Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method


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Transonic Nonlinear Aeroelastic Simulations
Using An Harmonic Balance Method

6th European CFD conference - ECCOMAS 2014

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Outline
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1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
Contents

- Overview
Contents

- Overview
- Motivation
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
Contents

- Overview
- Motivation
- Aeroelastic-Harmonic Balance
- Validation
- Results
Contents

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- Motivation
- Aeroelastic-Harmonic Balance
- Validation
- Results
- Conclusions
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4 Validation

5 Results

6 Conclusions
Motivation

Figure: F-16 Flight Test Variability\(^1\)

\(^1\) Thomas et al., AIAA paper 2010-2632
Hence, for such problems we need:

- Physics Based (nonlinear) Simulations $\rightarrow$ High-Fidelity Analysis
- Uncertainty & Variability $\rightarrow$ Probabilistic/Possibilistic
- Large Parameter Spaces $\rightarrow$ Efficient Methods
- Integration of Available Measurements $\rightarrow$ Model Updating and Calibration
Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations** → **High-Fidelity Analysis**
- Uncertainty & Variability → Probabilistic/Possibilistic
- **Large Parameter Spaces** → **Efficient Methods**
- Integration of Available Measurements → Model Updating and Calibration
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Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[ M\ddot{x} + C\dot{x} + Kx = F \]

\[ \dot{W} + R = 0 \]
Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[
\begin{align*}
M\ddot{x} + C\dot{x} + Kx &= F \\
\dot{\dot{W}} + R &= 0
\end{align*}
\]

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of \(N_H\) harmonics and fundamental frequency \(\omega\)

\[
x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1} \cos(n\omega t) + \hat{x}_{2n} \sin(n\omega t))
\]
Consider the general form of the fluid Euler equations:

\[
\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0
\]

\(^2\text{Hall et al., “Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique,” AIAA J., 40(5)}\)
Consider the general form of the fluid Euler equations:

\[ \frac{\partial W(t)}{\partial t} + R(t) = 0 \]

Apply Fourier expansion to \( W \) & \( R \) and re-write eq. in the frequency domain:

\[ \omega A \hat{W} + \hat{R} = 0 \]

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Consider the general form of the fluid Euler equations:

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Apply Fourier expansion to \( W \) & \( R \) and re-write eq. in the frequency domain:

\[
\omega \hat{A} \hat{W} + \hat{R} = 0
\]

Recast the Fourier coefficients in the time domain and solve:\(^2\)

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D \tilde{W} + \tilde{R} = 0
\]

\( \text{pseudo-time} \)

\( \text{Unsteady Source} \)

\( \text{residual} \)

---

where:

\[
D = E^{-1}AE, \quad E = \begin{bmatrix}
1/2 & 1/2 & \cdots & 1/2 \\
\cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\
\sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\
\cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\
\sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\
\vdots & \vdots & \cdots & \vdots \\
\cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\
\sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H}
\end{bmatrix}; \quad \tilde{W} = \begin{bmatrix}
W(t_0) \\
W(t_1) \\
\vdots \\
W(t_{2N_H})
\end{bmatrix}
\]

\[
t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \ldots, 2N_H\}
\]
Consider a generic, undamped, structural system described by the equation of motion:

\[ M\ddot{x} + Kx = F(W) \]
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\[ M\ddot{x} + Kx = F(W) \]

Recast in state-space form

\[ \dot{y} = A_s y + B_s F \]

where:

\[ A_s = \begin{bmatrix} 0 & I \\ M^{-1}K & 0 \end{bmatrix}; \quad B_s = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}; \quad y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

Even CFD unsteady problems can be computed efficiently, as long as \( N_H \) does not become excessive and \( \omega \) is known.
LCO Prediction

- However, we don’t know $\omega_{LCO}$
However, we don’t know $\omega_{LCO}$.
However, we don’t know $\omega_{LCO}$ 😞

New approach based on frequency updating
Define the following residual:

$$R(\omega, y) = \omega D_s y - (A_s y + B_s f)$$

From the residual, define the following figure of merit:

$$L = \frac{1}{2} R^T R = \frac{1}{2} [\omega D_s y - (A_s y + B_s f)]^T [\omega D_s y - (A_s y + B_s f)]$$
We then propose the following algorithm to determine LCOs:

Find $[\omega, F]$ that minimizes $R(\omega, y)$ using:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

\[
\frac{\partial L_n}{\partial \omega} = \left(Dy - B_s \frac{\partial f}{\partial \omega}\right)^T [\omega Dy - (A_s y + B_s f)]
\]
## Forced Motion Predictions

### AGARD CT-6

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_\infty$</th>
<th>$\alpha_m$</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>$x_m$</th>
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</thead>
<tbody>
<tr>
<td>CT6</td>
<td>0.796</td>
<td>0</td>
<td>1.01</td>
<td>0.202</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Figure:** NACA 64A010
Forced Motion Predictions

- Time marching_121x41
- HB_31x11
- HB_61x21
- HB_81x31
- Experiment
Forced Motion Predictions
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Problem widely studied in the community\(^3\):

\[ m\ddot{h} + S_\alpha \ddot{\alpha} + K_h = -q_\infty c C_l \]
\[ S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha = q_\infty c^2 C_m \]

\(^3\) Thomas et al., AIAA paper 2010-2632
Problem widely studied in the community:

\[
M = \begin{bmatrix}
1 & x_\alpha \\
x_\alpha & r_\alpha^2 \\
\end{bmatrix}, \quad 
K = \begin{bmatrix}
\left(\frac{\omega h}{\omega_\alpha}\right)^2 & 0 \\
0 & r_\alpha^2 \\
\end{bmatrix}
\]

\[
f = \begin{bmatrix}
-C_l \\
2C_m \\
\end{bmatrix}, \quad 
y = \begin{bmatrix}
h \\
b \\
x_\alpha \\
\end{bmatrix}, \quad 
V = \frac{U_\infty}{\sqrt{\mu \omega_\alpha b}}
\]
Frequency Convergence

(a) \( V_s = 0.725 \)

(b) \( V_s = 0.80 \)

A-HB initialised at flutter conditions
Frequency Convergence ($V_s = 0.80$)

- **Displacement $L_2$ Norm**
  - $\omega$ updating
  - $\omega$ fixed at LCO

- **Residual $L_2$ Norm**
  - $10^{-13}$

Number of Iteration: 0 to 600

Dr. Simão Marques — LCO Prediction with A-HB
Position-Velocity Diagram ($V_s = 0.80$)
LCO Amplitude

- Graphs showing LCO amplitude versus different parameters.
- Legend indicates HB_LCO and time marching data points.
Generic Fighter Delta Wing
Delta Wing

Structural Model based on 2D shell elements

3.94 Hz 12.88 Hz

15.82 Hz 27.56 Hz
Delta Wing

LCO Prediction - $M_\infty = 0.91; \alpha = 0^\circ; q_{\text{flutter}} = 0.759 q_{sl}$

Figure: $q_{LCO} = 0.85 q_{sl}$
Position-Velocity Diagram \((q_{LCO} = 0.85q_{sl})\)
Delta Wing
### Solver Efficiency

<table>
<thead>
<tr>
<th></th>
<th>Aerofoil Wall Clock [s]</th>
<th>Speed-Up</th>
<th>Delta Wing Wall Clock [h]</th>
<th>Speed-Up</th>
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</thead>
<tbody>
<tr>
<td>Time Marching</td>
<td>3990</td>
<td>1.0</td>
<td>204</td>
<td>1.0</td>
</tr>
<tr>
<td>1 Harmonic</td>
<td>282</td>
<td>14.1</td>
<td>8</td>
<td>25.5</td>
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<tr>
<td>2 Harmonics</td>
<td>480</td>
<td>8.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Harmonics</td>
<td>1050</td>
<td>3.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Harmonics</td>
<td>1224</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
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- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences.
Conclusions

• a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
• LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences
• the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy
Future Work

- Include stronger nonlinearities.
- Quantify uncertainties
Thank you for your attention

Questions Welcome
- Grid size: 160k
- NACA 65A004
Flutter Calculations:

**Eigenvalue Problem**

\[
\begin{bmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{bmatrix}
\begin{bmatrix}
p
\end{bmatrix}
= \lambda
\begin{bmatrix}
p
\end{bmatrix}
\]

this can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method\(^4\)

**Schur Formulation**

\[
S(\lambda)p_s = \lambda p_s
\]

where

\[
S(\lambda) = (A_{ss} - \lambda I) - A_{sf}(A_{ff} - \lambda I)^{-1}A_{fs}
\]

---

Flutter Calculations:

Figure: Delta Wing Flutter response with Volterra Series 808th ROM