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1 **PREDICTING AXIAL VELOCITY PROFILES WITHIN A DIFFUSING MARINE**
2 **PROPELLER JET**

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4
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8
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10
11 **ABSTRACT**

12
13 A full understanding of the hydrodynamic processes within the jet produced by a manoeuvring
14 ship’s propeller is essential in the development and maintenance of ports, docks and
15 harbours. In this investigation the predominant axial velocity component within a freely
16 expanding wash was studied. The flow fields formed by four propellers, each operating at four
17 power levels (speeds of rotation), were investigated under bollard pull conditions and in the
18 absence of a rudder, within a large free surface tank using Laser Doppler Anemometry. The
19 characteristics of these propellers extended the range over which high accuracy
20 measurements have been previously attempted. Comparison were made to existing
21 methodologies by which a prediction of the magnitudes of the axial velocity can be made, and
22 where deficient modifications to the methodologies have been developed and presented. The
23 jets were found to produce a maximum axial velocity along the initial efflux plane at a location
24 near the blade mid-span. The position and magnitude of the axial velocity was seen to
25 decrease as the jet entrained more flow and transitioned from the zone of flow establishment
26 into the zone of established flow.

27
28 **KEYWORDS**

29 Propeller Jets, Scour, Ports, Dock and Harbours, Hydraulics & Hydrodynamics

30

31 NOTATION

A	(-)	Coefficient defined in Equation 23
B	(-)	Coefficient defined in Equation 23
C	(-)	Experimentally determined constant (σ/X_0)
C_t	(-)	Thrust coefficient of propeller ($T/\rho n^2 D_p^4$)
c	(m)	Chord length
D_h	(m)	Diameter of hub
D_o	(m)	Initial diameter of slipstream
D_p	(m)	Diameter of propeller
h_d	(m)	Helical distance from the blade section leading edge to rake datum line
h_t	(m)	Helical distance from the blade section leading edge to position of maximum thickness
L_m	(m)	Characteristic length
N	(-)	Number of propeller blades
n	(rpm)	Propeller rotational speed
P'	(-)	Propeller pitch to diameter ratio
p	(m)	Propeller blade pitch
Re_{flow}	(-)	Reynolds number of jet flow ($V_o D_p / \nu$)
Re_{prop}	(-)	Reynolds number of propeller ($n D_p L_m / \nu$)
R_h	(m)	Radius of propeller hub ($D_p/2$)
R_m	(m)	Radial position of maximum axial velocity relative to the jet centreline at any section within the zone of flow establishment
R_{m0}	(m)	Radial distance from propeller axis to location of maximum axial velocity along efflux plane
R_p	(m)	Radius of propeller
R^2	(-)	Coefficient of determination
r	(m)	Radial distance across blade from propeller centreline
V_{max}	(m/s)	Maximum axial velocity

V_o	(m/s)	Maximum axial velocity along efflux plane
$V_{x,r}$	(m/s)	Axial velocity at position x, r
X	(m)	Cartesian co-ordinate measured laterally from face of propeller
X_o	(m)	Distance from propeller to end of zone of flow establishment
β		Blade Area Ratio
ν	(m ² /s)	Kinematic viscosity of fluid
π	(-)	Constant number pronounced pi ($\pi = 3.142$)
σ	(m)	Standard deviation of velocity

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53 **1.0 INTRODUCTION**

54 The problems within harbours and navigation channels associated with the close proximity of
55 manoeuvring vessels, have been well discussed in a range of both case studies and research
56 investigations, Fuehrer & Römisch (1977), Blaauw, H.G., and van de Kaa, E.J. (1978), Bergh
57 & Cederwall (1981), Berger *et al.* (1981), Fuehrer *et al.* (1981), Verhey *et al.* (1987), Hamill
58 (1987), Chait (1987), Stewart (1992), Hashmi (1993), Qurrain (1994), Froehlich & Shea
59 (2000), Sumer & Fredsoe (2002), Hong *et al.* (2013), Geisenhainer & Aberle (2013) and
60 Hamill *et al.* (2014). Guidelines for engineers have been developed (PIANC (2015), BAW
61 (2010) and CIRIA (2007)) incorporating the influence of engineering surfaces, beds and
62 slopes. In all cases these methodologies rely on an understanding of the fundamental
63 process that control the formation and diffusion of the jets formed.

64

65 Studies that have concentrated on the formation and diffusion of the jets created by the
66 manoeuvring vessels have been limited by the numbers of test propellers used in the studies,
67 Lam *et al.* (2012), and while providing a useful insight have not been in a position to provide
68 predictive methods that covered a meaningful range of operation as only one test propeller
69 was used. The formation process, and subsequent diffusion, of a ship's propeller jet must be
70 fully understood if an engineer is to be able to quantify any scouring damage that may occur,
71 and, more importantly, size protection systems to be deployed to prevent further damage.

72

73 The flow field produced by the action of rotating propeller blades is complex in nature. Near
74 to the propeller, the passing blades and rotating hub influence the characteristics of the flow.
75 As the jet diffuses downstream, the velocity characteristics become similar to a submerged
76 three-dimensional jet, Albertson *et al.* (1950).

77

78 Under normal operation the propeller flow is influenced by external characteristics such as
79 the hull of the ship or the presence of a rudder for directional purposes. While manoeuvring
80 or near to bollard pull conditions it has been found that such hull effects are negligible, Prosser
81 (1986). The jet produced by a rotating propeller under such conditions is a complex three-
82 dimensional flow with axial, radial and rotational velocity components, Hamill *et al.* (2003).

83 The axial velocity is the most significant component and is found along the propeller axis of
84 rotation. This component is used to impart a forward thrust to propel the ship in the direction
85 of movement. From the early work of Blauuw and van de Kaa (1978) to the recent PIANC
86 (2015) report, it has been cited that as the axial component is in the order of 10 times the
87 magnitude other components of velocity within the jet, those components “do not need to be
88 considered in the flow analysis of propeller or thruster jets” (PIANC 2015).

89

90 Experimental investigations by naval architects into the velocity fields produced by rotating
91 propeller blades have been focussed on the vicinity of the propeller: Min (1978), Cenedese *et*
92 *al.* (1988) and Felli *et al.*(2006). In contrast, most civil engineering designs of structures and
93 scour prevention systems require the downstream evolution characteristics of turbulent
94 propeller jets in order to determine the magnitude and position of propeller-induced scour.

95

96 This paper presents the findings from an extensive experimental investigation which tested
97 four propellers which were allowed to freely expand and whose characteristics covered a wide
98 range typical propeller types, with each propeller being tested at four speeds of rotation
99 (power settings) with velocity measurements of the time averaged components of velocity
100 being taken using Laser Doppler Anemometry (LDA).

101

102 **2.0 EXPERIMENTAL SETUP**

103 The propellers used in this investigation varied in size (D_p), numbers of blades (N), pitch to
104 diameter ratios (P'), thrust coefficients (C_t), rake and blade area ratios (β), as shown in **Table**
105 **1**. The number of propeller blades varied from three to six. The pitch to diameter ratio ranged
106 from a minimum of 0.735 up to a maximum of 1.0. The thrust coefficient, at zero advance
107 speeds, ranged from 0.2908 up to 0.558. The blade area ratios varied from 0.4525 to 0.922.
108 The blades of propeller 1, 3 and 4 had no forward inclination i.e. all blades are at 90° angles
109 to the hub while the blades of propeller 2 were inclined by a further 10° . In selecting these
110 differing propellers it was intended to test over a large practical variation of characteristics
111 typical of sea going vessels.

112

113 Froudian scaling was used to determine the speeds of rotation tested. It has been established
114 by Blaauw & van de Kaa (1978) that scale effects due to viscosity can be ignored if the
115 Reynolds number for the propeller exceeded 7×10^4 and the Reynolds number for the
116 propeller flow was greater than 3×10^3 . The Reynolds number for the jet flow is given by:

$$117 \quad R_{eflow} = \frac{V_0 D_p}{\nu} \quad \text{Equation 1}$$

118

119 The Reynolds number for the propeller is given by:

$$120 \quad R_{eprop} = \frac{n D_p L_m}{\nu} \quad \text{Equation 2}$$

121

122 The characteristic length, L_m depends on the blade area ratio, propeller and hub diameters as
123 well as the number of blades. Blaauw & van de Kaa (1978) defined this length term as follows:

$$124 \quad L_m = (\beta) D_p \pi \left(2N \left(1 - \frac{D_h}{D_p} \right) \right)^{-1} \quad \text{Equation 3}$$

125

126 The rotational speeds used in the programme of work were based on standard Froudian scale
127 of the efflux velocity within the jet and were based on calculations for a generic propeller
128 determined by Qurrain (1994) in a survey of typical ro-ro vessel operating from British ports.
129 This propeller had a diameter of 2.5m, power levels while manoeuvring gave rotations of 200
130 rpm and a typical thrust coefficient of 0.35 at bollard pull. The efflux velocity, calculated using
131 the equation given by Fuehrer and Römisch (1997), gave a value of $V_0=7.3\text{m/s}$. The
132 corresponding efflux velocity for each propeller was then scaled from this value and used to
133 back calculate the corresponding speed of rotation required to match this providing target
134 speeds for the experimental propellers (1 – 4) of 990, 1056, 865 and 640 rpm respectively.
135 The propellers were operated across a range of speeds that bounded these target values,
136 and these are listed in full in **Table 2**.

137

138 The Reynolds numbers for the propellers operating at these rotational speeds ranged from
139 1.4×10^4 to 7.7×10^4 , while the Reynolds numbers for the propeller jet ranged from 5.3×10^4
140 to 30×10^4 , **Table 2**. The Reynolds numbers for the propellers were, in some cases, slightly
141 less than 7×10^4 however, Blaauw & van de Kaa (1978) and Verhey *et al.* (1987) proposed

142 these scale effects would be insignificant. The Reynolds numbers for the jets were all greater
143 than 3×10^3 for the speeds of rotation investigated satisfying the criteria for Froudian scaling.
144 All experiments were carried out in a free-surface tank 7.5 x 4.4 x 1 m in size, partitioned to
145 allow the unhindered expansion of the propeller jets to be investigated (Qurrain 1994).

146

147 Velocity was measured using Laser Doppler Anemometry (LDA), which is a well-established
148 non-intrusive technique developed by Yeh & Cummins (1964). The 3D LDA adopted in this
149 research was a Dantec Dynamics three-component backscatter system with a water-cooled
150 Stabilite 2017 5W Argon-Ion laser manufactured by Spectra Physics as the illuminating light
151 source. Frequency shifting of 40 MHz using a Bragg cell was used to remove directional
152 ambiguity in the velocity measurements.

153

154 The optical probe was mounted on an automatic Dantec Dynamics 3D-traverse with
155 measurement accuracies within ± 0.05 mm in three orthogonal directions. The measurement
156 volume was located at a distance of 240 mm from the LDA probe. Three-dimensional LDA
157 configurations required the transformation of measurements made in a non-orthogonal
158 coordinate system into a Cartesian system. The transformation of measurements was carried
159 out each time the laser was set-up.

160

161 The LDA technique indirectly measured the velocity of the flow by measuring the speed of the
162 (seeding) particles suspended in the flow. The seeding material used in this study was non-
163 spherically shaped polyamide particles having a mean particle size of 20 μm and density of
164 1.03 g/cm^3 . All measurements were made in fully coincident mode i.e. all three processors
165 had to recognise a valid data point before accepting the data. The maximum data rates were
166 determined by the rates obtained with the lowest power channel. Data rates ranged between
167 a minimum of 30 and a maximum of 1000 particles per second.

168 An experimental measurement grid was established at which velocity readings were taken in
169 sections across the face of the propeller. The centre of the propeller hub, at the cutting edge
170 of the propeller blades, was taken as the zero location and measurements were taken on a Y

171 (horizontal), Z (vertical) grid in 2 – 5mm steps. The sections were repeated at 20mm intervals
172 moving away from the propeller in a horizontal plane, X.

173

174 **3.0 TIME-AVERAGED ANALYSIS OF THE AXIAL VELOCITY COMPONENT**

175 **3.1 Zone of Flow Establishment**

176 The maximum velocity, located on the initial plane of the jet, is termed the efflux velocity: V_0 .
177 Hamill *et al.* (2014) discuss the 3D nature of this velocity and concluded that for the axial
178 component, the magnitude could be obtained from:

$$179 \quad V_0 = 1.22 n^{1.01} D_p^{0.84} C_t^{0.62} \quad \text{Equation 4}$$

180

181 This equation presents an alternative means of calculating V_0 , which although still based on
182 the form of equation developed from the traditional actuator disc theory used in current design
183 guideline such as PIANC (2015), it attempts to provide corrections to the limiting assumptions
184 used in that theory which tend to overestimate the V_0 value. This deviation in predicted values
185 of V_0 is clearer for larger propellers.

186

187 All subsequent velocity values, at any location within the diffusing jet, have been shown to be
188 dependent on the magnitude of this initial value V_0 . The formation and diffusion process that
189 occur within the jet are also accepted to occur within two regions of transition as shown in
190 **Figure 1**. The first, where the jet forms and becomes established, is called the Zone of Flow
191 Establishment (ZFE). The second, where the jet subsequently decays to merge with any
192 background flow, is called the Zone of Established Flow (ZEF), Albertson *et al.* (1950). In
193 propeller jets the flow is said to be fully established when the maximum velocity location
194 moves from across the blade to act along the line of the propeller shaft axis. The differing
195 mechanisms that operate within these zones has resulted in previous researchers trying to
196 establish the location of the changeover so that different analytical techniques can be applied
197 to each zone.

198

199 Fuehrer & Römisch (1977) and Blaauw & van de Kaa (1978) found the end of the “ZFE”
200 occurred at a relative distance of $X_0/D_p = 2.6$. Verhey *et al.* (1987) suggested the zone length

201 was $X_o/D_p = 2.77$, while Stewart (1992) proposed the zone extended to approximately X_o/D_p
202 $= 3.25$ from the initial efflux plane.

203

204 **Figure 2** shows the measured velocity distributions obtained for propeller 2, at a test rotational
205 speed of 1000rpm. This profile is typical of all the tests conducted, for all the propellers tested.
206 The axial velocity distribution at $2D_p$ consisted of a low velocity core with the maximum peak
207 velocities located either side of the jet centreline. By $3D_p$, further entrainment of surrounding
208 fluid caused a decrease in the magnitude of the axial velocity distribution. The locations of the
209 peak velocities were still evident at positions along the propeller blades. However by $4D_p$, the
210 profiles have taken on the uniform normal distribution shape associated with the zone of
211 established flow. The central core was fully entrained and the maximum velocity reverting to
212 the centreline of the jet.

213

214 Investigations of the axial velocity profiles between $2D_p$ and $4D_p$, at 20 mm intervals, showed
215 that the transition location from the “ZFE” to the “ZEF” occurred at $X_o/D_p = 3.15, 3.26, 3.49$
216 and 2.9 for propellers 1, 2, 3 and 4. Over the range of propeller characteristics tested in this
217 study it is suggested that the extent of the initial zone can be approximated to be between 3
218 $\leq X_o/D_p \leq 3.5$, indicating significant difference from some of the earlier published work.
219 Stewart (1992) confirmed the extent of the zone of flow establishment occurred when the
220 maximum axial velocity was located along the propeller centreline at approximately $X_o/D_p =$
221 3.25. This compares favourably with the results of this investigation.

222

223 **3.1.1 Magnitude of the Maximum Axial Velocity**

224 Albertson *et al.* (1950) assumed there was no decay of the maximum axial velocity in the zone
225 of flow establishment as distance from the jet source increased. Blaauw & van de Kaa (1978),
226 Verhey (1983) and Fuehrer & Römisch (1977), working with propeller jets, also agreed with
227 this statement. Hamill (1987) however, found this hypothesis only held true up to a short
228 distance of approximately $X/D_p = 0.35$ behind the propeller. Beyond this distance, through
229 direct measurements, Hamill (1987) concluded the maximum axial velocities within the
230 propeller jet decreased with distance from the propeller as a result of lateral mixing i.e. the

231 jets expansion and its entrainment of ambient fluid, and was influenced by the blade area ratio
232 (β) as shown in equation 5:

$$233 \quad \frac{V_{max}}{V_0} = 0.87 \left(\frac{x}{D_p} \right)^{-\frac{\beta}{4}} \quad \text{Equation 5}$$

234

235 Stewart (1992) stated the application of equation 5 could not be generalised to any propeller
236 and developed the following linear decay equation:

$$237 \quad \frac{V_{max}}{V_0} = 1.0172 - 0.1835 \left(\frac{x}{D_p} \right) \quad \text{Equation 6}$$

238

239 The predictive solutions from the methods proposed by Albertson *et al.* (1950), Hamill (1987)
240 and Stewart (1992) were compared with the measured results from this investigation. **Figure**
241 **3** shows an exemplar of the comparison found between the current predictive methodologies
242 and the measurements taken. Decay in magnitude of the velocity with distance from the
243 propeller was found in all cases demonstrating that the suggestions based on the work by
244 Albertson *et al.* (1950) are invalid. Equation 7, proposed by Hamill (1987), was found to
245 overestimate the decay of the maximum axial velocity for propellers 2 and 4, with limited fit
246 being found from short regions with propellers 1 and 3. In the remainder of the zone, the
247 equation did not adequately determine the measured data. Equation 6 was developed from
248 tests conducted using propellers 1 and 4, which were also used in this investigation so it was
249 expected that the solutions of equation 6 would adequately predict the axial velocity decay
250 trends for those propellers. However, equation 6 was found to underestimate the axial
251 velocity decay trends, by up to 25%, for propellers 2 and 3 and therefore insufficiently
252 extrapolated outside the test range from which it was derived. Over all none of the current
253 methods provide an adequate method by which the maximum velocity at any axial distance
254 within the ZFE could be determined.

255

256 It was apparent from examining the measured data that the decay trends of the maximum
257 axial velocity follows a linear profile as was suggested by Stewart (1992). Based on a
258 stepwise variable selection process, of all available data for the four propellers tested at four
259 speeds of rotation, analysis determined that the variables that most influenced maximum axial

260 velocity (V_{max}) were the non-dimensionalised distance from the propeller source (X/D_p) and
 261 the propeller pitch to diameter ratio (P'). The following equation having a high coefficient of
 262 determination ($R^2 = 0.964$) was derived:

$$263 \quad \frac{V_{max}}{V_0} = 1.51 - 0.175 \left(\frac{X}{D_p} \right) - 0.46 P' \quad \text{Equation 7}$$

264

265 The output solutions of equation 7 were compared with the results of the empirical
 266 investigation and in all cases, the output solutions of this equation adequately predicted the
 267 decay trends of the maximum axial velocity from $X/D_p = 0.35$ to the end of the initial zone of
 268 flow establishment, **Figure 4**. It is therefore suggested for distances up to $X/D_p = 0.35$ no
 269 decay of the efflux velocity occurs as suggested by Hamill (1987) and that the maximum
 270 velocity with distance is equal to that found on the efflux plane. After this, the maximum axial
 271 velocity decays linearly throughout the remainder of the zone of flow establishment and can
 272 be determined using equation 7, given the efflux velocity (V_0), distance from the propeller (X),
 273 propeller diameter (D_p) and pitch to diameter ratio (P') as input variables.

274

275 **3.1.2 Axial Velocity Distributions within the Zone of Flow Establishment**

276 Along the initial efflux plane, and throughout the zone of flow establishment, the distribution
 277 of the axial velocity component was found to increase from the jet centreline towards a
 278 maximum value before then decreasing rapidly towards the tip of the blade, Hamill (1987).
 279 McGarvey (1996) derived an equation based on the physical properties of propeller blades to
 280 determine the distribution of the axial velocity component along the efflux plane:

$$281 \quad \frac{V_{x,r}}{Nnr} = 1.261 - 0.974 \left(\frac{p}{r} \right) + 0.733 \left(\frac{c}{r} \right) + 18.53 \left(\frac{t}{r} \right) + 5.028 \left(\frac{h_d}{r} \right) + 0.106 \left(\frac{p}{r} \right)^2 - 7.277 \left(\frac{h_d}{r} \right)^2 - 4.093 \left(\frac{h_t}{c} \right)^2$$

282

Equation 8

283 Albertson *et al.* (1950) found the velocity distribution at any section within a submerged jet to
 284 follow the general trend of the Gaussian normal probability function. Hamill (1987) made
 285 changes to the normal probability function and produced the following equation:

$$286 \quad \frac{V_{x,r}}{V_{max}} = EXP \left(-\frac{1}{2} \frac{(r - R_{m0})^2}{\sigma^2} \right) \quad \text{Equation 9}$$

287 Hamill (1987) measured the standard deviation, σ , as constant and equal to $0.5R_{m0}$ up to a
288 downstream distance of $X/D_p = 0.5$:

289
$$\sigma = \frac{1}{2}R_{m0} \quad \text{for } X/D_p < 0.5 \quad \text{Equation 10}$$

290

291 Beyond $X/D_p = 0.5$, to the end of the zone of flow establishment, the standard deviation was
292 defined as:

293
$$\sigma = \frac{1}{2}R_{m0} + 0.075 \left(X - \frac{D_p}{2} \right) \quad \text{for } X/D_p > 0.5 \quad \text{Equation 11}$$

294

295 The output results of equation 8, proposed by McGarvey (1996), were compared with the
296 experimental results in this study and **Figure 5** is a typical representation of the findings.

297 While the shape of the profile predicted does follow that expected the only propeller that gave
298 good agreement was propeller 1 (upon which the equation was developed). The method is
299 overly cumbersome and can be difficult to apply. This equation has therefore poor
300 generalisation capabilities when applied to any propeller.

301

302 Axial velocity distributions within the zone of flow establishment were measured and
303 compared with the output results of equations 9, 7 and 4 using non-dimensionalised values
304 of $V_{x,r}/V_{max}$ versus X/D_p . **Figures 6** shows a typical comparison, with good agreement being
305 predicted both in terms of magnitudes and profile shape. The use of equation 9, in conjunction
306 with equations 11, 10, 7 and 4, adequately determined the axial velocity distributions within
307 the Zone of Flow Establishment in the jets produced by each of the experimental propellers
308 tested, and removes the need to establish refined methods of analysis, and is recommended
309 for use in predicting the velocity distributions of the axial velocity within the zone.

310

311 **3.2 Zone of Established Flow**

312 *3.2.1 Magnitude of the Maximum Axial Velocity Decay within the Zone of Established Flow*

313 Differences exist in the decay between the zone of flow establishment and the zone of
314 established flow. This can be explained by the differences in the diffusion processes in these
315 two zones. In the first zone, diffusion is occurring both internally and externally. The jet is
316 entraining its low velocity core as well as the ambient fluid. The decay of maximum velocity is

317 therefore much more rapid than in the zone of established flow were the central core has
 318 already been entrained and only the external entrainment of the surrounding fluid is taking
 319 place, Stewart (1992).

320

321 Albertson *et al.* (1950) stated that for all jets, including propeller jets, the decay of velocity was
 322 proportional to the distance from the source could be found using:

$$323 \quad \frac{V_{max}}{V_0} = \frac{1}{2C} \left(\frac{X}{D_p} \right)^{-1} \quad \text{Equation 12}$$

324

325 where the constant C is the variation of the standard deviation of velocity with distance.

326

327 Other researchers also adopted the general form of equation 12: Fuehrer & Römisch (1977),
 328 Blaauw & van de Kaa (1978), Berger *et al.* (1981) and Verhey (1983). These equations are
 329 as follows for each author respectively:

$$330 \quad \frac{V_{max}}{V_0} = 2.6 \left(\frac{X}{D_p} \right)^{-1} \quad \text{Equation 13}$$

$$331 \quad \frac{V_{max}}{V_0} = 2.8 \left(\frac{X}{D_p} \right)^{-1} \quad \text{Equation 14}$$

$$332 \quad \frac{V_{max}}{V_0} = 1.025 \left(\frac{X}{D_p} \right)^{-0.6} \quad \text{Equation 15}$$

$$333 \quad \frac{V_{max}}{V_0} = 1.275 \left(\frac{X}{D_p} \right)^{-0.7} \quad \text{Equation 16}$$

334

335 Through direct experimental measurements Hamill (1987) suggested the decay of the
 336 maximum velocity can be described using the following equation, taking into account the
 337 propeller geometry:

$$338 \quad \frac{V_{max}}{V_0} = A \left(\frac{X}{D_p} \right)^B \quad \text{Equation 17}$$

339 where:

$$340 \quad A = -11.4 C_t + 6.65 \beta + 2.16 P'$$

$$341 \quad B = -C_t^{0.216} \cdot \beta^{1.024} \cdot P'^{-1.87}$$

342

343 Stewart (1992) reported the decay of the maximum axial velocity was independent of the
344 speed of rotation and propeller type used. A straight-line equation was proposed to determine
345 the decay within the zone of established flow:

$$346 \quad \frac{V_{max}}{V_0} = 0.543 - 0.0281 \left(\frac{X}{D_p} \right) \quad \text{Equation 18}$$

347

348 Hashmi (1993) found the maximum velocity in the wash was still measurable up to $X/D_p = 16$
349 downstream from the propeller. Hashmi (1993) therefore proposed the following equation in
350 exponential form to predict the decrease in V_{max} :

$$351 \quad \frac{V_{max}}{V_0} = 0.638 e^{-0.097 \left(\frac{X}{D_p} \right)} \quad \text{Equation 19}$$

352

353 Large differences therefore exist in the extensive range of semi-empirical equations available
354 to determine the decay of the maximum axial velocity within the zone of established flow. The
355 decay trends of the maximum axial velocity were therefore measured for each of the
356 experimental propellers tested to allow a comparison to be made between the measured and
357 predicted output solutions of the existing semi-empirical equations.

358

359 Equations 13 and 14 proposed by Fuehrer & Römisch (1977) and Blaauw & van de Kaa
360 (1978) overestimated the measured decay trends, **Figure 7**. Equations 15 and 16 suggested
361 by Berger *et al.* (1981) and Verhey (1983) produced similar decay trends throughout the zone
362 of established flow but showed under predictions of propeller 2 (and over predictions of
363 propeller 4) by some 20%, **Figure 8**. The linear equation 18 proposed by Stewart (1992)
364 adequately predicted the decay of propellers 1 and 4 from which it was derived, **Figure 9**.
365 However, the output solutions of equation 18 underestimated the decay trends of propellers
366 3 and 4, **Figure 9**. The generalisation capabilities of equation 18 were reduced when used to
367 predict the decay trends of propellers outside the test range of which it was derived. The
368 exponential form of equation 19 proposed by Hashmi (1993) also underestimated the decay
369 of all propellers at the beginning of this zone, **Figure 9**. It is obvious from these comparisons
370 that the simplified decay expressed by these equations is not sufficient to account for the
371 variations measured.

372

373 The power trend equation 17 suggested by Hamill (1987) is based on the main propeller
374 characteristics: propeller pitch to diameter ratio, blade area ratio and thrust coefficient. The
375 output solutions of equation 17 were found to adequately determine the experimental results
376 of propellers 1 and 2, giving low percentage differences of 20%, **Figures 10 a and b**. Equation
377 17 was also used to determine the maximum axial velocity within the ZEF of propellers 3 and
378 4. However, this equation overestimated the maximum axial velocity, **Figures 10 c and d**. It
379 does however, show that the variations can be better described by including the aspects of
380 the propeller geometry within the prediction.

381

382 In a manner similar to that adopted for the Zone of Flow Establishment, a stepwise variable
383 selection process was tested and it was found that the variables which most influenced the
384 determination of the maximum axial velocity (V_{max}) were the same, i.e. the non-
385 dimensionalised distance from the propeller source (X/D_p) and propeller pitch to diameter ratio
386 (P'). An equation having a high coefficient of determination ($R^2 = 0.924$) was derived.

387
$$\frac{V_{max}}{V_0} = 0.964 - 0.039 \left(\frac{X}{D_p} \right) - 0.344 P' \quad \text{Equation 20}$$

388

389 **Figure 11** shows an exemplar comparison of the output from equation 20 with the data
390 obtained from the tests using propeller 4. The measured decay trends were adequately
391 predicted using the distance from the initial efflux plane and pitch to diameter ratio as input
392 variables. Overall, equation 20 performs well in predicting the decay of the maximum axial
393 velocity within the zone of established flow, and it is recommended that it should be used in
394 place of the existing methodologies.

395

396 **3.2.2 Axial Velocity Distributions within the Zone of Established Flow**

397 Hamill (1987) investigated the methods available to determine the axial velocity distributions
398 within the zone of established flow proposed by Blaauw & van de Kaa (1978), Berger *et al.*
399 (1981), Verhey (1983) Fuehrer & Römisch (1977). The equations proposed by Berger *et al.*
400 (1981) and Verhey (1983) were found to be limited when applied to propeller jet flow. The

401 solutions proposed by Blaauw & van de Kaa (1978) and Fuehrer & Römisch (1977) are
402 respectively as follows:

403
$$\frac{V_{x,r}}{v_{max}} = EXP \left[-15.4 \left(\frac{r}{x} \right)^2 \right] \quad \text{Equation 21}$$

404
$$\frac{V_{x,r}}{v_{max}} = EXP \left[-22.2 \left(\frac{r}{x} \right)^2 \right] \quad \text{Equation 22}$$

405

406 The output solutions of equations 21 and 22, when calculated using equations 4 and 20, were
407 compared with measured non-dimensionalised axial velocity profiles for all the tested
408 propellers. Comparisons were made at downstream distances of $X/D_p = 4, 5$ and 6 within the
409 zone of established flow, and an example of the typical output obtained is shown in **Figure**
410 **12**. The output solutions of equation 22 proposed by Fuehrer & Römisch (1977) were found
411 to adequately predict the axial velocity distributions within the zone of established flow,
412 consistently, for all four experimental propellers investigated. These results agree with those
413 of Hamill (1987) and Stewart (1992), in that, equation 22 proposed by Fuehrer & Römisch
414 (1977) adequately predicts the axial velocity distributions within the zone of established flow.
415 It is suggested equation 22 needs no further modification and should be used in future
416 analysis.

417

418 **4.0 SUMMARY AND CONCLUSIONS**

419 A range of experimental propellers was tested at zero advance speeds, simulating the
420 manoeuvring operation when a ship departs from a port. The experiments simulated a freely
421 expanding jet, with no interference from any harbour configuration or the presence of any
422 rudder effect. The time-averaged (mean) velocity of these jets were investigated. This time-
423 averaged analysis can be used to assist engineers in designing suitable scour protection
424 systems to prevent damage of erodible seabed materials by expanding the envelope of
425 information available upon which engineering decisions can be based.

426

427 Semi-empirical equations have been derived, based on the main propeller characteristics and
428 rotational speed, to determine the location, magnitude and distribution of the axial velocity
429 within the freely expanding propeller jet produced by an un-ducted propeller.

430

431 When used in conjunction with Equation 4 for prediction the efflux velocity V_0 , (Hamill (2014))
432 the maximum axial velocity decayed linearly throughout the zone of flow establishment after
433 an initial distance of $X/D_p = 0.35$. The variables which most influenced the decay of the
434 maximum axial velocity were: the efflux velocity (V_0), distance from the propeller (X), propeller
435 diameter (D_p) and propeller pitch to diameter ratio (P'):

$$436 \quad \frac{V_{max}}{V_0} = 1.51 - 0.175 \left(\frac{X}{D_p} \right) - 0.46 P'$$

437 Similarly, within the zone of established flow a semi-empirical equation based on the propeller
438 characteristics determined the magnitude of the maximum axial velocity:

$$439 \quad \frac{V_{max}}{V_0} = 0.964 - 0.039 \left(\frac{X}{D_p} \right) - 0.344 P'$$

440 When used with equations 4, 7 and 20 the distribution of axial velocity within the Zone of Flow
441 Establishment was found to be adequately described by the equations developed by Hamill
442 (1987), while for distributions within the Zone of Established Flow the method reported by
443 Fuehrer (1977) is recommended.

444

445 The suite of equations presented and discussed within this paper relate to a free expanding
446 propeller jet and bring together the current knowledge available to the engineer. The testing
447 conducted, using state of the art LDA measurement in an expansive experimental, has
448 allowed knowledge gaps to be filled and an integrated axial velocity predictive method
449 published.

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