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On the Aperture Efficiency of Intelligent Reflecting Surfaces

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Abstract—The concept of intelligent reflecting surfaces (IRS) has recently gained significant attention due to their ability to manipulate the impinging electromagnetic signals and offer anomalous reflections. In this letter, we derive and address the missing piece of prior works, namely the *IRS aperture efficiency*. Our work provides practical guidelines for selecting the aperture size of the reflecting surface from an antenna designing point of view by taking *tapering level* into account, and also, revisits the pathloss expression based on a realistic physical model.

Index Terms—Aperture efficiency, intelligent reflecting surfaces, pathloss model.

I. INTRODUCTION

AS we are witnessing the steady roll-out of the fifth-generation (5G) cellular networks, the importance of massive multiple-input multiple-output (M-MIMO), becomes unquestionable. Looking ahead, a number of technologies have been proposed to extend the capabilities of M-MIMO, namely cell-free M-MIMO [1], millimeter wave (mmWave) communications [2]. These concepts are theoretically attractive, yet, they entail fundamental technological challenges, such as the increased cost of RF electronics at higher frequencies along with the short operating range, as well as, the excessive amount of signalling overhead requirements in cell-free M-MIMO to name but a few. The above discussion reveals that a technology which can harness recent advances in electromagnetics, communications and computing by maintaining the signal processing and hardware complexity at affordable levels is of pivotal importance. The focus of this paper is on such a technology, namely, *intelligent reflecting surfaces* (IRS), whose purpose is to perform operations such as reflect, refract, absorb, polarisation adjustment, and data modulation in order to guarantee high quality data delivery [3]. The potential of such a solution is outstanding, and it has recently come at the forefront of wireless communications research; not surprisingly, it is considered as one of the most promising technologies for the sixth generation (6G) communication systems [4]: these reconfigurable metasurfaces can be controlled either locally or at network level in order to customize the propagation of the radio waves, thereby

improving the network coverage through low-energy and low-complexity sensing [5], and basic mathematical operations [6]. This topological approach can offer seamless cellular coverage, as well as, surveillance and localization. The ability of enhancing the energy transmission efficiency and range with the large aperture of IRS has motivated diverse applications, namely simultaneous wireless information and power transfer (SWIPT) [7] and non-orthogonal multiple access (NOMA) [8] by compensating the significant power loss over long distance via passive beamforming. Furthermore, the physical layer security can be optimized when the active transmit beamforming and passive reflect beamforming at the IRS are jointly designed [9].

The common characteristic of the biggest body of IRS literature is that they address the problem of system modeling using idealistic information theoretic tools by often ignoring inherent electromagnetic phenomena. An illustrative example is [10, Eq. (9)], which proposes that the field strength of an IRS scales quadratically with the number of reflecting plates. However, this approach ignores the presence of phase errors and extra losses on an IRS (e.g., spillover loss from the missing part of the reference wave that is not captured on the aperture, conduction loss, dielectric loss etc). In this paper, we are moving away from the state-of-the-art, such as [7]–[9], [11], [12], and inform the theoretical modeling of IRS-based communication by starting from basic electromagnetic principles. The specific paper contributions are the following:

- We consider a rectangular IRS and provide both numerical and analytical approaches of a key factor in the IRS communication system, namely, aperture efficiency, which previous literature did not put into consideration.
- We provide a physical-based pathloss expression of the signal that traverses the path from the source and relayed by the IRS to the destination. Our model is an amelioration of previous results and enables to pursue a rigorous performance characterization of future IRS topologies.
- We investigate the size of IRS based from an antenna designing point of view, when we take the aperture efficiency into account, and demonstrate that there is a trade off between the distance from the source to IRS and the aperture size.

II. EFFICIENCY OF RECTANGULAR IRS

Ideally, the maximum achievable gain for a reflecting antenna array is obtained by assuming that the receiving amplitude and phase are uniformly distributed and that there is

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no spillover and taper losses at the same time. Mathematically speaking, it can be denoted as $G_{\max}$ and defined as [13]

$$G_{\max} = \frac{4\pi}{\lambda^2} A_p, \quad (1)$$

where λ and A_p are the wavelength of the incident waves and the aperture size of the reflective antenna array. However, in practice, an aperture antenna of finite size has an *effective gain* that is defined as

$$G_{\text{eff}} = \varepsilon_{\text{ap}} G_{\max}, \quad (2)$$

where ε_{ap} is the aperture efficiency, that is dependent on several factors, such as, the radiation pattern of the feeding antenna (or source), the excitation footprint of the source radiation pattern across the reflecting surface, the position of the source with respect to the reflecting surface and the operating frequency. The aperture efficiency of a reflecting surface can be defined as [13]

$$\varepsilon_{\text{ap}} \triangleq \varepsilon_{\text{cd}} \varepsilon_{\text{s}} \varepsilon_{\text{t}} \varepsilon_{\text{p}} \varepsilon_{\text{b}},$$

in which ε_{cd} is defined as the radiation efficiency (governed by intrinsic material losses, such as dielectric and conduction losses), ε_{p} as phase efficiency and ε_{b} is the blockage efficiency, respectively. In this paper, we ignore the intrinsic material losses and assume a point source to illuminate the IRS aperture, the aperture efficiency of a reflecting surface is mainly dominated by spillover efficiency (ε_{s}) and taper efficiency (ε_{t}) and the rest of efficiencies are assumed to be ideal (equal to 1) [14]. Thus, the aperture efficiency can be rewritten as

$$\varepsilon_{\text{ap}} = \varepsilon_{\text{s}} \varepsilon_{\text{t}}, \quad (3)$$

and since ε_{ap} is a real number equal to or less than 1, in real-world scenarios, we have $G_{\text{eff}} \leq G_{\max}$.

Note that in the past literature of IRS (e.g. [10], [15] among others), the aperture efficiency is unrealistically assumed to be 1, which means all the radiation from the source antenna is perfectly captured by the reflecting surface and the received waves have uniform distribution of amplitudes.

A. Spillover and Taper Efficiency

Spillover efficiency measures the amount of radiation from the source antenna that is reflected by the reflective antenna array. Because of the finite size of the array, some of the radiation from the source antenna will deviate from the main axis and eventually not being reflected. According to the definition of spillover efficiency, we have

$$\varepsilon_{\text{s}} = \frac{\iint_S |\vec{E}_{\text{F}}|^2 dS}{\iint_{S'} |\vec{E}_{\text{F}}|^2 dS'} \triangleq \frac{I_1}{I_2}, \quad (4)$$

where S and S' denote the surface of the IRS and the surface of the hemisphere in front of the source (see Fig. 1), while $\vec{E}_{\text{F}}$ is the electric field intensity of the source antenna. On the other hand, the taper efficiency (also known as illumination efficiency) is the representation of how uniform the electrical field is across the surface of reflective surface. In practice, the aperture fields will tend to diminish away from the main axis

of the reflector, which leads to lower gain, and this loss is captured by this parameter. Therefore, we can define the taper efficiency as

$$\varepsilon_{\text{t}} = \frac{1}{A_p} \frac{\left| \iint_{S''} \vec{E}_{\text{F}} \cdot \vec{E}_{\text{element}} dS'' \right|^2}{\iint_S |\vec{E}_{\text{F}}|^2 dS}, \quad (5)$$

where S'' is the surface of the sphere limited by the reflecting antenna array aperture, $\vec{E}_{\text{element}}$ is the electrical field, which is reflected by the element on the IRS, and $\cdot$ is the inner product of two vectors.

B. Derivation of Spillover and Taper Efficiency

We can easily see that the denominator of (5) is the same as the nominator of (4). Now, we define I_3 as

$$I_3 \triangleq \left| \iint_{S''} \vec{E}_{\text{F}} \cdot \vec{E}_{\text{element}} dS'' \right|, \quad (6)$$

therefore, we can have the taper efficiency rewritten as

$$\varepsilon_{\text{t}} = \frac{1}{A_p} \frac{I_3^2}{I_1}. \quad (7)$$

We now consider the radiation beam of the source antenna, which has the power pattern ($U_{\text{F}}(\theta, \phi)$), and it is modeled as

$$U_{\text{F}}(\theta, \phi) = E_0 \cos^{q_{\text{F}}}(\theta), \quad (8)$$

where E_0 is the magnitude of the electrical field and q_{F} is the parameter determining the directivity of the source antenna and the shape of the pattern: as q_{F} increases, the beam becomes narrower and delivers higher directivity, while θ and ϕ represent the elevation angle and azimuth angle. The Poynting vector of the source defined by the power pattern (8) can be written in terms of the source region spherical coordinates as

$$\vec{U}_{\text{F}}(\hat{r}) = \frac{E_0}{R} \cos^{q_{\text{F}}}(\theta) \hat{r}, \quad (9)$$

where $\hat{r}$ is the unit vector from the source antenna to the elements on the IRS, while R is the distance from the source antenna to the surface of the IRS. We assume that the incident wave from the source antenna is travelling along the z direction and polarized along the y axis. Substituting (9) into the electrical field intensity, we have $\vec{E}_{\text{F}}$ in spherical coordinates as

$$\vec{E}_{\text{F}} = \frac{E_0}{R} \cos^{q_{\text{F}}}(\theta) \sin(\phi) \cdot \hat{i}_{\theta}, \quad (10)$$

where $\hat{i}_{\theta}$ is the unit position vector from the source to IRS. As a result, I_1 , I_2 and I_3 can be rewritten as

$$I_1 = \iint_S \left| \frac{E_0 \cos^{q_{\text{F}}}(\theta)}{R} \sin(\phi) \right|^2 dS, \quad (11)$$

$$I_2 = \iint_{S'} \left| \frac{E_0 \cos^{q_{\text{F}}}(\theta)}{R} \sin(\phi) \right|^2 dS', \quad (12)$$

$$I_3 = \iint_S \left| \frac{E_0 \cos^{q_{\text{F}}+q_{\text{R}}+1}(\theta)}{R} \sin(\phi) \right| dS, \quad (13)$$

Fig. 1: Coordinate system and configuration parameters of a rectangular IRS.

where q_R is defined as the directivity of the reflective antenna array. Figure 1 shows the rectangular coordinates system used to determine the efficiencies of the rectangular IRS. Since we consider a reflecting surface with the aperture area shaped as rectangular, it is easier to model I_1 , I_2 and I_3 in a Cartesian coordinate system; thus, we can express $\cos(\theta)$ and $\sin(\phi)$ in Cartesian coordinates according to

$$\cos(\theta) = \frac{R}{\sqrt{R^2 + x^2 + y^2}}, \quad \sin(\phi) = \frac{y}{\sqrt{x^2 + y^2}}. \quad (14)$$

By substituting $\cos(\theta)$ and $\sin(\phi)$, we can have

$$I_1 = \frac{E_0^2}{R^2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} R^{2q_F+3} y^2 (R^2 + x^2 + y^2)^{-(q_F+\frac{3}{2})} \times (x^2 + y^2)^{-1} dx dy, \quad (15)$$

and

$$I_3 = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{E_0}{\sqrt{R^2 + x^2 + y^2}} \left(\frac{R}{\sqrt{R^2 + x^2 + y^2}} \right)^{q_F+q_R+1} \times \left(\frac{y}{\sqrt{x^2 + y^2}} \right)^2 dx dy, \quad (16)$$

where a and b are the lengths of IRS in the x and y direction, respectively, while I_2 can be derived analytically as follows:

$$I_2 = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \frac{E_0^2}{R^2} \cos^{2q_F}(\theta) \sin^2(\phi) R^2 \sin(\theta) d\theta d\phi \\ = E_0^2 \pi \left(\frac{1}{2q_F+1} \right), \quad \Re\{q_F\} > -\frac{1}{2}.$$

Unfortunately, I_1 and I_3 do not admit an analytical solution due to functions that are powered by $-(q_F + \frac{3}{2})$ and $q_F + q_R + 1$. Therefore, we now introduce two approximation methods, which are *Gaussian-Legendre Quadrature Rule (GLQR)* and *Taylor Expansion* to numerically evaluate these integrals.

1) *Gaussian-Legendre Quadrature Rule (GLQR)*: The reason we use GLQR is that it is generally more accurate than the Newton-Cotes quadrature technique.

Lemma 1: By using GLQR, we can obtain a numerical solution for I_1

$$I_1 \approx \frac{abE_0^2 R^{2q_F+1}}{4} \sum_{i=1}^n \sum_{j=1}^m \left[\left(\frac{a\xi_i}{2} \right)^2 + \left(\frac{b\eta_j}{2} \right)^2 \right]^{-1} \\ \times \left[R^2 + \left(\frac{a\xi_i}{2} \right)^2 + \left(\frac{b\eta_j}{2} \right)^2 \right]^{-q_F-\frac{3}{2}} \left(\frac{b\eta_j}{2} \right)^2 w_{1,i} w_{2,j}, \quad (17)$$

where n and m are the number of sample points, and $\{(\xi_i, \eta_i) | -1 \leq \xi_i \leq 1, -1 \leq \eta_i \leq 1\}$, while $w_{1,i}$ and $w_{2,j}$ are Gaussian-Legendre weighting functions:

$$w_{1,i} = \frac{2}{(1-x_i^2) [P'_n(x_i)]^2}, \quad w_{2,j} = \frac{2}{(1-y_j^2) [P'_m(y_j)]^2},$$

where x_i and y_j are the roots of the Legendre polynomial $P_n(\cdot)$.

Lemma 2: By using GLQR, we can obtain a numerical solution for I_3 as

$$I_3 \approx \frac{E_0^2 R^{q_F+q_R+1} ab}{4} \sum_{i=1}^n \sum_{j=1}^m \left[\left(\frac{a\xi_i}{2} \right)^2 + \left(\frac{b\eta_j}{2} \right)^2 \right]^{-1} \\ \times \left[R^2 + \left(\frac{a\xi_i}{2} \right)^2 + \left(\frac{b\eta_j}{2} \right)^2 \right]^{-\frac{q_F+q_R-2}{2}} \left(\frac{b\eta_j}{2} \right)^2 w_{1,i} w_{2,j}. \quad (18)$$

2) *Taylor Expansion*: We can approximate the integration problem (15) and (16) with Taylor expansion and further turn them into tractable closed-forms. We can observe from (15) and (16), that the term $R^2 + x^2 + y^2$ can be rewritten as

$$(R^2 + x^2 + y^2) = R^2 \left(1 + \frac{x^2 + y^2}{R^2} \right). \quad (19)$$

By defining z_s as

$$z_s = \frac{x^2 + y^2}{R^2}, \quad |z_s| \leq 1,$$

and by utilizing the property of Taylor expansion $\left(\frac{1}{1+z} \right)^p = \sum_{l=0}^{\infty} C_l z^l$, we can obtain the closed form of (15) and (16), by a third-order Taylor expansion:

$$I_1 \approx R^{-2} \left[\left(\frac{a^2 + b^2}{2} \right) \tan^{-1} \left(\frac{a}{b} \right) - \frac{q_F + \frac{3}{2}}{3} ab^3 + \frac{ab}{2} \right] \\ + R^{-4} \left[\frac{(q_F + \frac{3}{2})(q_F + \frac{1}{2}) a^2 b^3}{144} + \frac{ab^5}{160} \right], \quad (20)$$

$$I_3 \approx R^{-1} \left[\left(\frac{a^2 + b^2}{2} \right) \tan^{-1} \left(\frac{a}{b} \right) + \frac{ab}{2} \right] + R^{-3} ab^3 \\ \times \frac{(q_F + q_R + 2)}{24} \left[\frac{(q_F + q_R + 4)(5a^2 + 9b^2)}{240} - 1 \right]. \quad (21)$$

By observing (20) and (21), several parameters can be fine tuned to improve the aperture efficiency: for example, when

the distance between the source and the IRS is large and the IRS diameter is fixed, we can adjust the directivity of the source antennas to increase the aperture efficiency of the IRS.

III. PATHLOSS MODELING

When it comes to the pathloss in free space and within the far-field regime, we naturally consider the Friis formula

$$P_R = \frac{P_S G_S G_R \lambda^2}{(4\pi R_{S,IRS})^2} \quad (22)$$

where P_S and G_S are the power and gain of the source antenna respectively, while P_R and G_R are the power and gain of the IRS and $R_{S,IRS}$ is the distance from the source to the IRS. Here, we consider line-of-sight (LoS) setup, since it is reasonable for systems deployed in indoor or outdoor open spaces such as rural areas, and for mmWave wireless systems with small cell sizes.

Recall now the relationship between the reflecting array antenna gain and efficiency; we can define G_{eff} as G_R and substitute (1) and (2) into (22); then, we arrive at

$$\frac{P_R}{P_S} = \frac{G_S \varepsilon_s \varepsilon_t ab}{4\pi R_{S,IRS}^2}. \quad (23)$$

Unfortunately, (23) does not consider the fact that the effective aperture size changes as we steer the radiation from the optical axis such that the decay of the effective aperture size can be approximated by a cosine pattern. Thus, (23) can be rewritten by considering the direction of the incident beam [16]. The receiving gain of the IRS then becomes

$$G_{R,\text{beam}} = G_R \cos \theta_{\text{axis}}, \quad 0 \leq \theta_{\text{axis}} \leq \frac{\pi}{2}. \quad (24)$$

By substituting (24) into (22) we can have

$$\frac{P_R}{P_S} = \frac{G_S \varepsilon_s \varepsilon_t ab \cos \theta_{\text{axis}}}{4\pi R_{S,IRS}^2}. \quad (25)$$

As a result, the pathloss expression from source to destination through the reflecting IRS can be determined as

$$\beta_{\text{IRS}} = \frac{G_S G_D}{(4\pi)^2} \left(\frac{ab}{R_{S,IRS} R_{IRS,D}} \right)^2 \varepsilon_{\text{ap}}^2 \cos^2(\theta_{\text{axis}}), \quad (26)$$

where G_D is the gain of the receiving antenna, and $R_{\text{IRS},D}$ is the distance between the IRS and the destination. As we can see in (26), when we consider β_{IRS} as a function of θ_{axis} , it has peak value at $\theta_{\text{IRS}} = 0$. This implies that the pathloss expression from the source to the IRS becomes maximum when the source is located on the optical axis.

IV. NUMERICAL RESULTS

In this section, we present simulation results to validate our theoretical derivation of the aperture efficiency and our pathloss expression. The comparison of GLQR and Taylor expansion method is shown in Fig. 2, where we can see the behavior of spillover, taper and aperture efficiencies with the diameter equal to 10λ and the frequency is 3GHz, whilst the directivities are set as $q_F = 9$ and $q_R = 1$ [13]. We note that in Section II, we provided physical insights into the aperture efficiency definition in terms of the source radiated

Fig. 2: Spillover, taper and aperture efficiency performance.

fields projected onto the IRS aperture, thus, the result is valid regardless of the radiation zone.

As we can see from (26), the diameter of IRS is also an important parameter for calculating β_{IRS} . However, we cannot have an infinitely big aperture to capture all the signal power while satisfying the far-field constraint. Hence, we now consider a more practical scenario and provide a cost-effective way to define the size of the IRS which also enables us to evaluate the SNR performance. First, we consider a case that a source antenna serves as a point source located at $\mathbf{r}$; this assumption is reasonable when the source is located in the far-field region, and the amplitude of the spherical waves $\Phi(\mathbf{r})$ emitted from the source to the IRS, which is located at $\mathbf{r}'$, and it is denoted as $\Phi(\mathbf{r}'|\mathbf{r})$. Now, $\Phi(\mathbf{r}'|\mathbf{r})$ is the solution of Helmholtz equation

$$\nabla^2 \Phi(\mathbf{r}'|\mathbf{r}) + k^2 \Phi(\mathbf{r}'|\mathbf{r}) = -\delta(\mathbf{r}' - \mathbf{r}),$$

where ∇ denotes the gradient of a vector field and $\delta(\mathbf{r}' - \mathbf{r})$ is the three dimensional Dirac delta function. Thus, we obtain the solution as a scalar Green's function:

$$\Phi(\mathbf{r}'|\mathbf{r}) = \frac{e^{jk|\mathbf{r}'-\mathbf{r}|}}{4\pi|\mathbf{r}'-\mathbf{r}|} = \frac{e^{jkR}}{4\pi R},$$

where k is the wave number in the free space, and defined as $k = \frac{2\pi}{\lambda}$. The electrical field strength, E_s , captured by a rectangular aperture from a point source, can be expressed in Cartesian coordinates as

$$E_s = \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{e^{jkR}}{4\pi R} e^{-jk(\mathbf{r}' \cdot \hat{\mathbf{r}})} dx dy, \quad (27)$$

where $\mathbf{r}' = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ and $\hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}}$. As a result, (27) becomes:

$$E_s = \frac{e^{jkR}}{4\pi R} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-jk \sin \theta (\cos \phi x + \sin \phi y)} dx dy. \quad (28)$$

As the radiated power per unit area is proportional to the square of the electric field strength, the power pattern of the spherical wavefronts (28) reaching the rectangular aperture is

$$A(\theta, \phi) = \left(\frac{e^{jkR} ab \text{sinc}\left(\frac{ka}{2} \sin \theta \cos \phi\right) \text{sinc}\left(\frac{kb}{2} \sin \theta \sin \phi\right)}{4\pi R} \right)^2,$$

where $\text{sinc}(x) = \frac{\sin(x)}{x}$.

Now, we introduce the *tapering level*, which measures how much the electrical field is tapered at the aperture edge (A_{edge})

when compared to the centre of the IRS (A_{centre}). From this definition, we can express the tapering level as

$$TL = \frac{A(\theta, \phi)}{A(0, 0)} = \frac{A_{\text{edge}}}{A_{\text{centre}}}. \quad (29)$$

By setting the tapering level to a certain value, we can determine the corresponding size of the reflecting array, spillover and taper efficiency. Table I shows how these different parameters behave under different carrier frequencies (f_c), where R is the far-field distance ($R = 2 \max\{a^2, b^2\}/\lambda$) and $TL = -20$ dB, leading to different efficiencies. When the operating frequency is increased, the size of IRS can be reduced to maintain the same level of loss from the center toward the edge of the reflecting area. In this case, the far-field definition becomes more relevant when radiation from the IRS is considered. This is because when we consider the radiated field from the IRS aperture (under the calculated spillover and taper efficiencies), the IRS becomes the source of radiation for SNR calculations.

TABLE I: Spillover and taper efficiency with tapering level -20dB at different frequencies.

f_c (GHz)	IRS diameter (m)	ϵ_s	ϵ_t
3	0.072	0.4598	0.8933
5	0.043	0.1756	0.9751
28	0.02	0.0059	0.993

After elaborating on the diameter of the IRS, we can finally calculate the signal-to-noise ratio (SNR) with respect to the location of source, IRS and destination. We note that in [10], the SNR was defined as

$$\text{SNR} = \frac{(\sqrt{\beta_{\text{IRS}}} + \sqrt{\beta_{\text{SD}}})^2 P_S}{\sigma^2}, \quad (30)$$

where $\beta_{\text{SD}} = \frac{G_S G_D \lambda^2}{(4\pi R_{\text{S,D}})^2}$, the distance between the source and destination is denoted as $R_{\text{S,D}}$, and σ^2 denotes the noise power. Figure 3 shows the SNR performance for the proposed model and the model considered in [10]: we choose $R_{\text{S,D}} = 5$ m, $TL = -20$ dB, $P_S/\sigma^2 = 5$ dB, $G_S = G_D = 5$ dB and $f_c = 3$ GHz, while the location of IRS is moving from the source towards the destination. It is observed from the figure that the performance predicted in [10] overestimates the achievable SNR compared to our model, that takes into account the aperture efficiency. Moreover, when $R_{\text{S,IRS}}$ and θ_{axis} increase, the power captured by the IRS and the aperture efficiency decrease; this leads to even lower receiving power at the destination; thus, the SNR should be a non-increasing function with respect to $R_{\text{S,IRS}}$ and θ_{axis} , and it can be seen that our expression is able to capture this phenomenon, while the expression in [10] does not consider the affect of aperture efficiency and leads to slight overestimation of the SNR as the IRS approaches to the destination.

V. CONCLUSION

We studied an important parameter in designing an IRS-based communication system, i.e. aperture efficiency. We introduced two major contributions to the aperture efficiency concept, which are the spillover efficiency and the taper

Fig. 3: SNR comparison between the proposed model and [10].

efficiency, and presented that improving the efficiency can enable us to further boost the pathloss performance. When it comes to a real-world deployment of an IRS and because the size of the aperture is always a concern, we also studied the aperture size and proposed a practical way to derive the IRS diameter. Our results demonstrated that there is a trade-off between the distance from the source to the IRS and the diameter of the aperture size due to the far-field constraint. Finally, we proposed a pathloss expression in the far-field zone, which is connected with the tilt angle and how much of the electrical field is captured and reflected by an IRS.

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