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Cold-Atom-Induced Control of an Optomechanical Device

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We consider a cavity with a vibrating end mirror and coupled to a Bose-Einstein condensate. The cavity field mediates the interplay between mirror and collective oscillations of the atomic density. We study the implications of this dynamics and the possibility of an indirect diagnostic. Our predictions can be observed in a realistic setup that is central to the current quest for mesoscopic quantumness.

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Achieving quantum control over a system endowed with macroscopic degrees of freedom is a long-sought goal of modern physics. The accomplishment of such a task will help us to shift the domain of applicability and the exploitation of quantum technology from the context of microscopic systems fulfilling stringent criteria for quantumness to the mesoscopic world [1–3]. Important progress has been made along these lines: the gap between current experimental possibilities and the observation of genuine mesoscopic quantum effects is now only a few *quanta* wide [4]. In such a quest, a few systems allow for the observation of interesting quantum effects at a large scale: micro/nano-mechanical devices [5], collective excitations of ultracold atomic ensembles [6], and arrays of superconducting devices [7].

In this paper, we demonstrate mutual back-action dynamics of two macroscopic degrees of freedom embodied by physical systems of different nature. We consider the interplay between a Bose-Einstein condensate (BEC) and the vibrating end mirror of an optical cavity. We show a nontrivial intertwined dynamics between collective atomic modes, coupled to the cavity field, and the mechanical one, which experiences radiation-pressure forces. By focusing on noise properties, important signatures of one subsystem in the dynamics of the other can be revealed by looking at experimentally accessible quantities. We characterize the atom-induced back-action that modifies the cooling capabilities of the optomechanical system and show that our predictions can be tested with the current state of the art. Our study paves the way towards the use of the mutual interaction between atomic and mechanical subsystems for the sake of coherent quantum control at the mesoscopic scale. Such a coupling has the potential to implement effective schemes for quantum state engineering and dynamical manipulation of the (inaccessible) mechanical mode through the atomic subsystem.

The movable end mirror of the optical cavity of length $\mathcal{L}$ is assumed to perform harmonic oscillations at frequency ω_m along the cavity axis. The mirror is in contact with a background of phononic modes in equilibrium at tempera-

ture T . The cavity is pumped through its (steady) input mirror by a laser of tunable frequency. The BEC is confined in a large-volume trap within the cavity [6,8] [cf. Fig. 1(a)]. Alternatively, the BEC could be sitting in a 1D optical lattice generated by a trapping mode sustained by a bimodal cavity [9]. The atom-cavity interaction is insensitive to the details of the trapping, and our study holds in both cases. In the weakly interacting regime, the atomic field operator can be split into a classical part (the condensate wave function) and a quantum one (the fluctuations) expressed in terms of Bogoliubov modes. Recent experiments coupling a BEC to an optical resonator [8] suggest that the Bogoliubov modes interacting significantly with the cavity field are those with momentum $\pm 2k_c$ (k_c is the cavity-mode momentum), while the condensate is considered to be initially at zero temperature. The cavity end mirror experiences radiation pressure, while optical forces excite superpositions of atomic momentum modes. Interference between momentum-excited atoms and condensates creates a periodic density grating sensed via the cavity.

We write the Hamiltonian of the system made out of the cavity field, the movable mirror, and the BEC as $\hat{\mathcal{H}} = \sum_{j=M,C,A} \hat{\mathcal{H}}_j + \hat{\mathcal{H}}_{AC} + \hat{\mathcal{H}}_{MC}$, where $\hat{\mathcal{H}}_M = m\omega_m^2 \hat{q}^2/2 + \hat{p}^2/(2m)$, $\hat{\mathcal{H}}_C = \hbar(\omega_C - \omega_L)\hat{a}^\dagger\hat{a} - i\hbar\eta(\hat{a} - \hat{a}^\dagger)$, and $\hat{\mathcal{H}}_A = \hbar\tilde{\omega}\hat{c}^\dagger\hat{c}$. Here $\hat{q}$ ($\hat{p}$) is the mirror displacement (momentum), m is its effective mass, ω_C (ω_L) is the cavity (pump laser) frequency, and $\hat{a}$ ($\hat{a}^\dagger$) is the corresponding annihilation (creation) operator. Finally, $\tilde{\omega}$ and $\hat{c}$ ($\hat{c}^\dagger$) are the frequency and the bosonic annihilation (creation) operator of the Bogoliubov mode. We have incorporated a *displacing* term $-i\hbar\eta(\hat{a} - \hat{a}^\dagger)$ in the cavity Hamiltonian. This arises from the pump-cavity coupling, which shifts the cavity field in phase space (and, in turn, the equilibrium position of the vibrating end mirror) proportionally to the coupling parameter $\eta = \sqrt{2\kappa\mathcal{R}/\hbar\omega_L}$ ($\mathcal{R}$ is the laser power and κ is the cavity decay rate). For small mirror displacements and a large cavity-free spectral range with respect to ω_m , the mirror-cavity interaction can be put

in the form $\hat{\mathcal{H}}_{MC} = -\hbar\chi\hat{q}\hat{a}^\dagger\hat{a}$ with $\chi = \omega_C/\mathcal{L}$. On the other hand, the atom-cavity interaction reads

$$\hat{\mathcal{H}}_{AC} = (\hbar g^2 N_0)/(2\Delta_a)\hat{a}^\dagger\hat{a} + \hbar\sqrt{2}\zeta\hat{Q}\hat{a}^\dagger\hat{a}, \quad (1)$$

which contains two contributions: one is from the condensate only, while the second is related to the position-like operator $\hat{Q} = (\hat{c} + \hat{c}^\dagger)/\sqrt{2}$ of the Bogoliubov mode (we have assumed that the condensate wave function is not affected by the coupling to the cavity field). In Eq. (1), g is the vacuum Rabi frequency for the dipole-like transition connecting the atomic ground and excited states, N_0 is the number of condensed atoms, Δ_a is atom-cavity detuning, and the coupling rate $\zeta \propto \sqrt{N_0}g^2/\Delta_a$. As discussed in [10], a rigorous calculation shows that ζ also depends on the Bogoliubov mode function and can be conveniently tuned. While the first term in Eq. (1) embodies a cavity-frequency pull, the second is formally analogous to $\hat{\mathcal{H}}_{MC}$ and shows that, under the above working conditions, the BEC dynamics mimics that of a mechanical mode undergoing radiation-pressure effects. A similar result, for a BEC coupled to a static cavity, has been found in [8]. Our approach can be extended to include higher-order momentum modes.

The dynamics due to $\hat{\mathcal{H}}$ is complicated by the nonlinearity in $\hat{\mathcal{H}}_{MC}$ and $\hat{\mathcal{H}}_{AC}$. However, for an intense pump laser, the problem can be linearized by introducing quantum fluctuations as $\hat{O} \rightarrow \mathcal{O}_s + \delta\hat{O}$, with $\hat{O}$ any of the operators entering $\hat{\mathcal{H}}$, $\mathcal{O}_s$ its mean value, and $\delta\hat{O}$ the associated first-order quantum fluctuation [11]. We define the cavity quadratures as $\hat{x} = \hat{a} + \hat{a}^\dagger$ and $\hat{y} = i(\hat{a}^\dagger - \hat{a})$ and the operator $\hat{P} = i(\hat{c}^\dagger - \hat{c})/\sqrt{2}$ which is canonically conjugated to the position-like atomic operator $\hat{Q}$. Any realistic description of the problem at hand should include the most relevant sources of noise affecting the overall device, i.e., energy leakage from the cavity and thermal Brownian motion at temperature T undergone by the cavity mirror. We thus consider the Langevin equation $\partial_t\hat{\phi} = \mathcal{K}\hat{\phi} + \hat{\mathcal{N}}$, where we have introduced the vector $\hat{\phi}^T = (\delta\hat{x} \ \delta\hat{y} \ \delta\hat{q} \ \delta\hat{p} \ \delta\hat{Q} \ \delta\hat{P})$, the noise vector $\hat{\mathcal{N}}^T = (\sqrt{\kappa}(\delta\hat{a}_{in}^\dagger + \hat{a}_{in}) \ i\sqrt{\kappa}\delta(\hat{a}_{in}^\dagger - \hat{a}_{in}) \ 0 \ \hat{\xi} \ 0 \ 0)$, and the dynamical coupling matrix $\mathcal{K}$, which is given in Ref. [10]. The evolution of the system depends on a few crucial parameters, including the total detuning $\Delta = \omega_C - \omega_L - \chi q_s + \sqrt{2}\zeta Q_s + g^2 N_0/2\Delta_a$ between the cavity and the pump laser. This consists of the steady pull-off term in Eq. (1), as well as both the optomechanical contributions proportional to the displaced equilibrium positions of the mechanical and Bogoliubov modes. These are, respectively, given by the stationary values $q_s = \hbar\chi\alpha_s^2/m\omega_m^2$ and $Q_s = -\sqrt{2}\zeta\alpha_s^2/\tilde{\omega}$, which are in turn determined by the mean intracavity field amplitude $\alpha_s = \eta/\sqrt{\Delta^2 + \kappa^2}$. The interlaced nature of such stationary parameters (notice the dependence of α_s on the detuning) is at the origin of

bistability and chaotic effects [8,9,12]. As for the noise-related part of the dynamics, we have introduced $\delta\hat{a}_{in}$ and $\delta\hat{a}_{in}^\dagger$ as zero-average [$\langle\hat{a}_{in}(t)\rangle = \langle\hat{a}_{in}^\dagger(t)\rangle = 0$], delta-correlated [$\langle\hat{a}_{in}(t)\hat{a}_{in}^\dagger(t')\rangle = \delta(t-t')$] operators describing white noise entering the cavity from the leaky mirror. Dissipation of the mechanical mirror energy is, on the other hand, associated with the decay rate γ and the corresponding zero-mean Langevin-force operator $\hat{\xi}(t)$ having non-Markovian correlations ($\beta_B = \hbar/2k_B T$) [13] $\langle\hat{\xi}(t)\hat{\xi}(t')\rangle = (\hbar\gamma m/2\pi) \int \omega e^{-i\omega(t-t')} [\coth(\beta_B\omega) + 1] d\omega$. Although the non-Markovianity of the mechanical Brownian motion could be retained, for large mechanical quality factors ($\gamma \rightarrow 0$), a condition that is currently met [5], one can take $\langle\hat{\xi}(t)\hat{\xi}(t')\rangle \approx [\hbar\gamma m/\beta_B + i\partial_t]\delta(t-t')$ [13]. As our analysis relies on symmetrized two-time correlators, the term $\partial_t\delta(t-t')$ is ineffective and our description becomes fully Markovian. Here we show that the model above results in an interesting back-action dynamics, where the state of the mechanical mode is strongly intertwined with the BEC. The physical properties of the mirror are altered by the cavity-BEC coupling. Evidence of such an interaction, strong enough to inhibit the cooling capabilities of the radiation-pressure mechanism, is found in the noise properties of the mechanical mode.

We start by considering the modification in the mirror dynamics due to the coupling to the cavity and indirectly to the BEC. The Langevin equations are solved in the frequency domain, where we should ensure stability of the solutions. This implies negativity of the real part of the eigenvalues of $\mathcal{K}$. Numerically, we have found that stability is found for $\Delta > 0$ and weak BEC-to-cavity coupling, i.e., for $\{\chi\sqrt{\hbar/m\omega_m}, \zeta\} \ll \kappa$, which are conditions fulfilled throughout this work. We find the mirror displacement $\delta\hat{q}(\omega) = [\mathcal{A}_M(\omega)\delta\hat{y}_{in}(\omega) + \mathcal{B}_M(\omega)\delta\hat{x}_{in}(\omega) + \mathcal{C}_M(\omega)\hat{\xi}(\omega)]$, with $\mathcal{A}_M(\omega) = \mathcal{B}(\omega)\Delta/(\kappa - i\omega) = -\hbar\chi\alpha_s\sqrt{2\kappa}\Delta/d_M(\omega)$, $\mathcal{C}_M(\omega) = -\{(\omega^2 - \tilde{\omega}^2)[(\kappa - i\omega)^2 + \Delta^2] + 4\tilde{\omega}\Delta\alpha_s^2\zeta^2\}/d_M(\omega)$, and $d_M(\omega)$ is related to the effective susceptibility function of the mechanical mode [10]. We now compute the density noise spectrum (DNS) of $\delta\hat{q}(\omega)$. For a generic operator $\hat{O}(\omega)$ in the frequency domain, the DNS is defined as $S_O(\omega) = (1/4\pi) \times \int d\Omega e^{-i(\omega+\Omega)t} \langle\hat{O}(\omega)\hat{O}(\Omega) + \hat{O}(\Omega)\hat{O}(\omega)\rangle$. Using the correlation properties of the input and Brownian noise operators, after a little algebra one gets $S_q(\omega) = \sum_{\mathcal{J}=\mathcal{A},\mathcal{B}} |\mathcal{J}_M(\omega)|^2 + \hbar\gamma m[1 + \coth(\beta\omega)]|\mathcal{C}_M(\omega)|^2$. Some interesting features emerge from the study of $S_q(\omega)$. In Fig. 1 we compare the case of an empty optomechanical cavity [panel (b)] to one where a weak coupling with the atomic Bogoliubov mode of frequency $\tilde{\omega} = \omega_m$ is included [panel (c)]. For an empty cavity, the mechanical-mode spectrum is obviously identical to what has been found in Ref. [14]. Both the optical spring effect in a detuned optical cavity and a cooling/heating mechanism are evident: height, width, and peak frequency of $S_q(\omega)$ change with the detuning Δ . At $\Delta \simeq \kappa/2$ optimal cooling is

FIG. 1 (color online). (a) A laser is split by an unbalanced beam splitter. The transmitted part is phase modulated and enters the cavity. The (weak) reflected part of the laser probes the BEC. The signals from the cavity and the BEC go to a detection stage consisting of a switch (selecting the signal to analyze), a photodiode, and a spectrum analyzer (SA). (b) $S_q(\omega, \Delta)$ for an empty cavity having $L = 2.5$ cm, $m = 15$ ng, $\omega_m/2\pi = 275$ KHz, $\gamma = \omega_m/Q$ with $Q = 10^5$ and $T = 300$ K and $\kappa \simeq 5$ MHz. The pumping light has a wavelength of 1064 nm and an input power of 4 mW. (c) We include the effects of the atomic coupling by taking $\tilde{\omega} = \omega_m$ and $\zeta \simeq 0.7\chi\sqrt{\hbar/(m\omega_m)}$.

achieved with a considerable shrink in the height of the spectrum. However, as soon as the Bogoliubov mode enters the dynamics, major modifications appear. The optical spring effect is magnified [the redshift of the peak frequency of $S_q(\omega)$ is larger than at $\zeta = 0$] and a secondary structure appears in the spectrum, unaffected by any change of Δ . Such a structure is a second Lorentzian peak centered in $\omega \simeq \omega_m$ and it is a signature of the back-action induced by the atoms, an effect that comes from a three-mode coupling and, as discussed later, is *determined* by $\tilde{\omega}$ and ζ . In fact, by studying the dependence of $S_q(\omega)$ on the frequency of the Bogoliubov mode, we see that the secondary peak identified above is centered at $\tilde{\omega}$. For $\zeta \ll \chi\sqrt{\hbar/(m\omega_m)}$, the signature of the atomic mode in the spectrum of the mechanical one is small. Quantitatively, it only consists of a tiny structure subjected to negligible detuning-induced changes. The picture changes for $\tilde{\omega}$ close to the mechanical frequency. In this case, as seen in Fig. 1(c), the influence of the atomic medium is considerable and present at any value of Δ . While the mechanical mode experiences an enhanced optical spring effect (see [10]), the secondary structure persists even at $\Delta \sim \kappa/2$, the working point that, for our

choice of parameters, optimizes the mechanical cooling at the empty cavity. However, as we show later, here the strong optical spring effect is not accompanied by an effective mechanical cooling.

A better understanding is provided by studying $S_q(\omega, \Delta)$ against ζ [cf. Figure 2(a)]. At the optimal empty-cavity detuning and for $\tilde{\omega} \simeq \omega_m$, both the effects highlighted above are clearly seen: the *contribution* of the secondary structure centered at $\tilde{\omega}$ grows with ζ due to the increasing atomic back-action, while a large redshift and a shrinking of the mechanical-mode contribution to the DNS are visible. An intuitive explanation for this comes from a normal-mode description, where $\hat{\mathcal{H}}$ is diagonalized by linear combinations of mechanical and Bogoliubov modes. The weight of the latter grows with ζ , thus determining a strong influence of the atomic part of the system over the noise properties of the mechanical mode.

The consequences of the atomic back-action are not restricted to the effects highlighted above. Strikingly, the coupling between the atomic medium and the cavity field acts as a *switch* for the cooling experienced by the mechanical mode in an empty cavity [cf. Fig. 2(b)]. That is, the coupling to the collective oscillations of the atomic density determines the number of thermal excitations in the mechanical mode and thus its energy. A way to clearly see this is to consider the effective temperature $T_{\text{eff}} = \langle U \rangle / k_B$, where $\langle U \rangle = m\omega_m^2 \langle \delta \hat{q}^2 \rangle / 2 + \langle \delta \hat{p}^2 \rangle / (2m)$ is the mean energy of the mechanical mode. $\langle U \rangle$ is experimentally easily determined by measuring just the area underneath $S_q(\omega, \Delta)$. In fact, we have $\langle \delta \hat{p}^2 \rangle = \int d\omega S_r(\omega, \Delta) \times (r = q, p)$ with $S_p(\omega, \Delta) = m^2 \omega_m^2 S_q(\omega, \Delta)$. Such a temperature-regulating mechanism is explained in terms of a simple thermodynamic argument. The exchange of excitations behind passive mechanical cooling [5,14] occurs at the optical sideband centered at ω_m . When the frequency of the Bogoliubov mode does not match this sideband, the mirror-cavity interaction is only weakly disturbed by the BEC. Thus, mechanical cooling occurs as in an empty cavity: even for relatively large values of ζ , the cooling capabilities of the detuned optomechanical process are basically unaffected [see Fig. 2(b)]. However, by tuning $\tilde{\omega}$ on resonance with such optical sideband, we introduce a

FIG. 2 (color online). (a) $S_q(\omega, \Delta)$ against ω and ζ for $\tilde{\omega} \simeq \omega_m$ and $\Delta = \kappa/2$. The structure centered at ω_m and with amplitude growing with ζ is due to atomic back-action. Inset: same plot for $\tilde{\omega} = 0.8\omega_m$. (b) Temperature of the mechanical mode against Δ/κ . Solid (dashed) lines are for $\tilde{\omega} = 0.1\omega_m$ ($\tilde{\omega} \simeq \omega_m$). (c) $S_Q(\omega, \Delta)$ for $\tilde{\omega} \simeq \omega_m$ and $\zeta = 50$ Hz.

FIG. 3 (color online). Panels (a)–(c): $S_Q(\omega, \kappa/2)$ against ω and ζ for $\chi = k\omega_c/2\mathcal{L}(k = 0, 1, 2)$.

well-source mechanism for the recycling of phonons extracted from the mechanical mode and transferred to the cavity field. The BEC can now absorb some excitations taken from the mirror by the field, thus acting as a *phononic well*, and release them into the field at a frequency matched with ω_m . The mirror can take the excitations back, as in the presence of a *phononic source*: thermodynamical equilibrium is established at a temperature set by ζ . For strong atomic back-action, the mirror does not experience any cooling [Fig. 2(b)].

Analogously, one finds the atomic DNS associated with the position-like operator of the Bogoliubov mode, which reads $\delta\hat{Q}(\omega) = [\mathcal{A}_A(\omega)\delta\hat{y}_{\text{in}} + \mathcal{B}_A(\omega)\delta\hat{x}_{\text{in}} + \mathcal{C}_A(\omega)\hat{\xi}(\omega)]$ with $\mathcal{A}_A(\omega) = \Delta\mathcal{B}(\omega)/[\tilde{\omega}(\kappa - i\omega)] = 2i\alpha_s\zeta\Delta\sqrt{\kappa}\omega(i\gamma\omega + \omega^2 - \omega_m^2)/d_A$, $\mathcal{C}_A(\omega) = -2\sqrt{2}i\alpha_s^2\Delta\zeta\chi\omega\tilde{\omega}/md_A$, and d_A being rather lengthy. The spectrum $S_Q(\omega, \Delta)$ is then easily determined using the appropriate input-noise correlation functions (see Fig. 3). Clearly, in light of the formal equivalence of Eq. (1) with a radiation-pressure mechanism, by setting up the proper working point, the BEC undergoes a cooling dynamics similar to the one experienced by the mirror. The starting temperature of the Bogoliubov mode depends on the values taken by $\tilde{\omega}$ and ζ . At $\zeta = 0$, regardless of $\tilde{\omega}$, its effective temperature is very low. For a set value of ζ , the temperature arises as $\tilde{\omega} \rightarrow \omega_m$. Our investigation is such that weak coupling between the BEC and the cavity field is kept, so that the Bogoliubov expansion is valid. The mutually induced back-action at the center of our discussion is clearly visible in Figs. 2(c) and 3, where features similar to those present in the mechanical DNS appear. For $\chi = 0$, the atomics DNS at $\tilde{\omega} = \omega_m$ starts from zero (at $\zeta = 0$) and experiences redshifts and shrinking as the effective optomechanical coupling rate grows. Having switched off the coupling between the mechanical mode and the field, the spectrum is single peaked. This is not the case for $\chi \neq 0$, where a secondary structure appears, similar to the one in the mechanical DNS. The splitting between mechanical and atomic contributions to $S_Q(\omega, \kappa/2)$ grows with χ , a sign of the mechanically enhanced effect felt by the atomic mode.

We have discussed dynamical back-action in an experimentally viable setup. The indirect mutual cross talking between mechanical and atomic modes strongly modifies the respective dynamics and leaves a signature of the reciprocal influence in experimentally handy quantities. This opens up the way to novel diagnostic strategies, where

the relevant interaction parameters are determined by measuring the noise properties of only one subsystem and fitting them with the analytical expressions for the relevant spectra provided here. The BEC spectrum could be probed by homodyning a weak forward-scattered field, transversally fed into the cavity and coupled to the atoms, as done in similar contexts [6] [cf. Fig. 1(a)]. Our study demonstrates coherence and mutual control in the coupling between the BEC and the mirror. These features are crucial in designing strategies for state engineering of the mechanical mode by means of its interaction with the BEC, which is more accessible, easier to manipulate, and less prone to noise effects.

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