Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

6^{th} European CFD conference - ECCOMAS 2014

W. Yao
w.yao@qub.ac.uk

S. Marques
s.marques@qub.ac.uk

School of Mechanical and Aerospace Engineering
Queen's University Belfast

Queen's University Belfast

EPSRC
Engineering and Physical Sciences Research Council

Outline

Outline

Contents

- Overview

Contents

Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance

Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation

Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
- Results

Contents

Outline

Motivation

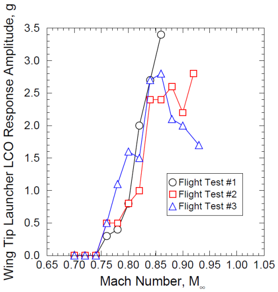


Figure : F-16 Flight Test Variability[1]

[1] Thomas *et al.*, AIAA paper 2010-2632

Motivation
Simulation Requirements

Hence, for such problems we need:

- Physics Based (nonlinear) Simulations → High-Fidelity Analysis
- Uncertainty & Variability → Probabilistic/Possibilistic
- Large Paramenter Spaces → Efficient Methods
- Integration of Available Measurements → Model Updating and Calibration

Motivation
Simulation Requirements

Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations → High-Fidelity Analysis**
- Uncertainty & Variability → Probabilistic/Possibilistic
- **Large Paramenter Spaces → Efficient Methods**
- Integration of Available Measurements → Model Updating and Calibration

Outline

Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \mathbf{F}$$
$$\dot{\mathbf{W}} + \mathbf{R} = 0$$

Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x = \mathbf{F}$$
$$\dot{\mathbf{W}} + \mathbf{R} = 0$$

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of N_H harmonics and fundamental frequency ω

$$x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1}\cos(n\omega t) + \hat{x}_{2n}\sin(n\omega t))$$

HB for CFD

Consider the general form of the fluid Euler equations:

$$\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0$$

[2] Hall *et al.* , "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," AIAA J., 40(5)

HB for CFD

Consider the general form of the fluid Euler equations:

$$\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0$$

Apply Fourier expansion to **W** & **R** and re-write eq. in the frequency domain:

$$\omega \mathbf{A}\hat{\mathbf{W}} + \hat{\mathbf{R}} = 0$$

[2] Hall *et al.*, "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," AIAA J., 40(5)

HB for CFD

Consider the general form of the fluid Euler equations:

$$\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0$$

Apply Fourier expansion to **W** & **R** and re-write eq. in the frequency domain:

$$\omega \mathbf{A}\hat{\mathbf{W}} + \hat{\mathbf{R}} = 0$$

Recast the Fourier coefficients in the time domain and solve:[2]

$$\underbrace{\frac{\partial \tilde{\mathbf{W}}}{\partial \tau}}_{pseudo-time} + \underbrace{\omega \mathbf{D}\tilde{\mathbf{W}}}_{Unsteady\ \ Source} + \underbrace{\tilde{\mathbf{R}}}_{residual} = 0$$

[2] Hall *et al.* , "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," AIAA J., 40(5)

HB for CFD

where:

$$\mathbf{D} = \mathbf{E}^{-1}\mathbf{A}\mathbf{E}, \quad \mathbf{E} = \begin{bmatrix} 1/2 & 1/2 & \cdots & 1/2 \\ \cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\ \sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\ \cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\ \sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\ \vdots & \vdots & \ldots & \vdots \\ \cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\ \sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H} \end{bmatrix}; \quad \tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{W}(t_0) \\ \mathbf{W}(t_1) \\ \vdots \\ \mathbf{W}(t_{2N_H}) \end{bmatrix}$$

$$t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \ldots, 2N_H\}$$

Aeroelastic - Harmonic Balance (A-HB)

Consider a generic, undamped, structural system described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(\mathbf{W})$$

Aeroelastic - Harmonic Balance (A-HB)

Consider a generic, undamped, structural system described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(\mathbf{W})$$

Recast in state-space form

$$\dot{\mathbf{y}} = \mathbf{A}_s\mathbf{y} + \mathbf{B}_s\mathbf{F}$$

where:

$$\mathbf{A}_s = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & 0 \end{bmatrix}; \quad \mathbf{B}_s = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}$$

HB for CFD

Applying the same transformations, we obtain a coupled system in the HB format:

$$\frac{\partial \tilde{\mathbf{y}}}{\partial \tau} + \omega \mathbf{D}_s \tilde{\mathbf{y}} + (\mathbf{A}_s \tilde{\mathbf{y}} + \mathbf{B}_s \mathbf{F}) = 0$$

$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$

HB for CFD

Applying the same transformations, we obtain a coupled system in the HB format:

$$\frac{\partial \tilde{\mathbf{y}}}{\partial \tau} + \omega \mathbf{D}_s \tilde{\mathbf{y}} + (\mathbf{A}_s \tilde{\mathbf{y}} + \mathbf{B}_s \mathbf{F}) = 0$$

$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$

Even CFD unsteady problems can be computed efficiently, as long as N_H does not become excessive and ω is known.

LCO Prediction

- However, we don't know ω_{LCO}

LCO Prediction

- However, we don't know ω_{LCO} 

LCO Prediction

- However, we don't know ω_{LCO} 
- New approach based on frequency updating

LCO Prediction
HB with Frequency Updating

Define the following residual:

$$\mathbf{R}(\omega, \mathbf{y}) = \omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})$$

From the residual, define the following figure of merit:

$$\mathbf{L} = \frac{1}{2}\mathbf{R}^T\mathbf{R} = \frac{1}{2}\left[\omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})\right]^T \left[\omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})\right]$$

LCO Prediction
HB with Frequency Updating

We then propose the following algorithm to determine LCOs:

Find $[\omega, \mathbf{F}]$ that minimizes $\mathbf{R}(\omega, \mathbf{y})$ using:

$$\frac{\partial \tilde{\mathbf{y}}}{\partial \tau} + \omega \mathbf{D}_s \tilde{\mathbf{y}} + (\mathbf{A}_s \tilde{\mathbf{y}} + \mathbf{B}_s \mathbf{F}) = 0$$

$$\downarrow$$

$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$

$$\downarrow$$

$$\frac{\partial \mathbf{L}_n}{\partial \omega} = \left(\mathbf{D}\mathbf{y} - \mathbf{B}_s \frac{\partial \mathbf{f}}{\partial \omega} \right)^T [\omega \mathbf{D}\mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})]$$

Outline

Forced Motion Predictions

AGARD CT-6

Case	M_∞	α_m	α_0	k	x_m
CT6	0.796	0	1.01	0.202	0.25

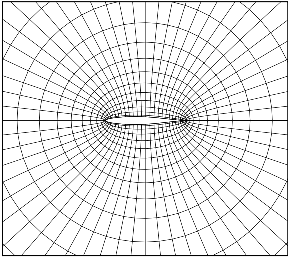
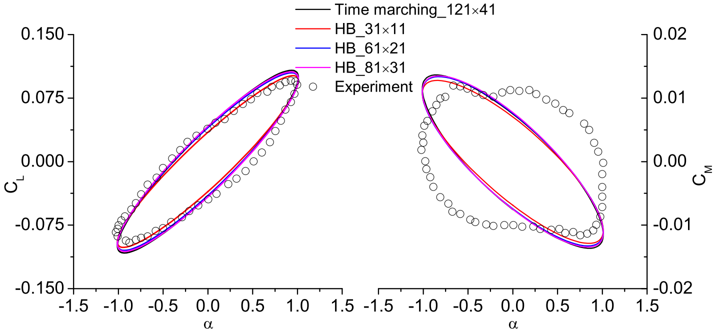
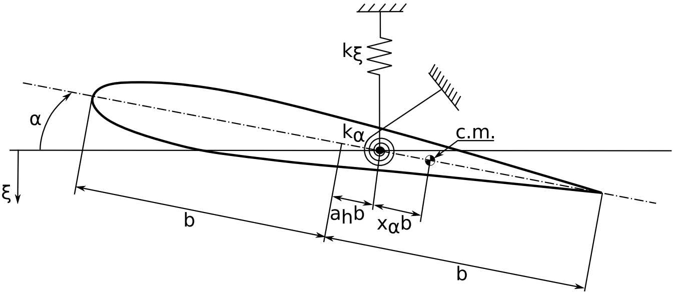


Figure : NACA 64A010

Forced Motion Predictions



Forced Motion Predictions

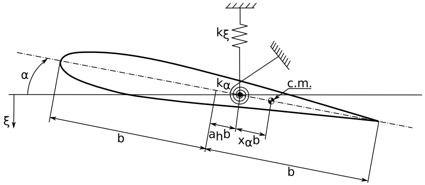


Outline

Pitch - Plunge Aerofoil

Problem widely studied in the community[3]:

$$
\begin{aligned}
m\ddot{h} + S_\alpha \ddot{\alpha} + K_h &= -q_\infty c C_l \\
S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha &= q_\infty c^2 C_m
\end{aligned}
$$



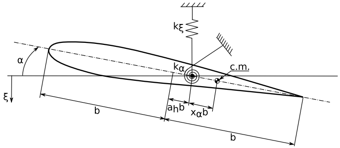
[3] Thomas *et al.* , AIAA paper 2010-2632

Pitch - Plunge Aerofoil

Problem widely studied in the community:

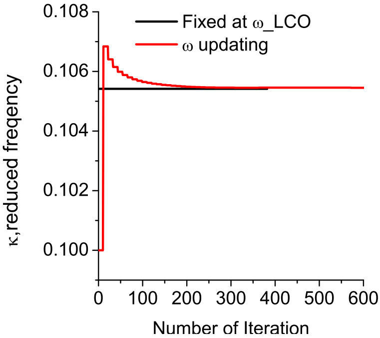
$$\mathbf{M} = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \left(\frac{\omega_h}{\omega_\alpha}\right)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} -C_l \\ 2C_m \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \frac{h}{b} \\ \alpha \end{bmatrix}, \quad V = \frac{U_\infty}{\sqrt{\mu \omega_\alpha b}}$$

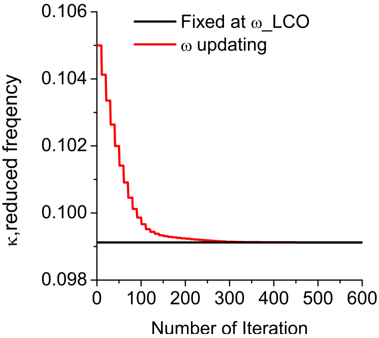


Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence



(a) $V_s = 0.725$

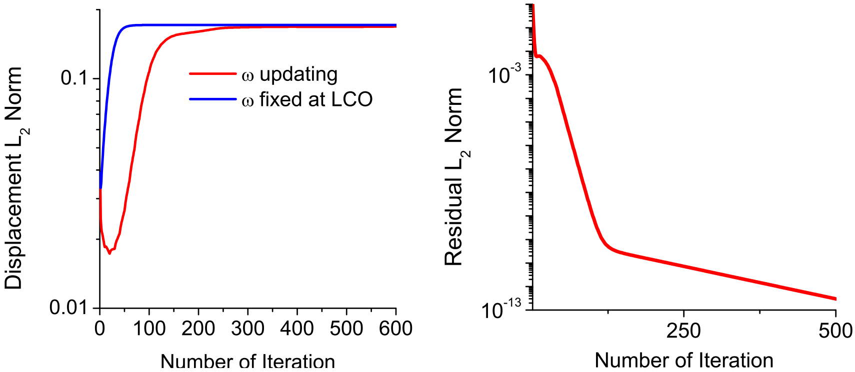


(b) $V_s = 0.80$

A-HB initialised at flutter conditions

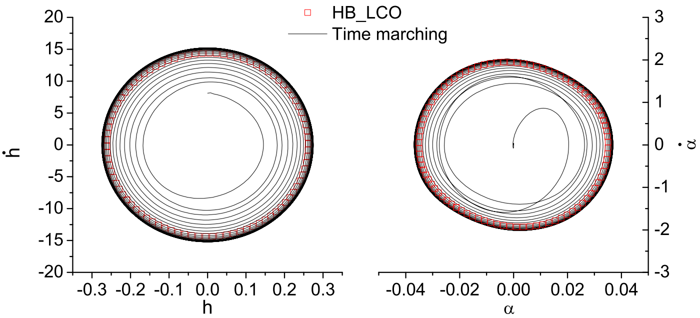
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence ($V_s = 0.80$)



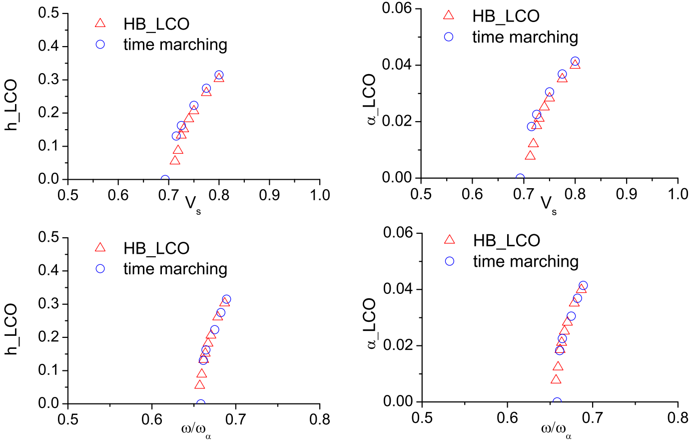
Pitch - Plunge Aerofoil
Solver Convergence

Position-Velocity Diagram ($V_s = 0.80$)



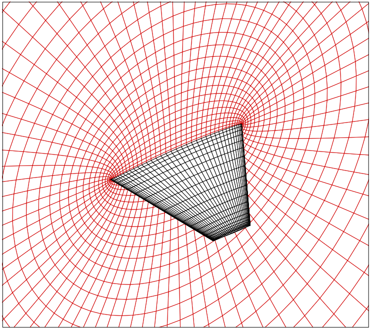
Pitch - Plunge Aerofoil Solver Convergence

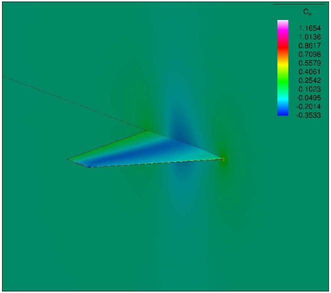
LCO Amplitude



Delta Wing

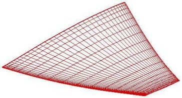
Generic Fighter Delta Wing



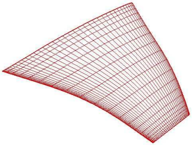


Delta Wing

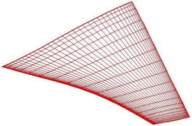
Structural Model based on 2D shell elements



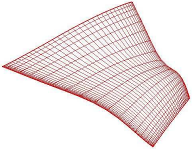
3.94Hz



12.88Hz



15.82



27.56Hz

Delta Wing

LCO Prediction - $M_\infty = 0.91;\ \alpha = 0^\circ;\ q_{flutter} = 0.759q_{sl}$

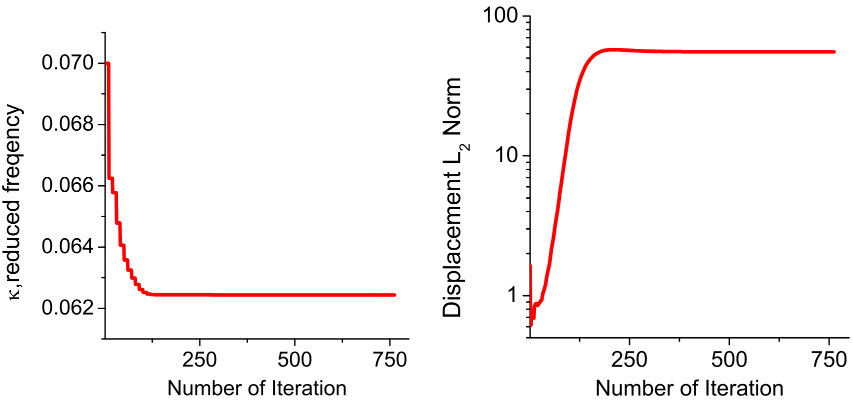
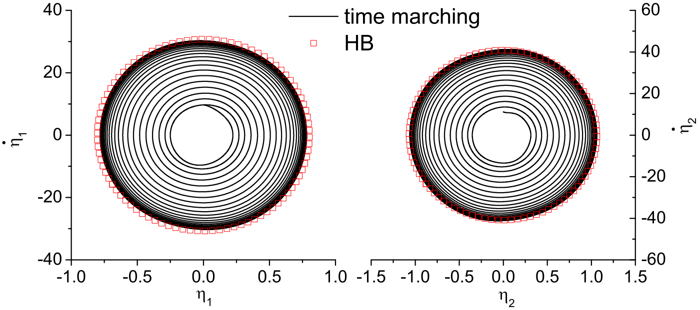


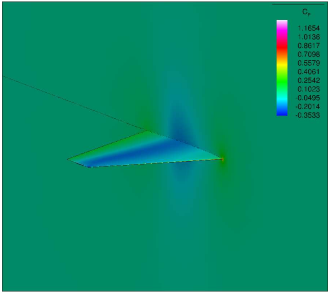
Figure : $q_{LCO} = 0.85q_{sl}$

Delta Wing

Position-Velocity Diagram ($q_{LCO} = 0.85 q_{sl}$)



Delta Wing


C_p
1.1654
1.0136
0.8617
0.7098
0.5579
0.4061
0.2542
0.1023
-0.0495
-0.2014
-0.3533

Solver Efficiency

	Aerofoil		Delta Wing	
	Wall Clock [s]	Speed-Up	Wall Clock [h]	Speed-Up
Time Marching	3990	1.0	204	1.0
1 Harmonic	282	14.1	8	25.5
2 Harmonics	480	8.3		
3 Harmonics	1050	3.8		
4 Harmonics	1224	3.3		

Outline

Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.

Conclusions

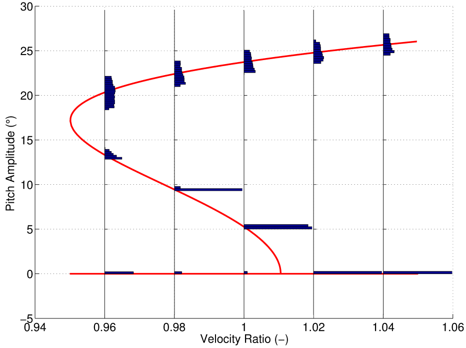
- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences

Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences
- the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy

Future Work

- Include stronger nonlinearities.
- Quantify uncertainties

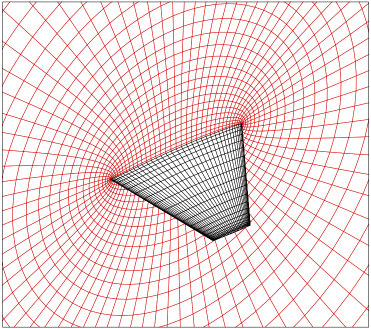


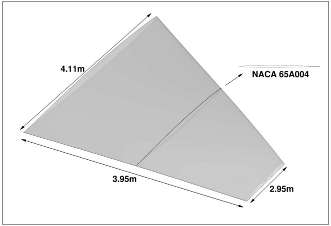
Q & A

Thank you for your attention

Questions Welcome

Q & A





- Grid size: 160k
- NACA 65A004

Q & A

Flutter Calculations:

Eigenvalue Problem

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \mathbf{p} = \lambda \mathbf{p}$$

this can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method[4]

Schur Formulation

$$S(\lambda)\mathbf{p}_s = \lambda \mathbf{p}_s$$

where

$$S(\lambda) = (A_{ss} - \lambda I) - A_{sf}(A_{ff} - \lambda I)^{-1} A_{fs}$$

[4] Badcock, Timme, Marques, *et al.* Envelope searches and uncertainty analysis with transonic aeroelastic simulation.Progress in Aerospace Sciences, 47(5), 2011.

Q & A

Flutter Calculations:

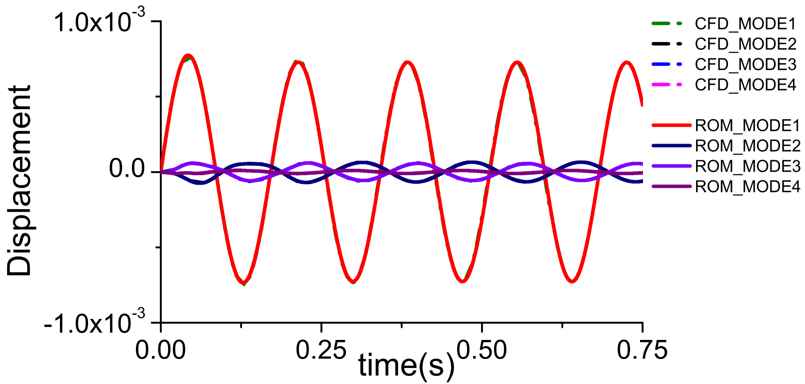


Figure : Delta Wing Flutter response with Volterra Series 808^{th} ROM