Massive MIMO Pilot Retransmission Strategies for Robustification against Jamming

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***Abstract*—This letter proposes anti-jamming strategies based on pilot retransmission for a single user uplink massive MIMO under jamming attack. A jammer is assumed to attack the system both in the training and data transmission phases. We first derive an achievable rate which enables us to analyze the effect of jamming attacks on the system performance. Counter-attack strategies are then proposed to mitigate this effect under two different scenarios: random and deterministic jamming attacks. Numerical results illustrate our analysis and benefit of the proposed schemes.**

I. Introduction

As an emerging candidate for 5G wireless communication networks, massive multiple-input multiple-output (MIMO) [1, 2] has drawn a lot of research interests recently. However, there are only a few works on physical layer security in this area [3–9]. Among the very few, only some of them have studied jamming aspects although jamming exists and has been identified as a critical problem for reliable communications, especially in massive MIMO systems, which are sensitive to pilot contamination [1]. For instance, the authors consider security transmission for a downlink massive MIMO system with presence of attackers capable of jamming and eavesdropping in [6, 7]. The problem of smart jamming is considered for an uplink massive MIMO system in [8], which shows that a smart jammer can cause pilot contamination that substantially degrades the system performance.

Most of the above works have been considered from a jammer point of view: study the jamming strategy, which is the most harmful for the legitimate user or for the eavesdropper. In this work, we motivate our study from the system perspective, in which we develop counter strategies to minimize the effect of jamming attacks. To this end, we first derive an achievable rate of a single user uplink massive MIMO with the presence of a jammer. Then, by exploiting asymptotic properties of massive MIMO systems, we propose two anti-jamming strategies based on pilot retransmission protocols for the cases of random jamming and deterministic jamming attacks. Numerical results show that the proposed anti-jamming strategies can significantly improve the system performance.

II. Problem Setup

We consider a single user massive MIMO uplink with the presence of a jammer as depicted in Fig. 1. Further, we assume that the base station (BS) has M antennas, the legitimate user and the jammer have a single antenna.

Let us denote $\mathbf{g}_\mathrm{u} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}_\mathrm{j} \in \mathbb{C}^{M\times 1}$ as the channel vectors from the user and the jammer to the BS, respectively. We assume that the elements of $\mathbf{g}_\mathrm{u}$ are independent and identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables, i.e., $\mathbf{g}_\mathrm{u} \sim \mathcal{CN}(0, \beta_\mathrm{u}\mathbf{I}_M)$, where β_u represents the large-scale fading (path loss and shadowing). Similarly, we assume that $\mathbf{g}_\mathrm{j} \sim \mathcal{CN}(0, \beta_\mathrm{j}\mathbf{I}_M)$ and is independent of $\mathbf{g}_\mathrm{u}$.

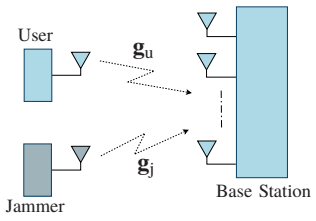

Fig. 1. Massive MIMO uplink with jamming attack.

We consider a block-fading model, in which the channel remains constant during a coherence block of T symbols, and varies independently from one coherence block to the next. To communicate with the BS, the legitimate user follows a two-phase TDD transmission protocol: i) Phase 1: the user sends pilot sequences to the BS for channel estimation, and ii) Phase 2: the user transmits the payload data to the BS. We assume that the jammer attacks the uplink transmission both in the training and in the data payload transmission phases.

A. Training phase

During the first τ channel uses ($\tau < T$), the user sends to the BS a pilot sequence $\sqrt{\tau p_t}\mathbf{s}_\mathrm{u}$, where p_t is the transmit pilot power and $\mathbf{s}_\mathrm{u} \in \mathbb{C}^{\tau\times 1}$ originates from a pilot codebook $\mathcal{S}$ containing τ orthogonal unit-power vectors. At the same time, the jammer sends $\sqrt{\tau q_t}\mathbf{s}_\mathrm{j}$ to interfere the channel estimation, where $\mathbf{s}_\mathrm{j} \in \mathbb{C}^{\tau\times 1}$ satisfies $\mathbb{E}\{\|\mathbf{s}_\mathrm{j}\|^2\} = 1$ and q_t is the transmit power of the jammer during the training phase. Accordingly, the received signals at the BS is given by

$$\mathbf{Y}_t = \sqrt{\tau p_t}\mathbf{g}_\mathrm{u}\mathbf{s}_\mathrm{u}^T + \sqrt{\tau q_t}\mathbf{g}_\mathrm{j}\mathbf{s}_\mathrm{j}^T + \mathbf{N}_t, \tag{1}$$

where $\mathbf{N}_t \in \mathbb{C}^{M\times\tau}$ is the additive noise matrix with unit power i.i.d. ZMCSCG elements.

The BS then performs a de-spreading operation as:

$$\mathbf{y}_t = \mathbf{Y}_t\mathbf{s}_\mathrm{u}^* = \sqrt{\tau p_t}\mathbf{g}_\mathrm{u} + \sqrt{\tau q_t}\mathbf{g}_\mathrm{j}\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^* + \tilde{\mathbf{n}}_t, \tag{2}$$

where $\tilde{\mathbf{n}}_t \triangleq \mathbf{N}_t\mathbf{s}_\mathrm{u}^*$ and $\tilde{\mathbf{n}}_t \sim \mathcal{CN}(0, \mathbf{I}_M)$. The minimum mean squared error (MMSE) estimate of $\mathbf{g}_\mathrm{u}$ given $\mathbf{y}_t$ is [10]

$$\hat{\mathbf{g}}_\mathrm{u} = c_\mathrm{u}\mathbf{y}_t, \tag{3}$$

where $c_\mathrm{u} = \frac{\sqrt{\tau p_t}\beta_\mathrm{u}}{\tau p_t\beta_\mathrm{u} + \tau q_t\beta_\mathrm{j}|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2 + 1}$.

The MMSE estimator (3) requires that the BS has to know β_u, β_j, and $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|$. Since β_u and β_j are large-scale fading coefficients which change very slowly with time (some 40 times slower than the small-scale fading coefficients), they can be estimated at the BS easily [11]. The quantity $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|$

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includes the jamming sequence $\mathbf{s}_\mathrm{j}$ which is unknown at the BS. However, by exploiting asymptotic properties of the massive MIMO, the BS can estimate $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|$ from the received pilot signal $\mathbf{Y}_t$. We will discuss about this in detail in Section IV-A.

Let $\mathbf{e_u} = \mathbf{g}_\mathrm{u} - \hat{\mathbf{g}}_\mathrm{u}$ be the channel estimation error. From the properties of MMSE estimation, $\hat{\mathbf{g}}_\mathrm{u}$ and $\mathbf{e_u}$ are independent. Furthermore, we have $\hat{\mathbf{g}}_\mathrm{u} \sim \mathcal{CN}(0, \gamma_\mathrm{u}\mathbf{I}_M)$ and $\mathbf{e_u} \sim \mathcal{CN}(0, (\beta_\mathrm{u} - \gamma_\mathrm{u})\mathbf{I}_M)$, where

$$\gamma_\mathrm{u} \triangleq \frac{\tau p_t \beta_\mathrm{u}^2}{\tau p_t \beta_\mathrm{u} + \tau q_t \beta_\mathrm{j} |\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2 + 1} = c_\mathrm{u}\sqrt{\tau p_t}\beta_\mathrm{u}. \tag{4}$$

B. Data transmission phase

During the last $(T - \tau)$ channel uses, the user transmits the payload data to the BS and the jammer continues to interfere with its jamming signal. Let x_u ($\mathbb{E}\{\|x_\mathrm{u}\|^2\} = 1$) and x_j ($\mathbb{E}\{\|x_\mathrm{j}\|^2\} = 1$) be the transmitted signals from the user and the jammer, respectively. The BS receives

$$\mathbf{y}_d = \sqrt{p_d}\mathbf{g}_\mathrm{u}x_\mathrm{u} + \sqrt{q_d}\mathbf{g}_\mathrm{j}x_\mathrm{j} + \mathbf{n}_d, \tag{5}$$

where p_d and q_d are the transmit powers from the user and jammer in the data transmission phase, respectively. The noise vector $\mathbf{n}_d$ is assumed to have i.i.d. $\mathcal{CN}(0, 1)$ elements.

To estimate x_u, the BS performs the maximal ratio combining based on the estimated channel $\hat{\mathbf{g}}_\mathrm{u}$ as follow

$$y = \hat{\mathbf{g}}_\mathrm{u}^H\mathbf{y}_d = \sqrt{p_d}\hat{\mathbf{g}}_\mathrm{u}^H\mathbf{g}_\mathrm{u}x_\mathrm{u} + \sqrt{q_d}\hat{\mathbf{g}}_\mathrm{u}^H\mathbf{g}_\mathrm{j}x_\mathrm{j} + \hat{\mathbf{g}}_\mathrm{u}^H\mathbf{n}_d. \tag{6}$$

III. Achievable Rate and Impact of Jamming Attack

In order to analyze the impact of jamming attack on the system, we derive a capacity lower bound (achievable rate) for the massive MIMO channel, described as in (6). Substituting $\mathbf{g}_\mathrm{u} = \hat{\mathbf{g}}_\mathrm{u} + \mathbf{e}_\mathrm{u}$ into (6), we have

$$y = \sqrt{p_d}\|\hat{\mathbf{g}}_\mathrm{u}\|^2 x_\mathrm{u} + \sqrt{p_d}\hat{\mathbf{g}}_\mathrm{u}^H\mathbf{e}_\mathrm{u}x_\mathrm{u} + \sqrt{q_d}\hat{\mathbf{g}}_\mathrm{u}^H\mathbf{g}_\mathrm{j}x_\mathrm{j} + \hat{\mathbf{g}}_\mathrm{u}^H\mathbf{n}_d. \tag{7}$$

Since y consists of the signals associated with the channel uncertainty and jamming, we derive an achievable rate using the method suggested in [12]. To this end, we decompose the received signal in (7) as

$$\begin{aligned} y = {} & \sqrt{p_d}\mathbb{E}\{\|\hat{\mathbf{g}}_\mathrm{u}\|^2\}x_\mathrm{u} + \\ & \underbrace{\sqrt{p_d}(\|\hat{\mathbf{g}}_\mathrm{u}\|^2 - \mathbb{E}\{\|\hat{\mathbf{g}}_\mathrm{u}\|^2\}) + \hat{\mathbf{g}}_\mathrm{u}^H\mathbf{e}_\mathrm{u})x_\mathrm{u} + \sqrt{q_d}\hat{\mathbf{g}}_\mathrm{u}^H\mathbf{g}_\mathrm{j}x_\mathrm{j} + \hat{\mathbf{g}}_\mathrm{u}^H\mathbf{n}_d}_{\triangleq\, n_\mathrm{eff}\ -\ \text{effective noise}}. \end{aligned} \tag{8}$$

Since n_eff and the desired signal are uncorrelated, we can obtain an achievable rate by treating n_eff as the worst-case Gaussian noise, which can be characterized as follows.

Proposition 1. *An achievable rate of the massive MIMO channel with jamming is*

$$R = (1 - \tau/T)\log_2(1 + \rho), \tag{9}$$

where ρ is the effective SINR, given by

$$\rho = \frac{M p_d \gamma_\mathrm{u}}{p_d\beta_\mathrm{u} + q_d\beta_\mathrm{j} + M\frac{q_d q_t}{p_t}\left(\frac{\beta_\mathrm{j}}{\beta_\mathrm{u}}\right)^2 |\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2\gamma_\mathrm{u} + 1}. \tag{10}$$

Proof. See Appendix A. □

Two interesting remarks can be made:

(i) If the jammer does not attack during the training phase ($q_t = 0$) or $\mathbf{s}_\mathrm{j}$ and $\mathbf{s}_\mathrm{u}$ are orthogonal ($\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^* = 0$), the achievable rate becomes

$$R = \left(1 - \frac{\tau}{T}\right)\log_2\left(1 + \frac{M p_d \gamma_\mathrm{u}}{p_d\beta_\mathrm{u} + q_d\beta_\mathrm{j} + 1}\right) \overset{M\to\infty}{\to} \infty. \tag{11}$$

The achievable rate increases without bound as $M \to \infty$, even when the jammer attacks during the data transmission phase.

(ii) If the jammer attacks during the training phase and $\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^* \neq 0$, we have

$$R \overset{M\to\infty}{\to} \left(1 - \frac{\tau}{T}\right)\log_2\left(\frac{p_t p_d}{q_t q_d}\frac{\beta_\mathrm{u}^2}{\beta_\mathrm{j}^2}\frac{1}{|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2}\right). \tag{12}$$

This implies that when the training phase is attacked, the achievable rate is rapidly saturated even when $M \to \infty$. This is the effect of *jamming-pilot contamination*.

IV. Pilot Retransmission Scheme

As discussed in Section III, the jamming attack during the training phase highly affects the system performance. Therefore, we focus on the training phase and construct counter strategies to mitigate the effect of jamming-pilot contamination. We propose pilot retransmission schemes where the pilot will be retransmitted when the jamming-pilot contamination is high ($|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|$ is large). Note that, some overheads for synchronization are necessary for the pilot retransmission protocols. However, those overheads are negligible compared to the payload data.

A. Mathematical Preliminaries

We show that by exploiting asymptotic properties of the massive MIMO system, the BS can estimate $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|$ and $\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T$ from the received pilot signals $\mathbf{y}_t$ and $\mathbf{Y}_t$ even $\mathbf{s}_\mathrm{j}$ is unknown.

1) Estimation of $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2$: By the law of large numbers,

$$\begin{aligned} \frac{1}{M}\|\mathbf{y}_t\|^2 = {} & \tau p_t \frac{\|\mathbf{g}_\mathrm{u}\|^2}{M} + \tau q_t |\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2\frac{\|\mathbf{g}_\mathrm{j}\|^2}{M} + \frac{\|\tilde{\mathbf{n}}_t\|^2}{M} \\ & + \sqrt{\tau p_t}\frac{\mathbf{g}_\mathrm{u}^H\left(\sqrt{\tau q_t}\mathbf{g}_\mathrm{j}\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^* + \tilde{\mathbf{n}}_t\right)}{M} \\ & + \sqrt{\tau q_t}\mathbf{s}_\mathrm{u}^T\mathbf{s}_\mathrm{j}^*\frac{\mathbf{g}_\mathrm{j}^H\left(\sqrt{\tau p_t}\mathbf{g}_\mathrm{u} + \tilde{\mathbf{n}}_t\right)}{M} \\ & + \frac{\tilde{\mathbf{n}}_t^H\left(\sqrt{\tau p_t}\mathbf{g}_\mathrm{u} + \sqrt{\tau q_t}\mathbf{g}_\mathrm{j}\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*\right)}{M} \\ \overset{a.s.}{\to} {} & \tau p_t\beta_\mathrm{u} + \tau q_t|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2\beta_\mathrm{j} + 1, \text{ as } M \to \infty, \end{aligned} \tag{13}$$

where $\overset{a.s.}{\to}$ denotes almost sure convergence. From (13), and under the assumption that the BS knows β_u and β_j, $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2$ can be estimated as

$$\widehat{|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2} = \frac{1}{\tau q_t M\beta_\mathrm{j}}\|\mathbf{y}_t\|^2 - \frac{p_t\beta_\mathrm{u}}{q_t\beta_\mathrm{j}} - \frac{1}{\tau q_t\beta_\mathrm{j}}. \tag{14}$$

2) Estimation of $\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T$: From (1) and again from the law of large numbers, as $M \to \infty$, we have

$$\frac{1}{M}\mathbf{Y}_t^H\mathbf{Y}_t \overset{a.s.}{\to} \tau p_t\beta_\mathrm{u}\mathbf{s}_\mathrm{u}^*\mathbf{s}_\mathrm{u}^T + \tau q_t\beta_\mathrm{j}\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T + \mathbf{I}_\tau. \tag{15}$$

Thus, the BS can estimate $\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T$ as

$$\widehat{\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T} = \frac{1}{\tau q_t\beta_\mathrm{j}M}\mathbf{Y}_t^H\mathbf{Y}_t - \frac{p_t\beta_\mathrm{u}}{q_t\beta_\mathrm{j}}\mathbf{s}_\mathrm{u}^*\mathbf{s}_\mathrm{u}^T - \frac{1}{\tau q_t\beta_\mathrm{j}}\mathbf{I}_\tau. \tag{16}$$

Based on the estimates of $|\mathbf{s}_\mathrm{j}^T\mathbf{s}_\mathrm{u}^*|^2$ and $\mathbf{s}_\mathrm{j}^*\mathbf{s}_\mathrm{j}^T$, in next sections, we propose two pilot retransmission schemes to deal with two common jamming cases: random and deterministic jamming.

B. Pilot Retransmission under Random Jamming

In practice, if the jammer does not have the prior knowledge of the pilot sequences used by the user, then it will send a random sequence to attack the system. During the training phase, the user sends a pilot sequence $\mathbf{s}_\mathrm{u} \in \mathcal{S}$, while the jammer sends a random jamming sequence. The BS estimates $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$ and requests the user to retransmit a new pilot sequence until $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$ is smaller than a threshold ε or the number of transmissions exceeds the maximum number $N_{\max}$. The pilot retransmission algorithm is summarized as follows:

Algorithm 1 (Under random jamming)**.**

1. *Initialization: set $N = 1$, choose the values of pilot length τ, threshold ε, and $N_{\max}$ ($N_{\max}\tau < T$).*
2. *User sends a random $\tau \times 1$ pilot sequence $\mathbf{s}_\mathrm{u} \in \mathcal{S}$.*
3. *The BS estimates $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$ using* (14)*. If $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2 \le \varepsilon$ or $N = N_{\max} \rightarrow$ Stop. Otherwise, go to step 4.*
4. *Set $N = N + 1$, go to step 2.*

Let $\mathbf{s}_\mathrm{u}(n)$ and $\mathbf{s}_\mathrm{j}(n)$ be the pilot and jamming sequences respectively, corresponding to the nth retransmission, $n = 1, \ldots, N$. Similar to (9), the achievable rate of the massive MIMO with anti-jamming for random jamming is given by

$$R_\mathrm{rj} = \left(1 - \frac{N\tau}{T}\right) \log_2\left(1 + \frac{M p_d \gamma_\mathrm{u}}{p_d \beta_\mathrm{u} + q_d \beta_\mathrm{j} + \alpha_\mathrm{rj} + 1}\right), \quad (17)$$

where $\alpha_\mathrm{rj} = M \frac{q_d q_t}{p_t} \frac{\beta_\mathrm{j}^2}{\beta_\mathrm{u}^2} \min_n |\mathbf{s}_\mathrm{j}(n)^T \mathbf{s}_\mathrm{u}(n)^*|^2 \gamma_\mathrm{u}$. Note that in order to realize the achievable rate R_rj in (17), the BS has to buffer the received pilot signal then processes with the best one (with minimal $|\mathbf{s}_\mathrm{j}(n)^T \mathbf{s}_\mathrm{u}(n)^*|^2$) after N pilot retransmissions. There exists case where $|\mathbf{s}_\mathrm{j}(1)^T \mathbf{s}_\mathrm{u}(1)^*|^2$ is minimum which degrades the system performance since it consumes more training resource without finding a better candidate. However, the pilot is only retransmitted when the first transmission is bad, and hence, there will be a high probability that the retransmission is better than the first one.

C. Pilot Retransmission under Deterministic Jamming

Next, we assume that the jamming sequences are deterministic during the training phase, i.e., $\mathbf{s}_\mathrm{j}(1) = \cdots = \mathbf{s}_\mathrm{j}(N)$. Such scenario can happen, for instance, in case the jammer has the prior knowledge of the pilot length and pilot sequence codebook and tries to attack using a deterministic function of those training sequences [7]. In this case, the massive MIMO system can outsmart the jammer by adapting the training sequences based on the knowledge on the current pilot transmission instead of just randomly retransmitting them as in the previous case.

We observe that $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$ can be decomposed as

$$|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2 = \mathbf{s}_\mathrm{u}^T \mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*. \quad (18)$$

So, if the BS knows $\mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T$, it can choose $\mathbf{s}_\mathrm{u}$ to minimize $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|$. In Section IV-A2, we know that the BS can estimate $\mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T$ from $\mathbf{Y}_t$. From this observation, we propose the following pilot retransmission scheme:

Algorithm 2 (Under deterministic jamming)**.**

1. *Initialization: choose the values of pilot length τ and threshold ε.*
2. *User sends a $\tau \times 1$ pilot sequence $\mathbf{s}_\mathrm{u} \in \mathcal{S}$.*
3. *The BS estimates $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$ using* (14)*. If $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2 \le \varepsilon \rightarrow$ Stop. Otherwise, go to step 4.*
4. *The BS estimates $\mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T$ using* (16)*. Then, the BS finds $\mathbf{s}_\mathrm{u}^{\mathrm{opt}}$ so that $\mathbf{s}_\mathrm{u}^{\mathrm{opt}T} \mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^{\mathrm{opt}*}$ is minimal. If $\mathbf{s}_\mathrm{u}^{\mathrm{opt}T} \mathbf{s}_\mathrm{j}^* \mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^{\mathrm{opt}*} < |\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2$, then the user will retransmit this new pilot.*

Since the BS requests the user to retransmit its pilot only if $|\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|$ of the first transmission exceeds the threshold ε, the achievable rate is

$$R_\mathrm{dj} = \left(1 - \frac{N\tau}{T}\right) \log_2\left(1 + \frac{M p_d \gamma_\mathrm{u}}{p_d \beta_\mathrm{u} + q_d \beta_\mathrm{j} + \alpha_\mathrm{dj} + 1}\right), \quad (19)$$

where $\begin{cases} \alpha_\mathrm{dj} = M \frac{q_d q_t}{p_t} \left(\frac{\beta_\mathrm{j}}{\beta_\mathrm{u}}\right)^2 |\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*|^2 \gamma_\mathrm{u}, N = 1, \text{ if } |\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^*| \le \varepsilon \\ \alpha_\mathrm{dj} = M \frac{q_d q_t}{p_t} \left(\frac{\beta_\mathrm{j}}{\beta_\mathrm{u}}\right)^2 |\mathbf{s}_\mathrm{j}^T \mathbf{s}_\mathrm{u}^{\mathrm{opt}*}|^2 \gamma_\mathrm{u}, N = 2, \text{ otherwise.} \end{cases}$

The maximal number of retransmissions for this case is one.

V. Numerical Results

In this section, we numerically evaluate the performance of the proposed anti-jamming schemes in term of the average achievable rate. The average is taken over 50000 realizations of $(\mathbf{s}_\mathrm{u}, \mathbf{s}_\mathrm{j})$. We assume that the transmit powers at the user and jammer satisfy $\tau p_t + (T-\tau) p_d \le TP$ and $\tau q_t + (T-\tau) q_d \le TQ$. We also assume $T = 200$ channel uses, maximum number of transmissions $N_{\max} = 2$.

Fig. 2 illustrates the average achievable rates for different anti-jamming schemes according to the training payload (τ/T). It shows that in order to achieve the best performances, the training payloads should be selected properly to balance the channel estimation quality (τ is large enough) and the resource allocated for data transmission (τ is not too large). As expected, the proposed schemes with pilot retransmission outperform the conventional scheme (without pilot retransmission). When the training sequence is very long, i.e., τ/T is large, the proposed schemes are close to the conventional one since the probability of pilot retransmission is very small as the channel estimation quality is often good enough after the first training transmission. Note that in this simulation, we choose $\varepsilon = 0.1$ which is not optimal in general. It is expected that the benefits of the our proposed schemes are even larger with optimal ε.

Figure 3 shows the average achievable rates versus the number of BS antennas. Without anti-jamming strategy, the pilot contamination can severely harm the system performance and obstruct the scaling of achievable rate with M. This is consistent with our analysis in Section III. The performance can be remarkably improved by using the pilot retransmission protocols. Particularly, for the case of deterministic jamming, the proposed scheme can overcome the pilot contamination bottleneck and allows the achievable rates scale with M even when M is large.

VI. Conclusion

The problem of anti-jamming for a single-user uplink massive MIMO has been considered. It showed that jamming attacks could severely degrade the system performance. By exploiting the asymptotic properties of large antenna array,

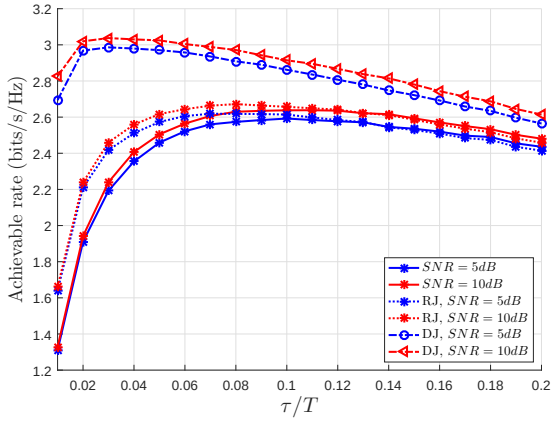

Fig. 2. Average achievable rates of different anti-jamming schemes for $\varepsilon = 0.1$, $q_t = p_t = q_d = p_d = SNR$, $M = 50$. The solid curves, dotted curves (with label "RJ"), and dashed curves (with label "DJ") denote the achievable rates without pilot retransmission (c.f. Proposition 1), with counter strategy for random jamming (c.f. Alg. 1), and with counter strategy for deterministic jamming (c.f. Alg. 2), respectively.

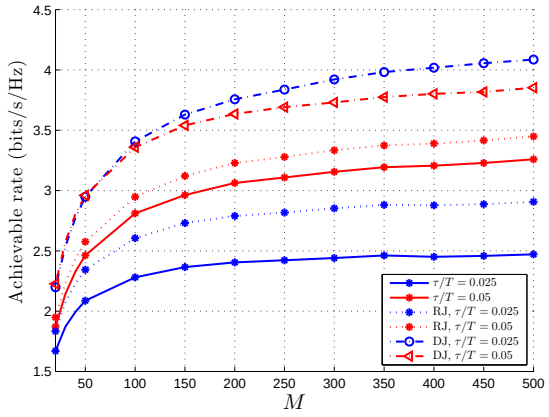

Fig. 3. Average achievable rates according to the number of antennas M for $\varepsilon = 0.1$, $p_t = q_t = p_d = q_d = SNR = 5dB$. The legend for curves is similar to Fig. 2.

we proposed two pilot retransmission protocols. With our proposed schemes, the pilot sequences and training payload could flexibly be adjusted to reduce the effect of jamming attack and improve the system performance.

Future work may study multi-user networks. For instance, our results can be readily extended if a max-min fairness criterion is used. Then the pilot retransmission protocol design is considering the worst user who has the smallest achievable rate.

Appendix A
Proof of Proposition 1

By treating n_{eff} as Gaussian additive noise, an achievable rate of the channel in (8) is given by

$$R = \left(1 - \frac{\tau}{T}\right)\log_2\left(1 + \frac{p_d|\mathbb{E}\{\|\hat{\mathbf{g}}_{\text{u}}\|^2\}|^2}{\mathbb{E}\{|n_{\text{eff}}|^2\}}\right). \tag{20}$$

Let us define

$$\rho \triangleq \frac{p_d|\mathbb{E}\{\|\hat{\mathbf{g}}_{\text{u}}\|^2\}|^2}{\mathbb{E}\{|n_{\text{eff}}|^2\}} \triangleq \frac{p_d M^2\gamma_{\text{u}}^2}{E_1 + E_2 + E_3}, \tag{21}$$

where $E_1 \triangleq p_d\mathbb{E}\left\{\left|\|\hat{\mathbf{g}}_{\text{u}}\|^2 - \mathbb{E}\left\{\|\hat{\mathbf{g}}_{\text{u}}\|^2\right\} + \hat{\mathbf{g}}_{\text{u}}^H\mathbf{e}_{\text{u}}\right|^2\right\}$, $E_2 \triangleq q_d\mathbb{E}\left\{\left|\hat{\mathbf{g}}_{\text{u}}^H\mathbf{g}_{\text{j}}\right|^2\right\}$, and $E_3 \triangleq \mathbb{E}\left\{|\hat{\mathbf{g}}_{\text{u}}^H\mathbf{n}_d|^2\right\}$. Since $\hat{\mathbf{g}}_{\text{u}}$ and $\mathbf{e}_{\text{u}}$ are independent zero mean random vectors, we have

$$\begin{aligned} E_1 &= p_d\mathbb{E}\{\|\hat{\mathbf{g}}_{\text{u}}\|^4\} - p_d(\mathbb{E}\{\|\hat{\mathbf{g}}_{\text{u}}\|^2\})^2 + q_d\mathbb{E}\{|\hat{\mathbf{g}}_{\text{u}}^H\mathbf{e}_{\text{u}}|^2\} \\ &= p_d M(M+1)\gamma_{\text{u}}^2 - p_d M^2\gamma_{\text{u}}^2 + p_d M\gamma_{\text{u}}(\beta_{\text{u}} - \gamma_{\text{u}}) \\ &= M\gamma_{\text{u}}p_d\beta_{\text{u}}. \end{aligned} \tag{22}$$

From (3), and using the fact that $\mathbf{g}_{\text{u}}, \mathbf{g}_{\text{j}}, \tilde{\mathbf{n}}_t$ are independent and zero mean random vectors, we have

$$\begin{aligned} E_2 &= q_d c_{\text{u}}^2\mathbb{E}\left\{\left|\sqrt{\tau p_t}\mathbf{g}_{\text{u}}^H\mathbf{g}_{\text{j}} + \sqrt{\tau q_t}\|\mathbf{g}_{\text{j}}\|^2\mathbf{s}_{\text{u}}^T\mathbf{s}_{\text{j}}^* + \tilde{\mathbf{n}}_t^H\mathbf{g}_{\text{j}}\right|^2\right\} \\ &\overset{(a)}{=} q_d c_{\text{u}}^2\left(\tau p_t\mathbb{E}\left\{\left|\mathbf{g}_{\text{u}}^H\mathbf{g}_{\text{j}}\right|^2\right\} + \tau q_t|\mathbf{s}_{\text{u}}^T\mathbf{s}_{\text{j}}^*|^2\mathbb{E}\{\|\mathbf{g}_{\text{j}}\|^4\} + \mathbb{E}\left\{\left|\tilde{\mathbf{n}}_t^H\mathbf{g}_{\text{j}}\right|^2\right\}\right) \\ &= q_d c_{\text{u}}^2(\tau p_t M\beta_{\text{u}}\beta_{\text{j}} + \tau q_t M(M+1)\beta_{\text{j}}^2|\mathbf{s}_{\text{u}}^T\mathbf{s}_{\text{j}}^*|^2 + M\beta_{\text{j}}). \end{aligned}$$

Then by using (4),

$$E_2 = Mq_d\gamma_u\left(\beta_{\text{j}} + M\gamma_u\frac{q_t}{p_t}\frac{\beta_{\text{j}}^2}{\beta_{\text{u}}^2}|\mathbf{s}_{\text{j}}^T\mathbf{s}_{\text{u}}^*|^2\right). \tag{23}$$

Similarly,

$$E_3 = \mathbb{E}\{|\hat{\mathbf{g}}_{\text{u}}^H\mathbf{n}_d|^2\} = M\gamma_{\text{u}}. \tag{24}$$

Substituting (22), (23), and (24) into (21) we obtain (9).

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