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Analysis of the fast electron scaling theory for the heating of a solid target

R. J. Garland,¹ M. Borghesi,¹ and A. P. L. Robinson^{2, a)}

¹⁾*School of Mathematics and Physics, Queens University Belfast, University Road, Belfast, BT7 1NN, Northern Ireland, UK*

²⁾*Central Laser Facility, Rutherford Appleton Laboratory, Didcot, Oxfordshire, OX11 0QX, UK*

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Simple scaling laws for laser-generated fast electron heating of solids that employ a Spitzer-like resistivity are unlikely to be universally adequate as this model does not produce an adequate description of a material's behaviour at low temperatures. This is demonstrated in this paper by using both numerical simulations and by comparing existing analytical scaling laws for low temperature resistivity. Generally we find that, in the low temperature regime, the scaling for the heating of the background material has a much stronger dependence on the key empirical parameters (laser intensity, pulse duration, etc.).

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I. INTRODUCTION

During a relativistic laser pulse ($>10^{18} \text{ Wcm}^{-2}$) interaction with a solid target, a small proportion of electrons are heated to high energy and accelerated into the target.^{1–21} This fast electron beam (FEB) is of critical importance to a variety of contexts including: fast-ignition ICF^{1,2,5–13,15,17–21}, laser-driven ion acceleration schemes^{2,4,9,12–16}, x-ray production^{2,4,7,9–14}, generation of warm dense matter (WDM)^{13–15,22} & lab astrophysics experiments³.

The propagation of this FEB through a target will generate large scale electric fields which will lead to the generation of a return current. This counter propagating return current is responsible for the dominant heating mechanism exhibited in solid targets, i.e. Ohmic heating^{3,15,18}. In principle, a typical fast electron beam can heat solid density material to 100–1000 eV. Developments in guiding and controlling (e.g.³) fast electron propagation have opened up the prospect of controllable enhanced heating using laser-generated fast electron beams, a highly useful technique for investigating the properties of both warm and hot dense matter.

The pursuit of fast-electron-based heating requires an understanding of how the temperature of the heated material scales with the key experimental parameters. Many of the theoretical analyses developed so far (e.g.³) use assumptions such as a Spitzer-like resistivity for the background plasma. Whilst nearly all materials will eventually exhibit Spitzer-like behaviour at sufficiently high temperature, for many materials there can be substantial deviations from Spitzer resistivity for temperatures below a few hundred eV. As we shall show in this paper, the assumption of Spitzer-like resistivity leads to the conclusion that the Ohmic heating of a target scales rather slowly with parameters such as laser intensity, pulse duration and target density. With different resistivity profiles the heating will scale much more strongly with these parameters. We will also present comparisons between hybrid simulation results and analytical scalings to verify these conclusions.

^{a)}Electronic mail: alex.robinson@stfc.ac.uk

This paper is structured as follows: Section II will discuss the relevant background theory and the derivation of the scaling relations. Section III–V will describe hybrid simulations in which we studied temperature scaling, discuss the results and their relation to the theoretical scalings.

II. THEORY

A. Spitzer-based Scaling

We consider solid targets a few micrometers thick, in which the dominant heating mechanism is Ohmic heating induced by the return current^{3,18}. The assumption is also made that the FEB follows Davies' "rigid beam" model, where the density of this beam is fixed throughout⁸.

For the calculations we assume that in a beam-heated plasma, the Spitzer resistivity applies, which is given by^{2,3}:

$$\eta = 10^{-4} \frac{Z \ln \Lambda}{T^{\frac{3}{2}}}, \quad (1)$$

as well as the ideal gas heat capacity:

$$C = \frac{3}{2} n_c e, \quad (2)$$

where n_c is the density of the background electrons, T is the temperature of the cold electrons, $\ln \lambda$ the coulomb logarithm & Z the atomic number.

This model is completed by the equation for the Ohmic heating of the background plasma^{2,8,11}:

$$\frac{\delta T}{\delta t} = \frac{\eta j_f^2}{C}, \quad (3)$$

where j_f is the fast electron current density.

Using the rigid beam model, we combine the equations above and integrate to arrive at the equation:

$$T(t) = \left[B t_h + T_0^{\frac{5}{2}} \right]^{\frac{2}{5}}, \quad (4)$$

where T is the background electron temperature(eV), T_0 the initial background temperature(eV), t_h the heating time and $B = \frac{5Z \ln \Lambda j_f^2}{3n_c e}$. Now if we consider the situation of strong heating, where $T \gg T_0$, Eq.4 becomes:

$$T(t) = (B t_h)^{\frac{2}{5}} = \left(\frac{5 \ln \Lambda}{e} \right)^{\frac{2}{5}} \frac{Z^{\frac{2}{5}} j_f^{\frac{4}{5}} t_h^{\frac{2}{5}}}{n_c^{\frac{2}{5}}}, \quad (5)$$

where t_h is the heating time.

We then rewrite the fast electron current density so that it is related to laser parameters:

$$j_f = \frac{e \beta I_L}{\epsilon_f} \quad (6)$$

where β is the conversion efficiency from laser energy to fast electrons, I_L the intensity of the laser and ϵ_f the mean energy of the FEB. As the fast electron temperature exhibits a ponderomotive scaling one has $\epsilon_f = A\sqrt{I_L\lambda_L}$, with A a proportionality constant and λ_L the laser wavelength. This leads to³:

$$T \propto \frac{\beta^{\frac{4}{5}} I_L^{\frac{2}{5}} t_h^{\frac{2}{5}}}{\lambda_L^{\frac{4}{5}} n_i^{\frac{2}{5}}}, \quad (7)$$

which shows the impact of the relevant parameters associated with Ohmic heating from the fast electron beam.

B. Scaling Accounting for Low Temperature Resistivity

The main crux of this paper, is that the scaling equation presented in Eq.7, based on the assumption of a pure Spitzer-like resistivity, does not accurately represent the relation of the empirical variables to the heating of the target. We now want to proceed to modify the above derivation to account for low temperature resistivity. The difficulty with this is that low temperature resistivity is generally quite a complex matter, and models do not, in general, produce neat closed-form expressions for resistivity that are amenable to simple analysis. What can be done, however, is to examine what happens if we assume certain simple analytic forms for the resistivity. Here we consider the cases of (a) resistivity that is constant with temperature, (b) resistivity that varies as $T^{1/2}$ (a.k.a. “square root” resistivity), and (c) resistivity that varies linearly with temperature. Although these do not necessarily conform precisely to a particular system they do mimic some of the possible variations that can occur in the low temperature regime, and the changes to the scalings that these produce are indicative of what low temperature resistivity can do in general.

With reference to Davies⁸, the resistivity of a given system can be defined as:

$$\eta = \eta_0 \left(\frac{T}{T_0} \right)^\alpha, \quad (8)$$

where α is an arbitrary constant.

Substituting this equation into Eq.3 results in a first order differential equation for the temperature:

$$T = T_0 \left(1 + (1 - \alpha) \frac{\eta_0 j_f^2 \tau}{C T_0} \right)^{1/(1-\alpha)}, \quad \alpha < 1. \quad (9)$$

1. Constant Resistivity

Considering $\alpha = 0$, the constant resistivity case, the solution can be written as⁸:

$$T \propto \frac{\beta^2 I_L \tau_L}{Z n_i \lambda_L} \quad (10)$$

By comparing this to Eq.7, we can see that the scaling in terms of laser intensity, pulse duration, conversion efficiency, and ion density is significantly faster (linear or reciprocal linear) than in the Spitzer resistivity case. There is also a Z dependence to consider: if the resistivity does not scale $\propto Z$ then, as Eq.10 indicates, the temperature scaling will acquire some Z dependence.

2. $T^{1/2}$ (Square Root) Resistivity

By considering $\alpha = \frac{1}{2}$, the "square root" resistivity case, as a low temperature model, one obtains:

$$T \propto \frac{\beta^4 I_L^2 \tau_L^2}{Z^2 n_i^2 \lambda_L^2} \quad (11)$$

This scaling also differs significantly from the Spitzer case and the dependence from the single parameters is even stronger than for the constant resistivity case. This also applies to the dependence on Z .

3. Linear Resistivity

The case of linear resistivity analysed in⁸ was found to lead to:

$$T = T_0 \exp\left(\frac{\eta_0 \beta^2 I_L \tau}{C T_0 \lambda^2}\right), \alpha = 1. \quad (12)$$

The heating for this case differs from ones previously discussed in that the temperature dependence exhibits an exponential dependence from the parameters instead of a polynomial.

4. Model Assumptions

The calculations of the previous section have been done under the strong heating assumption, i.e. that the temperature of the target raises to $T \gg T_0$ (the initial target temperature). By taking Eq. 9 and considering the case $\alpha = \frac{1}{2}$ the assumption leads to:

$$T = (A + T_0^{\frac{1}{2}})^2 = A^2 + 2AT_0^{\frac{1}{2}} + T_0. \quad (13)$$

where

$$A = \frac{2\eta_f^2 \tau_L}{3n_e e}, \quad (14)$$

Additionally, the ionisation, Z^* , and the heat capacity of the target (dependent on Z^* and the equation of state) are both assumed to be constant.

While the considerations done so far are based on highly simplified analytical approaches, numerical simulations can be performed for a more comprehensive assessment. Section IV will test the validity of these equations, with the parameters I_L , λ & n_i being varied.

III. NUMERICAL SIMULATIONS OF FAST ELECTRON TRANSPORT

The variation in the length and time scales required to describe the FEB & return current creates difficulties for pure PIC simulations, which have led to the development of hybrid-PIC codes¹. These hybrid codes allow for the physics of the simulation to be computed via two methods, with the FEB treated kinetically and the background plasma treated as a fluid.

The hybrid-code used in this paper is ZEPHYROS, which was developed by Dr. A. P. L. Robinson¹³. ZEPHYROS follows Davies hybrid model^{7,13} where the electric field is calculated from Ohm's law:

$$\mathbf{E} = -\eta \mathbf{j}_f, \quad (15)$$

where $\mathbf{j}_f$ is the fast electron current.

By considering current neutrality, one has:

$$\mathbf{j}_f + \mathbf{j}_b \approx \frac{\nabla \mathbf{B}}{\mu_0^{-1}}, \quad (16)$$

where $\mathbf{j}_b$ is the return current.

From equations 15 & 16, one can then arrive at the equations for $\mathbf{E}$ & $\mathbf{B}$:

$$\mathbf{E} = -\eta \mathbf{j}_f + \frac{\eta}{\mu_0} \nabla \times \mathbf{B} \quad (17)$$

and

$$\frac{\delta \mathbf{B}}{\delta t} = \eta \nabla \times \mathbf{j}_f + \nabla(\eta) \times \mathbf{j}_f + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} - \frac{1}{\mu_0} \nabla \eta \mathbf{B} \quad (18)$$

ZEPHYROS does not simulate a laser pulse but instead inputs a fast electron beam into the plasma. The parameters of this fast electron beam are calculated on the basis of the nominal laser input e.g. with an intensity given by $I_L \times \beta$. Furthermore, ZEPHYROS requires a resistivity model to be defined before the start of the simulation, with Lee-More^{1,24,25} & QMD^{1,23} being two possible models available. For the purpose of this work, we will only consider the Lee-More model.

The Lee-More model is a semi-analytical resistivity model which describes dense plasmas & non-plasma states via the Thomas-Fermi ionisation model. Transport coefficients are obtained from the Boltzmann equation in the relaxation time approximation, with the Fermi-Dirac distribution being used to account for the electron degeneracy effects^{1,24}. Values for these transport coefficients are approximated from Coulomb cross sections with appropriate cut-off parameters. These cut-off parameters are calculated from partial wave calculations and are used to account for electron degeneracies, with the minimum parameter being limited to the mean inter-atomic distance and the maximum being set as the Debye length²⁴.

Desjarlais²⁵ improved on this model by including a modified version of the Thomas-Fermi model which incorporates the Saha model, allowing for a more accurate representation of metal-insulator transitions^{1,25}. More accurate treatments of collisions were also included, allowing for e-e collisions to be included in the calculations^{1,23,25}.

The key result of the Lee-More model is the electron relaxation time, τ , which is described by electron-ion & electron-neutral collisions:

$$\frac{1}{\tau_{ec}} = \frac{1}{\tau_{ei}} + \frac{1}{\tau_{en}} \quad (19)$$

where τ_{ei} & τ_{en} are the collision times between ions and neutral atoms respectively. The resistivity for the model is:

$$\eta = \left(\frac{m_e}{n_e e^2} \right) v_m \quad (20)$$

where v_m is the electron momentum slowing-down rate and m_e the mass of an electron.

There are, however, some inherent issues with the Lee-Moore model. For instance, the model finds it difficult to capture the correct characteristics of material which is neither a conductor or resistor at room temperature¹. Robinson²³ also went on to show that the electron-impact excitations, which are neglected by the Lee-More model, play a significant role in resistivity.

IV. SIMULATIONS

Numerical simulations were devised to verify the scenario outlined in the previous section and obtain empirical dependences of target heating from the key target and laser parameters.

These simulations were split into 4 smaller subsets investigating 4 separate materials: Aluminium, Titanium, Gold and a CH foam. Each of these respective subsets were all benchmarked by a respective "test run". The test-run, denoted by Run A in table I, consisted of 4×10^7 particles in a simulation size of $200 \times 200 \times 200$ grids which each of these grids being $0.1 \mu\text{m}$, resulting in a simulation size of $20 \times 20 \times 20 \mu\text{m}$. Initial laser parameters were based on those of the TARANIS laser at Queens University Belfast¹⁴. The injected fast electron beam has a Gaussian profile with $r_{\text{spot}} = 10 \mu\text{m}$, and an intensity, I_L , of $2 \times 10^{19} \text{Wcm}^{-2}$. The laser to fast electron conversion efficiency, β , was taken to be 0.3, the laser heating time, t_h , was set as 10^{-13}s , the wavelength, λ , was taken as $1.053 \mu\text{m}$ and the density, n_i , was chosen according to the target being simulated (table II). The fast electron divergence angle was taken from a TARANIS experiment¹⁴, as $60^\circ (1.047 \text{rad})$, and was assumed to be constant throughout the simulations.

Using the parameters from Run A, the fast electron beam initial temperature can be calculated from the ponderomotive scaling proposed by Wilks³:

$$T(\text{MeV}) = 0.511 \left(\sqrt{1 + \frac{I_L \lambda_{\mu\text{m}}^2}{1.37}} - 1 \right), \quad (21)$$

and is found to be 1.61MeV .

The values of I_L , λ & n_i were all then varied from the initial test run, Run A, to verify how each of these respective variables affect the heating and how this compares to the Spitzer-based dependencies discussed previously. The input parameters for the respective runs can be found in table I.

	Run I (Wcm^{-2})	$\lambda (\mu\text{m})$	$n_i (\text{cm}^{-3})$	Fast e- Temp (MeV)
A	2×10^{19}	1.053	See table 2	1.61
B	9.25×10^{19}			
C	5.55×10^{19}			
D	3.33×10^{19}			
E	1.2×10^{19}			
F	7.2×10^{18}			
G	4.32×10^{18}			
H		4.875		9.02
I		2.925		5.22
J		1.755		2.95
K		0.6318		0.824
L		0.379		0.388
M		0.227		0.166
N			See table 2	
O			See table 2	
P			See table 2	
Q			See table 2	
R			See table 2	
S			See table 2	

TABLE I: Parameters used in each run, with Run A representing the standard run.

Run	$n_i(\text{Al})$	$n_i(\text{Ti})$	$n_i(\text{Au})$	$n_i(\text{CH})$
A	6	5.71	5.9	0.829
N	27.8	26.4	27.3	3.4
O	16.7	15.9	16.4	2.3
P	10	9.52	9.83	1.38
Q	3.6	3.43	3.54	0.497
R	2.16	2.06	2.124	0.298
S	1.296	1.2744	1.2334	0.179

TABLE II: Different ion densities used for the four targets given in units of $\times 10^{28} \text{ cm}^{-3}$.

The output taken from these simulations were the maximum heating values found in the each of the respective simulations were then taken at 2, 4 & 6 μm within the target.

V. RESULTS & DISCUSSION

Figure 1 presents the results from the simulations of table I together with analytical predictions based on Spitzer resistivity and a linear guideline to represent the constant resistivity for CH. A general discordance of the numerical values with the analytical estimate is clearly evident while there is good agreement with the linear guideline for CH. By fitting power laws to the results, it is possible to evaluate the heating dependence on the 3 respective variables in terms of exponent coefficients ($T \propto I_L^\alpha \lambda^\epsilon n_i^\rho$), as shown in table III, which refers to data taken at a depth of 2 μm into the target.

Run	I	λ	n_i
Al	1.17037	-1.39737	-0.94616
Ti	1.31882	-1.48537	-1.14573
Au	1.71775	-1.61837	-1.61676
CH	0.89455	-1.14409	-0.47616

TABLE III: Exponents from power laws for heating at 2 μm

Graphs were also produced for the depths of 4 & 6 μm into the target, which however follow the same trend as figure 1 and so are not included. Power laws were applied to these graphs and the dependencies found are presented in tables IV & V

Run	I	λ	n_i
Al	1.54547	-1.74238	-1.35978
Ti	1.71326	-1.74842	-1.48347
Au	1.71396	-1.31961	-1.61393
CH	0.89452	-1.14852	-0.49701

TABLE IV: Exponents from power laws 4 μm

Run	I	λ	n_i
Al	1.41015	-1.29582	-1.35218
Ti	1.52150	-1.36827	-1.34533
Au	1.23550	-0.60065	-1.28172
CH	0.88295	-1.11727	-0.50418

TABLE V: Exponents from power laws 6 μm

FIG. 1: (Colour online) Maximum heating values reached for all targets at $2\mu\text{m}$ with the line representing the variables dependence in Spitzer & Constant Regimes(see text).

If the heating scaling predictions based on Spitzer Resistivity of Eq. 7 were representative of the scaling with key experimental parameters, then exponents close to 0.4, -0.8 & -0.4 for Intensity, Wavelength & Density would be expected. The results presented in Fig. 1 & tables III - V clearly show that in almost all cases the observed exponents differ extensively from this. One can then compare the numerical exponents of tables III-V with those predicted by the different models in section II. For example the constant resistivity model, Eq.10, predicts exponents of 1, -1 & -1, which presents a better, but still imperfect, representation for some of the trends for the lower Z-targets eg. Al $2\mu\text{m}$. The square-root resistivity model, Eq. 11, predicts exponents of 2, -2 & -2, which is closer to the numerical exponents for some of the dependences with the higher Z targets e.g. $1.7 I_L$ exponent at $4\mu\text{m}$ for Au & Ti. The fact that the scaling for the higher Z targets departs further from the Spitzer scaling is consistent with these simulations being affected by low temperature resistivity. While it is obvious that, due to their simple nature, the scaling models of section B are not

wholly adequate to explain the trends observed in the simulations, they act however as a motivation towards the development of a more appropriate scaling theory.

By imposing a limit of strong heating on the $2\mu\text{m}$ will allows us to evaluate the strong heating assumption made in Section II. This is done by removing heating values which failed to reach 10, 50 & 100eV respectively. The CH target is excluded from these considerations since all values of T are in excess of 100eV.

Run	I	λ	n_i
Al(10eV)	1.17037	-1.3973	-0.9461
Al(50eV)	1.06293	-1.1934	-0.7301
Al(100eV)	0.96880	-1.1934	-0.5593
Ti(10eV)	1.31882	-1.4853	-1.1457
Ti(50eV)	1.07748	-1.177	-0.6947
Ti(100eV)	0.97502	-1.0041	-0.6947
Au(10eV)	1.60688	-1.1132	-1.2631
Au(50eV)	1.31637	-0.7412	-0.9605
Au(100eV)	1.24259	-0.2969	-0.7802

TABLE VI: Exponents from power laws for heating at $2\mu\text{m}$ in the strong heating limit. The number in the bracket represents the lowest heating value allowed.

Table VI shows that when the limit of strong heating has been imposed, the heating behaviour each of the three targets can be reasonably described by the constant resistivity model. It can also be seen that when heating the target to higher temperatures there is a drift towards the Spitzer regime for the n_i dependence. It is interesting to note that this is not the case for the I dependence for all materials & for λ dependence for Al & Ti. These targets still exhibit low temperature like dependencies even at $T > 100$ eV where one would expect dependencies closer to Spitzer. This suggests that the initial low temperature phase plays an important role even towards the alter evolution of the target, in its plasma phase.

A. Impact of Collisional Heating

While Ohmic heating is the dominant heating mechanism occurring within these simulations, one must not forget the role of collisional drag heating. Collisional heating occurs due to fast electrons collisions with the surrounding target, with the majority of this heating occurring at the stopping distance of these fast electrons^{1,26,27}. To investigate the role of this mechanism in the previous results, and to see if the conclusion still stands with the removal of this mechanism, another set of simulations, following the previous set, were undertaken for Al & CH targets with collisional drag heating turned off in the code. An extra simulation was also carried out for the CH target where there was no acceleration from the fast electrons which, when coupled with the drag being off, results in a completely rigid beam. This causes the FEB in the CH target to stay completely uniform throughout, with no loss in energy to the target. This simulation was added mainly to try and explain the deviation from the Spitzer scenario for the n_i dependence of the CH target. The exponents for the two sets of simulations can be found in tables VII - IX.

Run	I	λ	n_i
Al	1.17037	-1.3973	-0.9461
Al _{drag}	1.17371	-1.38557	-1.04787
CH	0.89455	-1.1440	-0.47617
CH _{drag}	0.89412	-1.14139	-0.47401
CH _{drag+accl}	0.89642	-1.14068	-0.47616

TABLE VII: Exponents for simulations described in section V.A.

Run	I	λ	n_i
Al	1.54547	-1.74238	-1.35978
Al _{drag}	1.65364	-1.84295	-1.57542
CH	0.89452	-1.14852	-0.49701
CH _{drag}	0.90193	-1.35040	-0.47695
CH _{drag+accl}	0.90336	-1.14129	-0.49079

TABLE VIII: Exponents for simulations described in section V.A.

Run	I	λ	n_i
Al	1.41015	-1.29582	-1.35218
Al _{drag}	1.69495	-1.5338	-1.53862
CH	0.88295	-1.11727	-0.50418
CH _{drag}	0.89675	-1.13157	-0.47506
CH _{drag+accl}	0.91498	-1.13147	-0.50102

TABLE IX: Exponents for simulations described in section V.A.

Exponents from tables VII-IX clearly show that the role of collisional heating has little impact in the heating of the target. The only difference found within the results are at $4 \mu\text{m}$ for n_i & $6 \mu\text{m}$ for all parameters, with the Al_{drag} simulation suggesting that the target is in the square root regime instead of the constant regime. These results, however, are all in agreement that these targets are still well within the lower temperature resistivity regime. Due to Collisional Drag Heating being dependent on n_e , CH experiences even less of heating losses when compared to Al which is clearly seen in tables VII - IX with the exponents producing very similar results. It can also be seen that turning off the accelerating feature for the fast electrons also has a very limited impact on these exponents too, suggesting that the role of this process is not significant in the regime being investigated.

B. Significance of low temperature resistivity

Something which must be quantified from these results is the significance of this low temperature resistivity regime during the heating of the targets. This was tested by running simulations for Al & CH, following ones outlined above, with the initial temperature set at 300eV. By doing this, it will allow us to see if the target presents a different resistivity regime and, if so, will show us that low temperature resistivity plays a major role in the heating of the target. The exponents from these simulations, along with the original results, can be found below in tables X - XII.

Run	I	λ	n_i
Al	1.17037	-1.39737	-0.94616
Al ₃₀₀	0.37674	-0.48238	-0.17757
CH	0.89455	-1.14409	-0.47616
CH ₃₀₀	0.69361	-0.89574	-0.39153

TABLE X: Exponents for targets starting at 300eV simulations at $2 \mu\text{m}$.

Run	I	λ	n_i
Al	1.54547	-1.74238	-1.35978
Al ₃₀₀	0.10347	-0.12865	-0.12965
CH	0.89452	-1.14852	-0.49701
CH ₃₀₀	0.35588	-0.52057	-0.17690

TABLE XI: Exponents for targets starting at 300eV simulations at $4 \mu\text{m}$.

Run	I	λ	n_i
Al	1.41015	-1.29582	-1.35218
Al ₃₀₀	0.03053	-0.03558	-0.00728
CH	0.88295	-1.11727	-0.50418
CH ₃₀₀	0.15600	-0.25548	-0.06068

TABLE XII: Exponents for targets starting at 300eV simulations at $6\mu\text{m}$.

Tables X - XII clearly show that both Al & CH targets are now operating in a completely different regime than the low temperature resistivity, with the CH target presenting Spitzer like characteristics. These results further confirm the importance of the initial low temperature phase in determining how the target is heated by hot electrons.

VI. SUMMARY & CONCLUSIONS

In this paper we have studied the scaling of the Ohmic heating of solid density material by laser-generated fast electron beams. Specifically we have attempted to elucidate the effects that accounting for the low temperature resistivity has on how fast electron heating scales with key empirical parameters. The low temperature resistivity of solid density materials is a complex subject, but in most experiments solid targets start out cold, and this is therefore an issue that cannot be avoided. By assuming some simple analytical forms, it becomes apparent that the scaling is likely to become substantially stronger than one would anticipate on the basis of using the Spitzer resistivity.

To test this approach, numerical simulations were carried out on 4 targets: Al, Ti & Au and a CH foam for a variety of different laser and target parameters. Results from these simulations show that the heating scalings of the metallic targets are clearly strongly affected by the low temperature resistivity. Even CH shows a fairly strong deviation from the scaling law derived on the basis of Spitzer resistivity. By examining the scaling in only cases where certain temperatures are exceeded, we find that there is a slow drift back towards a scaling closer to the Spitzer-based one provided that the target is sufficiently strongly heated.

A set of further numerical simulations were run on Al & CH targets to see the impact of collisional heating during these simulations by turning this effect off. The results from these simulations produced very similar scalings therefore showing that the collisional heating had very little impact on the overall heating.

Simulations were also run for Al & CH targets whose initial temperatures were set at 300eV to see if the low temperature resistivity is as significant as believed. Results from these simulations clearly show another resistivity regime, confirming that this low temperature resistivity plays an important role during the heating of these targets.

The combination of the analytical and numerical approaches taken here demonstrates that low temperature resistivity has a profound effect on the scaling of fast electron heating, and future efforts to exploit fast electron heating will have to account for this.

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