Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method


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Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

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W. Yao  
w.yao@qub.ac.uk

S. Marques  
s.marques@qub.ac.uk

School of Mechanical and Aerospace Engineering  
Queen’s University Belfast
Outline

1. Overview
2. Motivation
3. Aeroelastic-Harmonic Balance
4. Validation
5. Results
6. Conclusions
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Motivation

Figure: F-16 Flight Test Variability\textsuperscript{1}

\textsuperscript{1}Thomas et al., AIAA paper 2010-2632
Hence, for such problems we need:

- Physics Based (nonlinear) Simulations $\rightarrow$ High-Fidelity Analysis
- Uncertainty & Variability $\rightarrow$ Probabilistic/Possibilistic
- Large Parameter Spaces $\rightarrow$ Efficient Methods
- Integration of Available Measurements $\rightarrow$ Model Updating and Calibration
Hence, for such problems we need:

- **Physics Based (nonlinear) Simulations** → **High-Fidelity Analysis**
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- **Large Parameter Spaces** → **Efficient Methods**
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Consider a generic structural system described by the equation of motion:

\[ M\ddot{x} + C\dot{x} + Kx = F \]
\[ \dot{W} + R = 0 \]
Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

\[
M \ddot{x} + C \dot{x} + K x = F \\
\dot{W} + R = 0
\]

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of \( N_H \) harmonics and fundamental frequency \( \omega \)

\[
x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1} \cos(n\omega t) + \hat{x}_{2n} \sin(n\omega t))
\]
Consider the general form of the fluid Euler equations:

\[
\frac{\partial W(t)}{\partial t} + R(t) = 0
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\[ \frac{\partial W(t)}{\partial t} + R(t) = 0 \]

Apply Fourier expansion to \( W \) & \( R \) and re-write eq. in the frequency domain:

\[ \omega A\hat{W} + \hat{R} = 0 \]

\[ ^2 \text{Hall et al., “Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique,” AIAA J., 40(5)} \]
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Apply Fourier expansion to \( W \) & \( R \) and re-write eq. in the frequency domain:

\[
\omega \hat{A} \hat{W} + \hat{R} = 0
\]

Recast the Fourier coefficients in the time domain and solve:\(^2\)

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D \tilde{W} + \tilde{R} = 0
\]

\( \tau \) \text{ pseudo-time} \quad \text{Unsteady Source} \quad \text{residual}

\(^2\)Hall \ et \ al., “Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique,” AIAA J., 40(5)
HB for CFD

where:

\[ D = E^{-1} A E, \quad E = \begin{bmatrix} 1/2 & 1/2 & \cdots & 1/2 \\
\cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\
\sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\
\cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\
\sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\
\vdots & \vdots & \cdots & \vdots \\
\cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\
\sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H} \end{bmatrix}; \quad \tilde{W} = \begin{bmatrix} W(t_0) \\
W(t_1) \\
\vdots \\
W(t_{2N_H}) \end{bmatrix} \]

\[ t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \ldots, 2N_H\} \]
Consider a generic, undamped, structural system described by the equation of motion:

\[ M\ddot{x} + Kx = F(W) \]
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\[ M\ddot{x} + Kx = F(W) \]

Recast in state-space form

\[ \dot{y} = A_s y + B_s F \]

where:

\[ A_s = \begin{bmatrix} 0 & I \\ M^{-1}K & 0 \end{bmatrix}; \quad B_s = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}; \quad y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]
Applying the same transformations, we obtain a coupled system in the HB format:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

Even CFD unsteady problems can be computed efficiently, as long as \( N_H \) does not become excessive and \( \omega \) is known.
However, we don’t know $\omega_{LCO}$
However, we don’t know $\omega_{LCO}$.
However, we don’t know $\omega_{LCO}$.

New approach based on frequency updating.
Define the following residual:

\[ R(\omega, y) = \omega D_s y - (A_s y + B_s f) \]

From the residual, define the following figure of merit:

\[ L = \frac{1}{2} R^T R = \frac{1}{2} \left[ \omega D_s y - (A_s y + B_s f) \right]^T \left[ \omega D_s y - (A_s y + B_s f) \right] \]
We then propose the following algorithm to determine LCOs:

Find \([\omega, F]\) that minimizes \(R(\omega, y)\) using:

\[
\frac{\partial \tilde{y}}{\partial \tau} + \omega D_s \tilde{y} + (A_s \tilde{y} + B_s F) = 0
\]

\[
\frac{\partial \tilde{W}}{\partial \tau} + \omega D_f \tilde{W} + \tilde{R} = 0
\]

\[
\frac{\partial L_n}{\partial \omega} = \left( D_y - B_s \frac{\partial f}{\partial \omega} \right)^T [\omega D_y - (A_s y + B_s f)]
\]
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## Forced Motion Predictions

### AGARD CT-6

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_\infty$</th>
<th>$\alpha_m$</th>
<th>$\alpha_0$</th>
<th>$k$</th>
<th>$x_m$</th>
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<td>0</td>
<td>1.01</td>
<td>0.202</td>
<td>0.25</td>
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</table>

**Figure:** NACA 64A010
Forced Motion Predictions

![Graph showing forced motion predictions with different line styles and markers representing experimental data.]
Forced Motion Predictions
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Problem widely studied in the community\(^3\):

\[
\begin{align*}
  m\ddot{h} + S_\alpha \dot{\alpha} + K_h &= -q_\infty c C_l \\
  S_\alpha \ddot{h} + I_\alpha \dot{\alpha} + K_\alpha &= q_\infty c^2 C_m
\end{align*}
\]

\(^3\)Thomas et al., AIAA paper 2010-2632
Problem widely studied in the community:

\[
\begin{align*}
M &= \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r^2_\alpha \end{bmatrix}, & K &= \begin{bmatrix} \left( \frac{\omega h}{\omega_\alpha} \right)^2 & 0 \\ 0 & r^2_\alpha \end{bmatrix} \\
\mathbf{f} &= \begin{bmatrix} -C_l \\ 2C_m \end{bmatrix}, & \mathbf{y} &= \begin{bmatrix} \frac{h}{b} \\ \frac{\alpha}{\alpha} \end{bmatrix}, & V &= \frac{U_\infty}{\sqrt{\mu \omega_\alpha b}}
\end{align*}
\]
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence

(a) $V_s = 0.725$

(b) $V_s = 0.80$

A-HB initialised at flutter conditions
Pitch - Plunge Aerofoil
Solver Convergence

Frequency Convergence ($V_s = 0.80$)
Position-Velocity Diagram ($V_s = 0.80$)
LCO Amplitude

- Pitch - Plunge Aerofoil
- Solver Convergence

Dr. Simão Marques — LCO Prediction with A-HB
Generic Fighter Delta Wing
Delta Wing

Structural Model based on 2D shell elements

3.94Hz 12.88Hz 15.82 27.56Hz
LCO Prediction - $M_{\infty} = 0.91; \alpha = 0^\circ; q_{\text{flutter}} = 0.759q_{sl}$
Delta Wing

Position-Velocity Diagram \( (q_{LCO} = 0.85q_{sl}) \)
Delta Wing
## Solver Efficiency

<table>
<thead>
<tr>
<th></th>
<th>Aerofoil Wall Clock [s]</th>
<th>Speed-Up</th>
<th>Delta Wing Wall Clock [h]</th>
<th>Speed-Up</th>
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<td>1.0</td>
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<td>2 Harmonics</td>
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<td>3 Harmonics</td>
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<td>4 Harmonics</td>
<td>1224</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
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- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
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• a new formulation for LCO prediction based on a Harmonic Balance method has been presented.

• LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences.
Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences.
- the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy.
• Include stronger nonlinearities.
• Quantify uncertainties
Thank you for your attention

Questions Welcome
- Grid size: 160k
- NACA 65A004
Flutter Calculations:

**Eigenvalue Problem**

\[
\begin{bmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{bmatrix} \mathbf{p} = \lambda \mathbf{p}
\]

this can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method\(^4\)

**Schur Formulation**

\[
S(\lambda)\mathbf{p}_s = \lambda \mathbf{p}_s
\]

where

\[
S(\lambda) = (A_{ss} - \lambda I) - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}
\]

Flutter Calculations:

Figure: Delta Wing Flutter response with Volterra Series 808th ROM