Strength and ductility of RC jacketed columns: a simplified analytical method

Document Version:
Peer reviewed version

Strength and ductility of RC jacketed columns:
a simplified analytical method

Giovanni Minafò*, Fabio Di Trapani,

Dipartimento di Ingegneria Civile, Ambientale, aerospaziale e dei Materiali (DICAM), Università degli Studi di Palermo, Facoltà di Ingegneria, Viale delle Scienze, 90128 Palermo (Italy).

Giuseppina Amato

Queen's University Belfast, School of Planning, Architecture and Civil Engineering, David Keir Building, BT9 5AG Belfast (UK).

*Corresponding author, email: giovanni.minafo@unipa.it , ph. +39 09123896749

ABSTRACT

Reinforced concrete (RC) jacketing is a common method for retrofitting existing columns with poor structural performance. It can be applied in two different ways: if the continuity of the jacket is ensured, the axial load of the column can be transferred to the jacket, which will be directly loaded; conversely, if no continuity is provided, the jacket will induce only confinement action. In both cases the strength and ductility evaluation is rather complex, due to the different physical phenomena included, such as confinement, core-jacket composite action, preload and buckling of longitudinal bars.

Although different theoretical studies have been carried out to calculate the confinement effects, a practical approach to evaluate the flexural capacity and ductility is still missing. The calculation of these quantities is often related to the use of commercial software, taking advantage of numerical methods such as fibre method or finite element method.

This paper presents a simplified approach to calculate the flexural strength and ductility of square RC jacketed sections subjected to axial load and bending moment. In particular the proposed approach is based on the calibration of the stress-block parameters including the confinement effect. Equilibrium equations are determined and buckling of longitudinal bars is modelled with a suitable stress-strain law. Moment-curvature curves are derived with simple calculations. Finally, comparisons are made with numerical analyses carried out with the code OpenSees and with experimental data available in the literature, showing good agreement.

1 INTRODUCTION

Jacketing of reinforced concrete (RC) columns is a technique widely adopted in current engineering practice to retrofit existing weak members and increase their strength and ductility. The method consists in casting a RC layer (jacket) around the column, in order to increase the confinement effect on the member and/or enlarge the cross section. The effect provided by the jacket depends on whether or not it is directly loaded (i.e. when the jacket is continuous and well connected in correspondence of the slabs) or indirectly loaded (i.e. when a gap exists between the jacket and the slabs). In the first case, a core-jacket composite action as well as the confinement effect due to external stirrups, which enhance the axial capacity [1-4], takes place. Conversely, if the jacket is indirectly loaded the main effect of the technique is the confinement pressure induced by the external layer on the inner column core. In both cases, the amount of transverse and longitudinal steel is crucial for the overall efficacy of the technique, as well as the thickness of the jacket.

To evaluate the strength and deformation capacities of a jacketed element Eurocode 8 [5] allows to make three simplifying assumptions: (i) absence of slippage between old and new concrete; (ii) application of concrete properties over the full section of the element; (iii) neglecting of the confinement effects and buckling of longitudinal bars. Moreover EC8 assumes the full axial load acting on the jacketed element (core-jacket composite action). The strength and ductility capacity of the jacketed member obtained under these assumption (monolithic member) is then calibrated by applying suitable multipliers or *monolithic factors K_i*, commonly derived from empirical analysis [6-8]. While the Eurocode approach has the advantage of being quite expeditive for the engineering practice experimental studies have shown that monolithic factors values show a large dispersion as they are sensitive to the applied axial load, percentage of longitudinal reinforcement and relative strength of the core and jacket concrete [8, 9].

A different approach can be found in the literature where a number of experimental [2-4] and theoretical [10-12] researches have tried to evaluate the influence of different aspects such as preload, core-jacket interface treatment and rebar slippage in column-footing joint on the capacity of the RC jacketed member.

An iterative algorithm for calculating the lateral response curve of RC jacketed members, including the relative slip at interface between old and new concrete, was proposed in [10]. The authors proposed a model based on the estimation of crack spacing, taking into account the possible presence of dowels and the concrete frictional resistance at interface.

The case of jacketed columns subjected to axial load and bending moment is studied in [11] by means of non-linear finite element analyses validated through a set of experimental tests. The authors found that the influence of the old-new concrete interface cannot be neglected and that strength degradation at the interface can be modelled by reducing the coefficients of friction and adhesion. Other studies, however, showed that the interface influence is significantly reduced by roughing the existing column surface, or by using bonding agents or steel connectors before the jacket is applied [13, 14].

More recently a theoretical model to calculate proper constitutive laws for old and new concrete and steel was proposed and validated with experimental data available in the literature [12]. The analyses included confinement effect and buckling of longitudinal bars. The case of eccentrically loaded columns was studied through a numerical approach based on the discretization of the section by means of the fibre model.

Finally in [15] the author has proposed a stress block approach to model the different mechanical properties of concrete in the core and in the jacket which also takes into account the effect of confinement and buckling of bars.

This paper extends the approach presented in [15] and provides a simplified estimation of strength and ductility of RC jacketed columns subjected to axial load and uniaxial bending moment, under the assumption of absence of slippage at core-jacket interface. Stress block coefficients are evaluated for different values of pitch of stirrups and axial strain. Moment-curvature curves are derived with few points and ductility analysis is carried out for both directly loaded jackets. Results obtained are compared with numerical analyses performed with OpenSees [16]. In this case, sections are modelled with a square fibre discretization and the constitutive law of confined concrete available in the OpenSees library has been adopted.

The proposed method considers the effect of the different concrete properties between core and jacket and includes confinement effects and buckling of longitudinal bars. Consequently, it removes the previously mentioned hypotheses (ii) and (iii) of EC8 approach. The proposed methodology can be reliably used under the assumption of negligible interface bond degradation. However, it should be also noted that even if slippage is not considered, the use of new monolithic factors especially devoted to address its effect would be a possible solution for including slippage in the calculation.

2 ANALYTICAL INVESTIGATION

2.1 Constitutive law of materials

The concrete constitutive law adopted in this study takes into account the effect of confinement. In particular Mander et al. (1988) model [17] was adopted as it was shown in [12] to be suitable for both the compressive behaviour of jacket and core.

$$\sigma_c = \frac{\frac{\varepsilon}{\varepsilon_{cc}} \cdot f_{cc} \cdot r}{r - 1 + \left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^r} \tag{1}$$

with

$$r = \frac{E_c}{E_c - E_{sec}} \tag{2}$$

where $E_c = 5000\sqrt{f_c}$ in MPa and $E_{sec} = f_{cc}/\varepsilon_{cc}$.

As it is well-known, the peak stress f_{cc} and the peak strain ε_{cc} of confined concrete have to be calculated on the basis of the effective confinement pressure f_l.

This can be simply calculated based on rigid body equilibrium of the section in the plane of the stirrup, the latter assumed yielded. For the considered case study of square section (see Figure 1) the expressions of confinement pressure induced from external and internal stirrups in the core have the following form due to internal (Eq. 3) and external (Eq. 4) stirrups respectively:

$$f_{l,c} = \frac{2 f_{y,st,co} A_{st,co}}{(b - c_c) s_c} \tag{3}$$

$$f_{l,j} = \frac{2 f_{y,st,j} A_{st,j}}{(B - \delta) s_j} \tag{4}$$

$A_{st,co}$ and $A_{st,j}$ being the area of the legs in the core and jacket stirrups and $f_{y,st,co}$ and $f_{y,st,j}$ the yielding strength of the stirrup steel in the core and jacket respectively

Suitable efficiency coefficients have to be considered in order to take into account the effective confined concrete area in the section of transverse reinforcement and between two successive stirrups. These coefficients were proposed in [12] and reviewed also in [15], simply by adapting the expressions proposed by Mander et al. (1988) [17].

Figure 2 shows a plot of effectively confined concrete for two RC jacketed sections. The section in Figure 3-a has a normalized jacket thickness $\delta/b = 0.167$, while the one in Figure 2b is $\delta/b = 0.33$.

It should be noted that in general the in-plane efficiency coefficient of the jacket is quite low, especially if only four bars are placed and for common values of concrete cover. Therefore, for design/verification purposes the concrete jacket can be considered as unconfined. For practical applications, a minimum value of three bars for each side should be recommended.

It should also be noted that in order to simplify calculations, a simplified version of the Mander et al. (1988) [17] compressive stress-strain model valid for both confined and unconfined concrete was proposed in [18]. The model is defined by three branches

$$0 \le \xi < 1; \quad \sigma_c = K f_c (1 - |1 - \xi|^n) \tag{5a}$$

$$1 \le \xi < \xi_u; \quad \sigma_c = K f_c - \left(\frac{K f_c - f_{cu}}{\xi_u - 1}\right)(\xi - 1) \tag{5b}$$

$$\xi_u \le \xi < \xi_f; \quad \sigma_c = f_{cu}\left(\frac{\xi - \xi_f}{\xi_u - \xi_f}\right) \tag{5c}$$

In the above equations f_{cu} is the stress corresponding to stirrup fracture strain; $K = f_{cc}/f_c$ is the confinement ratio, $\xi = \varepsilon_c/\varepsilon_{cc}$ is the normalized strain, $\xi_u = \varepsilon_{cu}/\varepsilon_{cc}$; $n = E_c \varepsilon_{co}/f_c$ and $n = E_c \varepsilon_{cc}/f_{cc}$ are used for unconfined and confined concrete, respectively. This model is easy to be integrated and adopted for sectional analysis.

Figure 3 shows the comparison between the stress-strain law of confined concrete expressed by Eq. (1) and the simplified relation proposed in [18]. The example refers to a square RC jacketed section with $\delta/b = 0.33$ and for different pitch of stirrups. Confinement pressure are calculated as discussed above.

With reference to the stress-strain laws of bars, it has to be noted that for an exact calculation the constitutive law of steel in tension should take into account the strain-hardening effect, while that in compression should include the buckling effects, especially when stirrups are largely spaced. In the analysis here proposed it was adopted the constitutive model proposed in [19] which takes into account the effect of buckling

$$\frac{\varepsilon^*}{\varepsilon_y} = 55 - 2.3\sqrt{\frac{f_y}{100}\frac{L}{d_b}}; \; and \; \frac{\varepsilon^*}{\varepsilon_y} \geq 7$$

(6a)

$$\frac{\sigma^*}{\sigma_l^*} = \alpha\left(1.2 - 0.016\sqrt{\frac{f_y}{100}\frac{L}{d_b}}\right); \; and \; \frac{\sigma}{f_y} \geq 0.2$$

(6b)

$$\frac{\sigma}{\sigma_l^*} = 1 - \left(1 - \frac{\sigma^*}{\sigma_l^*}\right)\left(\frac{\varepsilon - \varepsilon_y}{\varepsilon^* - \varepsilon_y}\right) \qquad for \; \varepsilon_y < \varepsilon < \varepsilon^*$$

(6c)

$$\sigma > 0.2 f_y; \; \sigma = \sigma^* - 0.02 E_s(\varepsilon - \varepsilon^*) \qquad for \; \varepsilon > \varepsilon^*$$

(6d)

In the previous equations it is $\alpha = 1$ for linear hardening bars and $\alpha = 0.75$ for perfectly elastic–plastic bars, d_b is the diameter of the longitudinal bar in the core or in the jacket and L is the buckling length. As recalled in [12], the key parameter to evaluate the buckling behaviour of longitudinal bars is the critical length-to-diameter ratio L/d_b, which can be calculated with a simple model of elastic beam on elastic soil. It is also demonstrated in [12] that second order effects are negligible for pitch-to-diameter $\frac{s}{d_b} < 4.5$, consequently this value is recommended as design reference for stirrups for the jacket. In the following, elastoplastic behaviour of steel

7

is assumed for reinforcement of both core and jacket, in tension and in compression. However, it has to be stressed that a preliminary verification of the critical length of bars in the concrete core is necessary to confirm this assumption.

2.2 Equilibrium of the section

With reference to symbols and diagrams in Figure 1 and Figure 3 and under the assumptions of plane section, perfect concrete-slip bond and negligible concrete tension strength, the equilibrium equations of the jacketed section can be written as

$$N = C_j + C_{co} + F'_j + F'_{co} + F_j + F_{co} \tag{7a}$$

$$M - N\left(\frac{B}{2} - x_c\right) = C_j d_j + C_{co} d_{co} + F'_j(x_c - c_j) + F'_{co}(x_c - \delta - c_{co}) +$$

$$+F_j(b + \delta - x_c - c_{co}) + F_{co}(B - x_c - c_{co}) \tag{7b}$$

If the trend of compressive stresses is supposed to follow the stress-block assumption, the resultant compressive forces in concrete are calculated as

$$C_j = \alpha_j \beta_j \cdot f_{c,j} x_c - B - (\beta_j x_c - \delta) b \alpha_j f_{c,j} \tag{8}$$

$$C_{co} = \alpha_{co} \beta_{co} \cdot f_{cc,co}(x_c - \delta) b \tag{9}$$

C_j and C_{co} being respectively the compressive force in the concrete jacket and core.

Resultant forces in the bars of the jacket are

$$F_j = \sigma_{s,j} A_{s,j} = \gamma_j f_{y,j} A_{s,j} \tag{10}$$

$$F'_j = \sigma'_{s,j} A'_{s,j} = \gamma'_j f_{y,j} A'_{s,j} \tag{11}$$

$\gamma_j = \frac{\sigma_{s,j}}{f_{y,j}}$ and $\gamma'_j = \frac{\sigma'_{s,j}}{f_{y,j}}$ being the stress ratio in the jacket's bars. Similarly, forces in steel of the core are

$$F_{co} = \sigma_{s,co} A_{s,co} = \gamma_{co} f_{y,co} A_{s,co} \tag{12a}$$

$$F'_{co} = \sigma'_{s,co} A'_{s,co} = \gamma'_{co} f_{y,co} A'_{s,co} \tag{12b}$$

$\gamma_{co} = \frac{\sigma_{s,co}}{f_{y,co}}$ and $\gamma'_{co} = \frac{\sigma'_{s,co}}{f_{y,co}}$ being the stress ratio in core reinforcement.

The distance of the resultant compressive force in the concrete jacket from the neutral axis is evaluated as

$$d_j = \frac{B\,\beta_j x_c\left(x_c - \frac{\beta_j x_c}{2}\right)}{B\,\beta_j x_c - b(\beta_j x_c - \delta)} - (\beta_j x_c - \delta) b \frac{\left(x_c - \delta - \frac{(\beta_j x_c - \delta)}{2}\right)}{B\beta_j x_c - b(\beta_j x_c - \delta)} \tag{13}$$

while the distance of the resultant compressive force in the concrete core from the neutral axis is

$$d_{co} = x_c - \delta - \frac{\alpha_{co}\beta_{co}}{2}(x_c - \delta) \tag{14}$$

It has to be noted that if $\beta_j x_c < \delta$, the second terms in Eqs. (8) and (13) have to be set equal to zero, and furthermore if $x_c < \delta$, Eqs. (9) and (14) are as well equal to zero.

Once that the constitutive law of concrete in compression is defined, the stress block parameters α and β have to be calculated to be used for the calculation of the flexural capacity of the jacketed section.

For the generic known value of concrete strain ($\varepsilon^i_{c,j}$ for the jacket, ε^{*i}_{co} for the core), the stress-block parameters can be obtained taking the first and second moments of area of the stress-strain law expressed by Eqs. (5). The following expressions result:

$$\alpha\,\beta = \frac{\int_0^{\varepsilon_c} \sigma_c d\varepsilon_c}{f_c \varepsilon_c} \tag{15a}$$

$$\beta = 2 - 2\frac{\int_0^{\varepsilon_c} \sigma_c \varepsilon_c\, d\varepsilon_c}{\varepsilon_c \int_0^{\varepsilon_c} \sigma_c\, d\varepsilon_c} \tag{15b}$$

Figure 5 shows the variation of the stress-block parameters as a function of the axial strain for the core concrete and for different values of the core-concrete confinement factor $K_c = f_{cc,co}/f_{c,co}$. Both parameters depend strictly on the axial strain and confinement level. The first parameter $(\alpha\beta)$ tends to reach a constant value after the peak strain and greater values are expected by increasing confinement. The second parameter (β) shows a more distinct variation with the axial strain and lower values are reached for greater confinement ratios.

The charts in Figure 5 can be used for sectional calculation. The curves are obtained for values of K$_c$ ranging between 1.2 and 2.0 and for axial strains ε corresponding to the most compressed side of the cross section.

2.3 Definition of the Moment-Curvature Curves

The sectional calculation of RC jacketed sections is a difficult task and most studies propose formulations based on numerical algorithms, which require the use of a numerical software. The proposed model is based on the determination of a discrete arbitrary number of points of the moment-curvature curve.

The geometrical and mechanical properties of the section are assumed to be known and the axial force N is assumed constant during the calculation; using the symbols expressed in Figure 1 and the stress/strain diagrams in Figure 3, the following step-by-step procedure is adopted

- a value is assigned to the maximum compressive strain of jacket's concrete $\varepsilon_{c,j}^i$ and stress block parameters are calculated by means of Eqs. (5) and Eqs. (15) for the concrete jacket $\alpha_j^i \, \beta_j^i$ for the i-th step;

- the top strain in the core's concrete $\varepsilon_{co}^{*,i}$ can be expressed as a function of $\varepsilon_{c,j}^i$, in the form

$$\varepsilon_{co}^{*i} = \frac{x_c^i - \delta}{x_c^i} \varepsilon_{c,j}^i \tag{16}$$

Since the neutral axis depth is unknown, the stress block parameters α_{co}^i β_{co}^i for the core's concrete cannot be calculated numerically. However, a good approximation is obtained by calculating α_{co}^i β_{co}^i with reference to $\varepsilon_{c,j}^i$ instead of ε_{co}^{*i} due to the very similar values of the two strains. The accuracy of this approximation depends on the jacket thickness δ and axial force. However, for practical cases and for realistic range of δ ($0.167 < \delta/b < 0.33$), very low errors are expected, as shown in the next section;

- the strains in steel bars are calculated as

$$\varepsilon_{s,j}^{\prime i} = \frac{\varepsilon_{c,j}^i}{x_c^i}\left(x_c^i - c_j\right) \tag{17a}$$

$$\varepsilon_{s,j}^i = \frac{\varepsilon_{c,j}^i}{x_c^i}\left(B - c_j - x_c^i\right) \tag{17b}$$

$$\varepsilon_{s,co}^{\prime i} = \frac{\varepsilon_{c,j}^i}{x_c'}\left(x_c^i - \delta - c_{co}\right) \tag{17c}$$

$$\varepsilon_{s,co}^i = \frac{\varepsilon_{c,j}^i}{x_c'}\left(\delta + b - c_{co} - x_c^i\right) \tag{17d}$$

In the above equations $\varepsilon_{s,j}^{\prime i}$ and $\varepsilon_{s,j}^i$ are the strain in top and bottom steel of the jacket respectively and $\varepsilon_{s,co}^{\prime i}$ and $\varepsilon_{s,co}^i$ are the strain in top and bottom steel of the core.

- the steel stress ratios are evaluated under the assumption of elastoplastic stress strain law in both tension and compression

$$\gamma_j^{\prime i} = \begin{cases} if \; |\varepsilon_{s,j}^{\prime i}| > \varepsilon_{y,j} \to \gamma_j^{\prime i} = sign(\varepsilon_{s,j}^{\prime i}) \\ if \; |\varepsilon_{s,j}^{\prime i}| < \varepsilon_{y,j} \to \gamma_j^{\prime i} = \frac{\varepsilon_{s,j}^{\prime i} E_s}{f_{y,j}} \end{cases} \tag{18a}$$

$$\gamma_j^i = \begin{cases} if \; |\varepsilon_{s,j}^i| > \varepsilon_{y,j} \to \gamma_j^i = sign(\varepsilon_{s,j}^i) \\ if \; |\varepsilon_{s,j}^i| < \varepsilon_{y,j} \to \gamma_j^i = \frac{\varepsilon_{s,j}^i E_s}{f_{y,j}} \end{cases} \tag{18b}$$

$$\gamma_{co}^{\prime i} = \begin{cases} if \; |\varepsilon_{s,co}^{\prime i}| > \varepsilon_{y,co} \to \gamma_{co}^{\prime i} = sign(\varepsilon_{s,co}^{\prime i}) \\ if \; |\varepsilon_{s,co}^{\prime i}| < \varepsilon_{y,co} \to \gamma_{co}^{\prime i} = \frac{\varepsilon_{s,co}^{\prime i} E_s}{f_{y,co}} \end{cases} \tag{18c}$$

11

$$\gamma_{co}^i = \begin{cases} if \ |\varepsilon_{s,co}^i| > \varepsilon_{y,co} \rightarrow \gamma_{co}^i = sign(\varepsilon_{s,co}^i) \\ if \ |\varepsilon_{s,co}^i| < \varepsilon_{y,co} \rightarrow \gamma_{co}^i = \dfrac{\varepsilon_{s,j}^i E_s}{f_{y,co}} \end{cases} \tag{18d}$$

It has to be noted that for the first analysis step, steel is assumed to behave elastically and consequently the second expressions for each of Eqs. (18) can be assumed.

- Equilibrium equations Eqs. (7) are solved in terms of neutral axis depth x_c^i and bending moment M^i.

  The corresponding curvature is simply calculated as $\varphi^i = \varepsilon_{c,j}^i / x_c^i$.

- The procedure is repeated until compressive strain in the concrete $\varepsilon_{c,j}^i$ or tensile strain in steel $\varepsilon_{s,j}^i$ reaches its ultimate value.

Figure 5 shows the normalised moment-curvature curves derived with the proposed procedure. In particular, Figure 5-a refers to sections with same mechanical properties between jacket and core, while Figure 5 -b shows the case of sections with compressive strength of the core equal to $f_{c,j} = 1.5 \ f_{c,co}$. For the two cases three normalised jacket thickness are considered corresponding to $\delta/b = 0.13$, $\delta/b = 0.2$ and $\delta/b = 0.3$; longitudinal reinforcement ratio is $\rho_j = \dfrac{(A_{s,j} + A_{s,j}')}{(B_2 - b_2)} = 2\%$ for the jacket and $\rho_{co} = \dfrac{(A_{s,co} + A_{s,co}')}{b_2} = 1\%$ for the core, while the normalised axial force $n = N/(b^2 f_{c,co})$ is assumed equal to 0.25. As it can be observed a large increase of ultimate bending strength is obtained even for low values of δ/b, while ductility is not greatly affected by the values of jacket thickness.

It should be noted that the accuracy of results obtained with the proposed procedure depends on the size of the analysis step $\Delta\varepsilon_{c,j}^i$. In particular, a higher precision can be achieved with smaller analysis steps. Figure 6 shows the normalised moment-curvature curves for a section having $\delta/b = 0.17$, $\rho_j = 2\%$ and $\rho_{co} = 1\%$. The two curves have been produces with two

different strain steps $\Delta\varepsilon_{c,j}^i = 0.0005$ **and** $\Delta\varepsilon_{c,j}^i = 0.0003$**. The comparison shows that the bending moment capacity can be reliably obtained with only few steps.**

3   COMPARISONS WITH NUMERICAL ANALYSES AND EXPERIMENTAL DATA

The proposed model is validated with experimental data available in the literature [2] and with numerical analyses carried out with OpenSees [16]. OpenSees was chosen for the wide library of constitutive models that allows for a complete modelling of the case study, including confined and unconfined concrete.

In particular, a ZeroLengthSection element is created between two nodes (see Figure 7). Node 1 is fixed while loads are applied at node 2. The analysed section is subdivided with the classic fibres method and assigned to the zero length element. For the examined case the jacket was divided into 100 square fibres while the confined region was modelled with 400 square cells. Rebars were considered as points, and overlapping with the square concrete cells was considered by neglecting the single cell coincident with a bar location.

UniaxialMaterial objects are used to define the constitutive law of constituent materials, taking into account the different confinement ratio in the core and in the jacket. The Concrete01 model is used to model both the jacket and core concrete. This material model adopts the uniaxial Kent-Scott-Park (1982) stress-strain law for concrete in compression and no tensile strength [20]. Steel bar objects were characterized by the Steel02 material object, which correspond to the Menegotto-Pinto (1973) model with isotropic strain hardening [21]. The analysis procedure takes advantage of a step-by-step numerical algorithm (Newton-Raphson) for the solution of the non-linear system to calculate the moment-curvature curves. The required precision is achievable by setting the number of points defining the domain. In the present analysis, the number of points was assumed equal to 200.

Figure 9 shows the comparison between the analytical results obtained with the proposed model and those computed numerically for two values of thickness of the jacket δ and for two values of steel and concrete strength (200-400 MPa for steel and 30-40MPa for concrete) as reported in Table 1.

For each case studied two levels of axial force are considered, corresponding approximately to $n = 0.25$ and $n = 0.5$. As it can be observed, a good match between the adopted analytical procedure and the numerical computation are achieved. Few analysis steps of the proposed procedure are enough to predict quite accurately the moment-curvature response of the jacketed section.

A further comparison in Figure 10 shows the numerical and analytical moment-curvature curves together with the experimental results determined in [2]. **In particular, specimens RBR, MBR and SBR are considered, referring respectively to reinforced, monolithic and strengthened sections. The geometry of the specimens is reported in Table 2**. The concrete compressive strength varies for each case analysed and further details can be found in [2].

Also in this case the proposed analytical fits the numerical solutions with good accuracy and a relative error lower than 5%.

Since both the numerical and analytical approaches lead to similar results this difference can be attributed to experimental values, and especially the effective strength of the steel, not exactly matching the ones used in the model.

It should also be noted that while **the proposed model does not take into account the initial damage level, preload and reinforcement slippage, the three specimens differ for the different concrete properties and damage level at the time of the jacketing. In fact, as discussed in [2], specimens were loaded to a predetermined damage level, then unloaded, jacketed and retested. In particular, column RBR was jacketed after observing**

considerable concrete crushing, specimen SBR was jacketed after observing first signs of cover crushing and specimen MBR was reinforced before loading.

However, it has to be noted that the result is quite conservative with respect to safety in all examined cases. From this preliminary verification the model can be considered as a useful tool for design purposes of RC jacketed columns. Further experimental investigations should be addressed to verify in deep the suitability of the model.

4   DUCTILITY CALCULATION

One of the main advantages in adopting an easy hand-computing procedure is the evaluation of important design parameters.

In fact, if the top concrete compressive strain $\varepsilon_{c,j}$ is set equal to the ultimate value $\varepsilon_{cu,j}$ the procedure described in section 2 is able to provide the ultimate moment M_u and curvature ϕ_u of the section.

The computation of curvature corresponding to the yielding of the jacket steel ϕ_y is more difficult to be performed, since the concrete strain $\varepsilon_{c,j,y}$ at the top of the section is unknown. However, one of the several methods for solving numerically a non-linear equation can be adopted. As an example, the secant method is here used in the following manner:

- a first tentative value is assigned to the concrete top strain $\varepsilon_{c,j,y}^i$

- the first equilibrium equation (Eq. 7a) can be written in symbolic form $f(\varepsilon_{c,j,y}^i)$; the target value of $\varepsilon_{c,j,y}$ should be obtained when $f(\varepsilon_{c,j,y}) = 0$;

- for the generic i^{th} step the equilibrium equation provides an unbalance value and consequently $f(\varepsilon_{c,j,y}^i) \neq 0$;

- the new value of concrete top strain is obtained as

$$\varepsilon_{c,j,y}^{i+1} = \varepsilon_{c,j,y}^i - \frac{\varepsilon_{c,j,y}^i - \varepsilon_{c,j,y}^{i-1}}{f(\varepsilon_{c,j,y}^i) - f(\varepsilon_{c,j,y}^{i-1})} f(\varepsilon_{c,j,y}^i) \qquad (19)$$

15

the procedure is repeated until the unbalanced value is less than a fixed tolerance $f\left(\varepsilon_{c,j,y}\right) < |\delta|$ and the yield curvature is finally determined as $\varphi_u = \varepsilon_{c,j,y}/x_{c,y}$.

Figure 11 presents an example of iterative calculation of the yielding curvature φ_y by the above described method and adopting two different first-tentative values of $\varepsilon_{c,j,y}^i$. In particular the unbalanced value $f\left(\varepsilon_{c,j,y}^i\right)$ is plotted against the calculated curvature φ_y^i. As it could be noted, in both cases few iterations -less than ten- are enough to achieve the desired solution.

This calculation allows to perform a ductility evaluation for the jacketed member. Figure 12-a shows the curvature ductility ($\mu = \varphi_u/\varphi_y$) of a square RC jacketed section with $\delta/b = 0.13$ and $\delta/b = 0.33$ as a function of the mechanical ratio of longitudinal steel in the jacket $\omega = \frac{A_{s,j}f_{y,j}}{\left[(B^2-b^2)f_{c,j}\right]}$ **and for fixed values of normalised axial force in the core** $n = N/(b^2 f_{c,co})$. The trend of μ shows that it can be considered a function of the axial force and substantial values of ductility are achieved for great levels of axial force. Figure 12-b shows the trend of the normalised ultimate moment also as a function of ω. As expected, a linear variation of the bending strength is observed.

Figure 12 allows a reasonable ductility assigned design; for a fixed normalised axial force in the core column n, the required jacket thickness can be selected between the lower and upper boundaries of δ/b (0.13-0.33) to achieve the desired ductility by means of Figure 12-a, considering that higher values of desired ductility requires larger thicknesses. Finally, flexural strength is controlled by selecting an appropriate amount of steel by means of Figure 12-b. Then, ω is chosen for a fixed value of design bending moment M.

5   DESIGN CONSIDERATIONS AND LIMITS OF APPLICABILITY

The proposed approach allows for easy design calculation of the flexural behaviour of RC jacketed sections.

When adopting the model, it should be noted that the following assumptions are made: - absence of slippage between old and new concrete; - application of full axial load over the jacketed member; - perfect bond between concrete and steel bars.

However, since cracks open in the outer shell and slippage may occur between the inside core and the outside jacket if the two surfaces are not properly connected, a refined analysis of the rotational characteristics should include the effective crack spacing, slippage between core and jacket, and tension stiffening effect between cracks. Such a calculation is quite complex, and sophisticated numerical methods should be adopted e.g. iterative models were proposed in [10] and [8]. In fact, the presence of flexural cracks along the length of the column causes significant localization of strain damage at the cracked sections, whereas where cracks do not occur, the curvature can be assumed to be continuous and smooth. Based on these considerations, the proposed model focuses on the zones of the column where cracks do not occur, i.e. under the assumption of smooth and continuous curvature.

On other hand, the model has the advantage of taking into account different effects, such as: - confinement induced by both internal and external stirrups; - core-jacket composite action; - core and jacket having different concrete properties; - buckling of longitudinal bars.

It is worth noting that although the common design procedure according to technical codes (Eurocode 2 [22] and Eurocode 8 [5]) applies under the same aforementioned assumption on the curvature, it does neglect these latter aspects.

For the sake of clarity, an example of application is shown in Figure 13 and the calculations, obtained with a spreadsheet, have been reported on Appendix A. **The case study refers to a square RC section with the geometrical and mechanical properties reported in Table 3. The comparison shows the response obtained with the proposed model and that obtained according to the Eurocode 8 [5]. The constitutive laws of constituent materials are assumed as in Eurocode 2 [22].** Under these assumptions, the moment-curvature is obtained

17

by means of a three-linear curve defined by three stages: cracking of concrete, yielding of steel and ultimate state due to steel failure. **The results obtained with the two methods differ by about 12%.**

6   CONCLUSIONS

In this paper, a simplified analytical method is presented to calculate the moment-curvature curve for RC jacketed columns. The model is based on a step-by-step procedure, based on the stress-block approach. From a comparison with numerical analyses carried-out with OpenSees and experimental published data the following conclusions can be drawn:

- The stress-block approach is suitable to be applied to RC jacketed sections if the parameters are well-calibrated;

- Results derived with the proposed method are in good accordance with those obtained numerically. In addition, comparisons with a limited number of experimental data have shown good agreement. Further experimental investigations are needed to further verify the proposed model;

- The proposed model allows an easy calculation of ductility of RC jacketed sections. A combination of this method with monolithic coefficient and safety factors could provide a useful tool for practical engineering applications.

- **The proposed methodology can be reliably used under the assumption of negligible bond degradation at core-jacket interface. However, even if slippage is not considered, the use of monolithic factors especially devoted to address its effect would be a possible solution for including slippage in the calculation. For these reasons, further studies should be addressed to the calibration of new monolithic factors especially devoted to address the slippage at jacket-core concrete interface.**

REFERENCES

[1] Rodriguez M, Park R. Seismic load tests on reinforced concrete columns strengthened by jacketing. ACI Struct J 1994;91:150-159.

[2] Ersoy U, Tankut AT, Suleiman R. Behavior of jacketed columns. ACI Struct J 1993;90:288-293.

[3] Takeuti AR, de Hanai JB, Mirmiran A. Preloaded RC columns strengthened with high-strength concrete jackets under uniaxial compression. Mater Struct 2008;41:1251-62.

[4] Julio EN, Branco FA. Reinforced Concrete Jacketing--Interface Influence on Cyclic Loading Response. ACI Struct J 2008;105.

[5] European Committee for Standardization. Eurocode 8: Design of structures for earthquake resistance – Part 3: Assessment and retrofitting of buildings. Terminology and general criteria for test methods 2005:40.

[6] Thermou GE, Papanikolaou VK, Kappos AJ. Flexural behaviour of reinforced concrete jacketed columns under reversed cyclic loading. Eng Struct 2014;76:270-82.

[7] Vandoros KG, Dritsos SE. Axial preloading effects when reinforced concrete columns are strengthened by concrete jackets. Progress in Structural Engineering and Materials 2006;8:79-92.

[8] Thermou GE, Papanikolaou VK, Kappos AJ. Monolithicity factors for the design of R/C columns strengthened with R/C jackets. 2ECEES, Second European Conference on Earthquake Engineering and Seismology, Istanbul, Turkey 25-29 August 2014.

[9] Thermou GE. Strengthened Structural Members and Structures: Analytical Assessment. Encyclopedia of Earthquake Engineering 2014:1-20.

[10] Thermou G, Pantazopoulou S, Elnashai A. Flexural behavior of brittle RC members rehabilitated with concrete jacketing. J Struct Eng 2007;133:1373-84.

[11] Lampropoulos A, Dritsos SE. Modeling of RC columns strengthened with RC jackets. Earthquake Eng Struct Dyn 2011;40:1689-705.

[12] Campione G, Fossetti M, Giacchino C, Minafò G. RC columns externally strengthened with RC jackets. Mater Struct 2014;47:1715-28.

[13] Júlio EN, Branco FA, Silva VD. Reinforced concrete jacketing-interface influence on monotonic loading response. ACI Struct J 2005;102:252.

[14] Altun F. An experimental study of the jacketed reinforced-concrete beams under bending. Constr Build Mater 2004;18:611-8.

[15] Minafò G. A practical approach for the strength evaluation of RC columns reinforced with RC jackets. Eng Struct 2015;85:162-9.

[16] McKenna F, Fenves G, Scott M. Open system for earthquake engineering simulation. University of California, Berkeley, CA 2000.

[17] Mander JB, Priestley MJ, Park R. Theoretical stress-strain model for confined concrete. J Struct Eng 1988;114:1804-26.

[18] Karthik MM, Mander JB. Stress-block parameters for unconfined and confined concrete based on a unified stress-strain model. J Struct Eng 2010;137:270-273.

[19] Dhakal RP, Maekawa K. Modeling for postyield buckling of reinforcement. J Struct Eng 2002;128:1139-47.

[20] Scott B, Park R, Priestley M. Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates. ACI Structural Journal 1982;79:13-27.

[21] Menegotto M, Pinto PE. Method of analysis for cyclically loaded reinforced concrete plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending . IABSE Symposium on Resistance and ultimate deformability of structures acted on by well-defined repeated loads, Lisbon 1973;13:15-22.

[22] European Committee for Standardization. CEN (2004b) Eurocode 2: Design of concrete structures – Part 1: General rules and rules for buildings. Product requirements and evaluation of conformity for vehicle restraint systems 2004:36.

[23] Collins MP, Mitchell D. Prestressed concrete structures: Response Publications, 1997.

Appendix A

Numerical example

	Core	Jacket
A.1 Data		
Axial load	N=600 kN.	
Geometry	b=300 mm	δ=100 mm
	$c_{co} = 20\ mm$	$c_j = 20\ mm$
Reinforcement	$A'_{s,co} = A_{s,co} = 462\ mm^2$	$A'_{s,j} = A_{s,j} = 1600\ mm^{22}$
	$f_{y,co} = 200\ MPa$	$f_{y,j} = 391.3\ MPa$
	$E_s = 206000\ MPa$	$E_s = 206000\ MPa$
Concrete	$f_{c,co} = 20\ MPa$	$f_{c,j} = 40\ MPa$
	$K_{c,co} = 1.3$	$K_{c,j} = 1$ (unconfined)
A.2 Calculation of concrete properties		
	$f_{cc,co} = K_{c,co}f_{c,co} = 26$ MPa	
See [23]	$\varepsilon_{c0,co} = 0.0015 + \dfrac{f_{c,co}}{70000}$ $= 0.00179$	$\varepsilon_{c0,j} = 0.0015 + \dfrac{f_{c,j}}{70000} = 0.0021$
See [17]	$\varepsilon_{cc,co} = \varepsilon_{c0,co}[1 + 5(K_{c,co} - 1)]$ $= 0.0045$	
See [17]	$\varepsilon_{cu,co} = 5\varepsilon_{cc,co} = 0.0223$	$\varepsilon_{cu,j} = 0.0036$ See [23]
See [23]		$f_{cu,j} = 12\ MPa$
A.3 Calculation of moment-curvature response for $\varepsilon_{cj} = \varepsilon_{cu,j}/3 = 0.0012$		
Eqs. (15a-b)	$\alpha_{co} = 0.73;\ \beta_{co} = 0.71$	$\alpha_j = 0.59;\ \beta_j = 0.69$
Eq.(7a)	$x_c = 97.81\ mm$	
	$\varphi = \dfrac{\varepsilon_{c,j}}{x_c} = 0.00001227\ mm^{-1}$	
Eqs.17a-d	$\varepsilon'_{s,co} = -0.000272$	$\varepsilon'_{s,j} = -0.000955$
	$\varepsilon_{s,co} = -0.003462$	$\varepsilon_{s,j} = -0.004689$
Eqs.18a-d	$\gamma'_{s,co} = -0.28;\ \gamma_{s,co} = -1$	$\gamma'_{s,co} = 0.50;\ \gamma_{s,j} = -1;$
Eq. (7b); Eqs.(8-14).	$M = 431\ kNm$	

List of main symbols

$A'_{s,co}$: total area of longitudinal steel in the bottom part of the core's section;
$A'_{s,j}$: total area of longitudinal steel in the bottom part of the jacket's section;
$A_{s,co}$: total area of longitudinal steel in the upper part of the core's section;
$A_{s,j}$: total area of longitudinal steel in the upper part of the jacket's section;
$A_{st,co}$: area of stirrup's legs in the core;
$A_{st,co}$: area of stirrup's legs in the jacket;
b: side length of the square core section;
B: side length of the square jacketed section;
C_c: internal compressive force in the concrete core;
c_{co}: thickness of concrete cover in the core;
C_j: internal compressive force in the concrete jacket;
c_j: thickness of concrete cover in the jacket;
d_b: diameter of longitudinal bar in the jacket or core;
E_c: Young modulus of concrete;
E_{sec}: secant modulus of concrete;
F'_{co}: force in upper steel of core;
F^i_j: force in upper steel of jacket;
$f_{c,co}$: unconfined compressive strength of core's concrete;
$c_{c,j}$: unconfined compressive strength of jacket's concrete;
F_c: force in bottom steel of core;
$f_{cc,co}$: peak stress of confined concrete of core;
f_{cc}: peak stress of confined concrete;
f_{cu}: concrete stress corresponding to stirrup fracture strain;
F_j: force in bottom steel of jacket;
$f_{l,co}$: confinement pressure due to core stirrups;
$f_{l,j}$: confinement pressure due to jacket stirrups;
f_l: effective confinement pressure;
$f_{y,co}$: yield stress of steel bars in the core;
$f_{y,j}$: yield stress of steel bars in the jacket;
K: confinement ratio;
K_{co}: confinement ratio of core's concrete;
$k_{e,p,j}$: in-plane confinement efficiency coefficient for the jacket's section;
L: buckling length
s_{co}: pitch of stirrups in the core;
s_j: pitch of stirrups in the jacket;
x_c: neutral axis depth;
α_{co}: parameter defining the stress-block breadth for the core's concrete;
α_j: parameter defining the stress-block breadth for the jacket's concrete;
b_{co}: parameter defining the stress-block depth for the core's concrete;
b_j: parameter defining the stress-block depth for the jacket's concrete;
δ: thickness of the jacket;
ε_{cc}: compressive axial strain corresponding to peak stress in confined concrete;
μ: curvature ductility;
ξ: normalized axial strain of concrete
φ_u: curvature at ultimate state;

φ_y: curvature at yielding;

List of Figures

TABLES

Table 1. Geometrical and mechanical characteristics of columns (a)-(d) in Figure 9

Column type	b (mm)	c_{co} (mm)	δ (mm)	c_j (mm)	ρ_j	ρ_{co}	$f_{y,j} = f_{y,co}$ (MPa)	$f_{c,j} = f_{c,co}$ (MPa)
(a)	200	20	60	20	2%	1%	200	30
(b)	200	20	60	20	2%	1%	400	40
(c)	200	20	30	20	2%	1%	200	30

(d)	200	20	30	20	2%	1%	400	40

Table 2. Geometrical and mechanical characteristics of specimens **RBR, MBR and SBR**

Core	b (mm)	c_{co} (mm)	$f_{y,co}$ (MPa)	$d_{b,co}$ (mm)	$d_{st,co}$ (mm)	s_{co} (mm)
	160	5	300	12	4	100
Jacket	δ (mm)	c_j (mm)	$f_{y,j}$ (MPa)	$d_{b,j}$ (mm)	$d_{st,co}$ (mm)	s_j (mm)
	35	5	280	12	8	100
	MBR		RBR		SBR	
	$f_{c,co}$ (MPa)	$f_{c,j}$ (MPa)	$f_{c,co}$ (MPa)	$f_{c,j}$ (MPa)	$f_{c,co}$ (MPa)	$f_{c,j}$ (MPa)
	31.5	31.5	30.7	34.5	33	40.3

Table 3. Geometrical and mechanical characteristics of case study **in Figure 12**

Core	b (mm)	c_{co} (mm)	$A_{s,co} = A'_{s,co}$ (mm^2)	$f_{y,co}$ (MPa)	$f_{c,co}$ (MPa)	K_c
	300	5	462	200	20	1.3
Jacket	δ (mm)	N (kN)	$A'_{s,j} = A_{s,j}$ (mm^2)	$f_{y,j}$ (MPa)	$f_{c,j}$ (MPa)	
(a)	100	360	1600	391.3	40	
(b)	50	360	700	391.3	40	
(c)	50	720	700	391.3	40	

25
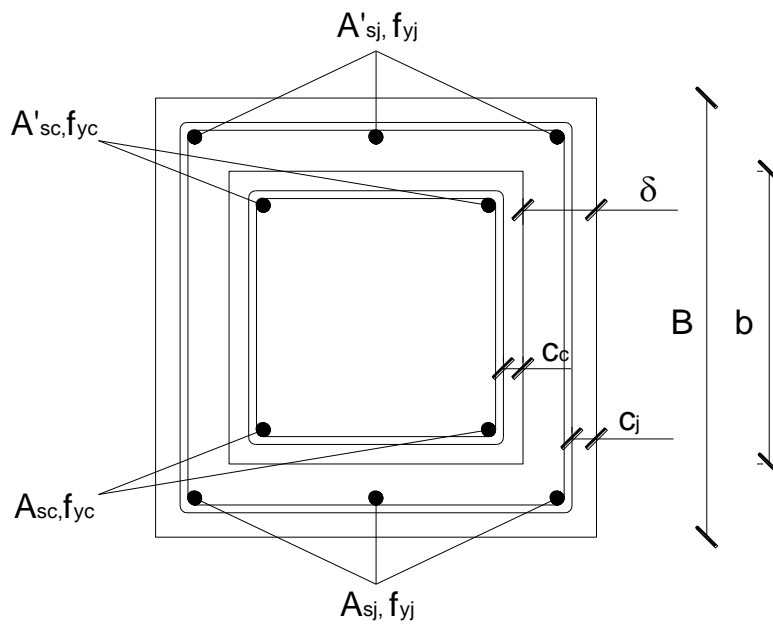
Figure 14. Case study: RC square jacketed column.
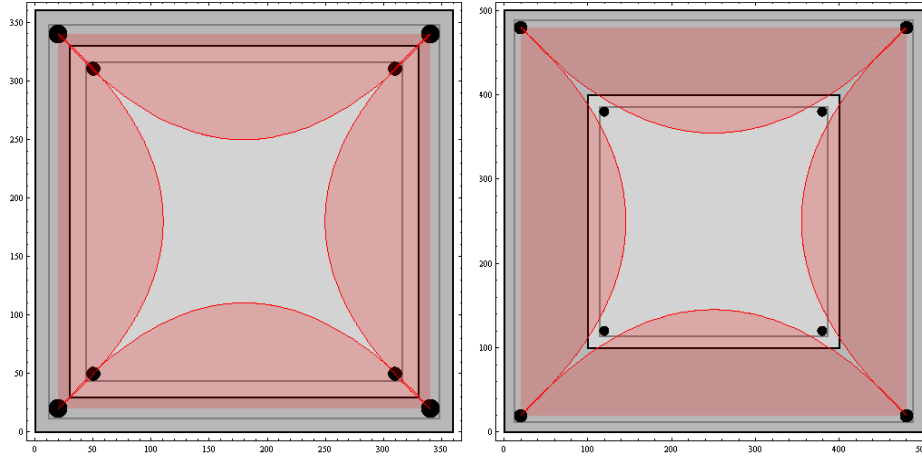
Figure 15. Confinement efficiency in square RC jacketed sections. a) $\delta/b = 0.167$, $k_{e,p,j} = 0.63$; b) $\delta/b = 0.33$, $k_{e,p,j} = 0.28$.
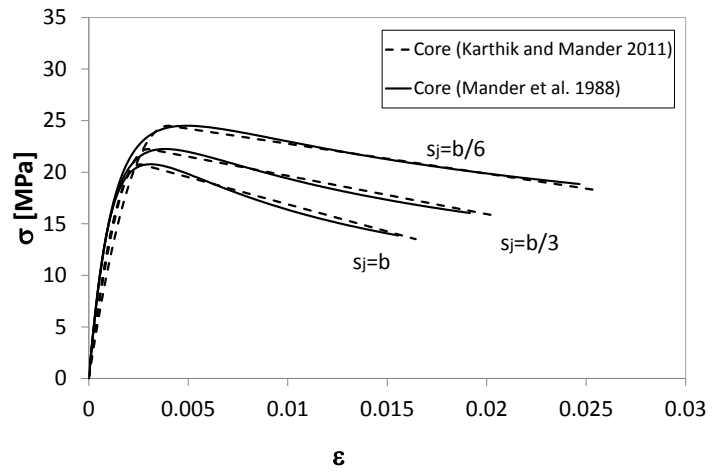
Figure 16. Stress-strain relationships for confined concrete of the core for different stirrup pitch of ($\delta/b = 0.33$).
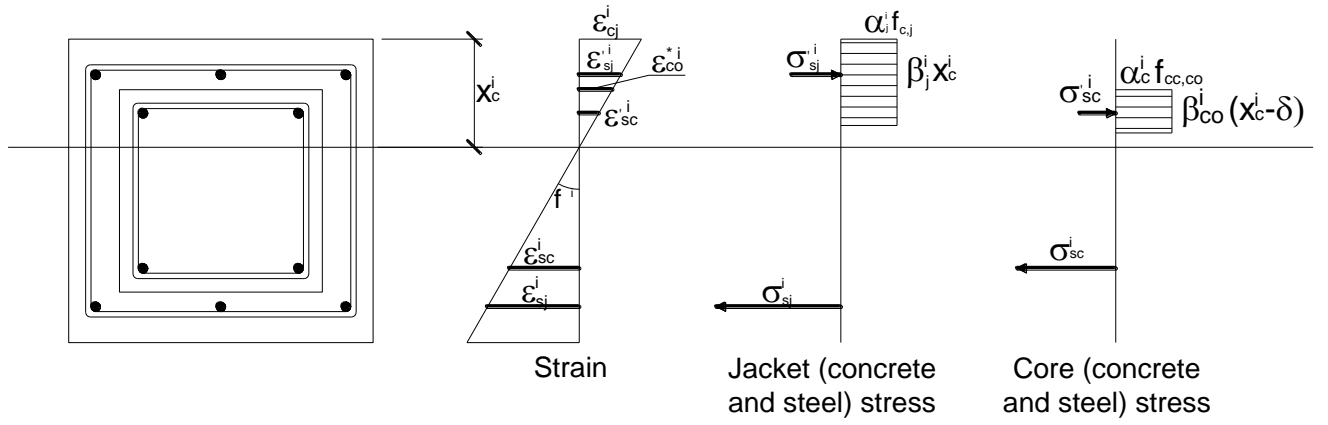
Figure 17. Strain and stress distribution and corresponding force at the generic i^{th} step of the procedure for Moment-Curvature curves.
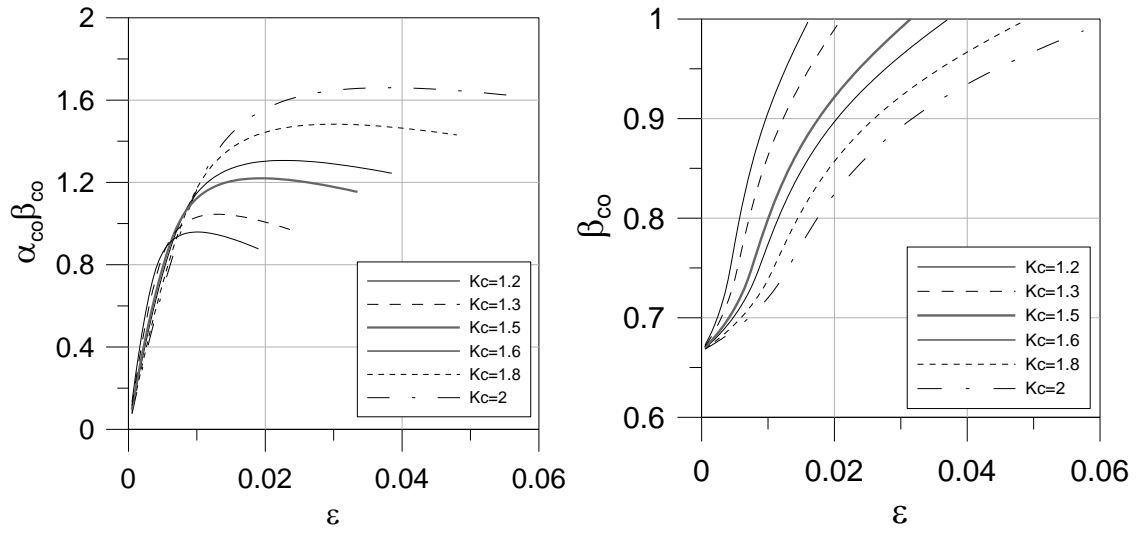
Figure 18. Stress block parameters for confined concrete

a)

b)
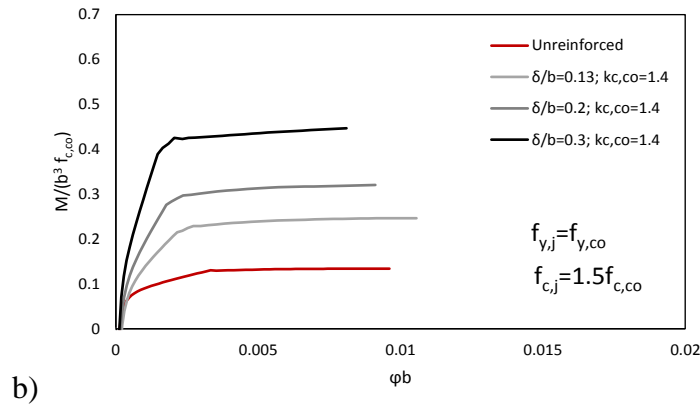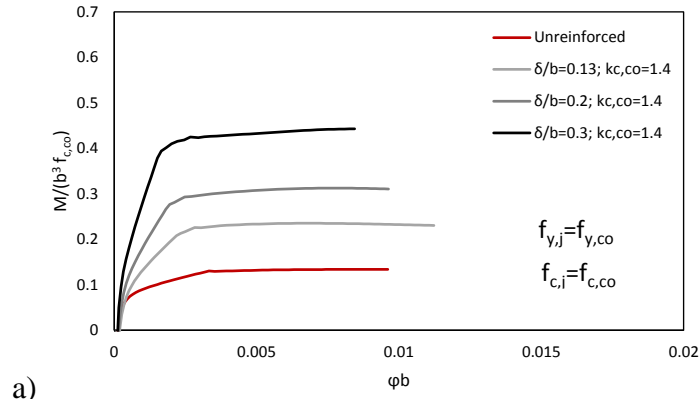
Figure 19. Normalised moment-curvature curves for jacketed sections ($\rho_j = 2\%, \rho_{co} = 1\%, n = 0.25$); a) $f_{y,j} = f_{y,co}$; $f_{c,j} = f_{c,co}$. b) $f_{y,j} = f_{y,co}$; $f_{c,j} = 1.5\, f_{c,co}$.

31
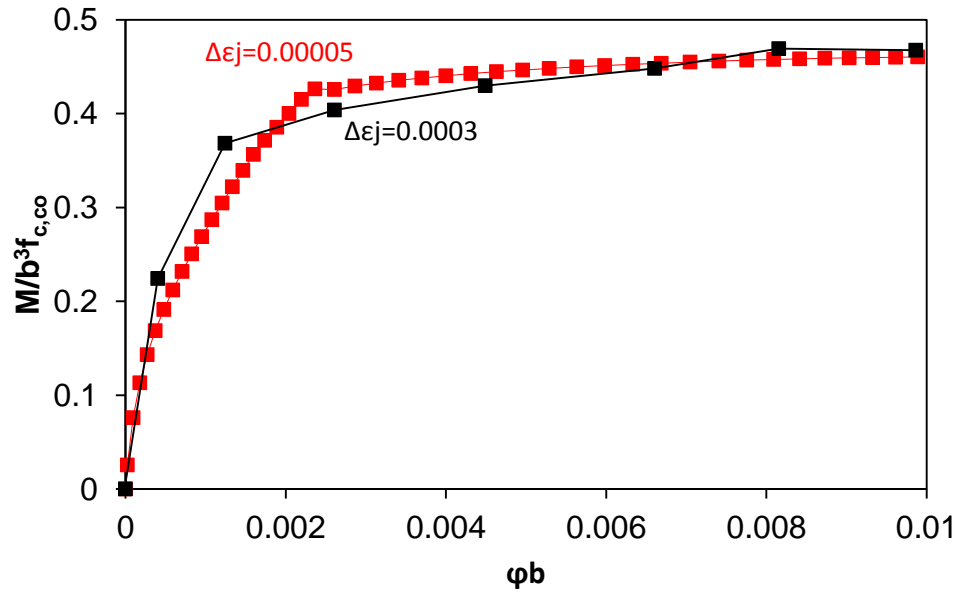
Figure 20. Effect of analysis step size on moment-curvature curve
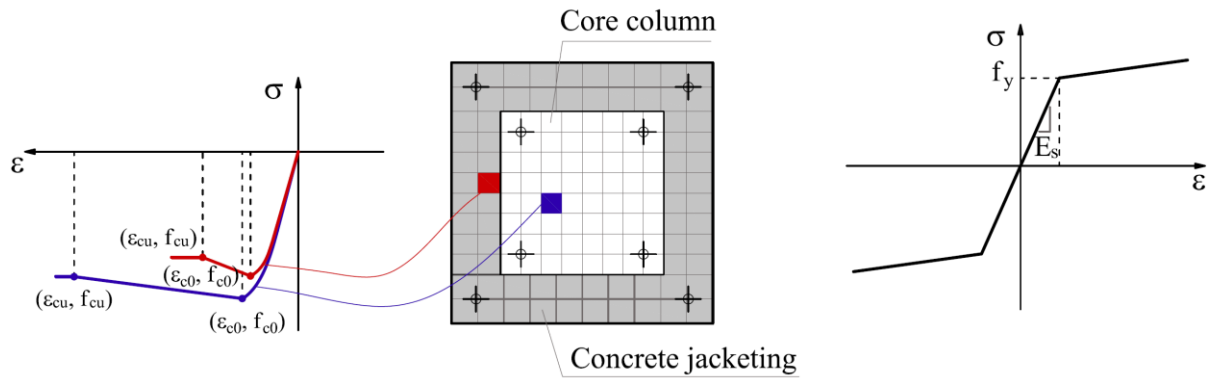
Figure 21. Sectional model in OpenSees.
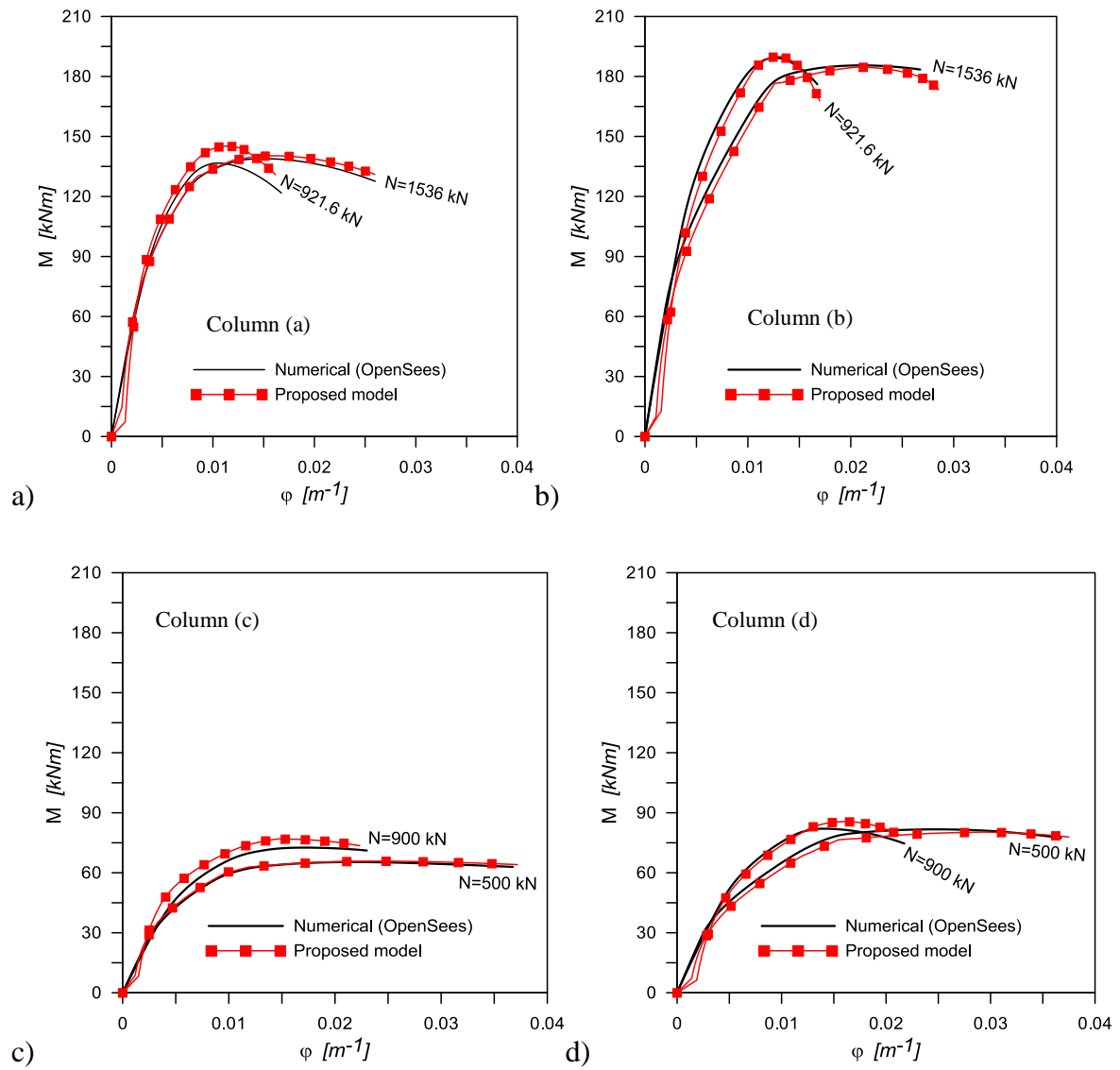
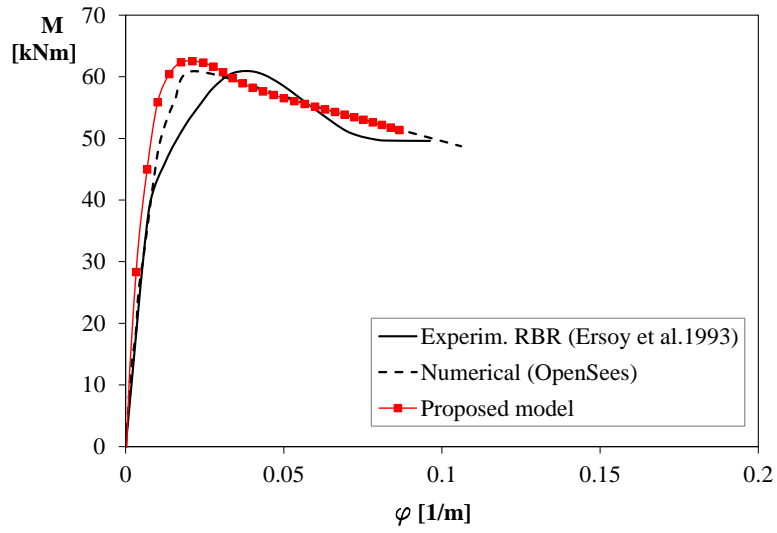Figure 22. Comparison between analytical and numerical results.
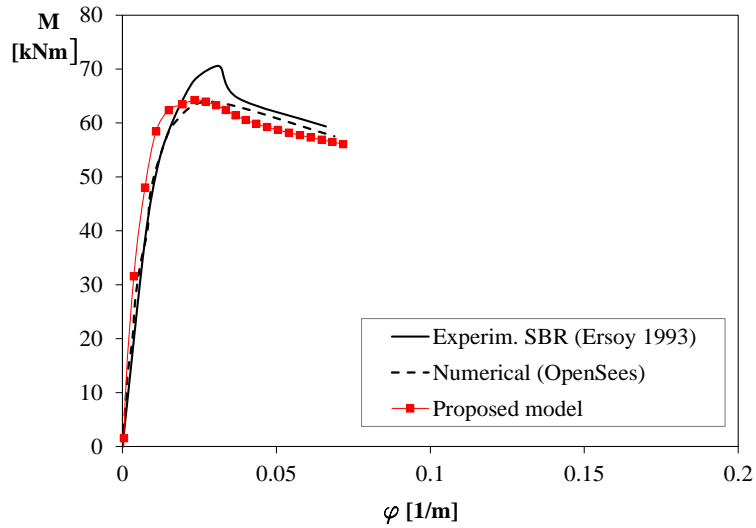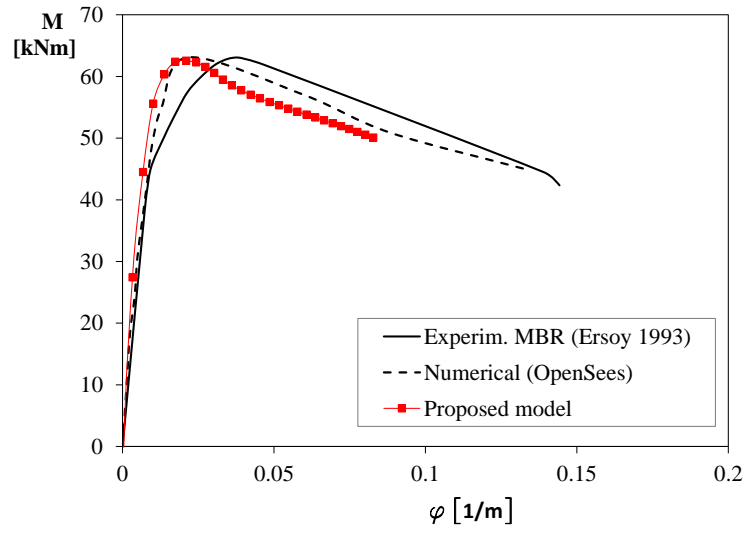
Figure 23. Comparison between analytical, numerical and experimental results.
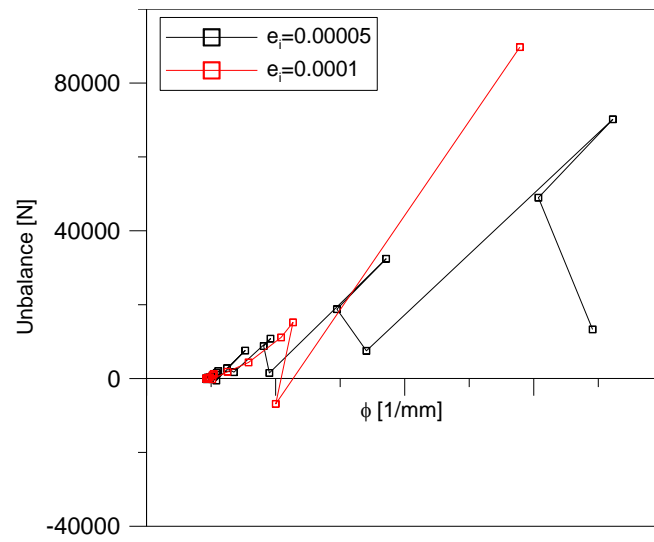
Figure 24. Iterative calculation of yielding curvature by secant method.
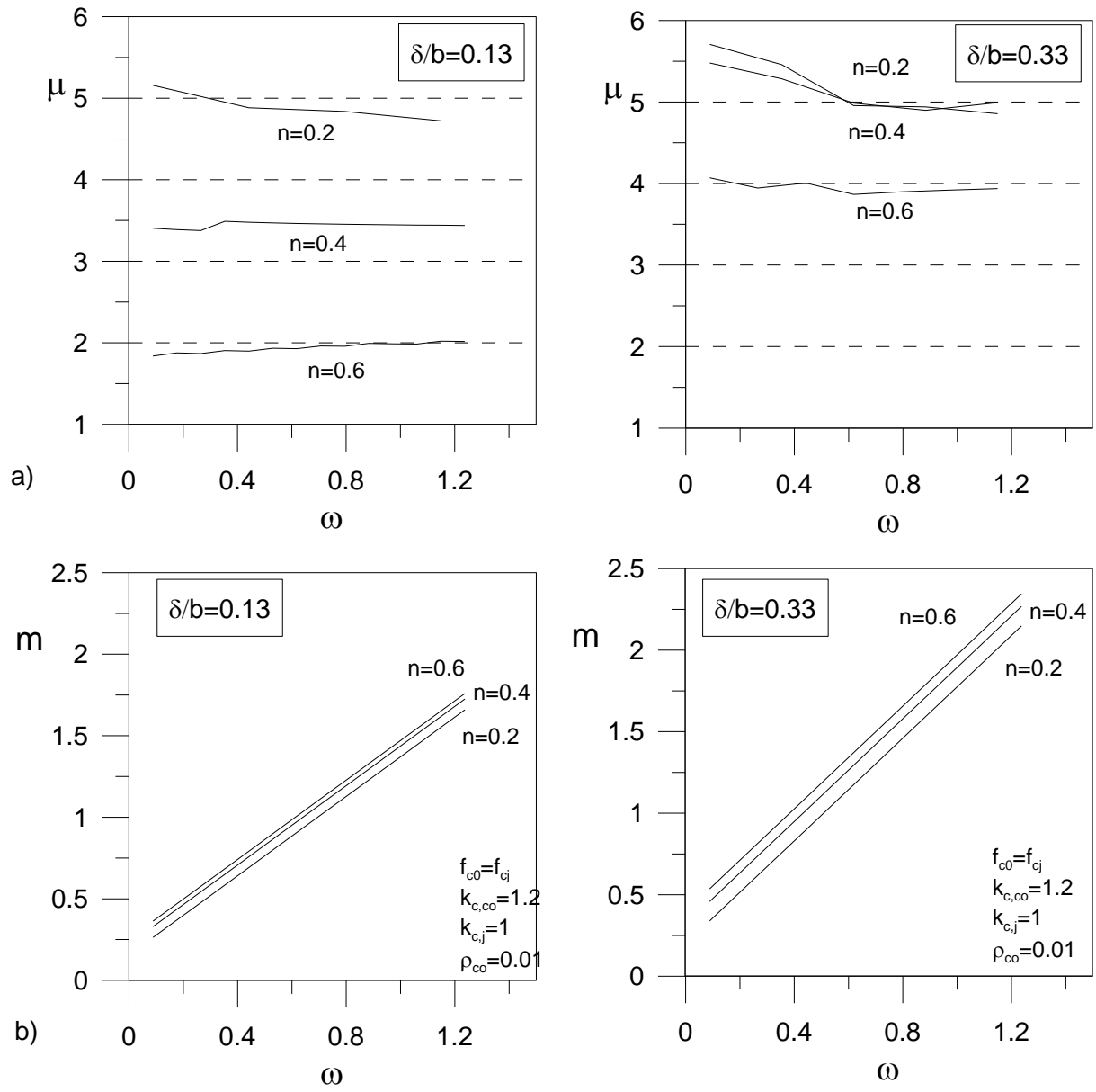
Figure 25. Non-dimensional parameters. a) Curvature ductility as a function of the mechanical ratio of reinforcement in the jacket; b) ultimate moment as a function of the mechanical ratio of reinforcement in the jacket;
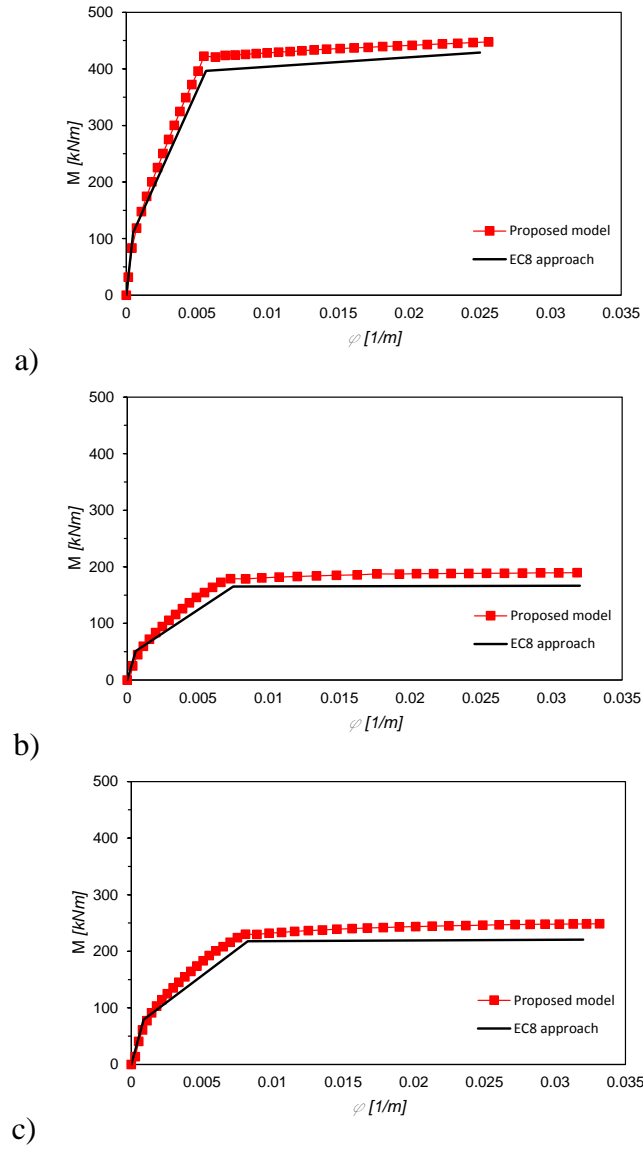
Figure 26. Comparison between proposed model and Eurocode approach. a) δ=100 mm; N=600 kN; b) δ=50 mm; N=360 kN; c) δ=50 mm; N=720 kN;