Collaborative cognitive-activation strategies as an emancipatory force in promoting girls’ interest in and enjoyment of mathematics: A cross-national case study

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Abstract

The results of international large-scale assessments have revealed the emergence of gender disparities in attitudes to mathematics, with girls generally demonstrating lower levels of interest in and enjoyment of mathematics than boys. Given that attitudes to mathematics are key determinants of future STEM participation, collaborative cognitive-activation teaching strategies, which harmonise with the core tenets of feminist mathematical pedagogy, are proposed as a possible approach to improving girls’ relationships with mathematics. The results of a small-scale cross-national case study that incorporated this approach through a six week intervention are reported. The findings show a significant increase in girls’ enjoyment of mathematics but there was no significant change in boys’ attitudes. Potential implications for mathematics education policy and practice are elucidated.

Highlights

- Girls generally tend to have more negative attitudes to mathematics than boys
- Attitudes to mathematics are important determinants of future STEM participation
- Collaborative cognitive-activation strategies may improve girls’ attitudes

Keywords

Mathematics education; gender equity; enjoyment; collaboration; cognitive-activation
1. Introduction

To maximise the probability of successfully competing in the current global economy, it is important for individual countries to give all of their citizens appropriate opportunities to realise their educational potential. In particular, it is imperative that gender equality is promoted by encouraging both males and females to gain the knowledge, skills and dispositions that will allow them to make contributions to the economic growth of their nations. Internationally, there has been some success in addressing gender disparities in educational outcomes, but there are growing concerns about the under-representation of women in employment areas allied to science, technology, engineering and mathematics (STEM). For example, in relation to countries within the Organisation for Economic Co-operation and Development (OECD), just 14% of first-time female university students in 2012 opted to study STEM disciplines such as computing, engineering, manufacturing or construction, compared to an equivalent figure of 39% for males (OECD, 2015). Given that STEM courses can potentially lead to well-paid and intellectually fulfilling careers, the significant differential in their uptake by women and men is particularly worrying from a gender equality perspective.

Mathematics is often an important prerequisite for the study of STEM courses at tertiary level and for pursuing STEM-related careers. On a triennial cycle, the OECD conducts the Programme for International Assessment (PISA), which is a large-scale study that assesses pupils from OECD nations and other partner countries in mathematics, science and reading. It is alarming that, for the majority of regions participating in PISA 2012 (which had a particular focus on mathematics), girls did worse on average than boys in mathematics. Across OECD countries, the mean mathematics score for boys exceeded that for girls by 11 scale points and the advantage in favour of boys was generally more pronounced for higher achieving pupils (OECD, 2014). For example, in the United Kingdom, the mean PISA 2012 mathematics score for boys was 500 while that for girls was 488, and the difference in the means was statistically significant at the 5% level. In Ireland, there was also a statistically significant difference between boys’ mean mathematics score (509) and that for girls (494) (OECD, 2015). The gender disparity in mathematics achievement is a possible contributory factor to the differential uptake of STEM-related tertiary level courses and careers.

PISA 2012 measured pupils’ intrinsic motivation to learn mathematics according to how they responded (“strongly agree”, “agree”, “disagree” or “strongly disagree”) to a number of statements about their interest in, and enjoyment of, mathematics. For high-achieving pupils,
there was a strong relationship between intrinsic motivation to learn mathematics and mathematics performance, but intrinsic motivation appeared to have little bearing on the performance of the lowest-achieving pupils (OECD, 2014). Given that affective factors such as interest in and enjoyment of mathematics are strongly correlated with performance for higher-achieving pupils in the subject, it is again regretful that, on average, girls demonstrated lower levels of intrinsic motivation than boys in the majority of countries. For example, in the United Kingdom (Ireland), 43.4% (38.8%) of boys but just 38.3% (35.2%) of girls either agreed or strongly agreed with the statement “I do mathematics because I enjoy it”. A similar pattern emerged when pupils were asked to indicate the extent to which they agreed with the statement “I am interested in the things I learn in mathematics”. In the United Kingdom (Ireland), 59.3% (52.1%) of boys and 53.8% (47.1%) of girls either agreed or strongly agreed with the statement (OECD, 2015). Unfortunately these recent figures corroborate the findings of Frenzel et al. (2007) and Else-Quest et al. (2010), who concluded that girls demonstrate considerably less enjoyment of mathematics than boys.

There is evidence in the academic literature to support some form of reciprocal relationship between mathematical achievement and affect but there is debate over the type of relationship, with some researchers suggesting the dominant direction is likely to be from emotional engagement to achievement (e.g. Ma & Kishor, 1997) and others suggesting the opposite (e.g. Hannula et al., 2014). It is probable that the relationship between affect and achievement in mathematics is extremely complicated and that it is mediated by a number of factors pertaining, for example, to pupils’ attitudes to both mathematics and learning (Grootenboer & Marshman, 2016). Although the directionality of the relationship between affective outcomes and achievement in mathematics is contentious, factors such as enjoyment and interest are important determinants of future study in mathematics and participation in careers involving mathematics (Frenzel et al., 2007; Harackiewitz et al., 2000; Wigfield et al., 2002). According to Hannula et al. (2014):

> Attitudes and motivation are important, because they determine how much people choose to study mathematics after it becomes optional and in many countries the society has a shortage of mathematically educated persons in scientific and technical fields. Moreover, the needs of society increasingly emphasize creativity, problem solving, and other higher-level cognitive processes, which are intrinsically intertwined with emotions. (p. 249)
This suggests that, to improve the uptake of mathematics-related courses at university level and mathematics-related careers by female pupils, it will be necessary to increase efforts to promote more positive attitudes to mathematics amongst females.

A number of researchers have identified that, in general, pupils’ attitudes to mathematics become more negative after they transition from primary to secondary school (e.g. Galton et al. 2003; Grootenboer & Marshman, 2016). Some studies have shown that girls tend to enjoy mathematics slightly more than boys at age 11 but that, by age 15, the situation has reversed, with girls indicating lower levels of enjoyment of the subject (Bevan, 2001; Prendergast & O’Donoghue, 2014). This deterioration in pupils’ attitudes to mathematics with increasing age was mirrored in findings reported by the TIMSS (Trends in International Mathematics and Science Study) large-scale international comparative study conducted by the International Association for the Evaluation of Educational Achievement in 2011. On average, for nations that participated in the TIMSS 2011 fourth grade study, 84% of pupils reported that they either liked or somewhat liked learning mathematics, while 16% indicated that they did not like learning mathematics. However, for the countries participating in the eighth grade study, 68% of pupils on average either liked or somewhat liked learning mathematics, and 31% did not like learning mathematics (Mullis et al., 2012). Therefore the transition from primary to secondary mathematics would appear to be a potential time when pupils, and girls in particular, begin to develop more negative dispositions to mathematics. A particular focus on identifying policies and practices for ameliorating the emergence of more negative attitudes to mathematics during this critical phase of children’s education is thus warranted.

Various options have been suggested for narrowing the differential in levels of affective engagement with mathematics exhibited by males and females including, for example, approaches that take cognisance of parental attitudes to mathematics, teacher attitudes, curriculum orientations and pedagogical strategies employed by teachers (OECD, 2015; OECD, 2016; Vale, 2010). In particular, OECD (2015) indicated that cognitive-activation teaching strategies, whereby pupils are encouraged to think more deeply to find solutions to mathematical problems and to concentrate on the methods used to arrive at solutions rather than focusing solely on the answers, are associated with higher achievement of girls in mathematics. Burge et al. (2015) also suggested that frequency of use of cognitive-activation teaching strategies is positively correlated with pupils’ intrinsic motivation to learn mathematics. Cognitive-activation pedagogical approaches offer opportunities for problem-
solving, reflection, guided discovery and collaborative learning, and a brief overview of the relevance of each of these to developing pupils’ mathematical competencies is now provided.

1.1 Problem-solving

Problem-solving is a key aspect of mathematics, but it has been stressed in many national curricula in recent years (Schoenfeld, 2014). According to the NCCA (2005, p.5), by solving mathematical problems pupils “acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom”. An international survey of teaching and learning conducted by the OECD (TALIS) noted that, to promote cognitive activity, teachers should use challenging material that incorporates appropriate opportunities for problem-solving (Burge et al., 2015).

1.2 Reflection

One of the main obstacles when solving mathematical problems is often a lack of reflection. A study conducted by Goldberg & Bush (2003) found that, when pupils are given a problem, they frequently launch into it with a particular strategy, persist with the strategy without “looking back” and finish the solution without re-examining it. This often results in a solution that does not work, and it is thus important to encourage pupils to reflect on the appropriateness of their solutions to mathematical problems.

1.3 Guided discovery

With guided discovery, the mathematics is familiar to teachers but this does not preclude the option of pupils constructing their own mathematical understanding through discovery learning. Such an approach encourages a pupil-centred lesson in which the teacher acts as more of a facilitator of discussion and debate in the classroom.

1.4 Collaborative learning

In keeping with a pupil-centred, cognitive-activation approach, incorporating appropriate opportunities for collaborative group work is one of the dominant approaches to knowledge construction (Damsa & Ludvigsen, 2016). Group work offers a framework in which pupils
can support each other, share ideas and engage in valuable discussion (Backhouse et al., 1992). In line with guided discovery, the principal role of the teacher within group work is to ensure appropriate group composition and management.

During the past decade, feminist mathematical pedagogy, which came to prominence in the 1990s, has experienced some resurgence in the literature (e.g. Spielman, 2008; Zohar, 2006). Given the disparities in achievement and attitudes to mathematics between the genders that are again emerging, we believe that it is timely to re-visit feminist perspectives on mathematical pedagogy. Jacobs (1994/2010) suggested that feminist mathematical pedagogy, which advocates approaches to teaching mathematics where knowledge is contextualised and predicated on personal or shared experiences in a manner that is favourable for females, has the potential to be inclusive since it promotes and supports learning for males and females alike. Against the backdrop of this claim, we report on research underpinning the development of a collaborative learning resource, Izak9, which epitomises the core tenets of feminist mathematical pedagogy and facilitates the use of cognitive-activation teaching strategies. We also summarise the findings of a small-scale cross-national case study to evaluate its potential for promoting social justice by improving girls’ enjoyment of mathematics. Given the importance of attitudes to mathematics as determinants of future STEM participation, and the general deterioration in pupils’ attitudes to mathematics after transition from primary to post-primary education, the evaluation focused on comparing the effect of Izak9 on female and male pupils’ interest in, and enjoyment of, mathematics during the first year of secondary education in Northern Ireland (NI) and the Republic of Ireland (RoI). Implications of our findings for policy and practice in mathematics education are elucidated.

The following section outlines the theoretical framework, which was derived from feminist mathematical pedagogy and Pekrun’s “control-value theory of achievement emotions” (Pekrun, 2000, 2006), that informed the development of the Izak9 resource.

2. Feminist mathematical pedagogy: a route to empowerment and enjoyment

There is an extensive body of international research on gender differences in relation to both mathematical achievement and attitudes to mathematics. The differences are complex and vary over time, but repeated reports of gender biases in favour of male pupils throughout the
20th century led to an upsurge in research on gender equity issues in mathematics during the 1980s and 1990s. Prior to this, gender differences in mathematics had been construed in terms of a deficit model, which viewed male outcomes in mathematics as the norm and urged females to become more like males in respect of their mathematical behaviour (Jacobs, 1994/2010). As Jacobs (1994/2010) noted, such models overlooked “any substantive difference in the ways females and males are” (p. 435). Accordingly, they were challenged by numerous researchers who proposed feminist models that took cognisance of the fact females may learn in different ways from males (Becker, 1995; Belenky et al., 1986/1997; Damarin, 1995). The Izak9 resource that we consider in this article was specifically designed to take cognisance of the model of feminist mathematical pedagogy proposed by Jacobs (1994/2010), which is based largely on the findings of Belenky et al. (1986/1997).

On the basis of detailed interviews with 135 women, Belenky et al. (1986/1997) provided useful insights into the preferred learning styles of females and, importantly, contrasted them with those favoured by males. Drawing on the work of Gilligan (1982), Belenky et al. (1986/1997) posited that females usually tend to be ‘connected knowers’ whereas males have a tendency to be ‘separate knowers’, although they acknowledged the non-exclusivity of their claim. Connected knowing is deemed to entail acquiring knowledge in appropriate contexts that are based on personal or shared experiences, and permit emotional and intellectual attachment to the objects of study. Separate knowing, on the other hand, involves using impersonal procedures that exclude personal feelings or beliefs to obtain absolute truths. Jacobs (1994/2010) made the point that traditional modes of mathematics instruction relied heavily on expository teaching and pupils working individually on problems and, as such, they advantaged separate knowers over connected knowers. To accommodate the potential differences in learning styles between the genders, Jacobs (1994/2010) recommended that mathematics educators should embrace ‘connected teaching’ whereby teachers and pupils would solve problems and discover mathematics collaboratively in a supportive environment that encourages alternative methods of solution, i.e. that collaborative cognitive-activation strategies should be embraced:

In this community of learners, an instructor can design learning activities that enable students to use their experiences, either “real” world or classroom based, to enable them to learn … These should actively involve the learners, causing them to engage in inquiry and reflect on their work … Alternate methods of solutions would be encouraged, where finding another way to solve a problem would be more valued than
solving a similar problem in the same way. The emphasis would be on generating hypotheses rather than proving stated theorems. (p. 444)

Peterson & Fennema (1985) found that connecting with other people has a central role in promoting the performance and decision-making skills of females, and Jacobs (1994/2010) used this finding to assert that it is imperative for learning activities to be done collaboratively rather than competitively or individually.

Those who oppose gender difference theories in mathematics education have criticised the association of females with predominantly connected and males with mainly separate learning styles. For example, Mendick (2005) argued that such a dichotomous categorisation of gender traits in mathematics only serves to fix and perpetuate the differences, thus reproducing gender disparities in mathematical outcomes: “By aligning separate-ness with masculinity and connected-ness with femininity, these approaches feed the oppositional binary patterning of our thinking and, in the final analysis, reiterate it” (p. 163). However, we contend that, despite such criticisms, the type of connected, cognitively-active teaching approaches that emanate from the feminist mathematical pedagogy paradigm proposed by Jacobs (1994/2010) are actually very closely aligned with the gender neutral pedagogical strategies associated with sociocultural learning theories in mathematics education.

Pupil-centred approaches to mathematical learning such as collaborative cognitive-activation tasks are often justified with reference to a sociocultural theoretical framework, which is strongly associated with Vygotsky’s claim that the origins of human cognition are inherently social (Sfard, Forman & Kieran, 2001). According to sociocultural theory, mathematical learning necessarily involves communication in social contexts, either between pupils and teachers or pupils and their peers (Lerman, 2014). Vygotsky (1978) believed that all learning occurs on two levels: initially on a social level through interaction with other people, and that the learning is then integrated into the learner’s cognitive architecture:

Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relationships between individuals. (p.57)

Vygotsky did not distinguish between females and males in his theorising and therefore, when connected, cognitively-active teaching strategies are viewed through a sociocultural
theoretical lens, there is no reason to suspect they would necessarily lead to the perpetuation of gender inequalities in mathematics posited by Mendick (2005). On the contrary, if the findings of Belenky et al. (1986/1997) are accepted, the pedagogical strategies espoused by Jacobs (1994/2010) should promote social justice by improving mathematical outcomes for females without necessarily compromising the achievements of males.

According to the theoretical framework proposed by Pekrun (2000, 2006), individual differences in achievement emotions such as interest and enjoyment are causally related to variations in interpretations of particular scenarios. In his so-called “control-value theory of achievement emotions”, Pekrun (2000, 2006) posited that control-related appraisals, such as pupils’ assessments of their own competence or of the impact of their effort in a problem-solving scenario, and value-related appraisals, such as pupils’ assessments of the intrinsic value of a particular subject or achievement outcomes in the subject, are instrumental in determining achievement emotions. For example, higher levels of perceived control and/or value will lead to higher levels of positive achievement emotions such as enjoyment. Conversely, lower levels of perceived control/value in a learning situation would be related to negative achievement outcomes such as anxiety. Pekrun’s theory implies that relationships between control/value assessments and achievement emotions should be isomorphic for males and females since achievement emotions are causally related to control/value assessments for both genders. However, we argue that, when collaborative cognitive-activation teaching strategies such as those aligned with feminist mathematical pedagogy (Jacobs, 1994/2010) are utilised, females are likely to feel more in control of problem-solving situations due to the support of their peers, i.e. exhibit higher levels of perceived control. Accordingly, our theoretical framework posits a causal relationship between use of collaborative cognitive-activation teaching approaches and girls’ achievement emotions in mathematics.

The design of the Izak9 resource that features in the case study incorporates many facets of feminist mathematical pedagogy and the resource facilitates use of collaborative cognitive-activation teaching strategies. For example, Jacobs (1994/2010) recommended that mathematics educators should embrace ‘connected teaching’ whereby teachers and pupils would solve problems and discover mathematics collaboratively in a supportive environment that encourages reflection and alternative methods of solution. This is an accurate description of the pedagogies supported through the use of Izak9, as outlined in section 3.1. The case study described in this article was conducted to test the efficacy of Izak9 and its design
principles for improving girls’ interest in and enjoyment of mathematics. We hypothesise that the collaborative cognitive-activation strategies facilitated by Izak9 should improve girls’ attitudes to mathematics. Therefore the purpose of the study is to answer the following research question:

How does gender influence interest in and enjoyment of mathematics when collaborative cognitive-activation strategies are used in teaching the subject?

3. **Methodology for case study**

A quantitative approach was taken to determine how collaborative cognitive-activation teaching strategies influence male and female pupils’ attitudes towards mathematics. Participants’ interest in and enjoyment of mathematics were measured before and after a six week intervention, in which they were taught the subject using Izak9 for one lesson per week.

3.1 **The Izak9 resource**

Izak9 is a learning tool consisting of a system of cubes that can be used by pupils to solve particular mathematical tasks. The concrete resource consists of 27 individual cubes which combine to form a larger 3×3×3 cube structure, as shown in Figure 1.

**Figure 1: Izak9 cubes**

When separated, this large cube can be organised into three colour-coded sets of nine smaller cubes, and each set of small cubes can be used by a group of pupils to solve mathematical tasks. A key aspect of the Izak9 resource is that the physical cubes are used in tandem with animated e-learning resources (Schlindwein, 2015). Much research has identified the benefits
associated with appropriate use of technology-based resources in supporting mathematical pedagogy (Dick & Hollebrands, 2011; Gadanidis & Geiger, 2010). Through a suite of e-learning materials, pupils are introduced to mathematical tasks by two animated characters, Abacus and Helix. An identical set of virtual cubes is used by Abacus and Helix to facilitate pre-designed tasks and relevant questioning to extend the tasks. The use of this technology, in conjunction with the concrete resource, reflects a philosophical shift in the mathematics classroom where traditional ‘chalk and talk’ methods of teaching the subject are being replaced by innovative, practical methods, many of which are supplemented by a digitalised, interactive approach (Parmar & Rathod, 2014). Mixed ability pupils are provided with opportunities for multi-sensory collaborative learning, using a combination of the virtual and physical resources to solve mathematical problems (Schlindwein, 2015).

With regard to the physical, each face of an individual cube displays different shapes, whole numbers, fractions, percentages or patterns. Pupils work in groups and complete ready-made tasks which are shown to the class via a data projector or using tablet devices. Each of these tasks involves pupils stepping into a virtual world to receive instructions from Abacus and Helix, and then constructing a wall of cubes accordingly. When pupils have arrived at a solution to the task by arranging the cubes appropriately, teams review the work they have done and the task is extended through questioning and discussion, which are facilitated by Abacus and Helix or the classroom teacher. As mentioned previously, the design of Izak9 and the associated tasks, epitomise the core tenets of feminist mathematical pedagogy (Jacobs, 1994/2010) and facilitates the use of cognitive-activation teaching strategies (Burge et al., 2015) such as problem-solving, reflection, guided discovery and collaborative learning, as outlined in section 1. Consideration will now be given to how the design of Izak9 accommodates the use of these strategies.

3.1.1 Problem-solving

In contrast to the repetitive nature of many exercise-filled textbooks (O’Keeffe & O’Donoghue, 2011), the majority of tasks presented by Izak9 allow a range of solutions to a given problem. For example, in one of the tasks pupils are asked to build a 3 x 3 wall using multiples of three (see Figure 2).

Figure 2: 3 x 3 Demo Task
As a follow-on activity, pupils may be invited to calculate the ‘sum of the numbers inside circles’. The typical range of responses to this question exemplify the different thought processes of individual pupils. Some may use a visual approach by exploiting number bonds i.e. “I added 3 and 27, then 9 and 21, finally adding 15, which makes 75” (Schlindwein, 2015, p. 12). Some may take a more traditional approach by adding the numbers in increasing order of magnitude. Others may favour having the numbers called out to assist them with performing the addition. In some cases, pupils may physically remove the relevant cubes from the wall and organise them into an appropriate arrangement to facilitate calculation of the sum, before performing the operation. These approaches only represent a snapshot of the multiplicity of strategies that may be used to solve this problem (Schlindwein, 2015). Such variety highlights that in mathematics, as in life, there are alternative ways of solving problems. This aligns with cognitive-activation pedagogical approaches which require pupils to forge links between new information and previously-learned information (Burge et al., 2015). In turn, this gives pupils a taste “for making knowledge rather than just receiving it” (Neyland, 1995, p. 43).

3.1.2 Reflection

Izak9 promotes reflection on solutions to mathematical problems by urging pupils to make use of an evaluative loop when solving each task. This loop urges pupils to ‘Plan, Do, Review and Make Sense’ when carrying out each task. In line with cognitive-activation, such approaches promote deeper thought by pupils when they are solving problems, and encourage them to give prominence to the methods utilised in arriving at a solution, rather than just focusing on the actual answer (Burge et al., 2015).
3.1.3 Guided discovery

During the construction phase of each Izak9 task, the teacher can engage pupils in discussion about the different approaches they took when completing the task. After this phase, the importance of effective questioning by the teacher to facilitate and guide further learning comes to the fore. For example, in one of the tasks, pupils are instructed to build a wall containing yellow circles as shown in Figure 3.

Figure 3: Odd One Out Task

From here, the teacher can guide the pupils’ learning and could, for example, initiate a discussion about properties of prime numbers. He/she could provide a task that is more open-ended, e.g. “If we use this selection of numbers and the number operations of addition and subtraction, can we generate more prime numbers?” (Schlindwein, 2015, p. 11). It is important for teachers to vary their questioning strategies and also to develop the ability to pivot and modify the lesson depending upon the responses of pupils in their class (Schlindwein, 2015). Pupils will have a sense of ownership of the learning outcomes since they will perceive them as originating from their own work (Schlindwein, 2015).

3.1.4 Collaborative learning

Through Izak9, pupils work together in groups on ready-made tasks. The role of team captain is rotated for different tasks to give each pupil the chance to provide leadership for a task in addition to the chance to take direction from their peers. The development of these interpersonal skills through the use of Izak9 is very important, as pupils are given the
opportunity, with the support of their peers, to openly articulate their thoughts, mathematical processes and problem-solving strategies (Schlindwein, 2015). This support is particularly important in promoting the performance and decision-making skills of females, in learning mathematics (Peterson & Fennema, 1985).

3.2 The study

The Izak9 intervention was administered by student teachers of mathematics enrolled in a university in the RoI and a university in NI. Volunteers were sought from a postgraduate mathematics teacher education programme in each institution to deliver an Izak9 intervention to first year post-primary mathematics classes while on school placement. It was decided to apply the intervention with the assistance of student teachers as opposed to qualified teachers as the researchers felt that this was also an excellent opportunity for these prospective mathematics teachers to gain experience in using an innovative resource in their teaching and experiment with different teaching strategies. Research suggests that many prospective mathematics teachers hold sets of pedagogical beliefs that are more traditional than progressive (Handal, 2003). Such beliefs are often the result of student teachers’ individual educational experiences based on observations of their own teachers at school (Lortie & Clement, 1975). These beliefs are difficult to change and very often conflict with educational innovations (Prendergast et al., 2014). The authors of this study felt that this was an opportunity to challenge any predefined views of teaching by exposing the prospective teachers to alternative teaching strategies that differ from the approaches they themselves may have encountered during their previous education. It was anticipated that this decision would have minimal impact on the success of the intervention since all of the volunteers were given a two hour training session on Izak9 by the developer of the resource. Furthermore, the majority of the tasks were led by Abacus and Helix and the teachers acted more as facilitators.

The intervention itself involved the delivery of prescribed Izak9 sessions for one lesson (approximately 40 minutes) per week over a six week period in each of the 12 schools involved in the study. At the outset and at the end of the intervention period, measurements were made of pupils’ interest in and enjoyment of mathematics.

3.3 Participants
The participants were 253 mixed ability first year post-primary school pupils (151 female and 102 male) from both the RoI and NI. These pupils were generally aged 11-13 years old in both jurisdictions, but age was not systematically controlled for since it was recorded in complete years immediately prior to the commencement of the intervention period. During the past decade, both the RoI and NI have introduced significant reforms to their post-primary mathematics curricula, with the advent of Project Maths in the RoI in 2008 and a revised NI curriculum in 2007. Both curricula promote a pupil-centred approach to mathematical learning, predicated on a sociocultural theoretical framework, which is strongly associated with Vygotsky’s claim that the origins of human cognition are inherently social (Sfard, Forman & Kieran, 2001). Thus many aspects of the reformed curricula of both jurisdictions are in line with the collaborative cognitive-activation strategies promoted by Izak9.

### 3.4 Measure of pupil interest in and enjoyment of mathematics

Aiken’s pre-validated, subject-specific enjoyment scale (Aiken, 1974), which consists of 11 items assessing attitudes to mathematics, was used to obtain a quantitative measure of pupil interest in and enjoyment of mathematics. Many of the items on the Aiken scale can be directly linked to pupil interest and the scale has good reliability (with Cronbach’s alpha of 0.95). Approximately 50% of the scale items consist of statements consistent with a positive attitude, while the remainder are aligned with a negative attitude towards mathematics. Pupils were asked to indicate the extent to which they agreed or disagreed with each statement: 0 = strongly disagree, 1 = disagree, 2 = undecided, 3 = agree, 4 = strongly agree. Scoring was reversed for negatively-worded items. Therefore a higher score indicates a more positive attitude towards mathematics and the maximum attainable score is 44.

### 3.5 Procedure

The intervention involved the delivery of Izak9 sessions for one lesson (approximately 40 minutes) per week over a six-week period in the student teachers main teaching block placement. The work was carried out in line with the research governance regulations of each university, and the study was approved by the research ethics committees of both institutions.

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1 Appendix A
Although the student teachers’ block placements took place at two different periods of the 2015/16 academic year in each university, the exact timeline was followed in each participating school. All student teachers had commenced teaching in their schools four weeks before the start of the intervention. This ensured that there was enough time to pursue informed consent from the principals of participating schools, parents and pupils before the start of the intervention.

As denoted in Figure 4, at the outset and at the end of the intervention period, pupils completed Aiken’s (1974) enjoyment scale, thereby permitting calculation of a measure of any change in pupils’ interest in and enjoyment of mathematics engendered by using collaborative cognitive-activation teaching strategies.

**Figure 4: Timeline of study**

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<tr>
<th>Timeline Event</th>
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<tbody>
<tr>
<td>Two weeks prior to School Placement Block</td>
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<tr>
<td>Student teachers trained in use of Izak9</td>
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<tr>
<td>Weeks 1-4 of School Placement Block</td>
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<tr>
<td>Informed consent sought from all participants</td>
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<tr>
<td>Week 5 of School Placement Block</td>
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<td>Pre-intervention Aiken scale questionnaire</td>
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<td>Weeks 6 - 11 of School Placement Block</td>
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<tr>
<td>Delivery of six lesson Izak9 intervention</td>
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<tr>
<td>Week 12 of School Placement Block</td>
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<tr>
<td>Post-intervention Aiken scale questionnaire</td>
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The intervention itself consisted of six tasks, one per lesson. These tasks were chosen to correspond with any commonalities in the mathematics curriculum for first year post-primary pupils in both jurisdictions. For each task, pupils were split into groups of four or five and each group was assigned a colour, red, green or purple, corresponding to the colours of the cubes in Izak9.

**Task one: 3 by 3 Demo** – This task was described previously when outlining the design and development of Izak9 (see Figure 2). One of the faces of each cube contains a multiple of
three. The task was to find the multiple of three on every cube and build a wall displaying the multiples of three from smallest to largest. The task was then extended through a set of ready-made questions.

**Task two**: ABC followed by Eliminator – One of the faces of each cube contains a numeral between one and nine. The task was to arrange the numbers one to nine in alphabetical order. Once the wall of numbers from one to nine was built, pupils answered a series of questions called “Eliminator”. Pupils eliminated one number at a time by removing it from the wall until they had only four numbers left. They then used the remaining four numbers to answer the final part of the question. The questions were a general mix of mental mathematics topics.

**Task three**: Stairs – For this task pupils were required to use the faces of the cubes with single digit numbers to build three columns, one with two cubes, one with three cubes and one with four cubes. The task was to build these columns so that the sum of the numbers in each column was the same.

**Task four**: Target Countdown – For this task pupils were shown an arrangement of numbers for three seconds only. They then had just one minute to build a wall identical to the selection of numbers they had just seen. After this preliminary activity, pupils worked collaboratively on associated multistep questions that used the wall as a stimulus, e.g. find the product of the numbers in the orange squares divided by the sum of the numbers in the other squares.

**Task five**: The Great Wall – For this task pupils were required to come together and work as one large team with all 27 cubes. On one face of each of the cubes, there is either a fraction, a percentage or a visual representation of a fraction. The task was to build a giant wall using all 27 cubes, nine cubes long and three cubes high. The percentages were arranged in order along the bottom layer, with the pictorial representations sitting on top of each equivalent percentage, and finally the equivalent fractions in the corresponding position on the upper layer.

**Task six**: Further Eliminator – For this task pupils were required to design eliminator rounds of their own and each group posed their round for the other groups to solve.

To ensure the reliability of the research, each class group participating in the study completed the same Izak9 tasks in the same order over the six week study.
3.6 Data analysis

Summary statistics, in the form of means and standard deviations, for the pre- and post-intervention enjoyment scores derived from the Aiken scale were calculated for males and females separately, and for the whole sample. An independent samples t-test was used to test for a difference between males’ and females’ pre-intervention Aiken enjoyment scores. A two-way repeated measures analysis of variance was conducted on the enjoyment scores, with time (pre- or post-intervention) as the within-subjects variable and gender (female or male) as the between-subjects factor, to determine if the Izak9 intervention had a significant effect on enjoyment overall and to ascertain if any effect was gender-dependent. Follow-up paired-samples t-tests were used to calculate Cohen’s d effect sizes for the whole sample and for both genders. Given the relatively small sample size used in the case study, the influence of jurisdiction was investigated by depicting pre- and post-intervention mean enjoyment scores for both genders on separate plots for each country. All calculations were performed using IBM SPSS Statistics version 22.

4. Results of case study

Table 1 shows pre- and post-intervention means and standard deviations of enjoyment scores for the total sample of pupils and for each gender. An independent-samples t-test revealed that the two genders did not differ significantly from each other on the pre-intervention enjoyment scores: t(251) = .923, p = .357 (two-tailed).

<table>
<thead>
<tr>
<th></th>
<th>Overall (n = 253)</th>
<th>Female (n = 151)</th>
<th>Male (n = 102)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
</tr>
<tr>
<td>Mean</td>
<td>28.5</td>
<td>29.9</td>
<td>28.0</td>
</tr>
<tr>
<td>SD</td>
<td>9.1</td>
<td>8.5</td>
<td>9.2</td>
</tr>
</tbody>
</table>

The two-way repeated measures analysis of variance confirmed that Izak9 had a significant effect on pupils’ pre- and post-intervention enjoyment scores: Wilks’ Lambda = .967, F(1,251) = 8.581, p = .004, partial $\eta^2 = .033$. The time x gender interaction was also
significant: Wilks’ Lambda = .954, F(1,251) = 12.111, p = .001, partial $\eta^2 = .046$, indicating that gender had a significant effect on the change in enjoyment scores from pre- to post-intervention.

Table 2 contains the results of the follow-up paired-samples t-tests and the calculated Cohen’s d effect sizes for the whole sample and for both genders. Cohen’s d is considered to be small if $d \leq .3$, medium if $d \approx .5$ and large if $d \geq .8$ (Cohen, 1988).

**Table 2: Results of paired-samples t-tests (two-tailed) and effect sizes for total sample and by gender**

<table>
<thead>
<tr>
<th></th>
<th>t-statistic</th>
<th>r</th>
<th>p</th>
<th>Cohen’s d effect size†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>t(252) = 3.594</td>
<td>.749</td>
<td>&lt; .001</td>
<td>0.16</td>
</tr>
<tr>
<td>Female</td>
<td>t(150) = 4.783</td>
<td>.726</td>
<td>&lt; .001</td>
<td>0.29</td>
</tr>
<tr>
<td>Male</td>
<td>t(101) = -.391</td>
<td>.810</td>
<td>.697</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

† Effect sizes were calculated using the formula suggested by Dunlap et al. (1996, p. 171) for paired-samples t-tests, i.e. $d = t \sqrt{\frac{2(1-r)}{n}}$ where $t$ = value of t-statistic, $r$ = correlation coefficient between pre- and post-intervention scores, and $n$ = number of cases.

It is noteworthy that, overall, there was a significant but very small positive effect on enjoyment levels, with scores increasing from M = 28.5, SD = 9.1 to M = 29.9, SD = 8.5. However, girls demonstrated a significant, small positive effect (M = 28.0, SD = 9.2 to M = 30.5, SD = 8.0), while the boys actually showed a non-significant, very small negative effect (M = 29.1, SD = 9.0 to M = 28.9, SD = 9.1).

Graphs depicting pre- and post-intervention mean enjoyment scores for both genders in the RoI and NI are shown in Figures 5 and 6 respectively. In both jurisdictions, girls’ enjoyment scores increased from pre- to post-intervention (M = 28.7, SD = 9.8 to M = 31.3, SD = 8.7 in the RoI; M = 27.0, SD = 8.2 to M = 29.4, SD = 6.7 in NI). However, while boys’ scores increased marginally from M = 28.9, SD = 9.4 to M = 29.0, SD = 9.6 in the RoI, boys’ scores in NI actually decreased very slightly from M = 29.3, SD = 8.6 to M = 28.8, SD = 8.8 in NI.
5. Discussion

The aim of the research described in the current article was to test the impact of a pedagogical approach which, according to the underpinning theoretical framework, would
improve girls’ interest in and enjoyment of mathematics, particularly during the early stages of post-primary education. Based on research that was conducted in the aftermath of PISA 2012, and the core tenets of feminist mathematical pedagogy, which came to prominence during the 1990s, a resource was designed that facilitated the use of collaborative cognitive-activation strategies in teaching mathematics. Our hypothesis was that the use of such strategies would improve girls’ attitudes to mathematics by promoting their interest in and enjoyment of the subject. Our findings from a small-scale cross-national case study lend some support to this hypothesis. There was a significant, small increase in girls’ enjoyment of mathematics over the six week intervention period. On the other hand, there was an extremely small, but not statistically significant, decrease in boys’ enjoyment of the subject, despite the fact there was no significant difference between girls’ and boys’ pre-intervention enjoyment scores. The increase in girls’ enjoyment of mathematics over the course of the study was similar in both the RoI and NI. However, boys’ enjoyment increased marginally in the RoI but actually decreased marginally in NI, although the change was not significant in either jurisdiction.

Our findings in the case study would appear to corroborate Jacobs’ contention that girls are more positively disposed to pedagogical approaches that incorporate what she termed ‘connected teaching’ (Jacobs, 1994/2010). Jacobs (1994/2010) posited that, for girls to flourish in mathematics, it is important for them to “generate their own knowledge and connect with the knowledge of other students” (p. 443), and in so doing they should be allowed to share their learning in small groups. However, it is important to stress that the approach adopted in our research went beyond the mere gimmick of asking pupils to work collaboratively in small groups to solve mathematical problems. We placed a particular focus on the use of strategies that required pupils to collaboratively consider alternative methods of solving mathematical problems and to delineate their relative merits. Furthermore, pupils were required to use the Izak9 resource to actually formulate problems for their peers to work on, and multiple methods of solution were also encouraged for the pupil-generated problems. Thus the pedagogical approach entailed the use of what we term ‘collaborative cognitive-activation’ teaching and learning strategies since pupils were being explicitly taught approaches such as reflecting, summarising, questioning and conjecturing that would empower them when solving mathematical problems, but within the supportive framework of peer groups (where risk-taking was promoted). Our findings suggest that this type of pedagogy seems to enhance girls’ control-related appraisals of problem-solving scenarios.
which, according to Pekrun (2000, 2006), tends to precipitate more positive achievement emotions such as enjoyment of learning.

The results obtained in this small-scale case study resonate with the findings reported in the large body of empirical research that indicates how collaborative learning experiences positively impact upon girls’ affective responses to mathematics. For example, Boaler (1997a; b; c), Barnes (2000) and Anderson (2005) all confirm the benefits of collaboration in promoting girls’ enjoyment of mathematics. The affective advantages of cognitive-activation approaches to mathematics instruction have also been documented (OECD, 2016) and, in line with our findings, it therefore seems plausible that the concept of collaborative cognitive-activation holds promise for improving girls’ attitudes to mathematics.

A potential criticism of collaborative cognitive-activation strategies may be that, based on the results of the small-scale study outlined in this article, such approaches are inequitable because they do not appear to improve boys’ interest in and enjoyment of mathematics. This finding may be contingent on the fact that, by nature, boys tend to be ‘separate knowers’ and, as such, they derive less benefit from the type of connected teaching implicated in collaborative cognitive-activation. Consequently, they may not experience the enhanced feelings of control and empowerment that seem to be implicit for girls. At first sight, this may cast doubt on Jacobs’ assertion that “using feminist pedagogy should benefit not only female students but also other students and society at large and in no way denies the power or beauty of mathematics” (Jacobs, 1994/2010, p. 445). Conversely, we suggest that Jacobs is justified in making such a claim since, if used judiciously and in appropriate contexts, the collaborative cognitive-activation strategies that epitomise fundamental aspects of feminist mathematical pedagogy may have an important role to play in promoting social justice by reducing gender disparities in attitudes to mathematics.

Whilst the research reported in this article appears to augur well for reducing gender inequalities in mathematics, it is important to be cognisant of its limitations. The case study did not involve a control group to mitigate against the possibility of drawing erroneous conclusions if the observed effects were actually due to factors other than the use of collaborative cognitive-activation teaching strategies. However, it is important to note that the findings were consistent in both jurisdictions, which engenders a degree of confidence in the conclusions drawn. The sample size involved in the case study was small, and this may adversely affect the generalisability of our findings. Furthermore, for practical reasons, the intervention was delivered over a very restrictive time frame of six weeks and, to safeguard
against the observed positive effects for girls being attributable to situational interest (Mitchell, 1993) rather than a genuine improvement in their intrinsic motivation to learn mathematics, it would have been preferable to use a longer intervention period, and to incorporate a delayed post-intervention measurement of pupils’ enjoyment of mathematics. A further objection may be that there has been no attempt to systematically control for the effect of pupils’ age and, consequently, the observed effects may not generalise to other age groups. Neither was there an attempt to systematically study how the composition of pupil groups used in the collaborative activities influenced the conclusions. To address these issues, a much larger scale study would be required. However, our results would suggest that further research into the potential benefits of collaborative cognitive-activation teaching strategies is warranted and, if this article serves as a catalyst for triggering it, our aim will have been realised.

6. Conclusion

The results of international large-scale assessments such as PISA and TIMSS have shown the emergence of gender disparities in attitudes to mathematics, with girls generally demonstrating lower levels of interest in and enjoyment of mathematics than boys. Given the intimate link between attitudes to mathematics and the uptake of university courses or careers in STEM-related disciplines, and the under-representation of women in these areas, the differential in girls’ and boys’ affective relationships with mathematics is problematic. This article has reported on the research that underpinned the development of a teaching resource that facilitates what we term collaborative cognitive-activation in mathematics lessons. This pedagogical approach engages pupils in collaborative activities that reinforce the key skills involved in solving mathematical problems, such as conjecturing and critically appraising alternative methods of solution.

A small-scale cross-national case study was conducted to determine how gender influences pupils’ interest in and enjoyment of mathematics when collaborative cognitive-activation strategies are used in teaching the subject. The results of this case study suggest that the use of such pedagogical approaches leads to small improvements in girls’ interest in and enjoyment of mathematics, but no significant effect was observed for boys. Notwithstanding the limitations of the research alluded to previously, this implies that the judicious use of collaborative cognitive-activation strategies, in conjunction with other pedagogical
approaches, may have the potential to contribute to promoting gender equity in mathematics and, ultimately, in STEM-related careers. However, effective use of collaborative cognitive-activation approaches requires teachers to devote sufficient time to the activities to realise their benefits. It may therefore be appropriate for curriculum architects to slightly reduce curriculum content, as a precursor to achieving more positive attitudes to mathematics generally. As alluded to in the introduction, a range of other factors, such as parental attitudes, also influence pupils’ relationships with mathematics. However, our initial work suggests that collaborative cognitive-activation strategies seem to be a possible emancipatory force that may contribute to promoting girls’ interest in and enjoyment of mathematics. As a potential form of liberatory pedagogy, they are worthy of further research.

Acknowledgements

This work was supported by a grant from the Standing Conference on Teacher Education, North and South (SCoTENS).
Appendix A: Aiken’s Enjoyment of Mathematics Scale (Aiken, 1974)

Age: _____________________________

Gender: __________________________

Directions: Draw a circle around the option that shows how closely you agree or disagree with each statement: SD (Strongly Disagree), D (Disagree), U (Undecided), A (Agree), SA (Strongly Agree).

<p>| | | | | |</p>
<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>1. I enjoy going beyond the assigned work and trying to solve new problems in mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>2. Mathematics is enjoyable and stimulating to me.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>3. Mathematics makes me feel uneasy and confused.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>4. I am interested and willing to use mathematics outside school and on the job.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>5. I have never liked mathematics, and it is my most dreaded subject.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>6. I have always enjoyed studying mathematics in school.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>7. I would like to develop my mathematical skills and study this subject more.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>8. Mathematics makes me feel uncomfortable and nervous.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>9. I am interested and willing to acquire further knowledge of mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>10. Mathematics is dull and boring because it leaves no room for personal opinion.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>11. Mathematics is very interesting, and I have usually enjoyed classes in the subject.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
</tbody>
</table>
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