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Sparse Heteroscedastic Multiple Spline Regression Models for Wind Turbine Power Curve Modeling

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Abstract—An accurate wind turbine power curve (WTPC) plays a vital role in wind power forecasting and wind turbine condition monitoring. There are two major shortcomings of current WTPC models that prevent more accurate WTPC estimation, limited nonlinear fitting ability and the lack of in-depth understanding of the complex characteristics of WTPC. This paper proposes two novel regression models to overcome these two disadvantages simultaneously. First, they make use of multiple spline regression models (MSRM) with different basis functions and different numbers of knots to describe the complex nonlinear relationship between wind speed and wind power. Moreover, sparse prior distributions help avoid the adverse effects of redundant mapping features and useless basis functions on the model performance. Second, they embed the heteroscedasticity of WTPC modeling into MSRM based on Gaussian and Student's t -distributions, respectively. Finally, two sparse heteroscedastic MSRM with Gaussian and Student's t -distributions will be constructed and named as SHMSRM-G and SHMSRM-T, respectively. We compare the proposed models with fifteen benchmark models, and find that they can generate more accurate WTPCs than the others in different seasons and different wind farms. Thus, it is important to consider the complex nonlinear fitting ability and heteroscedasticity together in constructing accurate WTPC models.

Index Terms—Power curve modeling, multiple spline regression models, sparsity, heteroscedasticity, variational Bayesian.

I. INTRODUCTION

RECENTLY, the development and utilization of renewable energy has become a common way to solve the problem of energy shortage and environmental pollution worldwide [1]. Wind energy, as a clean, environmentally friendly, inexhaustible and inexpensive renewable energy, has attracted more and more attention [2]. An accurate WTPC plays a vital role in many applications of wind power [3].

Usually, wind turbine manufacturers will provide theoretical WTPCs for their own wind turbine, which describes the performance of a wind turbine at a particular air density [4]. However, the air density varies with the change of time

and geographic location. As a result, the same wind turbine will also perform differently during different seasons and at different wind farms. It is essential to model the actual WTPCs in practice. In literature, according to the modeling theory, the deterministic WTPC models can be divided into two categories, curve fitting models and artificial intelligence (AI)-based models [3].

Curve fitting models, which are usually S-shaped, can be grouped into two parts: the segmented curve fitting models and the integrated curve fitting models [3]. Examples of the former contain linearized segmented model [5], polynomial model [6], Sigmoid function-based [7] and Gaussian cumulative distribution function-based [8] power curves. The popular models in the latter are three-parameter logistic function (3-PLF) [9], four-parameter logistic function (4-PLF) [10], five-parameter logistic function (5-PLF) [10], six-parameter logistic function (6-PLF) [11] and the modified hyperbolic tangent (MHTan) [12]. As for AI-based models, they can learn the complex nonlinear relationship between wind speed and wind power automatically. The popular ones are support vector machine (SVM) [13], Gaussian process (GP) [14], extreme learning machine (ELM) [15], spline regression-based models [1] [4], adaptive neuro-fuzzy inference system (ANFIS) [16], monotonic regression [17] and copula model [18] etc.

Currently, two strategies are usually considered to obtain more accurate WTPCs. First, improving the quality of raw wind data. In the open environment, the collected data contain many uncertainties, such as outliers and missing data, which will affect the process of model training [19]. A man-made approach was designed to detect outliers in wind data [4]. Three-sigma rule [13] and K-means clustering [20] were utilized to filter the original wind data. In [19], quartile method and density-based clustering were employed to process different types of outliers. Second, developing novel WTPC models by considering the task characteristics. It was reported that WTPC modeling was characterized by heteroscedasticity and non-Gaussian error distribution. Based on the above findings, Wang et al. proposed heteroscedastic spline regression model (HSRM) [4], robust spline regression mode (RSRM) [4], asymmetric spline regression with mixture of asymmetric Gaussian distributions (MoAG-ASR) [1] and asymmetric spline regression with mixture of asymmetric exponential power distributions (MoAEP-ASR) [1]. Also, Rogers et al. utilized a heteroscedastic GP regression to model WTPCs [21].

In practice, due to the complexity of climate and environment, the actual WTPCs will be also very complex. In this case, they cannot be modelled by single models

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comprehensively. So, developing novel WTPC models with superior nonlinear fitting ability is also a useful strategy for generating accurate WTPCs. This paper takes account of some spline regression models with different spline basis functions (SBFs) and different numbers of knots in describing the complex nonlinear WTPCs. Thus, a framework of multiple spline regression models (MSRM) was constructed. However, some redundant mapping features and SBFs may have adverse effects on the regression performance of MSRM. To solve this problem, sparse prior distributions were given to the regression coefficients and the weights of SBFs to get sparse solutions of model parameters in MSRM. Besides, based on Gaussian distribution and Student's t -distribution, the characteristic of WTPC modeling, heteroscedasticity, was also considered in MSRM, thus resulting in two sparse heteroscedastic MSRM, SHMSRM-G and SHMSRM-T, respectively.

In order to optimize the proposed models, two methods, expectation maximization (EM) and variational Bayesian (VB), come into our mind. EM is a partially non-Bayesian, maximum likelihood method [22]. It gives probability distributions over the latent variables together with point estimates for the other parameters [22]. The computed point estimates are the modes of these parameters; no other information is available. As to VB, it estimates the actual posterior distributions of all variables, both parameters and latent variables. When point estimates need to be derived, generally the mean is used rather than the mode, as is normal in Bayesian inference [23]. Due to more information is provided for model parameters, VB will be used to optimize the proposed models. The main contributions of the paper can be summarized as follows.

- A framework of MSRM was constructed to enhance the nonlinear fitting ability of the proposed models in modeling the actual complex WTPCs.
- Sparse prior distributions were used to get the sparse model solutions, which help avoid the bad effects of the redundant mapping features and useless SBFs.
- The heteroscedasticity of WTPC modeling was embedded into the proposed models based on Gaussian distribution and Student's t -distribution, respectively.

The rest of the paper is organized as follows. Section II introduces the spline regression model (SRM) briefly. The two proposed models are described in detail in Section III. Section IV presents the processes of WTPC modeling. Section V shows the experiments and the corresponding experimental analysis, while Section VI concludes the whole paper.

II. SPLINE REGRESSION MODEL

SRM has been widely used in WTPC modeling due to its superior nonlinear fitting ability [1] [4] [24]. In WTPC modeling, supposing x_i and y_i denote wind speed and the corresponding power output, respectively. Given N training samples $\{x_i, y_i\}_{i=1, \dots, N}$, a general SRM can be expressed as

$$y_i = f_{basis}(x_i) + \varepsilon_i, \quad (i = 1, \dots, N) \quad (1)$$

where ε_i is the regression error, $f_{basis}(\cdot)$ is the estimated function related with a SBF. Equation (1) can be expressed in a matrix form,

$$\mathbf{Y} = \mathbf{H}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

where $\mathbf{Y} = [y_1, \dots, y_N]^T \in \mathbb{R}^N$ is the vector of output, $\boldsymbol{\varepsilon} = [\varepsilon_1, \dots, \varepsilon_N]^T \in \mathbb{R}^N$ is the vector of regression error, $\boldsymbol{\beta} = [\beta_1, \dots, \beta_M]^T \in \mathbb{R}^M$ is the vector of regression coefficients, $\mathbf{h}_m \in \mathbb{R}^N$ is the m th mapping feature generated by the selected $f_{basis}(\cdot)$, $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_M] \in \mathbb{R}^{N \times M}$ is the matrix of all mapping features, M is the number of mapping features, which are related with the number of knots set in $f_{basis}(\cdot)$ [24].

In literature, two popular SBFs, truncated power basis and B-spline basis, are widely used to construct a SRM. Their expressions can be referred to [1], [4] and [24]. The least squares solution for (2) is $\hat{\boldsymbol{\beta}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{Y}$ [24].

III. SPARSE HETEROSCEDASTIC MULTIPLE SPLINE REGRESSION MODELS

There are three major drawbacks of the original SRM: (a) sometimes the performance of SRM is sensitive to the number of knots, and the optimal one varies with the task [24]; (b) the SRM with single SBF has the limited ability in describing the complex nonlinear relationship; (c) the underlying assumption in SRM is not consistent with the characteristics of the observed task, resulting in the biased results. To solve these drawbacks, this paper proposed two types of sparse heteroscedastic MSRM, SHMSRM-G and SHMSRM-T.

The proposed models take advantage of several spline regression models with different types of SBFs and various numbers of knots, and the weight of each model and regression coefficients are optimized automatically by sparse Bayesian learning. So, they can not only describe the complex nonlinear relationship with MSRM, but also avoid the adverse effects of redundant mapping features and useless SBFs automatically, thus overcoming the first and second drawbacks of SRM. For a wind turbine, at the same wind speed, different environmental conditions may result in different power output. So, it can be found that the variance of wind power will vary with wind speed. The errors of WTPC modeling will be heteroscedastic through analysing [4]. The proposed models employed the Gaussian distribution-based and Student's t -distribution-based heteroscedastic error assumptions to replace the original Gaussian error assumption, respectively, thus overcoming the third drawback of SRM. The framework of MSRM and the proposed models are introduced in detail below.

A. Framework of multiple spline regression models (MSRM)

Given the input data $\{x_i\}_{i=1, \dots, N}$, K SBF matrices $\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_K$ can be calculated with different numbers of knots and different SBFs. The numbers of mapping features in these matrices are $M_1, M_2, \dots, M_K$, respectively. The size of $\mathbf{H}_i$ may be different from $\mathbf{H}_j$ when $i \neq j$. They can be transformed into the same size by (3)

$$\tilde{\mathbf{H}}_k = [\mathbf{H}_k, \mathbf{0}_{N \times (\tilde{M} - M_k)}], \quad k = 1, \dots, K \quad (3)$$

where $\tilde{\mathbf{H}} \in \mathbb{R}^{N \times \tilde{M}}$, $\mathbf{0}_{N \times (\tilde{M} - M_k)}$ is the $N \times (\tilde{M} - M_k)$ zero matrix, and $\tilde{M} = \max\{M_1, M_2, \dots, M_K\}$. So, in order to avoid selecting optimal mapping features and SBFs, and to

take advantages of different single SRMs, the framework of MSRM is proposed and given by

$$y_i = \omega^\top [(\tilde{\mathbf{H}}_1^{(i,\cdot)})^\top, \dots, (\tilde{\mathbf{H}}_K^{(i,\cdot)})^\top]^\top \beta + \varepsilon_i \quad (4)$$

where $\omega = [\omega_1, \dots, \omega_K]^\top \in \mathbb{R}^K$ is the weight vector of different SBF matrices, $\beta = [\beta_1, \dots, \beta_{\tilde{M}}]^\top \in \mathbb{R}^{\tilde{M}}$ is the vector of regression coefficients, $\tilde{\mathbf{H}}_k^{(i,\cdot)}$ is the i th row of $\tilde{\mathbf{H}}_k$. The matrix form of (4) is

$$\mathbf{Y} = \tilde{\mathbf{H}}_\omega \beta + \varepsilon \quad (5)$$

where $\tilde{\mathbf{H}}_\omega = \sum_{k=1}^K \omega_k \tilde{\mathbf{H}}_k \in \mathbb{R}^{N \times \tilde{M}}$.

Within the framework of MSRM, β and ω can be learnt simultaneously with the training data. In the following subsection, sparse heteroscedastic models will be developed based on MSRM from Bayesian perspective.

B. Sparse heteroscedastic multiple spline regression models with Gaussian distribution (SHMSRM-G)

To develop the proposed model, the prior distributions of all parameters in SHMSRM-G are given and the corresponding posterior distributions are also derived in detail as follows.

1) *Prior distributions of all parameters in SHMSRM-G*: A sparse representation has several advantages, e.g. over-training is often avoided [25] and inferred sparse coefficients also often have biological/physical meaning, of interest for model interpretation [26]. In this paper, sparse solutions for the model weight vector (ω) and the regression coefficients (β) can help reduce the adverse effects of some useless SBFs and mapping features. So, considering the above benefits, it is essential to build a sparse MSRM.

Generally, a sparse model can be obtained through prior distributions that have fat tails and sharp peaks at zero [27], [28]. Such priors are also denoted as sparsity-inducing priors. So, based on sparse Bayesian learning, $p(\beta_m)$ and $p(\varrho_m)$ are given a Gaussian distribution $\mathcal{N}(\cdot)$ and a Gamma distribution $\mathcal{G}(\cdot)$ with two hyperparameters b_0, c_0 , respectively [29],

$$p(\beta_m) = \mathcal{N}(\beta_m | 0, \varrho_m^{-1}), \quad p(\varrho_m) = \mathcal{G}(\varrho_m | b_0, c_0). \quad (6)$$

Similar to β , the weight vector of all SBF matrices, ω , should also have sparse characteristic to reduce the adverse effects of some unreasonable numbers of knots and SBFs on the final regression results. Thus,

$$p(\omega_k) = \mathcal{N}(\omega_k | 0, \eta_k^{-1}), \quad p(\eta_k) = \mathcal{G}(\eta_k | d_0, e_0). \quad (7)$$

where d_0, e_0 are two hyperparameters. In certain applications, due to the complex data collection environment and different collection devices, or the various source of data generation, the error often does not obey Gaussian distributions. For example, it was reported that the error distribution of WTPC modeling was heteroscedastic. To overcome the inconsistency between the error assumption in the model and the real error characteristic in practice, and considering that Central Limit Theorem makes Gaussian distribution a good choice for modeling the regression errors or noise [30], the error term ε_i in MSRM is given by the following Gaussian distribution,

$$p(\varepsilon_i) = \mathcal{N}(\varepsilon_i | 0, \gamma_i^{-1}). \quad (8)$$

Fig. 1. Hierarchic priors for all parameters in SHMSRM-G.

From (8), for different samples, the error obey different Gaussian distributions. It means that the error is heteroscedastic, which meets the task characteristic of WTPC modeling. To form a full probability model, the prior distribution of γ_i is assumed to obey a Gamma distribution with the hyperparameters f_0, g_0 ,

$$p(\gamma_i) = \mathcal{G}(\gamma_i | f_0, g_0). \quad (9)$$

According to (8), the likelihood of the observation $\mathbf{Y}$ can be expressed as

$$p(\mathbf{Y}) = \prod_{i=1}^N \mathcal{N}(y_i | \tilde{\mathbf{H}}_\omega^{(i,\cdot)} \beta, \gamma_i^{-1}) = \prod_{i=1}^N \mathcal{N}(y_i | \omega^\top \tilde{\mathbf{H}}^{(i,\cdot)} \beta, \gamma_i^{-1}) \quad (10)$$

where $\tilde{\mathbf{H}}_\omega^{(i,\cdot)}$ denotes the i th row of the matrix $\tilde{\mathbf{H}}_\omega$ and $\tilde{\mathbf{H}}^{(i,\cdot)} = [(\tilde{\mathbf{H}}_1^{(i,\cdot)})^\top, \dots, (\tilde{\mathbf{H}}_K^{(i,\cdot)})^\top]^\top \in \mathbb{R}^{K \times \tilde{M}}$.

Finally, the Bayesian model for SHMSRM-G can be constructed with the prior distributions from (6) to (10). The joint probability density function of the observed data $\mathbf{Y}$ and the corresponding parameters Θ is given by

$$p(\mathbf{Y}, \Theta) = p(\mathbf{Y}) \prod_{m=1}^{\tilde{M}} p(\varrho_m) p(\beta_m) \prod_{k=1}^K p(\eta_k) p(\omega_k) \prod_{i=1}^N p(\gamma_i) \quad (11)$$

where $\varrho = \{\varrho_1, \dots, \varrho_{\tilde{M}}\}$, $\eta = \{\eta_1, \dots, \eta_K\}$, $\gamma = \{\gamma_1, \dots, \gamma_N\}$, and $\Theta = \{\varrho, \beta, \omega, \eta, \gamma\}$. Then, our goal turns to infer the posteriors of all involved variables in (11).

2) *Posterior distributions of all parameters in SHMSRM-G*: In this paper, VB is employed to estimate all parameters in the joint probability $p(\mathbf{Y}, \Theta)$. The posterior distribution $q(\Theta)$ is supposed to approximate $p(\mathbf{Y}, \Theta)$, the factorized form of $q(\Theta)$ is expressed as

$$q(\Theta) = q(\omega) q(\eta) q(\beta) q(\varrho) q(\gamma). \quad (12)$$

According to the theory of VB, the posteriors of all model parameters can be calculated as follows.

First, taking the expectation of the logarithmic joint probability $\log p(\mathbf{Y}, \Theta)$ with respect to Θ except for β , it can be concluded that the posterior of β , $q(\beta)$, is a multivariate Gaussian distribution

$$q(\beta) = \mathcal{N}(\beta | \mu_\beta, \Sigma_\beta) \quad (13)$$

with the mean μ_β and covariance Σ_β updated by

$$\Sigma_\beta = \left[\sum_{i=1}^N \langle \gamma_i \rangle \tilde{\mathbf{H}}^{(i,\cdot)\top} \langle \omega \omega^\top \rangle \tilde{\mathbf{H}}^{(i,\cdot)} + \mathbf{A} \right]^{-1} \quad (14)$$

$$\mu_\beta = \sum_{i=1}^N \langle \gamma_i \rangle y_i \Sigma_\beta \tilde{\mathbf{H}}^{(i,\cdot)\top} \langle \omega \rangle$$

where $\mathbf{A} = \text{diag}(\langle \varrho_1 \rangle, \dots, \langle \varrho_M \rangle)$, $\langle \cdot \rangle$ means the expectation calculation, and the function $\text{diag}(\cdot)$ is used to make a diagonal matrix.

Subsequently, by calculating the expectation of $\log p(\mathbf{Y}, \Theta)$ with respect to Θ except for ϱ , it can be found that the posterior of ϱ_m is a Gamma distribution, namely

$$q(\varrho_m) = \mathcal{G}(\varrho_m | b_{\varrho_m}, c_{\varrho_m}) \quad (15)$$

with hyperparameters $b_{\varrho_m}, c_{\varrho_m}$ updated by

$$b_{\varrho_m} = b_0 + \frac{1}{2}, \quad c_{\varrho_m} = c_0 + \frac{1}{2} \langle \beta_m^2 \rangle. \quad (16)$$

By calculating $\langle \log p(\mathbf{Y}, \Theta) \rangle$ with respect to Θ except for ω , it can be inferred that $q(\omega_k)$ is a multivariate Gaussian distribution,

$$q(\omega) = \mathcal{N}(\omega | \mu_\omega, \Sigma_\omega) \quad (17)$$

with the mean μ_ω and covariance Σ_ω updated by

$$\Sigma_\omega = \left[\sum_{i=1}^N \langle \gamma_i \rangle \tilde{\mathbf{H}}^{(i,\cdot)} \langle \beta \beta^\top \rangle \tilde{\mathbf{H}}^{(i,\cdot)\top} + \mathbf{B} \right]^{-1} \quad (18)$$

$$\mu_\omega = \sum_{i=1}^N \langle \gamma_i \rangle y_i \Sigma_\omega \tilde{\mathbf{H}}^{(i,\cdot)} \langle \beta \rangle$$

where $\mathbf{B} = \text{diag}(\langle \eta_1 \rangle, \dots, \langle \eta_K \rangle)$.

For the parameter η , the posterior $q(\eta_k)$ is a Gamma distribution parameterized with g_{η_k} and h_{η_k} ,

$$q(\eta_k) = \mathcal{G}(\eta_k | d_{\eta_k}, e_{\eta_k}), \quad (19)$$

$$d_{\eta_k} = d_0 + \frac{1}{2}, \quad e_{\eta_k} = e_0 + \frac{1}{2} \langle \omega_k^2 \rangle.$$

Similarly, for the posterior of γ_i , it obeys a Gamma distribution,

$$q(\gamma_i) = \mathcal{G}(\gamma_i | f_{\gamma_i}, g_{\gamma_i}), \quad (20)$$

$$f_{\gamma_i} = f_0 + \frac{1}{2},$$

$$g_{\gamma_i} = g_0 + \frac{1}{2} \langle (y_i - \omega^\top \tilde{\mathbf{H}}^{(i,\cdot)} \beta)^2 \rangle.$$

The posterior distributions of all parameters in SHMSRM-G are inferred and presented from (13) to (20). The optimization procedure of SHMSRM-G is summarized in Algorithm 1.

C. Sparse heteroscedastic multiple spline regression models with Student's t -distribution (SHMSRM-T)

Different from SHMSRM-G, the error is assumed to obey a Student's t -distribution, thus resulting in a novel regression model, namely SHMSRM-T. The main reasons for the utilization of Student's t -distribution error assumption are presented as follows. First, Student's t -distribution is more general than Gaussian distribution. Specifically, when the degree of

Algorithm 1: Optimization procedure of SHMSRM-G

Input: Samples $\{x_i, y_i\}_{i=1, \dots, N}$, K SBF matrices, $b_0, c_0, d_0, e_0, f_0, g_0, Maxi$.

Output: μ_ω, μ_β

```

1 Initialize  $\langle r_{ik} \rangle, \langle \rho \rangle, \langle \nu \rangle, \langle \sigma_k^{-1} \rangle, \mu_\beta, \Sigma_\beta, \mu_\omega, \Sigma_\omega \langle \eta \rangle, \langle \varrho \rangle$ ;
2 while  $i < Maxi$  do
3   Update  $\mu_\beta, \Sigma_\beta$  according to (13);
4   Update  $b_{\varrho_m}, c_{\varrho_m}$  according to (16);
5   Update  $\mu_\omega, \Sigma_\omega$  according to (18);
6   Update  $d_{\eta_k}, e_{\eta_k}$  according to (19);
7   Update  $f_{\gamma_i}, g_{\gamma_i}$  according to (20);
8 end
9 return  $\mu_\omega, \mu_\beta$ ;
```

freedom in the probability density function of Student's t -distribution is equal to 1, it reduces to a Cauchy distribution; it approaches a Gaussian distribution when the degree of freedom tends to infinity [31]. Second, it is a heavy-tailed distribution, which is more robust than Gaussian distribution [32]. The priors and posteriors of all model parameters are described as follows.

1) *Prior distributions of all parameters in SHMSRM-T:* Generally, the probability density function of a Student's t -distribution with zero mean and ν degree of freedom can be expressed as

$$\mathcal{T}(\varepsilon | 0, \sigma, \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2}) \sqrt{\nu\pi}\sigma} \left(1 + \frac{\varepsilon^2}{\nu\sigma^2} \right)^{-\frac{\nu+1}{2}} \quad (21)$$

where σ is the scale parameter. In order to describe the heteroscedasticity of the regression errors, they are assumed to obey Student's t -distributions, whose parameters vary with the sample,

$$p(\varepsilon_i) = \mathcal{T}(\varepsilon_i | 0, \sigma_i, \nu_i). \quad (22)$$

It is reported that Student's t -distribution is a type of scale mixture of Gaussians [32]. So, it can be rewritten as [32], [33]

$$\mathcal{T}(\varepsilon | 0, \sigma, \nu) = \int_0^\infty \mathcal{N}(\varepsilon | 0, \phi^{-1}\sigma) \mathcal{G}\left(\phi | \frac{\nu}{2}, \frac{\nu}{2}\right) d\phi. \quad (23)$$

According to (23), a two-level generative model can be used to represent the heteroscedastic Student's t -error distribution in (22), namely

$$p(\varepsilon_i) = \mathcal{N}(\varepsilon_i | 0, \phi_i^{-1}\sigma_i), \quad p(\phi_i) = \mathcal{G}\left(\phi_i | \frac{\nu_i}{2}, \frac{\nu_i}{2}\right). \quad (24)$$

Thus, the likelihood function $p(\mathbf{Y})$ for the observed data is

$$p(\mathbf{Y}) = \prod_{i=1}^N \mathcal{N}(y_i | \omega^\top \tilde{\mathbf{H}}^{(i,\cdot)} \beta, \phi_i^{-1}\sigma_i) \quad (25)$$

To complete a full probability model for SHMSRM-T, the scale parameter σ_i and the degrees of freedom ν_i are assumed to obey an inverse Gamma distribution and a Gamma distribution, respectively,

$$p(\sigma_i) = \mathcal{IG}(\sigma_i | h_0, i_0), \quad p(\nu_i) = \mathcal{G}(\nu_i | r_0, s_0) \quad (26)$$

where h_0, i_0, r_0, s_0 are hyperparameters.

Fig. 2. Hierarchic priors for all parameters in SHMSRM-T.

The prior distributions for the other parameters in SHMSRM-T, such as β , ρ , ω and η , are the same with the priors in SHMSRM-G. Therefore, the corresponding joint probability density function can be given by

$$p(\mathbf{Y}, \Xi) = p(\mathbf{Y}) \prod_{m=1}^{\tilde{M}} p(\rho_m) p(\beta_m) \prod_{k=1}^K p(\eta_k) p(\omega_k) \cdot \prod_{i=1}^N p(\phi_i) p(\sigma_i) p(\nu_i) \quad (27)$$

where $\phi = \{\phi_1, \dots, \phi_N\}$, $\sigma = \{\sigma_1, \dots, \sigma_N\}$, $\nu = \{\nu_1, \dots, \nu_N\}$, and $\Xi = \{\rho, \beta, \omega, \eta, \phi, \sigma, \nu\}$. Then, our goal turns to infer the posteriors of all involved variables in (27).

2) *Posterior distributions of all parameters in SHMSRM-T*: VB is also employed to estimate all parameters in SHMSRM-T. The posterior of β in SHMSRM-T is also a multivariate Gaussian distribution, whose parameters are similar to (14). Its expression is

$$q(\beta) = \mathcal{N}(\beta | \mu_\beta, \Sigma_\beta), \quad \Sigma_\beta = \left[\sum_{i=1}^N \langle \phi_i \rangle \langle \sigma_i^{-1} \rangle \tilde{\mathbf{H}}^{(i, \cdot)^\top} \langle \omega \omega^\top \rangle \tilde{\mathbf{H}}^{(i, \cdot)} + \mathbf{A} \right]^{-1}, \quad \mu_\beta = \sum_{i=1}^N \langle \phi_i \rangle \langle \sigma_i^{-1} \rangle y_i \Sigma_\beta \tilde{\mathbf{H}}^{(i, \cdot)^\top} \langle \omega \rangle. \quad (28)$$

As to the parameter ω , its posterior distribution is

$$q(\omega) = \mathcal{N}(\omega | \mu_\omega, \Sigma_\omega), \quad \Sigma_\omega = \left[\sum_{i=1}^N \langle \phi_i \rangle \langle \sigma_i^{-1} \rangle \tilde{\mathbf{H}}^{(i, \cdot)^\top} \langle \beta \beta^\top \rangle \tilde{\mathbf{H}}^{(i, \cdot)} + \mathbf{B} \right]^{-1}, \quad \mu_\omega = \sum_{i=1}^N \langle \phi_i \rangle \langle \sigma_i^{-1} \rangle y_i \Sigma_\omega \tilde{\mathbf{H}}^{(i, \cdot)^\top} \langle \beta \rangle. \quad (29)$$

Taking the expectation of $\log p(\mathbf{Y}, \Xi)$ with respect to Ξ except for ϕ_i , the posterior of ϕ_i is a Gamma distribution,

$$q(\phi_i) = \mathcal{G}(\phi_i | m_{\phi_i}, n_{\phi_i}), \quad m_{\phi_i} = \frac{\langle \nu_i \rangle}{2} + \frac{1}{2}, \quad n_{\phi_i} = \frac{\langle \nu_i \rangle}{2} + \frac{1}{2} \langle \sigma_i^{-1} \rangle \langle (y_i - \omega^\top \tilde{\mathbf{H}}^{(i, \cdot)} \beta)^2 \rangle. \quad (30)$$

For the parameter σ_i , its posterior distribution $q(\sigma_i)$ is an inverse Gamma distribution parameterized by $h_{\sigma_i}, i_{\sigma_i}$,

$$q(\sigma_i) = \mathcal{IG}(\sigma_i | h_{\sigma_i}, i_{\sigma_i}), \quad h_{\sigma_i} = h_0 + \frac{1}{2}, \quad i_{\sigma_i} = i_0 + \frac{1}{2} \langle \phi_i \rangle \langle (y_i - \omega^\top \tilde{\mathbf{H}}^{(i, \cdot)} \beta)^2 \rangle. \quad (31)$$

According to the expectation of $\log p(\mathbf{Y}, \Xi)$ with respect to Ξ except for ν_i , the logarithmic posterior of ν_i can be expressed as (dropping the constant term)

$$\log q(\nu_i) = \frac{\nu_i}{2} \log \frac{\nu_i}{2} - \log \Gamma\left(\frac{\nu_i}{2}\right) - \frac{\nu_i}{2} \langle \phi_i \rangle + \left(\frac{\nu_i}{2} - 1\right) \langle \log \phi_i \rangle + (j_0 - 1) \log \nu_i - k_0 \nu_i. \quad (32)$$

From (32), $q(\nu_i)$ does not obey any common distributions. According to the theory of Stirling's approximation, $\log(\nu_i/2)$ can be approximated by the following equation [34],

$$\log \Gamma\left(\frac{\nu_i}{2}\right) = \frac{1}{2} \left(\log 2\pi - \log \frac{\nu_i}{2} \right) + \frac{\nu_i}{2} \left(\log \frac{\nu_i}{2} - 1 \right). \quad (33)$$

Therefore, it can be inferred that $q(\nu_i)$ is Gamma distributed by replacing the $\log(\nu_i/2)$ in (32) by (33),

$$q(\nu_i) = \mathcal{G}(\nu_i | r_{\nu_i}, s_{\nu_i}), \quad (34)$$

where

$$r_{\nu_k} = r_0 + \frac{1}{2}, \quad s_{\nu_k} = s_0 - \frac{1}{2} [1 + \langle \log \phi_i \rangle - \langle \phi_i \rangle]. \quad (35)$$

As to the other parameters η_k and ρ_m , it can be seen from Fig. 1 and Fig. 2 that they are only related with their own priors and the priors of their related parameters (β_m, ω_k). Due to the priors for β, ρ, ω and η here are the same as the priors in SHMSRM-G. So, the posterior distributions of η_k and ρ_m are the same with (19) and (15), respectively. The optimization procedure of SHMSRM-T is summarized in Algorithm 2.

Algorithm 2: Optimization procedure of SHMSRM-T

Input: Samples $\{x_i, y_i\}_{i=1, \dots, N}$, K SBF matrices, $b_0, c_0, d_0, e_0, h_0, i_0, r_0, s_0, Maxi$.

Output: μ_ω, μ_β

```

1 Initialize  $\langle \rho \rangle, \langle \nu \rangle, \langle \sigma_k^{-1} \rangle, \mu_\beta, \Sigma_\beta, \mu_\omega, \Sigma_\omega \langle \eta \rangle, \langle \rho \rangle$ ;
2 while  $i < Maxi$  do
3   Update  $\mu_\beta, \Sigma_\beta$  according to (28);
4   Update  $\mu_\omega, \Sigma_\omega$  according to (29);
5   Update  $m_{\phi_i}, n_{\phi_i}$  according to (30);
6   Update  $h_{\sigma_i}, i_{\sigma_i}$  according to (31);
7   Update  $r_{\nu_i}, s_{\nu_i}$  according to (35);
8   Update  $d_{\eta_k}, e_{\eta_k}$  according to (19);
9   Update  $b_{\rho_m}, c_{\rho_m}$  according to (15);
10 end
11 return  $\mu_\omega, \mu_\beta$ ;

```

IV. PROCESSES OF WTPC MODELING

Due to the complicated environment or a wind turbine fault, there are many outliers in the collected wind data. In general, an accurate WTPC cannot be obtained with the original unprocessed data. So, it is an essential step to preprocess the collected wind data [1] [12].

In this paper, a fuzzy c-means (FCM) clustering-based approach is used to detect outliers automatically. First, FCM is used to divide the data into T clusters. The i th sample in the t th cluster can be expressed as $\mathbf{x}_i^t$ ($i = 1, \dots, N_t, t = 1, \dots, T$),

TABLE I
DATA COLLECTED POSITION, NUMBER OF SAMPLES IN TRAINING AND TESTING SETS, AND VALUES OF SOME STATISTICS.

Dataset	Latitude	Longitude	All samples	Training set	Testing set	Mean (kW)	Min (kW)	Max (kW)	Std (kW)
A (Spring)	N 43°15'54"	E 117°32'44"	6000	5000	1000	136.5125	0.0000	854.9500	217.4973
B (Winter)	N 43°15'54"	E 117°32'44"	6000	5000	1000	244.8589	0.0000	856.9252	274.2028
C (Xichang)	N 42°55'56"	E 119°1'43"	5500	4500	1000	459.3550	0.2527	1564.9000	428.0186
D (Hunan)	N 27°14'40"	E 113°10'54"	9500	7500	2000	656.6667	0.0000	2069.9500	791.0424

TABLE II
FIVE CLUSTER CENTERS GENERATED BY FCM ON FOUR DATASETS.

Dataset	Cluster Center 1	Cluster Center 2	Cluster Center 3	Cluster Center 4	Cluster Center 5
A (Spring)	(3.2111, 12.6162)	(6.1589, 1.1904×10 ²)	(8.2129, 2.7816×10 ²)	(10.9976, 5.4984×10 ²)	(14.8659, 7.8673×10 ²)
B (Winter)	(3.9793, 19.8650)	(6.8292, 1.6493×10 ²)	(8.7311, 3.4846×10 ²)	(10.8429, 5.7656×10 ²)	(14.4399, 8.0324×10 ²)
C (Xichang)	(3.3939, 80.9016)	(5.5498, 2.6569×10 ²)	(6.5433, 4.2758×10 ²)	(9.5472, 1.0555×10 ³)	(13.3308, 1.4909×10 ³)
D (Hunan)	(1.6113, 13.5565)	(4.3617, 3.9997×10 ²)	(6.2745, 9.6424×10 ²)	(7.7831, 1.5132×10 ³)	(10.4993, 1.9761×10 ³)

and $\sum_{t=1}^T N_t = N$. Then, the mean μ_t (also known as cluster center) and covariance Σ_t of all samples in the t th cluster can be computed. Third, the Mahalanobis distance dis_i^t between the t th cluster center μ_t and the sample x_i^t should be calculated based on $dis_i^t = (x_i^t - \mu_t) \Sigma_t^{-1} (x_i^t - \mu_t)^T$. Fourth, if the distance dis_i^t is larger than the given threshold, the sample x_i^t can be considered as an outlier; otherwise, it is a normal sample. Finally, the selected normal samples are used to the train the WTPC models, and get the final WTPCs.

V. CASE STUDIES

In this section, the performances of different WTPC models are evaluated in different seasons and different wind farms.

A. Data description

Saihanba wind farm, China, provided two datasets, Dataset A and Dataset B. They were all collected from the same wind turbine but in spring and winter, respectively. The other two datasets, Dataset C and Dataset D, were from Xichang and Hunan wind farms, China, respectively. For the first two datasets, all samples were recorded every one minute. But in the remaining two datasets, the time resolution of the data was ten minutes. Only two variables, wind speed and wind power, were measured and preserved in all datasets. The entire dataset was divided into two subsets, a training set and a testing set. The training set is used to train the selected WTPC model, while the testing set is utilized to evaluate the model performance. Table I shows the information of all datasets, including data collected positions, number of samples in training and testing sets, and values of some statistics (mean, minimum, maximum and standard deviation) for all wind power data in the whole dataset.

B. Experiment setting

To test the effectiveness of SHMSRM-G and SHMSRM-T, five integrated curve fitting models and ten AI-based models are employed as benchmark models. The former contain 3-PLF, 4-PLF, 5-PLF, 6-PLF and MHTan. The later include GP,

SVM, ELM, ANFIS, SRM, weighted ELM (WELM) [35], HSRM, RSRM, MoAG-ASR and MoAEP-ASR.

As to the integrated curve fitting models, similar to [3], [4] and [12], the optimal model parameters can be optimized by the backtracking search optimization algorithm (BSA)¹. In SVM, the RBF kernel function is selected, and the all model parameters are tuned by grid search. For ELM and WELM, the activation function is Sigmoid. And the optimal number of hidden layers is chosen from the set $\{10, 20, \dots, 180, 200\}$. The optimization procures and initialized model parameters of HSRM, RSRM, MoAG-ASR and MoAEP-ASR are presented in [4] and [1]. As to spline regression-based models, two candidate SBFs are truncated power basis and B-spline basis, the candidate number of knots is in the set $\{2, 4, 6, \dots, 20\}$, and the order of spline is set as 3. The proposed two models take the above two SBFs and ten numbers of knots into consideration. Thus, there will be twenty SBF matrices, namely $K = 20$. As to all hyperparameters $\{b_0, c_0, d_0, e_0, f_0, g_0, h_0, i_0, r_0, s_0\}$, they are set as 0.001. And, the maximum number of iterations $Maxi$ is set as 50.

Three error indicators, mean absolute error (MAE) [4], root mean squared error (RMSE) [20] [24] and normalized mean absolute percentage error (NMAPE) [3] [24], are used to measure the performances of different WTPC models. To make a fair model comparison, all models are trained with the same training set, and they all have only one input (wind speed) and one output (wind power).

C. WTPC modeling

Here, the results of data preprocessing and WTPC modeling are analyzed.

1) *Data preprocessing*: Outliers prevent obtaining more accurate WTPCs. It is essential to preprocess the original wind data. According to the content in section IV, a FCM-based method is used for detecting outliers in training sets automatically. Here, the number of clusters and the threshold are set as 5 and 15, respectively. For different datasets, five cluster centers, presented in Table II, can be obtained when sending

¹<https://www.mathworks.com/matlabcentral/fileexchange/44842-backtracking-search-optimization-algorithm>

TABLE III
THE RESULTS OF SHMSRM-G, SHMSRM-T AND BENCHMARK MODELS.

Models	Dataset A (Spring)			Dataset B (Winter)			Dataset C (Xichang)			Dataset D (Hunan)		
	MAE(kW)	RMSE(kW)	NMAPE	MAE(kW)	RMSE(kW)	NMAPE	MAE(kW)	RMSE(kW)	NMAPE	MAE(kW)	RMSE(kW)	NMAPE
3-PLF	15.1827	19.1000	1.8946	20.4266	25.3246	2.4529	20.8619	31.0964	1.3331	49.5778	64.1729	2.4043
4-PLF	15.1971	19.1161	1.8964	20.5274	25.4023	2.4650	21.1574	31.1785	1.3520	49.8178	64.3210	2.4159
5-PLF	10.1283	16.4637	1.2639	14.2197	20.9155	1.7076	19.0592	30.6037	1.2179	37.2068	55.9600	1.8043
6-PLF	10.1337	16.1827	1.2646	14.9511	21.2848	1.7954	18.5268	30.2350	1.1839	40.6550	57.7892	1.9716
MTHan	11.5151	16.7755	1.4370	14.9748	21.6661	1.7982	25.6294	38.9473	1.6378	42.9278	59.8989	2.0818
SVM	9.7656	16.0846	1.2186	12.5110	20.4217	1.5024	17.1861	30.1403	1.0982	32.0216	52.9679	1.5529
GP	9.7715	16.0860	1.2194	12.6274	20.3898	1.5164	17.4269	29.7732	1.1136	32.6556	53.1104	1.5836
ELM	9.9923	16.6761	1.2469	12.7763	20.5045	1.5342	18.4905	30.3686	1.1816	32.0073	53.2754	1.5522
ANFIS	11.1184	19.4247	1.3874	12.3891	20.4108	1.4877	19.4030	31.0595	1.2399	31.9720	52.9914	1.5505
SRM	9.8720	16.5259	1.2319	12.8612	20.5692	1.5444	18.5585	30.6420	1.1859	34.5597	54.3409	1.6760
WELM	9.8852	16.6264	1.2336	12.5660	20.2934	1.5090	16.8134	29.9042	1.0744	31.2720	52.8500	1.5165
HSRM	9.7706	16.2107	1.2193	12.4909	20.1686	1.5000	16.5676	28.9852	1.0587	30.8560	52.1964	1.4964
RSRM	9.7575	15.9058	1.2176	12.2733	20.3556	1.4738	16.4125	29.6338	1.0488	30.6879	53.3324	1.4882
MoAG-ASR	9.8289	16.2620	1.2265	12.2699	19.9507	1.4734	16.3805	29.5380	1.0467	30.4604	54.6448	1.4772
MoAEP-ASR	9.8199	16.1723	1.2254	12.2332	19.9879	1.4690	16.2176	29.6430	1.0363	30.4849	52.5489	1.4784
SHMSRM-G	9.7473	16.0979	1.2164	12.0004	19.8486	1.4411	15.6231	28.7664	0.9983	30.6741	52.3980	1.4875
SHMSRM-T	9.7402	16.0757	1.2155	12.1552	20.1066	1.4597	15.1624	28.6887	0.9689	29.8551	52.0828	1.4478

the wind speed and wind power data into FCM. Meanwhile, all training samples can be classified into 5 clusters. Then, in each cluster, the Mahalanobis distance between the cluster center and the samples belonging to the cluster can be computed separately. Fig. 3 presents the calculated Mahalanobis distance in Cluster 1 and Cluster 5 of Dataset D. In Fig. 3, the red point represents an outlier, whose Mahalanobis distance is greater than the defined threshold 15. For different clusters, some outliers can be detected and removed. Finally, the processed wind data in Dataset A and Dataset D are presented in Fig. 4.

Fig. 3. The calculated Mahalanobis distance in Cluster 1 and Cluster 5.

Fig. 4. The processed wind data and detected outliers.

From Fig. 4, most obvious outliers can be detected. But no one can guarantee that the wind data preprocessing methods

can recognize all outliers. Similar to [12], [13] and [24], there will be still some hidden outliers in the processed data. So, developing novel WTPC models that are robust to outliers is necessary.

2) *Results of WTPC modeling*: This paper proposes two novel WTPC models, which consider the characteristic of WTPC modeling in the presence of outliers. The processed wind data are used to train those WTPC models (including the proposed models and benchmark models), the model performances on testing set are summarized in Table III.

Among all benchmark models, the AI-based models can be divided into two groups. The first group consists of some popular WTPC models, e.g., GP, SVM, ELM, ANFIS and SRM. The remaining AI-based models comprise the second group. And the proposed models also belong the second group. All AI-based models in the second group can be considered as the models that consider the characteristic of WTPC modeling. Specifically, WELM, HSRM, SHMSRM-G and SHMSRM-T focus on the difference of estimated errors between samples, namely the heteroscedasticity, while RSRM, MoAG-ASR and MoAEP-ASR consider the overall distribution of estimated errors in all samples, namely the non-Gaussian characteristic.

From Table III, the majority of AI-based models perform better than the integrated curve fitting models in different datasets. In Dataset A, Dataset B and Dataset D, 5-PLF is the best integrated curve fitting model, while 6-PLF performs better than the other integrated curve fitting models in Dataset C. Generally, the models that consider the characteristics of WTPC modeling will outperform the models without considering these characteristics. So, among all AI-based benchmark models, it is not surprising to see that the majority of models in the second group are superior to the models in the first group. In all datasets, the best AI-based benchmark models are RSRM, MoAEP-ASR, MoAEP-ASR and MoAG-ASR, respectively, in terms of MAE and NMAPE.

There is no universal WTPC model that can always perform

TABLE IV
AVERAGE IMPROVEMENT PERCENTAGES OF THE PROPOSED MODELS AND THE COMPUTATIONAL TIME OF ALL MODELS.

Models	Improvement of SHMSRM-G (%)			Improvement of SHMSRM-T (%)			Computational time (s)				
	MAE	RMSE	NMAPE	MAE	RMSE	NMAPE	Dataset A	Dataset B	Dataset C	Dataset D	Average
3-PLF	35.0731	15.7956	35.0729	35.8604	15.7553	35.8594	24.0469	23.9531	22.2188	30.3750	25.1485
4-PLF	35.4964	15.9812	35.4962	36.2749	15.9412	36.2739	36.7188	33.2969	33.4531	49.5938	38.2657
5-PLF	13.7388	4.9229	13.7385	14.6388	4.8525	14.6376	199.5625	165.8750	163.2500	283.1250	202.9531
6-PLF	15.9430	5.3645	15.9440	16.8269	5.2963	16.8270	231.3906	203.2656	199.5625	328.4219	240.66015
MTHan	25.7005	12.7727	25.7010	26.3838	12.6895	26.3836	132.5938	221.0781	100.3281	151.7656	151.4414
SVM	4.3928	2.0895	4.3922	5.4112	2.0214	5.4096	1906.2813	1707.1357	1606.5432	5831.6109	2762.8928
GP	5.4079	1.8258	5.4085	6.4075	1.7576	6.4071	297.3125	275.4375	327.8125	1301.5937	550.5391
ELM	7.0494	3.3972	7.0489	8.0268	3.3278	8.0253	0.1250	0.1719	0.1406	0.1250	0.1406
ANFIS	9.7524	7.0960	9.7516	10.6900	7.0198	10.6881	3.3750	4.5938	3.7188	6.2500	4.4844
SRM	8.7541	3.9474	8.7533	9.6842	3.8758	9.6824	0.1250	0.1875	0.0156	0.1719	0.1250
WELM	3.7219	2.5076	3.7223	4.7716	2.4373	4.7710	1.5000	0.1406	1.9219	2.9531	1.6289
HSRM	2.6139	0.6628	2.6161	3.6810	0.5952	3.6821	72.8438	63.7344	62.5938	162.2344	90.3516
RSRM	1.7957	1.4905	1.7948	2.8675	1.4218	2.8655	75.1094	65.7969	63.9375	166.2188	92.7657
MoAG-ASR	1.7372	2.0612	1.7356	2.8152	1.9820	2.8125	35.6563	25.1406	10.8438	45.2031	29.21095
MoAEP-ASR	1.4219	1.1003	1.4213	2.5054	1.0274	2.5037	819.0469	869.6406	583.5625	922.5156	798.6914
SHMSRM-G	0.0000	0.0000	0.0000	/	/	/	8.3438	8.1797	8.0313	11.5469	9.0254
SHMSRM-T	/	/	/	0.0000	0.0000	0.0000	8.2344	9.5550	7.8750	10.8438	9.1271

better than the others under different conditions. Overall, the proposed two models outperform the other benchmark models. In spring (Dataset A), in terms of MAE and NMAPE, SHMSRM-T performs the best among all WTPC models. But RSRM produces a WTPC with the lowest RMSE. For the same wind turbine in winter (Dataset B), SHMSRM-G is the best WTPC model. As for the wind turbines in Xichang (Dataset C) and Hunan (Dataset D) wind farms, SHMSRM-T is superior to all benchmark models and SHMSRM-G in terms of three error indicators.

In different datasets, compared with fifteen benchmark models, the improvement percentages of MAE, RMSE and NMAPE for SHMSRM-G and SHMSRM-T are computed, respectively, as shown in [36]. The average improvement percentages for all datasets are presented in Table IV. Generally, it can be found that, for the proposed models, the average improvement percentages of MAE and NMAPE reach the maximum and minimum when compared with 4-PLF and MoAEP-ASR, respectively. Compared with HSRM, the lowest average improvement percentages of RMSE can be obtained.

Fig. 5. The estimated WTPCs in Dataset B and Dataset D.

The estimated WTPCs generated by the optimal WTPC models in Dataset B and Dataset D are shown in Fig. 5. From Fig. 5, the estimated WTPCs can fit the wind data properly, thus describing the actual performances of wind turbines in

practice. Moreover, different from the smooth ideal WTPC provided by manufacturer, the estimated actual WTPC may be a nonsmooth line, as can be seen in Dataset B.

D. Discussion

In terms of different error metrics, the ranks of all WTPC models can be computed. Then, the average ranks of different models on all datasets are employed to measure their overall performances in the task of WTPC modeling. The corresponding results are shown in Fig. 6.

Fig. 6. Average ranks of WTPC models on all datasets.

From Fig. 6, among the seventeen models, SHMSRM-T and SHMSRM-G rank first and second, respectively, in WTPC modeling. All AI-based models rank higher than five integrated curve fitting benchmark models. The reason may be that the former is more flexible than the later, thus can simulate more complex WTPCs. In twelve AI-based models, except WELM, the models in the second group perform better than the models in the first group. This phenomenon indicates that developing novel regression models that consider the task characteristic of WTPC modeling can help generate more accurate WTPCs. The ranks of HSRM, RSRM, MoAG-ASR and MoAEP-ASR are close to each other. It shows that whether considering the heteroscedasticity or the non-Gaussian error distribution, the constructed regression models show the similar robust performance in the presence of outliers.

There are three major reasons why the performances of the proposed two models are better than that of the other benchmark models. First, consideration of the heteroscedastic

characteristic of WTPC modeling, which is also the biggest difference among the proposed models, five integrated curve fitting models and the models in the first group of twelve AI-based methods. Second, utilization of multiple SBFs with different numbers of knots, which helps to describe the complex nonlinear relationship between wind speed and wind power. Third, the usage of sparse prior distributions on the regression coefficients and the weights of SBF matrices. The sparsity, as shown in Fig. 7, can help reduce the bad effects of some redundant SBFs and some useless mapping features.

Fig. 7. The estimated weights and regression coefficients on Dataset D.

From Fig. 7 (b) and (d), the closer the value of the regression coefficient is to 0, the smaller the effect of the corresponding feature on the final regression results. Fig. 7 (a) and (c) show the similar result. Some values of weights are very close 0, that means the corresponding SBFs have less effects on the final regression performance.

The computational time (CPU time) of different models is also compared. The results are shown in Table IV. It can be seen from Table IV that the computational time of the proposed models is less than the majority of compared models, including five integrated curve fitting models and six AI-based models. However, for the other four AI-based models (SRM, ELM, WELM and ANFIS), their computational time is less than the proposed models because they have fewer model parameters to be tuned. As to SVM and GP, they usually have higher computational complexity due to the large kernel matrix, thus resulting in more computational time. Compared with the other four spline-based regression models (HSRM, RSRM, MoAG-ASR and MoAEP-ASR), the advantage of the proposed models in computational time is also obvious. On the whole, the proposed models have higher power curve modeling accuracy with suitable computational time.

VI. CONCLUSIONS

WTPC modeling is crucial in the development and utilization of wind energy. Some current WTPC models have the

limited fitting ability because they do not consider the characteristic of WTPC modeling and they have the limited fitting ability in describing the complex nonlinear relationship between wind speed and wind power. To overcome these disadvantages, this paper proposes two novel WTPC models, SHMSRM-G and SHMSRM-T. Compared with the performances of fifteen benchmark WTPC models in different seasons and different wind farms, the results show that (a) as long as the characteristics of WTPC modeling are considered, whether it is heteroscedasticity or non-Gaussian error distribution, the accuracy of WTPC modeling can be enhanced; (b) the actual WTPCs are not always smooth, they may be non-smooth under complex environmental conditions; (c) a combination of multiple spline regression models can help describe more complex nonlinear WTPCs than single models; (d) sparse prior distributions on regression coefficients and weights of SBF matrices can help select useful SBFs and mapping features. In addition to the high accuracy, the two proposed models also have high computational efficiency.

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