Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method


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Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

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Outline

1 Overview

2 Motivation

3 Aeroelastic-Harmonic Balance

4 Validation

5 Results

6 Conclusions
Overview
Contents

- Overview
- Motivation
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
- Results
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
- Results
- Conclusions
Outline

1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
Motivation

Figure: F-16 Flight Test Variability\(^1\)

\(^1\) Thomas et al., AIAA paper 2010-2632
Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations** → High-Fidelity Analysis
- **Uncertainty & Variability** → Probabilistic/Possibilistic
- **Large Parameter Spaces** → Efficient Methods
- **Integration of Available Measurements** → Model Updating and Calibration
Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations** → **High-Fidelity Analysis**
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2. Motivation

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4. Validation

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6. Conclusions
Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[
M\ddot{x} + C\dot{x} + Kx = F \\
\dot{W} + R = 0
\]
Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[ M\ddot{x} + C\dot{x} + Kx = F \]
\[ \dot{W} + R = 0 \]

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of \( N_H \) harmonics and fundamental frequency \( \omega \)

\[ x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1} \cos(n\omega t) + \hat{x}_{2n} \sin(n\omega t)) \]
Consider the general form of the fluid Euler equations:

\[
\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0
\]

\cite{2Hall et al., "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," AIAA J., 40(5)}
Consider the general form of the fluid Euler equations:

$$\frac{\partial W(t)}{\partial t} + R(t) = 0$$

Apply Fourier expansion to $W$ & $R$ and re-write eq. in the frequency domain:

$$\omega A \hat{W} + \hat{R} = 0$$

Consider the general form of the fluid Euler equations:

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Apply Fourier expansion to $W$ & $R$ and re-write eq. in the frequency domain:

$$\omega A\hat{W} + \hat{R} = 0$$

Recast the Fourier coefficients in the time domain and solve: \(^2\)

$$\frac{\partial \tilde{W}}{\partial \tau} + \omega D \tilde{W} + \tilde{R} = 0$$

\( pseudo-time \)

\( Unsteady \ Source \)

\( residual \)

---

where:

\[ D = E^{-1}AE, \quad E = \begin{bmatrix}
    1/2 & 1/2 & \cdots & 1/2 \\
    \cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\
    \sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\
    \cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\
    \sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\
    \vdots & \vdots & \cdots & \vdots \\
    \cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\
    \sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H}
\end{bmatrix} \]

\[ \tilde{W} = \begin{bmatrix}
    W(t_0) \\
    W(t_1) \\
    \vdots \\
    W(t_{2N_H})
\end{bmatrix} \]

\[ t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \ldots, 2N_H\} \]
Aeroelastic - Harmonic Balance (A-HB)

Consider a generic, undamped, structural system described by the equation of motion:

\[ M\ddot{x} + Kx = F(W) \]
Consider a generic, undamped, structural system described by the equation of motion:

$$M \ddot{x} + Kx = F(W)$$

Recast in state-space form

$$\dot{y} = A_s y + B_s F$$

where:

$$A_s = \begin{bmatrix} 0 & I \\ M^{-1}K & 0 \end{bmatrix}; \quad B_s = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}; \quad y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

Even CFD unsteady problems can be computed efficiently, as long as \( N_H \) does not become excessive and \( \omega \) is known.
However, we don’t know $\omega_{LCO}$
However, we don’t know $\omega_{LCO}$ 😞
However, we don’t know $\omega_{LCO}$.

New approach based on frequency updating.
Define the following residual:

\[ R(\omega, y) = \omega D_s y - (A_s y + B_s f) \]

From the residual, define the following figure of merit:

\[ L = \frac{1}{2} R^T R = \frac{1}{2} [\omega D_s y - (A_s y + B_s f)]^T [\omega D_s y - (A_s y + B_s f)] \]
We then propose the following algorithm to determine LCOs:

Find \([\omega, F]\) that minimizes \(R(\omega, y)\) using:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

\[
\frac{\partial L_n}{\partial \omega} = \left( Dy - B_s \frac{\partial f}{\partial \omega} \right)^T \left[ \omega Dy - (A_s y + B_s f) \right]
\]
Outline

1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
Forced Motion Predictions

AGARD CT-6

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_\infty$</th>
<th>$\alpha_m$</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>$x_m$</th>
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<tbody>
<tr>
<td>CT6</td>
<td>0.796</td>
<td>0</td>
<td>1.01</td>
<td>0.202</td>
<td>0.25</td>
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</tbody>
</table>

Figure: NACA 64A010
Forced Motion Predictions
Forced Motion Predictions
Outline

1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
Problem widely studied in the community\(^3\):

\[
\begin{align*}
    m\ddot{h} + S_\alpha \ddot{\alpha} + K_h &= -q_\infty cC_l \\
    S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha &= q_\infty c^2 C_m
\end{align*}
\]

\(^3\) Thomas et al., AIAA paper 2010-2632
Pitch - Plunge Aerofoil

Problem widely studied in the community:

\[
M = \begin{bmatrix}
1 & x_{\alpha} \\
x_{\alpha} & r_{\alpha}^2
\end{bmatrix}, \quad K = \begin{bmatrix}
\left(\frac{\omega h}{\omega_{\alpha}}\right)^2 & 0 \\
0 & r_{\alpha}^2
\end{bmatrix}
\]

\[
f = \begin{bmatrix}
-C_l \\
2C_m
\end{bmatrix}, \quad y = \begin{bmatrix}
h \\
\frac{h}{b} \frac{1}{\alpha}
\end{bmatrix}, \quad V = \frac{U_{\infty}}{\sqrt{\mu \omega_{\alpha} b}}
\]
Frequency Convergence

(a) $V_s = 0.725$

(b) $V_s = 0.80$

A-HB initialised at flutter conditions
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence ($V_s = 0.80$)

Displacement $L_2$ Norm

Residual $L_2$ Norm

Number of Iteration

Number of Iteration

Dr. Simão Marques — LCO Prediction with A-HB
Position-Velocity Diagram \( (V_s = 0.80) \)
LCO Amplitude

- HB_LCO
- time marching

- $h_{LCO}$ vs $V_s$
- $\alpha_{LCO}$ vs $V_s$
- $h_{LCO}$ vs $\omega/\omega_\alpha$
- $\alpha_{LCO}$ vs $\omega/\omega_\alpha$
Generic Fighter Delta Wing
Delta Wing

Structural Model based on 2D shell elements

3.94Hz
12.88Hz
15.82
27.56Hz
Delta Wing

LCO Prediction - $M_\infty = 0.91; \ \alpha = 0^\circ; \ q_{\text{flutter}} = 0.759q_{sl}$

Figure: $q_{LCO} = 0.85q_{sl}$
Delta Wing

Position-Velocity Diagram ($q_{LCO} = 0.85q_{sl}$)
Delta Wing
## Solver Efficiency

<table>
<thead>
<tr>
<th>Aerofoil</th>
<th>Wall Clock [s]</th>
<th>Speed-Up</th>
<th>Delta Wing</th>
<th>Wall Clock [h]</th>
<th>Speed-Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Marching</td>
<td></td>
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<tr>
<td>1 Harmonic</td>
<td>282</td>
<td>14.1</td>
<td>8</td>
<td>8</td>
<td>25.5</td>
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<tr>
<td>2 Harmonics</td>
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<td>8.3</td>
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<td>3 Harmonics</td>
<td>1050</td>
<td>3.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Harmonics</td>
<td>1224</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
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Outline

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2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
• a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
Conclusions

- A new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences.
Conclusions

• a new formulation for LCO prediction based on a Harmonic Balance method has been presented.

• LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences

• the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy
Future Work

- Include stronger nonlinearities.
- Quantify uncertainties
Thank you for your attention

Questions Welcome
- Grid size: 160k
- NACA 65A004
Flutter Calculations:

Eigenvalue Problem

\[
\begin{bmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{bmatrix} p = \lambda p
\]

this can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method\(^4\)

Schur Formulation

\[S(\lambda)p_s = \lambda p_s\]

where

\[S(\lambda) = (A_{ss} - \lambda I) - A_{sf}(A_{ff} - \lambda I)^{-1}A_{fs}\]

Flutter Calculations:

Figure: Delta Wing Flutter response with Volterra Series $808^{th}$ ROM