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# Stochastic analysis of seepage under water-retaining structures

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This paper investigated the problem of confined flow under dams and water-retaining structures using stochastic modelling. The approach advocated in the study combined a finite-element method based on the equation governing the dynamics of incompressible fluid flow through a porous medium with a random field generator that generates random hydraulic conductivity based on log normal probability distribution. The resulting model was then used to analyse the confined flow under a hydraulic structure. Cases for a structure provided with cut-off wall and when the wall did not exist were both tested. Various statistical parameters that reflected different degrees of heterogeneity were examined and the changes in the mean seepage flow, the mean uplift force and the mean exit gradient observed under the structure were analysed. Results reveal that under heterogeneous conditions, the reduction made by the sheet pile in the uplift force and exit hydraulic gradient may be underestimated when deterministic solutions are used.

## Notation

$C_b$	Bligh's coefficient
$d$	depth of the sheet pile
$g_i$	local average of a standard Gaussian random field $g$ over the domain of the $i$ th element
$h$	uplift pressure
$[K_s]$	overall hydraulic conductivity matrix at saturation
$k$	hydraulic conductivity of the medium
$k_i$	hydraulic conductivity assigned to the $i$ th element
$[k_s]^e$	element hydraulic conductivity matrix at saturation
$P/\gamma$	pressure head
$Q$	steady-state flow
$\{Q_r\}^e$	element residual flow vector
$\{q\}$	vector of the fluid heads of the elements
$\{R_r\}$	overall residual flow vector
$\{r\}$	overall nodal fluid head vector
$t$	floor thickness
$Z$	elevation head
$\gamma$	unit weight of the fluid
$\theta$	scale of fluctuation
$\phi$	total fluid head
$\mu_k$	mean of hydraulic conductivity
$\mu_{\ln k}$	mean of logarithmic hydraulic conductivity
$\sigma_k$	standard deviation of hydraulic conductivity
$\sigma_{\ln k}$	standard deviation of the logarithmic hydraulic conductivity
$ \tau $	distance between points in the field

## 1. Introduction

The spatial variability in hydraulic conductivity and other soil properties has frequently been observed in real field sites. Through numerous detailed hydraulic conductivity measurements at the Borden site in Ontario, Sudicky (1986) observed that the hydraulic conductivity varies irregularly in three-dimensional space. The study tested 32 cores, each of which is  $\sim 2$  m long and found that the hydraulic conductivity ranged between  $6.0 \times 10^{-4}$  and  $2.0 \times 10^{-2}$  cm/s – that is, by more than a factor of 30. In another field study, Hicks and Onisiphorou (2005) analysed data from 71 cone penetration tests on sands and found large variability in the statistics and spatial correlation coefficient of shear strength parameters. Other examples of spatial variability of soils have been reported by Phoon and Kuthway (1999).

Traditional deterministic approaches, for analysing flow problems under water-retaining structures, generally represent soil properties using one single value. At most, the designer assigns different soil properties to individual layers of the soil. Hence, the obtained solutions do not account for the inherent variability of soils. This leads to the conclusion that results obtained using deterministic solutions, which disregard the variability in soil properties, may suffer from serious deficiencies in many cases (Ahmed, 2009, 2013).

Stochastic approaches based on probabilistic distributions of soil properties provide a framework for addressing more

effectively the aforementioned major deficiencies of deterministic methods. Freeze (1975) was among the pioneers of stochastic analysis of flow problems in porous media. This seminal work has inspired many other studies dedicated to the analysis of water flow problems using stochastic approaches (e.g. Ahmed, 2009, 2013; Griffiths and Fenton, 1997, 1998).

The most commonly used approach to account for soil heterogeneity is to assume homogeneous soil formations of several layers, each having its own soil properties. In contrast to some previous studies, which assessed the effectiveness of the cut-off walls using such an approach or by assuming homogeneous soil formations (e.g. Ahmed, 2011; Ahmed and Bazaraa, 2009), the research work in this paper assumed heterogeneous random soil to investigate the effectiveness of cut-off walls. It is therefore the objective of this paper to account for the effectiveness of the sheet pile or cut-off walls in reducing the seepage losses, the downstream uplift force and the exit hydraulic gradient based on heterogeneous random soil. The methodology adopted here reflects the variability in soil properties that exist in real-world problems. More specifically, the application problem consisted of a hydraulic structure subjected to two different scenarios, namely with and without the sheet pile. Various coefficients of variation and correlation lengths were examined to simulate sites with different degrees of heterogeneity. The hydraulic conductivity resulted from the random field generator was then mapped to a finite-element model, which estimates the seepage flow parameters. The corresponding changes in the mean seepage flow, the mean uplift force and the mean exit gradient were analysed. Furthermore, the results produced using the stochastic approach were contrasted with those obtained using a deterministic method. The obtained results provide some valuable insights into the nature of water flow problems, hence enabling a framework for improvement in the design of water-retaining structures.

## 2. Finite-element model

The finite-element part of the model used in this study is based on the partial differential equation governing steady incompressible fluid flow through porous media for both confined and free surface flow problems. For a detailed presentation of this part of the model as well as its validation and applications, the authors refer the reader to Ahmed (2008, 2011) and Ahmed and Bazaraa (2009). The corresponding partial differential equation can be written as

$$1. \quad \text{div}(k \text{ grad } \phi) = 0,$$

where  $k$  is the hydraulic conductivity of the medium;  $\phi = (P/\gamma) + Z$  is the total fluid head;  $P/\gamma$  is the pressure head;  $Z$  is the elevation head and  $\gamma$  is the unit weight of the fluid. The pseudo-functional for the steady-state flow, denoted as  $U$ ,

can be expressed as follows

$$2. \quad U = \frac{1}{2} \iiint_V k \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 \right] dx dy dz.$$

Applying the residual flow procedure (Desai and Baseghi, 1988) yields the following element equations

$$3. \quad [\mathbf{k}_s]^e \{q\} = \{Q_r\}^e,$$

where  $[\mathbf{k}_s]^e$  is the element hydraulic conductivity matrix at saturation;  $\{q\}$  is the vector of nodal fluid heads of the elements and  $\{Q_r\}^e$  is the element residual flow vector.

The assembly over all elements yields the following equation on the entire domain

$$4. \quad [\mathbf{K}_s] \{r\} = \{R_r\},$$

where  $[\mathbf{K}_s]$  is the overall hydraulic conductivity matrix at saturation;  $\{r\}$  is the overall nodal fluid head vector and  $\{R_r\}$  is the overall residual flow vector.

## 3. Random field model

A log normal distribution is commonly adopted to describe the probability density function of the soil hydraulic conductivity (e.g. Ahmed, 2009; Sudicky, 1986). The saturated hydraulic conductivity field was obtained through the transformation (Griffiths and Fenton, 1997)

$$5. \quad k_i = \exp(\mu_{\ln k} + \sigma_{\ln k} g_i),$$

where  $k_i$  is the hydraulic conductivity assigned to the  $i$ th element,  $g_i$  is the local average of a standard Gaussian random field  $g$  over the domain of the  $i$ th element, and  $\mu_{\ln k}$  and  $\sigma_{\ln k}$  are the mean and standard deviation of the logarithm of  $k$ , respectively, obtained through the transformations (Griffiths and Fenton, 1997)

$$6. \quad \sigma_{\ln k}^2 = \ln \left( 1 + \frac{\sigma_k^2}{\mu_k^2} \right),$$

$$7. \quad \mu_{\ln k} = \ln(\mu_k) - \frac{1}{2} \sigma_{\ln k}^2,$$

where  $\mu_k$  and  $\sigma_k$  denote the mean and standard deviation of  $k$ , respectively.

The local average subdivision technique (Ahmed, 2009, 2013; Fenton and Vanmarcke, 1990) was adopted to generate correlated local averages,  $g_i$ , based on a Gaussian probability distribution function, which has zero mean and unit variance, and a Gauss–Markov spatial correlation function

$$8. \quad \rho(\tau) = \exp\left(-\frac{2}{\theta}|\tau|\right),$$

where  $|\tau|$  is the distance between points in the field. The scale of fluctuation  $\theta$  is a measure of the distance between the adjacent strong or weak zones. Greater  $\theta$  means a more spatially uniform hydraulic conductivity field, while smaller  $\theta$  means rapid variations in the hydraulic conductivity from point to point in the field.

On the basis of the above equations, a random field generator was used to generate hydraulic conductivity distributions.

#### 4. Description of the application problem and the analysis procedure

The application problem deals with the confined flow under a hydraulic structure having a 15 m floor length (Figure 1). The structure rests on a pervious stratum of 5 m depth. The upstream water head was 1 m, while the downstream head was zero, which gives a 1 m differential head. The modelled zone was 45 m long between two vertical impervious boundaries located 15 m upstream and downstream the floor. The mesh comprised square elements of 0.25 m. This small element size was used to model accurately random fields with a small scale of fluctuation. The mean hydraulic conductivity was held at  $1 \times 10^{-5}$  m/s, the coefficient of variation  $COV = \sigma_k/\mu_k$  ranged from 0.125 to 8 and the scale of fluctuation ranged from 1 to 16 m (Figure 2). There was a 3 m deep cut-off wall driven under the floor at its midpoint for the part of the analysis that includes the cut-off wall.

### 5. Results and discussion

#### 5.1 Case 1: sheet pile is driven under the floor

Figure 3 shows the change of seepage flow as a function of the coefficient of variation for different scales of fluctuation.

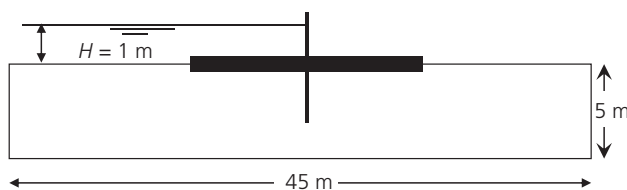


Figure 1. Confined seepage under a hydraulic structure



Figure 2. Distribution of hydraulic conductivity for a typical realisation; each element has its own hydraulic conductivity

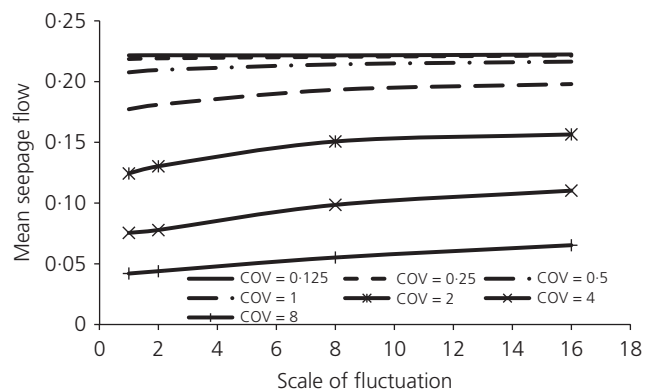


Figure 3. Influence of the scale of fluctuation  $\theta$  on mean seepage flow (case of sheet pile)

The deterministic solution of this problem produced normalised seepage flow  $Q/kH=0.222$ , which agrees well with the analytical solution of Harr (1962).

#### 5.1.1 Mean seepage flow

The mean seepage flow was significantly reduced as the coefficient of variation increased (Figure 3). Likewise, smaller scales of fluctuation reduced the seepage flow. The explanation of this reduction in seepage flow lies in the fact that, for a weakly correlated field having a small scale of fluctuation, the hydraulic conductivity is extremely changeable. Cells that have low hydraulic conductivity behave like blocks in the way of seeping water; and because they are spread all over the domain, the overall seepage flow is reduced. It can be noticed that as  $\theta$  became higher, the seepage flow moved towards the deterministic analysis. This is expected, since for a higher scale of fluctuation the field tends to become uniform; hence the mean seepage flow value of the 2000 realisations moves towards the deterministic value. When  $\theta$  varied from 8 to 16 m, the change in the seepage flow was slight.

#### 5.1.2 Mean uplift pressure

In contrast to the seepage flow, as the scale of fluctuation became larger, the uplift force decreased (Figure 4). This reduction in the uplift force was more pronounced for larger values of the coefficient of variation. This can be explained in a way similar to the seepage flow; for smaller scales of

fluctuations, the hydraulic conductivity is extremely changeable over the field; hence lower hydraulic conductivity cells work as blocks in the way of the flow. As a result, the seeping water accumulates in front of these blocks, which builds up the uplift pressure.

This last point may be clarified with reference to the uplift pressure distribution according to the method of Bligh (Leliavesly, 1965) as shown in Figure 5. The plot is for a case of hydraulic structure with the sheet pile at the end toe of the floor. This particular case was chosen because it will clarify the point easily. In Figure 5, the value of uplift pressure at the end of the downstream toe was  $h = (t + 2d)/C_b$ , where  $h$  is the uplift pressure,  $t$  the floor thickness,  $d$  the sheet pile depth and  $C_b$  is the coefficient of Bligh. When the sheet pile did not exist, the value of the uplift pressure at the same point was  $h = t/C_b$ . The uplift pressure at the start point of the floor was  $H$  for both cases. As a result, the uplift pressure at any point on the floor is greater when the sheet pile existed compared with the case where the sheet pile did not exist. A similar scenario happens when low hydraulic conductivity cells in the field block the way of seeping water; these cells play the role of sheet pile in Figure 5, and this led to a greater uplift force compared with the case without the sheet pile.

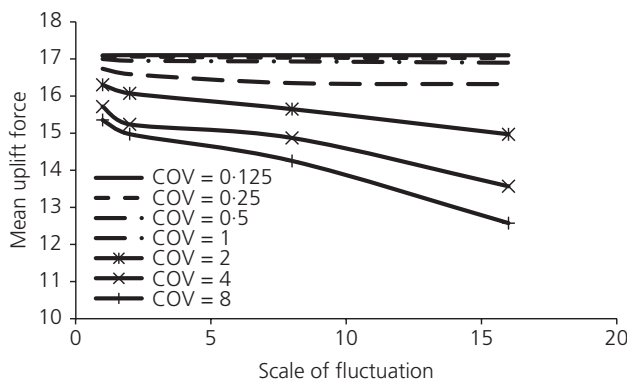


Figure 4. Influence of the scale of fluctuation  $\theta$  on downstream uplift force (case of sheet pile)

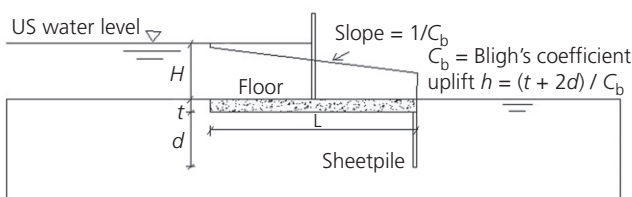


Figure 5. Distribution of the uplift pressure according to the method of Bligh

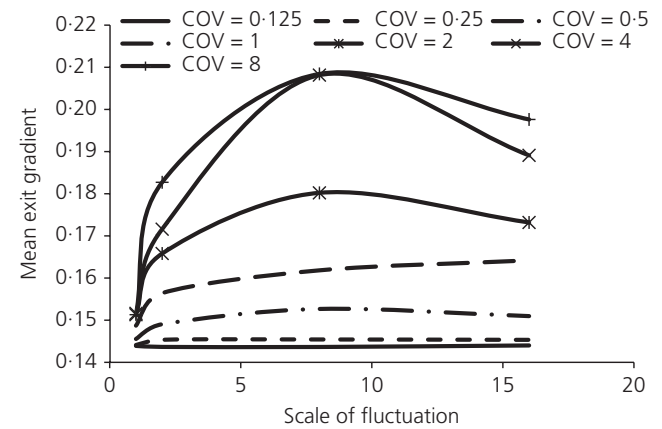


Figure 6. Influence of the scale of fluctuation  $\theta$  on exit hydraulic gradient (case of sheet pile)

### 5.1.3 Mean exit hydraulic gradient

As the scale of fluctuation increased, the exit gradient became greater (Figure 6). However, this happened only for  $\theta \leq 8$  m after which the exit gradient declined. Hence, the maximum exit gradient was attained at  $\theta = 8$  m. Therefore, it appears that there is a value of the scale of fluctuation at which the mean exit hydraulic gradient attains its maximum value.

This result is in agreement with results from other studies (e.g. Ahmed, 2013; Griffiths and Fenton, 1998). Ahmed (2013) observed similar results for anisotropic heterogeneous soil, in which the exit hydraulic gradient attained its maximum value at an anisotropic heterogeneity ratio of 3. However, in the work of Griffiths and Fenton (1998), the exit gradient was the greatest at  $\theta = 2$  m. However, the dimensions of the current problem being investigated were different from that of Griffiths and Fenton (1998). In addition, Griffiths and Fenton (1998) investigated a problem without the floor (it was just a simple sheet pile problem with a penetration depth equal to half of the depth of the pervious stratum). The authors therefore solved their problem for the case when there was no floor and found  $\theta = 8$  m also produced the greatest exit gradient when there was no floor. This means that the discrepancy between the authors' results and the results of Griffiths and Fenton (1998) is mainly due to the difference in the geometry and dimensions of the investigated problems. It is interesting to note that the ratio of the long dimension to the scale of fluctuation that produced the highest exit gradient was  $12.8/2 = 6.4$  in Griffiths and Fenton's (1998) problem, whereas it was  $45.0/8 = 5.6$  in the authors' problem.

The results of the exit gradient, in general, showed that any deviation from the homogeneous medium produced a greater exit gradient (Figure 6). The exit gradient was higher, for any

value of COV and  $\theta$ , than its value obtained from a deterministic solution which was 0.1435.

## 5.2 Case 2: No sheet pile is driven under the floor

### 5.2.1 Mean seepage flow

Results of seepage under hydraulic structure with no sheet pile under the floor (case 2) showed a different behaviour from the case when the sheet pile was enabled (case 1). In case 1, the mean seepage flow increased steadily with the increase of the scale of fluctuation. However in case 2, the value  $\theta=4$  produced the greatest seepage flow under the structure (Figure 7). As in case 1, increasing the coefficient of variation decreased the seepage flow.

The deterministic solution of the problem shows a reduction of the flow rate  $Q/kH$  from 0.26 for case 2 to 0.22 for case 1 – that is, a reduction of about 15%. This means the sheet pile reduced the flow by somewhat 15%. When both the coefficient of variation and the scale of fluctuation equalled 4, the flow rate dropped from 0.12 to 0.08 – that is, a reduction of about 33% due to the sheet pile derivation. This means that the reductions in seepage losses, due to the sheet pile derivation under the floor of the structure, may not be accurate when the soil is regarded as homogeneous.

### 5.2.2 Mean uplift force

The uplift force showed a different behaviour in case 2 compared with case 1. In case 1, the uplift steadily decreased as the scale of fluctuation  $\theta$  became greater. The difference in the uplift force for different values of coefficient of variation was more pronounced at larger values of the coefficient of fluctuation. In case 2, the value  $\theta=2$  showed greater mean uplift force than other values of  $\theta$  (Figure 8). The only exception from this is when  $\theta=16$ , which produced greater mean uplift force.

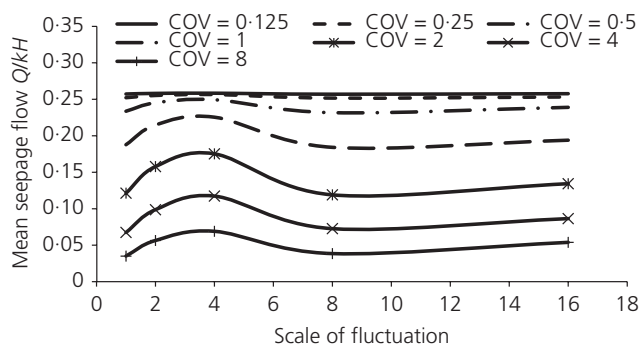


Figure 7. Influence of the scale of fluctuation  $\theta$  on mean seepage flow (no sheet pile)

It appears for case 2 that the mean seepage flow and uplift force reached their maximum values at some particular values of the scale of fluctuation. In the authors' problem, these values were in the range  $\theta=2-4$ .

### 5.2.3 Mean exit hydraulic gradient

Results of the mean exit hydraulic gradient for case 2 (Figure 9) showed a different behaviour compared with those in case 1 (Figure 6). For each coefficient of variation, the value of the scale of fluctuation  $\theta$  that resulted in the maximum mean exit hydraulic gradient was within the range  $\theta=2-4$  m. This is different from case 1, in which the maximum exit hydraulic gradient was attained at the scale of fluctuation  $\theta=8$ .

The above results confirm the need to consider soil variability when designing water-retaining structures, as recommended by Euro code 7. The case when a sheet pile was driven out below

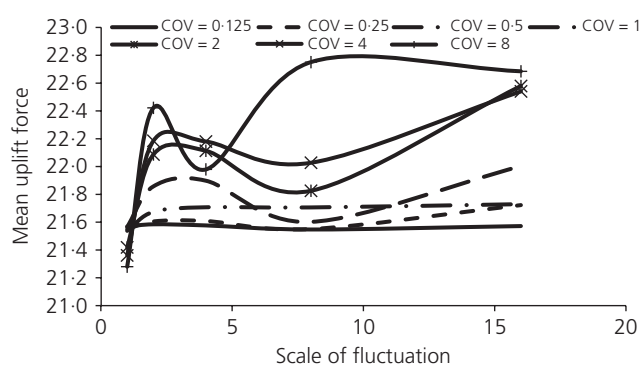


Figure 8. Influence of the scale of fluctuation  $\theta$  on downstream uplift force (no sheet pile)

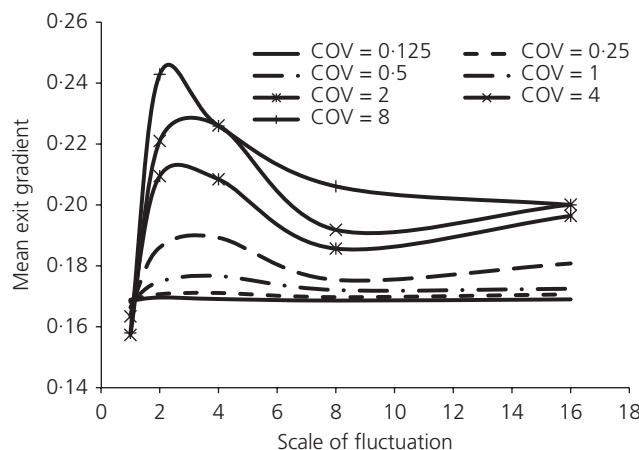


Figure 9. Influence of the scale of fluctuation  $\theta$  on exit hydraulic gradient (no sheet pile)

the structure has produced a different structural response from the case when there was no sheet pile. This happened even for the same problem.

### 5.3 Effectiveness of the cut-off wall

The deterministic exit hydraulic gradient below the structure was reduced by about 15% when a cut-off wall was installed at the middle of the floor. This happened for the case of homogeneous soil formation. However, the stochastic solution of the problem produced different reductions, and was heavily dependent on the scale of fluctuation. For example, when the coefficient of variation equalled 8, different scales of fluctuations showed different reductions in exit hydraulic gradient caused by the sheet pile. The reduction in the hydraulic gradient varied from zero for  $\theta=8$  to 25% for  $\theta=2$ . The value  $\theta=2$  produced the greatest reductions in the exit hydraulic gradient for all values of coefficient of variation. In contrast, the value  $\theta=8$  produced the smallest reduction regardless of the variation coefficient. As expected, smaller coefficients of variation produced nearly the same reduction as in the case of homogeneous soil, which was about 15%.

The reductions in the uplift force due to the cut-off wall were also found to be significantly dependent on the degree of heterogeneity. A homogeneous soil produced about 21% reduction in the uplift force as a result of cut-off wall installation below the floor, as shown by deterministic results. As the soil became heterogeneous, reductions in the uplift force due to the cut-off wall appeared to increase. Increasing the coefficient of variation consistently increased the reductions caused by the cut-off wall on the uplift pressure, and these can reach up to about 45% for  $COV=8$  and  $\theta=16$ . This means that the results obtained from the deterministic solution provide extra factor of safety because it produces lower reductions in the uplift force due to the cut-off wall compared with the real heterogeneous soil. The case  $\theta=16$  produced the greatest uplift reductions for all values of the coefficient of variation.

The seepage losses flowing below the structure for the homogeneous soil was reduced by 15% as a result of the cut-off wall. For heterogeneous soil, the flow rate's greatest reduction happened when  $\theta=2$ . It is the same value of  $\theta$  that resulted in the greatest reduction in the exit hydraulic gradient. Interestingly, values of  $\theta=1, 8$  and  $16$  produced seepage losses less than the homogeneous deterministic solution. This happened for larger coefficient of variations – that is,  $COV \geq 2$ . The value  $\theta=8$  gave the greatest seepage losses. Obviously, for larger values of  $\theta$ , the domain is strongly correlated and this creates preferential paths of high permeability for the water to flow.

The above results demonstrate that site heterogeneity has a great influence on the design parameters of hydraulic

structures such as the uplift force. Ignoring the effect of site heterogeneity at the design stage would result in the use of a high safety factor to account for the uncertainty in the design parameters, which leads to more costs for the structure. For this reason, Euro code 7 has recommended that variability in soil properties to be taken into consideration in the geotechnical design. Probabilistic analysis provides the most appropriate framework to account for this variability in soil properties.

## 6. Summary and conclusions

Inherent variability of soils is inevitable and the representation of this variability is important to have a more realistic understanding of water flow problems. The present study investigated the problem of confined seepage under hydraulic structures using a stochastic approach, which combined the random field theory and the finite-element method. Wide ranges of coefficient of variation as well as the scale of fluctuation were examined. A distinctive feature of the current study is that it enables consideration of high values of the coefficient of variation, which is not the case in other probabilistic methods such as the perturbation method. The perturbation method is only suitable for cases having a small coefficient of variation  $< 20\%$ .

Results of the present study have shown that the seepage flow became lower as the coefficient of variation increased. Likewise, smaller scales of fluctuation have also reduced the seepage flow under the structure. This happened when a cut-off wall was driven under the structure. When there was no cut-off wall, the maximum flow occurred when the scale of fluctuation equalled 4.

A different behaviour was observed in the case of uplift force – that is, as the scale of fluctuation became higher, the uplift force decreased. Increasing the coefficient of variation also lowered the uplift force under the structure. Larger values of the coefficient of variation had more impact on the uplift force than smaller coefficients of variation.

Likewise, the exit hydraulic gradient attained its maximum value at different scales of fluctuation for the case when a cut-off wall existed compared with the case without cut-off. In the first case, the exit gradient was greatest when the scale of fluctuation equalled 8 while in the second case, this corresponds to the range  $\theta=2-4$ .

The effectiveness of the cut-off walls obtained from the deterministic solution was found to be greatly different from the stochastic solution of the problem. The latter can handle the soil heterogeneity that exists in real-world problems. In heterogeneous soil, the effectiveness of the cut-off wall was found to be heavily dependent on the coefficient of variation and the scale of fluctuation, which represent different degrees of heterogeneity. Such heterogeneity cannot be reflected when

deterministic methods are used. This shows the importance of using probabilistic methods when analysing seepage flow problems through dams and under hydraulic structures.

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