the kinematic index, dynamic index, elastodynamic index and stiffness index.

4.1. Global kinematic index

The condition number of a Jacobian matrix is a commonly used index to evaluate motion/force transfer characteristics, which can be described as the degree close to singularity. It is noted that the homogenous Jacobian matrix should be derived first if one PKM owns both translation and rotation mobility [28,29].

Making separation of Eq. (15) results in

$$\mathbf{J}_{pa} \mathbf{v}_p = \dot{\mathbf{q}}_a, \quad (21)$$

where $\mathbf{J}_{pa} = \mathbf{J}_{pv} - \mathbf{J}_{vw} \mathbf{J}_{ww} \mathbf{J}_{wv}$, $\mathbf{J}_{pv} = [\mathbf{w}_1 \quad \mathbf{w}_2 \quad \mathbf{w}_3]$, $\mathbf{J}_{vw} = [\mathbf{a}_1 \times \mathbf{w}_1 \quad \mathbf{a}_2 \times \mathbf{w}_2 \quad \mathbf{a}_3 \times \mathbf{w}_3]$, $\mathbf{J}_{wv} = [\mathbf{u} \quad \mathbf{v} \quad \mathbf{0}]$, $\mathbf{J}_{ww} = [q_4(\mathbf{u} \times \mathbf{w}_4) \quad q_4(\mathbf{v} \times \mathbf{w}_4) \quad \mathbf{n}]$, $\dot{\mathbf{q}}_a = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3]^T$.

As can be found from Eq. (21), units of $\mathbf{v}_p$ and $\dot{\mathbf{q}}_a$ are the same. Therefore, $\mathbf{J}_{pa}$ is a homogeneous Jacobian matrix. The condition number of the homogeneous Jacobian matrix $\mathbf{J}_{pa}$ can be expressed as

$$\eta = \text{cond}(\mathbf{J}_{pa}). \quad (22)$$

Taking the Trimule with parameters in [30,31] as an example, distributions of the condition number over a working plane are depicted in Fig. 4. As shown, distributions of the condition number are strongly position-dependent and demonstrate plane symmetry.

Fig. 4. Distributions of the condition number over the working plane $z=0.7$ m.

In order to evaluate the kinematic index throughout the whole task workspace, a global kinematic index can be expressed as

$$F_1 = \frac{\int_V \eta dV}{\int_V dV}, \quad (23)$$

where V is the volume of the task workspace. Obviously, the smaller the F_1 , the better the PKM's kinematic performance.

4.2. Global dynamic index

For PKMs applied to HSM, the acceleration ability is required to improve the PKM's working efficiency. The acceleration ability is referred to the driving torque required to make the platform realize a unit velocity/acceleration with a unit load. Therefore, a rigid dynamic model is required.

Based on the principle of virtual work, the rigid dynamic model of the system can be derived

$$\mathbf{M}^* \ddot{\mathbf{U}}^* + \mathbf{C}^* \dot{\mathbf{U}}^* + \mathbf{G}^* + \mathbf{Q}^* - \mathbf{J}^T \mathbf{F}^* = \mathbf{f}^*, \quad (24)$$

where $\dot{\mathbf{U}}^*$ and $\ddot{\mathbf{U}}^*$ are the velocity and acceleration of the platform, respectively. $\mathbf{M}^*$ is the mass matrix, $\mathbf{C}^*$ is the coefficient matrix of $\dot{\mathbf{U}}^*$, $\mathbf{G}^*$ and $\mathbf{Q}^*$ are the gravity and Coriolis force of the system. $\mathbf{F}^*$ is the external force applied at the platform. $\mathbf{f}^*$ is the driving force of the system. For content limitation, the detailed derivation will not be addressed in this paper.

In general, effects of $\mathbf{C}^* \dot{\mathbf{U}}^*$ and $\mathbf{Q}^*$ on the driving force are relatively small, while that of the gravity can be compensated by the feedforward control. Therefore, the driving force is mainly caused by the inertia force when the external force is zero. Accordingly, the driving torque of leg_k can be expressed

$$\tau_{a,k} = \frac{p}{2\pi} \mathbf{M}_k \ddot{\mathbf{U}}^*, \quad (25)$$

where $\mathbf{M}_k$ is the k th row of the mass matrix $\mathbf{M}^*$, p is the pitch of the lead-screw.

Based on the principle of singular value decomposition, the maximum of $2\pi\tau_{a,k}/p$ required to realize the unit acceleration of the platform ($\|\ddot{\mathbf{U}}^*\|=1$) can be expressed as

$$\tau_{a,k \max} = \sqrt{\mathbf{M}_k \mathbf{M}_k^T}. \quad (26)$$

Based on mass parameters in [30], distributions of the maximum driving force among the three actuators $\max_{k=1-3} \tau_{a,k \max}$ over a working plane are calculated and shown in Fig. 5. As shown, the maximum driving force among the three actuators is strongly position-dependent and symmetric to the plane of $x=0$. Moreover, variations of the maximum driving force among the three actuators with respect to y are not monotonous. To be specific, the maximum driving force among the three actuators decreases first then increases with the increment of y .

In order to enable the PKM to achieve good dynamic performance throughout the whole task workspace, the maximum value of $\max_{k=1-3} \tau_{a,k \max}$ throughout the whole workspace is defined as a global dynamic index

$$F_2 = \max_{x \in V} \max_{k=1-3} \tau_{a,k \max}. \quad (27)$$

Fig. 5. Distributions of $\max_{k=1\sim3} \tau_{a,k,\max}$ over the working plane $z=0.7$ m.

4.3. Global elastodynamic index

PKMs applied to HSM need to have good inherent behaviors to decrease vibration in high-speed machining to guarantee machining accuracy. Therefore, an elastodynamic model of this PKM should be proposed first. A stiffness model is presented in our previous work based on the substructure synthesis technique [31], which is proved with good accuracy when compared to finite element analysis. Inspired by the previous work [31], an elastodynamic model of this PKM is built in this paper.

4.3.1. Dynamic model of leg subsystems

Structural diagrams of the UPS leg and UP leg are shown in Fig. 6. In order to decrease mass of the PKM, limbs are made of carbon fiber. Considering tensile-compressional, torsional and bending deformation, limbs are modeled as Euler-Bernoulli beams. Accordingly, universal joints, spherical joints as well as the fixed joint are simplified into 6-dof springs with equivalent stiffness and mass at their geometrical centers.

Fig. 6. Structural diagrams of the UPS leg and UP leg.

Based on the above description, the UPS leg and UP leg can be simplified into two spatial beam systems constrained by two groups of springs, as shown in Fig. 7. Herein, k_{Aix} , k_{Aiy} , k_{Aiz} , k_{Aiu} , k_{Aiv} and k_{Aiw} are three linear stiffness and three angular stiffness coefficients of joint A_i ; k_{Bix} , k_{Biy} , k_{Biz} , k_{Biu} , k_{Biv} and k_{Biw} are three linear stiffness and three angular stiffness coefficients of joint B_i ($i=1\sim4$).

Fig. 7. Force diagrams of the UPS leg and UP leg.

For the convenience of derivation, the UPS leg and the UP leg are both divided into n elements, with A_i being the 1st node and C_i being the $(n+1)$ th node respectively. The dynamic equation of leg_i subsystem can be expressed in B_i - $x_i y_i z_i$ as

$$\mathbf{m}_i \ddot{\mathbf{u}}_i + \mathbf{k}_i \mathbf{u}_i = \mathbf{f}_i, \quad (28)$$

where $\mathbf{m}_i$ and $\mathbf{k}_i$ are the mass and stiffness matrices of the leg_i ; $\mathbf{u}_i$ and $\mathbf{f}_i$ are the coordinate vector and external force vector of the leg_i , respectively.

Eq. (28) can be expressed in the global coordinate system B_4 -xyz as

$$\mathbf{M}_i \ddot{\mathbf{U}}_i + \mathbf{K}_i \mathbf{U}_i = \mathbf{F}_i, \quad (29)$$

where $\mathbf{M}_i = \mathbf{T}_i \mathbf{m}_i \mathbf{T}_i^T$, $\mathbf{K}_i = \mathbf{T}_i \mathbf{k}_i \mathbf{T}_i^T$, $\mathbf{U}_i = \mathbf{T}_i \mathbf{u}_i$, $\mathbf{F}_i = \mathbf{T}_i \mathbf{f}_i$. $\mathbf{T}_i$ is the nodal transformation matrix of B_i -xyz with respect to the global coordinate system B_4 -xyz.

4.3.2. Dynamic model of the platform subsystem

Comparing to rigidity of $legs$ 1~4, that of the platform is relatively large. Thus, the platform is treated as a rigid body. The force diagram of the platform is shown in Fig. 8. Herein, $\mathbf{f}_p$ and $\boldsymbol{\tau}_p$ are the external force and moment including the platform's gravity; $\mathbf{F}_{Ai}$ and $\mathbf{T}_{Ai}$ are reaction force and moment of joint A_i , $\mathbf{r}_{Ai}$ is the vector pointing from B_4 to point A_i .

Fig. 8. Force diagram of the platform.

Assuming that $\boldsymbol{\varepsilon}_p$ and $\boldsymbol{\xi}_p$ stand for linear displacement and angular displacement of the platform respectively, the dynamic equations of the platform subsystem can be expressed in B_4 -xyz as

$$-\sum_{i=1}^4 \mathbf{F}_{Ai} + \mathbf{f}_p = \mathbf{m}_p \ddot{\boldsymbol{\varepsilon}}_p, \quad -\sum_{i=1}^4 \mathbf{r}_{Ai} \times \mathbf{F}_{Ai} + \boldsymbol{\tau}_p - \sum_{i=1}^4 \mathbf{T}_{Ai} = \mathbf{I}_p \ddot{\boldsymbol{\xi}}_p, \quad (30)$$

where $\mathbf{m}_p$ and $\mathbf{I}_p$ are the mass and inertia matrices of the platform measured in B_4 -xyz.

4.3.3. Deformation compatibility conditions

Displacement relationship between the platform and leg_i is demonstrated in Fig. 9, where A_{ip} and A_{il} are interface points on the platform and leg_i . $\boldsymbol{\delta}_{Ai}$ and $\boldsymbol{\rho}_{Ai}$ are linear and angular displacements of the interface point A_{ip} while $\boldsymbol{\varepsilon}_{Ai}$ and $\boldsymbol{\xi}_{Ai}$ are linear and angular displacements of the interface point A_{il} . $\mathbf{k}_{A1i}$ and $\mathbf{k}_{A2i}$ are the linear stiffness matrix and angular stiffness matrix of joint A_i .

Fig. 9. Displacement relationship between the platform and leg_i .

Linear and angular displacements of the interface point A_{ip} can be expressed as

$$\boldsymbol{\delta}_{Ai} = \mathbf{R}_i^T \mathbf{D}^l \mathbf{U}_p, \quad \boldsymbol{\rho}_{Ai} = \mathbf{R}_i^T \mathbf{D}^a \mathbf{U}_p, \quad (31)$$

where $\mathbf{R}_i$ is the transformation matrix of B_i -xyz with respect to B_4 -xyz; $\mathbf{U}_p = [\boldsymbol{\varepsilon}_p \quad \boldsymbol{\xi}_p]^T$; $\mathbf{D}^l$ and $\mathbf{D}^a$ are transformation matrices of $\mathbf{U}_p$ with respect to $\boldsymbol{\delta}_{Ai}$ and $\boldsymbol{\rho}_{Ai}$.

Therefore, reactions applied at joint A_i can be expressed in the coordinate system B_i -xyz as

$$\mathbf{f}_{Ai} = -\mathbf{k}_{A1i} (\boldsymbol{\varepsilon}_{Ai} - \boldsymbol{\delta}_{Ai}), \quad \boldsymbol{\tau}_{Ai} = -\mathbf{k}_{A2i} (\boldsymbol{\xi}_{Ai} - \boldsymbol{\rho}_{Ai}). \quad (32)$$

Eq. (32) can be expressed in the global coordinate system B_4 -xyz as

$$\mathbf{F}_{Ai} = \mathbf{R}_i \mathbf{f}_{Ai}, \quad \mathbf{T}_{Ai} = \mathbf{R}_i \boldsymbol{\tau}_{Ai}. \quad (33)$$

Similarly, reactions applied at joint B_i can be expressed as

$$\mathbf{f}_{Bi} = -\mathbf{k}_{B1i} \boldsymbol{\varepsilon}_{Bi}, \quad \boldsymbol{\tau}_{Bi} = -\mathbf{k}_{B2i} \boldsymbol{\xi}_{Bi}, \quad (34)$$

where $\mathbf{k}_{B1i}$ and $\mathbf{k}_{B2i}$ are the linear stiffness matrix and angular stiffness matrix of the universal joint in

leg_i, ϵ_{Bi} and ζ_{Bi} are linear and angular displacements of joint B_i .

Eq. (34) can be expressed in the global coordinate system B_4 -xyz as

$$\mathbf{F}_{Bi} = \mathbf{R}_i \mathbf{f}_{Bi}, \quad \mathbf{T}_{Bi} = \mathbf{R}_i \mathbf{\tau}_{Bi}. \quad (35)$$

4.3.4. Dynamic Equation of the PKM Module

Substituting Eqs. (33) and (35) into (29) and (30), the dynamic equation of the PKM module can be obtained

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}. \quad (36)$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass matrix and stiffness matrix of the PKM; $\mathbf{U}$ and $\mathbf{F}$ are the global coordinate vector and external force vector of the PKM.

4.3.5. Definition of the elastodynamic index

Supposing that the external force $\mathbf{F}$ is equal to zero, natural frequencies of the system can be derived by solving Eq. (36). Distributions of the first two order natural frequencies over a working plane are shown in Fig. 10. Obviously, the first two order natural frequencies are strongly position-dependent and symmetric to the plane of $x=0$.

Fig. 10. Distributions of lower natural frequencies over the working plane $z=0.7$ m.

As shown in Fig. 10, the first two order natural frequencies are very close and distributions of which are the same over the task workspace. Therefore, a global elastodynamic index based on the first order natural frequency is expressed

$$F_3 = \sqrt{\frac{\int_V (f_1 - \bar{f}_1)^2 dV}{\int_V dV}}, \quad (37)$$

where $\bar{f}_1$ is the average of the first order natural frequency f_1 throughout the whole task workspace. F_3 stands for the fluctuation of the first order natural frequency over the whole task workspace. The smaller the F_3 , the more uniform the dynamic performance, which is helpful to the design of controller.

4.4. Global Stiffness index

It can be concluded that stiffness of a PKM is relatively high if its lower natural frequencies are high. However, it cannot make sure that distributions of the stiffness are uniform. Uniform stiffness distributions are helpful to the selection of machining area, error compensation and accuracy control.

There is a typical feature in the PKM that legs are coupled. Therefore, the system's stiffness matrix $\mathbf{K}$ in Eq. (36) is off-diagonal, making it difficult to propose an individual stiffness index. Meanwhile, units in the stiffness matrix $\mathbf{K}$ are not unified. Based on the above discussions, decomposition based on the screw theory will be addressed as follows.

In order to derive stiffness of the platform, the compliance matrix of the PKM must be obtained by conducting inverse operation of the stiffness matrix $\mathbf{K}$. Extracting the lower right corner 6×6 block matrix from the PKM's compliance matrix as $\mathbf{C}_p$, stiffness of the platform can be expressed by

$$\mathbf{K}_p = \text{diag}(\mathbf{R}^T, \mathbf{R}^T) \mathbf{C}_p^{-1} \text{diag}(\mathbf{R}, \mathbf{R}). \quad (38)$$

The eigenscrew problem of $\mathbf{K}_p$ can be described in ray coordinates as

$$\mathbf{K}_p \mathbf{A} \boldsymbol{\zeta} = \lambda \boldsymbol{\zeta}, \quad (39)$$

where the eigenvalue λ and the screw $\boldsymbol{\zeta}$ are defined as the eigenstiffness and eigenscrew of the stiffness matrix $\mathbf{K}_p$; $\mathbf{A}$ denotes the conversion between two types of coordinates, which can be expressed

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad (40)$$

where $\mathbf{0}$ and $\mathbf{I}$ denote a zero matrix and an identity matrix in 3×3 , respectively.

Then the eigenscrew decomposition of the stiffness matrix can be expressed as

$$\mathbf{K}_p = \sum_{j=1}^6 k_j \mathbf{s}_j \mathbf{s}_j^T, \quad (41)$$

where $\mathbf{s}_j$ is the unit eigenscrew corresponding to eigenstiffness λ_j ; k_j is the spring constant. And there exist

$$\mathbf{s}_j = \begin{pmatrix} \mathbf{n}_j \\ \mathbf{r}_j \times \mathbf{n}_j + h_j \mathbf{n}_j \end{pmatrix}, k_j = \frac{\lambda_j}{2h_j}, h_j = \frac{1}{2} \mathbf{s}_j^T \Delta \mathbf{s}_j, \quad (42)$$

where $\mathbf{n}_j$ is a unit vector representing the direction of the spring axis; $\mathbf{r}_j$ identifies the direction and distance to the line of action; h_j is the pitch of $\mathbf{s}_j$ ($j=1\sim6$).

Therefore, the PKM at any configuration can be regarded as a rigid body supported by six screw springs $\mathbf{n}_j$ with spring constant k_j . Distributions of the decomposed stiffness over a working plane are shown in Fig. 11. Obviously, the six spring constants are position-dependent. Furthermore, distributions of the 1st spring constant k_1 are symmetric to those of the 6th spring constant k_6 about the plane of $x=0$. This phenomenon can be found in distributions of the 2nd spring constant k_2 and the 5th spring constant k_5 as well as the 3rd spring constant k_3 and the 4th spring constant k_4 . Therefore, the first three spring constants can be used to evaluate stiffness of the system in a certain situation.

Fig. 11. Distributions of decomposed stiffness over the working plane $z=0.7$ m.

Based on the above discussions, a global stiffness index is proposed

$$F_4 = \sqrt{\frac{\int_V (k^* - \bar{k}^*)^2 dV}{\int_V dV}}, \quad (43)$$

where $k^* = \frac{1}{6} \sum_{j=1}^6 k_j$ is the average stiffness of the six spring constants at any configuration;

$\bar{k}^* = \frac{\int_V k^* dV}{\int_V dV}$ is the average of k^* throughout the whole task workspace. F_4 demonstrates the

fluctuation of the system's stiffness. Obviously, if the global stiffness index becomes smaller, distributions of the PKM's stiffness throughout are more uniform, which can make machining accuracy easy to control.

5. Optimal design

The radius R and height h of the task workspace are set to be 300 mm and 150 mm, respectively. The design parameters include driving parameters (mainly refer to the inertia of a motor rotor), dimensional parameters (a , b_x , b_y , H , l_1 , l_2 , l_3 and l_4) and structural parameters (external diameters: D_1 , D_2 , D_3 and D_4 ; thickness: t_1 , t_2 , t_3 and t_4). Since there are many design variables, it would be a huge workload to identify all variables after one time optimization. Therefore, a hierarchical optimization design process is depicted in Fig. 12.

Fig. 12. A hierarchical procedure for optimization design.

The hierarchical procedure can be described as follows. Firstly, driving motors are selected initially according to engineering experience. Secondly, structural parameters are discrete in a reasonable range and a group of structural parameters are selected initially to provide inertial parameters. Thirdly, dimensional synthesis can be conducted based on constraint conditions and objective functions. Then, based on the derived dimensional parameters from Step 3, structural parameters optimization can be conducted. If the derived structural parameters are not consistent with the initial ones in Step 2, another group of structural parameters should be chosen and dimensional synthesis in Step 3 should be repeated until the terminating condition TC 1 is satisfied. When the terminating condition TC 1 is satisfied, the selected motors in Step 1 should be judged whether they are matched with the optimal structural and dimensional parameters. If so, one group of optimal design parameters can be obtained. Otherwise, another motor should be chosen and the procedure should be repeated.

The initial structural parameters are listed in Table 1. Herein, D_1 , D_2 , D_3 , D_4 , t_1 , t_2 , t_3 and t_4 are external diameters and thickness of the stretchable limb, oscillating limb, thick limb and thin limb. The power of the initial selected servomotor is 1000 W, and inertia of the motor rotor is $2.45 \times 10^{-4} \text{ kg} \cdot \text{m}^2$.

Table 1 Initial structural parameters of the PKM (unit: mm).

D_1	t_1	D_2	t_2	D_3	t_3	D_4	t_4
40	6	62	8	144	7	92	7

Based on the proposed procedure, the following analysis can be conducted.

5.1. Dimensional synthesis

In order to achieve good kinematic and dynamic behaviors, dimensional synthesis of the PKM can be described as a nonlinear multi-objective programming problem with constraints

$$\chi \rightarrow \min, \quad (44)$$

$$F - (\chi + 1)F^* \leq 0, F^* = (\min F_1 \quad \min F_2 \quad \min F_3)^T,$$

$$F_1 = \frac{\int_V \eta dV}{\int_V dV}, \quad F_2 = \max_{x \in V} \max_{k=1 \sim 3} \tau_{a,k} \max, \quad F_3 = \sqrt{\frac{\int_V (f_1 - \bar{f}_1)^2 dV}{\int_V dV}}.$$

s.t.

$$\mu = \frac{q_{\max} - q_{\min}}{q_{\min}} \leq \mu_0, |\theta_i|_{\max} \leq \theta_{i0}, |\psi_i|_{\max} \leq \psi_{i0}.$$

In order to analyze effects of $\lambda_{a/b}$, $\lambda_{H/b}$ and $\lambda_{R/b}$ on the global kinematic index F_1 , dynamic index F_2 and elastodynamic index F_3 , variations of F_1 , F_2 and F_3 with respect to $\lambda_{a/b}$, $\lambda_{H/b}$ and $\lambda_{R/b}$ are depicted in Fig. 13. It is noted that values of $\lambda_{a/b}$, $\lambda_{H/b}$ and $\lambda_{R/b}$ are referred to those in [32]. As can be observed clearly, variations of F_1 , F_2 and F_3 are not absolutely the same. To be specific, the global kinematic index F_1 increases monotonously with the increment of $\lambda_{H/b}$ and $\lambda_{R/b}$. It seems that effects of $\lambda_{a/b}$ on the global kinematic index F_1 are very ‘weak’ even though the global kinematic index F_1 decreases monotonously with the increment of $\lambda_{a/b}$. The global dynamic index F_2 decreases first then increases with the increment of $\lambda_{H/b}$ while increasing monotonously with that of $\lambda_{a/b}$ and $\lambda_{R/b}$. However, effects of $\lambda_{a/b}$ on the global dynamic index F_2 become weak with the increment of $\lambda_{H/b}$. Variations of the global elastodynamic index F_3 are much different from F_1 and F_2 in that it increases first then decreases with the increment of $\lambda_{H/b}$ while decreasing first then increasing with that of $\lambda_{R/b}$ and $\lambda_{a/b}$. Furthermore, effects of $\lambda_{a/b}$ on the global index F_3 are not very large when the global index F_3 increases with the increment of $\lambda_{a/b}$. Thus, by combing the discussion above, the optimal parameter $\lambda_{a/b}^*$ is chosen to be 0.4 to guarantee design size of the platform.

Fig. 13. Variations of F_1 , F_2 and F_3 with respect to $\lambda_{a/b}$, $\lambda_{H/b}$ and $\lambda_{R/b}$.

According to Fig. 13, it is known that the three indices are conflicting, which proves that optimization based on an individual index is not enough. In order to solve this problem, the following synthesis index with weight factors is adopted

$$F = \sqrt{F_2'^2 + (\eta_1 F_1')^2 + (\eta_2 F_3')^2}, \quad (45)$$

where $\eta_1 = \frac{\tilde{F}_2'}{\tilde{F}_1'}$, $\eta_2 = \frac{\tilde{F}_3'}{\tilde{F}_1'}$. $\tilde{F}_1'$, $\tilde{F}_2'$ and $\tilde{F}_3'$ denote the average values of F_1 , F_2' and F_3' , respectively.

And F_2' and F_3' are dimensionless quantities, which can be expressed as

$$F_2' = \frac{F_2}{\tilde{F}_2}, \quad F_3' = \frac{F_3}{\tilde{F}_3}, \quad (46)$$

where $\tilde{F}_2$ and $\tilde{F}_3$ denote the minimum values of F_2 and F_3 , respectively.

Variations of the extension ratio μ , the synthesis index F and maximum rotation angles of universal joints with respect to $\lambda_{H/b}$ and $\lambda_{R/b}$ are addressed in Fig. 14.

Fig. 14. Variations of the extension ratio μ , the synthesis index F and maximum rotation angles of universal joints with respect to $\lambda_{H/b}$ and $\lambda_{R/b}$.

As shown, the extension ratio μ as well as the maximum rotation angles of universal joints increases monotonously with the increment of $\lambda_{R/b}$ while decreasing with that of $\lambda_{H/b}$. Relatively speaking, variations of the synthesis index F with respect to $\lambda_{H/b}$ are complicated. Specifically, the synthesis index F decreases first then increases then decreases then increases with the increment of $\lambda_{H/b}$. In general, the synthesis index F increases with the increment of $\lambda_{R/b}$. Supposing the allowable extension ratio μ_0 is 0.8, the range of the optimal parameter $\lambda_{H/b}$ is $[2.56 \ 3]$. And the synthesis index F increases with the increment of $\lambda_{R/b}$ when $\lambda_{H/b} \in [2.56 \ 3]$. Observing variations of the synthesis index F with respect to $\lambda_{R/b}$, we can get the optimal parameter $\lambda_{R/b}^*$ ($\lambda_{R/b}^* = 1.3$). Moreover, the synthesis index F decreases first then increases with the increment of $\lambda_{H/b}$ when $\lambda_{H/b} \in [2.56 \ 3]$. Therefore, making the synthesis index F be the minimize value, we can get the optimal parameter $\lambda_{H/b}^*$ ($\lambda_{H/b}^* = 2.60$). Accordingly, the allowable rotation angles of universal joints can be derived as shown in Fig. 14, which is useful to the mechanical design.

5.2. Structural parameters optimization

Based on the optimal parameters $\lambda_{a/b}^*$, $\lambda_{R/b}^*$ and $\lambda_{H/b}^*$, optimization of structural parameters are conducted. In this section, the global stiffness index F_4 is chosen as the corresponding objection function.

Variations of the global stiffness index F_4 with respect to structural parameters are shown in Fig. 15.

In general, the global stiffness index F_4 decreases with the increment of D_1 , D_4 , t_1 and t_4 while increasing with that of D_2 , D_3 , t_2 and t_3 . Relatively speaking, the thickness of the oscillating limb t_2 has a weak impact on the system's rigidity. In order to make the global stiffness index F_4 to be the minimum, the optimal structural parameters D_1 , t_1 , D_2 , t_2 , D_3 , t_3 , D_4 and t_4 can be chosen as 42 mm, 8 mm, 58 mm, 6 mm, 140 mm, 6 mm, 96 mm and 8 mm, respectively.

Fig. 15. Variations of the global stiffness index F_4 with respect to structural parameters.

It is noted that the optimal structural parameters are not consistent with the initial ones in Table 1. Therefore, dimensional synthesis needs to be conducted with another group of structural parameters. Finally, considering the design margin, the optimal dimensional and structural parameters are obtained as listed in Tables 2 and 3.

Table 2 Optimal dimensional parameters (unit: mm).

a	b_x	b_y	H	l_1	l_2	l_3	l_4
92.3	230.8	399.7	623	670	673	267	573

Table 3 Optimal structural parameters (unit: mm).

D_1	d_1	D_2	d_2	D_3	d_3	D_4	d_4
40	24	58	46	140	128	96	80

5.3. Matching design of driving parameters

Based the optimal design parameters listed in Tables 2 and 3, driving parameters of the selected servomotor needs to be checked whether they are matched to the optimal system. In this section, the principle of inertia matching is selected. If the principle of inertia matching is satisfied, the whole optimization process is done. Otherwise, another servomotor needs to be selected and the optimization process needs to be repeated.

Transform Eq. (24) in the joint space and denote $\tilde{\mathbf{M}}$ as the mass matrix expressed in the joint space. The equivalent load moment of inertia applied to the servomotor can be expressed as

$$I_k = \frac{p^2}{4\pi^2} \tilde{\mathbf{M}}(k, k) - I_M, k = 1 \sim 3, \quad (47)$$

where $\tilde{\mathbf{M}}(k, k)$ is the k th diagonal element of $\tilde{\mathbf{M}}$; I_M is the rotor inertia of the servomotor.

Similar to the kinematic performance, dynamic performance and stiffness performance discussed in Section 4, the equivalent load moment of inertia I_k is position-dependent. Therefore, the average value throughout the whole task workspace $I_{k\text{mean}}$ is taken into calculation. According to the principle of

inertia matching, there exists

$$1 \leq \frac{I_{kmean}}{I_M} \leq 3. \quad (48)$$

The average equivalent load inertia of the leg_1 I_{1mean} is $3.02 \times 10^{-4} \text{ kg} \cdot \text{m}^2$, while that of leg_2 (leg_3) I_{2mean} (I_{3mean}) is $2.87 \times 10^{-4} \text{ kg} \cdot \text{m}^2$. According to Eq. (46), there are

$$\frac{I_{1mean}}{I_M} = 1.23 \leq 3, \frac{I_{2mean}}{I_M} = 1.17 \leq 3.$$

The above results demonstrate that the initial selected servomotor is suitable. Therefore, we can get an optimal robot based on the initial selected servomotors and optimal dimensional and structural parameters as listed in Tables 2 and 3.

5.4. Performance comparison

Based on the optimal design variables in this paper and parameters in [30,31], performance comparison of the two Trimule robots is listed in Table 4. To make it reasonable, performance comparison is conducted in their whole task workspace. As shown in Table 4, the Trimule robot in [30,31] presents better kinematic performance throughout its whole task workspace. On the contrary, the optimal one in this paper has better dynamic behaviors and stiffness throughout the whole task workspace even though its kinematic performance is not good as the one in [30,31]. Especially, there is a relatively large difference in the dynamic index (acceleration ability) F_2 and stiffness index F_4 , which demonstrates that the optimal Trimule in this paper can achieve better working efficiency and rigidity. This would be helpful in HSM, which may result in high performance machining. This can verify that the proposed hierarchical design method is useful for this kind of PKMs.

Table 4 Performance comparison throughout the whole task workspace

	F_1	F_2	F_3	F_4
Trimule in [30,31]	3.38	242.04	4.49	1.42×10^6
Optimal Trimule	4.12	102.09	3.88	7.56×10^5

6. Conclusions

An optimal design method for the Trimule robot is proposed based on a comprehensive performance index system. The main contributions can be drawn as follows.

(1) Considering kinematic, dynamic and elastodynamic performance as well as stiffness, a performance evaluation system for PKMs in HSM is built. Results show that performance of the system is position-dependent and symmetric due to its symmetry in structural features.

(2) Considering there are many design variables, a hierarchical optimization design process is proposed, which is helpful to obtain the optimal driving, dimensional and structural parameters. Meanwhile, the optimal system has good kinematic, dynamic and stiffness performance through out the whole task workspace.

(3) Results show that variations of the proposed performance indices with respect to the design variables are conflicting, which demonstrates that optimization based on a single and two indices cannot make the robot own good kinematic performance, rigid dynamic performance and elastodynamic performance simultaneously. A synthesis index with weighted factors is proposed to make comprises on different performance indices.

(4) Although only the rotor inertia of the servomotor is considered in this paper for demonstrating the proposed approach, the required maximum torque and speed in a typical task can also be considered when necessary.

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