Multi-Way Massive MIMO with Maximum-Ratio Processing and Imperfect CSI


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Multi-Way Massive MIMO with Maximum-Ratio Processing and Imperfect CSI

Chung Duc Ho∗, Hien Quoc Ngo∗†, Michail Matthaiou∗, and Trung Q. Duong∗

∗Institute of Electronics, Communications and Information Technology (EICT), Queens University Belfast, Belfast, BT3 9DT, U.K.
†Department of Electrical Engineering (ISY), Linköping University, 581 83 Linköping, Sweden
Email:{chduc01, m.matthaiou, trung.q.duong}@qub.ac.uk, hien.ngo@liu.se

Abstract—This paper considers a multi-way massive multiple-input multiple-output amplify-and-forward relaying system, where single-antenna users exchange their information-bearing signals with the assistance of one relay station equipped with unconventionally many antennas. The relay first estimates the channels to all users through the pilot signals transmitted from them. Then, the relay uses maximum-ratio processing (i.e. maximum-ratio combining in the multiple-access pilot signals transmitted from them). Then, the relay uses maximum-ratio processing in the broadcast phase) to process the signals. A rigorous closed-form expression for the spectral efficiency is derived. We show that by deploying massive antenna arrays at the relay and simple maximum-ratio processing, we can serve many users in the same time-frequency resource, while maintaining a given quality-of-service for each user.

Index Terms—Channel state information, massive MIMO, multi-way relay networks.

I. INTRODUCTION

In multi-way relay networks, many users simultaneously exchange their bearing information among them via the help of a single sharing relay at the same time-frequency resource [1]. Multi-way relay networks provide spatial diversity, and hence, they can scale up the spectral efficiency of the system without increasing the system complexity. In [2], the authors showed that the spectral efficiency of multi-way relay networks is much higher than that of one-way or two-way networks. Hence, these systems have been considered for diverse applications, such as wireless conference and power control in heterogeneous cellular networks, to name a few

Massive multiple-input multiple-output (MIMO), where a base station equipped with hundreds of antennas serves many active users in the same time-frequency resource, is considered as one of the key candidates for next-generation wireless systems [3], [4]. In [3], the authors showed that massive MIMO can substantially reduce the effects of noise, small-scale fading and inter-user interference by using simple linear processing, including maximum-ratio (MR) or/and zero-forcing (ZF) techniques. In particular, the transmit power of each user can be made inversely proportional to the number of antennas at the base station. Furthermore, the performance of the system can be scaled up noticeably without increasing the system complexity.

Multi-way massive MIMO relay networks, which combine massive MIMO and multi-way relaying technologies, have received research interest recently [5]. This is because they can leverage the benefits of both massive MIMO and multi-way relaying. Therefore, they are considered as a strong candidate to offer a noticeable improvement of spectral and energy efficiency [6], [7]. However, these works consider ZF processing at the relay which involves a complicated matrix inversion. Moreover, in massive MIMO, channel acquisition is a critical problem, and hence, the issue of imperfect channel state information should be taken into account.

Inspired by the above discussion, in this paper we consider a multi-way massive MIMO with MR processing and imperfect channel state information (CSI) under the amplify-and-forward (AF) protocol. We note that when the number of antennas is large, ZF processing scheme is much more complicated than MR processing technique [8]. Most importantly, MR processing can be performed in a distributed fashion without large backhaul requirements. A complete transmission protocol under time-division duplex (TDD) operation is proposed. We derive a corresponding expression for the spectral efficiency in closed-form. Based on this closed-form expression, the effect of the number of relay antennas, imperfect channel estimation, and the number of users is analyzed.

Notations: The superscripts (·)T, (·)∗, and (·)H denote the transpose, conjugate, and Hermitian, respectively. The symbol ∥·∥ indicates the norm of a vector. The notations E{·} and Var{·} are the expectation and the variance operators, respectively; [X]mn or xmn denotes the (m, n)-th entry of matrix X, and I_K is the K×K identity matrix. Moreover, [X]k or x_k denotes the k-th column of matrix X.

II. SYSTEM MODEL

We consider a multi-way relaying massive MIMO system which consists of one relay equipped with M antennas, and K single-antenna users (M >> K). The K users exchange information with each other with the help of the relay by sharing the same time-frequency resource. The k-th user wants to decode all K−1 signals transmitted from other users. We make the assumption that all nodes operate in the half-duplex mode and the direct links (user-to-user links) do not exist due to the large obstacles and/or severe shadowing. Denote by G the M × K channel matrix between the relay and the K users. In addition, G models independent small-scale fading (Rayleigh fading) and large-scale fading (geometric attenuation and log-normal shadow fading). Also, we have that the channel coefficient between the m-th antenna of the relay and the k-th user is defined as

\[ g_{mk} = h_{mk} \sqrt{\beta_k}, \]

where \( h_{mk} \sim CN(0, 1) \) represents the small-scale fading, and \( \beta_k \) represents the large-scale fading. In matrix form,

\[ G = HD^{1/2}, \]

where H is an M × K matrix, \( [H]_{mk} = h_{mk} \) and D is a K × K diagonal matrix, where \( [D]_{kk} = \beta_k \).

The transmission leverages TDD operation, and is divided into three phases: i) channel estimation; ii) multiple-access (MA); and iii) broadcast (BC) phases.

A. Channel Estimation Phase

The relay node needs to know the channel for performing digital signal processing. To do this, a part of coherence interval is used for channel estimation. For each coherence interval of length T symbols, all users simultaneously transmit pilot sequences of length T symbols.
to the relay. Let $\phi_k \in \mathbb{C}^{* \times 1}$ be the pilot sequence sent from the $k$-th user. We assume that $\phi_1, \phi_2, \ldots, \phi_K$ are unit norm vectors and pairwise orthogonal, i.e., $\phi_k^H \phi_{k'} = 0$ for $k \neq k'$. This requires that $\tau \geq K$.

The $M \times \tau$ received pilot matrix at relay is given by

$$Y_p = \sum_{k=1}^{K} \sqrt{\tau P_p} \phi_k H + N_p = \sqrt{\tau P_p} G \Phi^H + N_p,$$  \hspace{1cm} (3)

where $\Phi \triangleq [\phi_1, \phi_2, \ldots, \phi_K]$, $P_p$ is the transmit power of each pilot symbol, $g_k$ is the $k$-th column of $G$, and $N_p$ is the additive white Gaussian noise (AWGN) matrix with i.i.d. $CN(0, 1)$ components.

At the relay, we apply the minimum mean-square-error (MMSE) technique to estimate the channel matrix $G$ [10]. The MMSE channel estimate of $G$ is

$$\hat{\Phi} = \frac{1}{\sqrt{\tau P_p}} Y_p \Phi = \left(G + \frac{1}{\sqrt{\tau P_p}} \tilde{N}_p\right) \hat{\Phi},$$  \hspace{1cm} (4)

where $\hat{\Phi} \triangleq \left(D^{-1} + \tilde{I}_K\right)^{-1}$ and $\tilde{N}_p \triangleq N_p \Phi$. From the property of $\Phi$, the elements of $\tilde{N}_p$ are i.i.d. $CN(0, 1)$ random variables (RVs). Let $E$ be the estimation error matrix. Then,

$$G = \hat{G} + E.$$  \hspace{1cm} (5)

From the property of MMSE estimation, $\hat{G}$ and $E$ are independent. We have $G \sim CN(0, D)$ and $E \sim CN(0, D_E)$, where $D$ and $D_E$ are diagonal matrices with

$$[D]_{kk} = \sigma_k^2 = \frac{\tau P_p \beta_k^2}{\tau P_p \beta_k + 1}, \quad \text{and} \quad [D_E]_{kk} = \sigma_k^2, \ k = 1, \ldots, K.$$  \hspace{1cm} (6)

B. Multiple-Access Phase

In this phase, data is transmitted to the relay in the same time-frequency resource from all users. The $M \times 1$ received vector at the relay is

$$y_R = \sqrt{P_u} G x + n,$$  \hspace{1cm} (7)

where $x = [x_1, \ldots, x_K]^T$, with $\sum_i \{x_i\} = 1$, is the $K \times 1$ signal vector transmitted from the $K$ users, $n$ is an $M \times 1$ AWGN vector with i.i.d. $CN(0, 1)$ components, and $P_u$ is the transmit power of each user. Then, the relay uses the channel estimate in the channel estimation phase and employs the MR combining scheme as:

$$\hat{y}_R = G^H y_R.$$  \hspace{1cm} (8)

C. Broadcast Phase

In this phase, the relay spends $K - 1$ time-slots to transmit all signals to $K$ users. The relay employs the MR scheme to broadcast a permuted version of $y_R$ at each time-slot [6]. The transmit signal vector at the relay for the $t$-th ($t = 1, 2, \ldots, K - 1$) time-slot can be expressed as

$$s_R^{(t)} = \sqrt{\alpha(t)} \hat{G} \Pi^{(t)} y_R = \sqrt{P \alpha(t)} A^{(t)} x + \sqrt{\alpha(t)} B^{(t)} n,$$  \hspace{1cm} (9)

where $\Pi^{(t)} \in \mathbb{C}^{K \times K}$ is the permutation matrix for time slot $t$ given as [6],

$$\Pi^{(t)} = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 \\
1 & 0 & 0 & \cdots & 0 & 0 
\end{bmatrix}.$$  \hspace{1cm} (10)

$$A^{(t)} = \hat{G} \Pi^{(t)} \hat{G}^H G, \quad B^{(t)} = \hat{G} \Pi^{(t)} \hat{G}^H.$$  \hspace{1cm} (11)

From (7), (8), (9), and (11) the normalization factor $\alpha(t)$ can be expressed as

$$\alpha(t) = \frac{P_u}{P_u \sum_{k=1}^{K} \left\{ ||A^{(t)}||_2^2 \right\} + \sum_{m=1}^{M} \left\{ ||B^{(t)}||_2^2 \right\}}.$$  \hspace{1cm} (12)

Then, the $K \times 1$ received signal vector at the $K$ users in the $t$-th time slot can be described as follows:

$$y_k^{(t)} = G^H \Phi_k n^{(t)} + w^{(t)},$$  \hspace{1cm} (13)

and substituting (9) into (13), we obtain

$$y_k^{(t)} = \sqrt{\alpha(t)} P_u g_k^T A_k^{(t)} x + \sqrt{\alpha(t)} G^T B_k^{(t)} n + w^{(t)}.$$  \hspace{1cm} (14)

III. SPECTRAL EFFICIENCY ANALYSIS

In this section, we analyse the spectral efficiency of the system. More specifically, we derive a closed-form expression for the spectral efficiency. Without loss of generality, we analyze the performance of the system in the first time-slot. The performance analysis for other time-slots follows the same methodology. Note that, hereafter, if $k = K$, then we set $k + 1 = 1$ and $k + 2 = 2$; if $k = 1$, then we set $k - 1 = K$; and if $k = 2$, we set $k - 2 = K$.

In the first time-slot, the $k$-th user wants to detect the signal $x_{k+1}$ transmitted from the $(k+1)$-th user. From (14), the received signal in the first time slot for the $k$-th user is described by

$$y_k^{(1)} = \sqrt{\alpha(1)} P_u g_k^T a_k^{(1)} x_k + \sqrt{\alpha(1)} P_u \sum_{i \neq (k+1)} g_i^T a_i^{(1)} x_i + \sqrt{\alpha(1)} \left( \sum_{m=1}^{M} \right) b_m^{(1)} n_m + w_k^{(1)} + \tilde{N}_k^{(1)},$$  \hspace{1cm} (15)

where $\tilde{N}_k^{(1)}$ is considered as the effective noise and given by

$$\tilde{N}_k^{(1)} = \sqrt{\alpha(1)} P_u \left( g_k^T a_k^{(1)} + E \left( g_k^T a_k^{(1)} \right) \right) x_{k+1} + \sqrt{\alpha(1)} \left( \sum_{m=1}^{M} \right) b_m^{(1)} n_m + w_k^{(1)}.$$  \hspace{1cm} (16)

From (16), it can be clearly seen that the “desired signal” term is uncorrelated with the “effective noise” term. Therefore, the signal-to-interference-plus-noise ratio (SINR) for the $k$-th user in the first time-slot can be written as

$$\gamma_k^{(1)} = \frac{\alpha(1) P_u \left( g_k^T a_k^{(1)} \right)^2}{\alpha(1) P_u \text{Var} \left( g_k^T a_k^{(1)} \right) + \text{IUs} + \text{AN}}.$$  \hspace{1cm} (17)
\[ \alpha^{(1)} = \frac{P_i}{M^3 P_u \sum_{k'=1}^{K} \sigma^2_{k',1} \sigma^2_{k',1} + M^2 \left( \sum_{k'=1}^{K} \sigma^2_{k',1} \right) \left( P_u \sum_{k'=1}^{K} \beta_{k'} + 1 \right) + M P_u \sum_{k'=1}^{K} \sigma^2_{k',1} \sigma^2_{k',1}}, \]  

(20)

where

\[ UI_k = \alpha^{(1)} P_u \sum_{\substack{i \neq \pm(k+1) \neq \pm \pm k \neq \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \pm \p
the spectral efficiency of the two-way massive MIMO relaying system is given by

\[
\text{SE}_{\text{two-way}, k}^{(1)} = \left( \frac{T - \tau}{T} \right) \left( \frac{K - 1}{K(K - 1)} \right) \log_2 (1 + \text{SINR}_k)
\]

\[
= \left( \frac{T - \tau}{T} \right) \left( \frac{1}{K} \right) \log_2 (1 + \text{SINR}_k),
\]

where SINR$_k$ corresponds to the SINR in (22) for $K = 2$.

In this example, we choose $D_d = 1000$ m, $\sigma_z = 8$ dB, $\nu = 4$, $d_0 = 200$ m, $P_u = P_t = P_0 = 0$ dB. Figure 2 demonstrates the cumulative distribution of the sum spectral efficiencies for two cases: $(K = 20, M = 200)$ and $(K = 10, M = 50)$. Compared to the two-way massive MIMO system, the multi-way massive MIMO system reduces the pre-log penalty (from $\frac{1}{\pi}$ to $\frac{1}{K}$), however, it exhibits more interference since many users simultaneously transmit data in the same frequency band. When $M$ is large, the interference is small. Therefore, the multi-way massive MIMO system outperforms the two-way massive MIMO system, especially at large $M$.

V. Conclusion

We studied multi-way massive MIMO relay networks with MR processing and imperfect CSI. We derived a closed-form expression for the spectral efficiency. Our work showed that, by using a large antenna array at the relay, many users can simultaneously exchange their information in the same frequency band without any performance degradation for each user. As a result, multi-way massive MIMO offers much higher sum spectral efficiency than conventional multi-way MIMO or two-way massive MIMO systems do. Furthermore, as the number of relay antennas grows large, we can reduce the transmitted power at the relay and/or each user proportionally to $1/M$, while maintaining a given quality-of-service.

VI. Appendices

A. Derivation of (20)

The normalization factor $\alpha^{(1)}$ in (12) can be rewritten as

\[
\alpha^{(1)} = \frac{P_t}{P_k \sum_{k=1}^{K} \mathcal{Q}_{1k} + P_k \sum_{k=1}^{K} \mathcal{Q}_{2k} + \sum_{m=1}^{M} \mathcal{Q}_{3m}},
\]

where

\[
\mathcal{Q}_{1k} = E\left\{ \left\| \left[ \mathbf{G}^\dagger \mathbf{T}_1 \mathbf{G}^H \right]_{k,k} \right\|_2^{2} \right\},
\]

\[
\mathcal{Q}_{2k} = E\left\{ \left\| \left[ \mathbf{G}^\dagger \mathbf{T}_1 \mathbf{G}^H \mathbf{E} \right]_{k} \right\|_2^{2} \right\},
\]

\[
\mathcal{Q}_{3m} = E\left\{ \left\| \left[ \mathbf{G}^\dagger \mathbf{T}_1 \mathbf{G}^H \right]_{m} \right\|_2^{2} \right\}.
\]

First, we compute $\mathcal{Q}_{1k}$. We have,

\[
\mathcal{Q}_{1k} = E\left\{ \left\| \left[ \mathbf{G}^\dagger \mathbf{T}_1 \mathbf{G}^H \right]_{k,k} \right\|_2^{2} \right\}
\]

\[
= E\left\{ \left\| \mathbf{g}_k^H \mathbf{g}_k^* \right\|_2 \left\| \mathbf{g}_k \right\|_2 \left\| \mathbf{g}_k^* \right\|_2 \right\} + E\left\{ \left\| \mathbf{g}_k^* \right\|_2 \left\| \mathbf{g}_k \right\|_2 \right\}
\]

\[
+ \sum_{k' = 1}^{K} \left\{ \left\| \mathbf{g}_{k'}^H \mathbf{g}_k \right\|_2 \left\| \mathbf{g}_{k'} \right\|_2 \right\},
\]

\[
= \sigma_k^2 + E\left\{ \left\| \mathbf{g}_k \right\|_2 \right\} + M \sigma_{k-1}^2 E\left\{ \left\| \mathbf{g}_k \right\|_2 \right\}
\]

\[
+ \sum_{k' \neq (k-1)}^{K} \left\{ \left\| \mathbf{g}_{k'}^H \mathbf{g}_k \right\|_2 \left\| \mathbf{g}_{k'} \right\|_2 \right\}.
\]

1 Alternatively, one could consider a multi-pair two-way relaying protocol as in [11], where $K$ sources exchange data with $K$ destinations over two orthogonal time-slots (i.e., the number of users is always an even number). For this case, the pre-log factor is $1/2$. Compared with our multi-way relaying protocol, the multi-pair two-way relaying protocol has a smaller pre-log factor (when $K > 2$), but suffers a similar interference effect since multiple users transmit their data in the same time-frequency resource.
where in the last equality we have used the fact that $\tilde{g}_{k}^{H}\tilde{g}_{k} \sim \mathcal{CN}(0, \sigma_k^2 I)$ is independent of $g_k$ [3]. By using Lemma 2.9 in [9], (40) becomes

$$q_{kk} = (M + 1)\sigma_k^4 + M^2(\beta_k - \bar{\beta}^2) \sum_{k'\neq k} \sigma_{k'}^2 \sigma_{k'k+1} + M^2 \bar{\beta}_k^2 \sigma_{k+1}^2$$

Similarly, we obtain $q_{2k} = M^3(\beta_k - \bar{\beta}^2) \sum_{k'\neq k} \sigma_{k'}^2 \sigma_{k'k+1}$ and $q_{3m} = M \sum_{k'=1}^{K} \sigma_{k'}^2 \sigma_{k'k+1}$. Substituting $q_{kk}$, $q_{2k}$ and $q_{3m}$ into (12), we obtain (20).

**B. Proof of Theorem 1**

1) Compute $\mathbb{E}\left\{|\tilde{g}_k^T a_k(1)\|^2\right\}$: Since $\tilde{G}$ and $E$ are independent, Then, we obtain

$$\mathbb{E}\left\{|\tilde{g}_k^T a_k(1)\|^2\right\} = \mathbb{E}\left\{|\tilde{g}_k^T \tilde{G}\tilde{\Pi}(1)\tilde{G}^H \tilde{G}_{k+1}\right\}$$

$$= \mathbb{E}\left\{|g_k\|^2 |g_{k+1}\|^2\right\} + \sum_{k'\neq k}^{K} \mathbb{E}\left\{|\tilde{g}_k^T \tilde{G}^H \tilde{g}_{k+1} |g_k\|^2\right\}$$

$$= M^2 \sigma_k^2 \sigma_{k+1}^2$$

(42)

2) Compute $\text{Var}(\tilde{g}_k^T a_k(1))$: From (42), the variance of $\tilde{g}_k^T a_k(1)$ is given by

$$\text{Var}(\tilde{g}_k^T a_k(1)) = \mathbb{E}\left\{|\tilde{g}_k^T a_k(1)\|^2\right\} - \mathbb{E}\left\{|\tilde{g}_k^T a_k(1)\|^2\right\}^2$$

$$= \mathbb{E}\left\{|\tilde{g}_k^T a_k(1)\|^2\right\} - M^2 \sigma_k^4 \sigma_{k+1}^4$$

$$= \mathbb{E}\left\{|(\tilde{g}_k^T + s_k^T)\tilde{G}\tilde{\Pi}(1)\tilde{G}^H (g_{k+1} + e_{k+1})|^2\right\}$$

$$- M^2 \sigma_k^2 \sigma_{k+1}^2$$

(43)

Since $\tilde{G}$ and $E$ are independent, (43) can be rewritten as

$$\text{Var}(\tilde{g}_k^T a_k(1)) = T_1 + T_2 + T_3 + T_4 - M^4 \sigma_k^2 \sigma_{k+1}^2$$

(44)

where

$$T_1 = \mathbb{E}\left\{|\tilde{g}_k^T G\tilde{\Pi}(1)G^H g_{k+1}|^2\right\}$$

$$T_2 = \mathbb{E}\left\{|\tilde{g}_k^T \tilde{G}^H e_{k+1}|^2\right\}$$

$$T_3 = \mathbb{E}\left\{|s_k^T \tilde{G}\tilde{\Pi}(1)\tilde{G}^H g_{k+1}|^2\right\}$$

$$T_4 = \mathbb{E}\left\{|s_k^T \tilde{G}^H e_{k+1}|^2\right\}$$

(45)-(48)

To compute $T_1$, we rewrite (45) as

$$T_1 = \sum_{k'=1}^{K} \mathbb{E}\left\{|\tilde{g}_k^T \tilde{g}_{k'-1}^H g_{k+1}|^2\right\}$$

$$= \mathbb{E}\left\{|g_k|^4\right\} \mathbb{E}\left\{|g_{k+1}|^4\right\} + \mathbb{E}\left\{|\tilde{g}_k^T \tilde{g}_{k+1}^H g_{k+1}|^2\right\}$$

$$+ \mathbb{E}\left\{|\tilde{g}_k^T \tilde{g}_{k+1}^H g_{k+1}|^2\right\} + \sum_{k' \neq k, k'=1}^{K} \mathbb{E}\left\{|\tilde{g}_k^T \tilde{g}_{k+1}^H g_{k+1}|^2\right\}$$

(49)

Again, by using Lemma 2.9 in [9], we get

$$T_1 = M^3(\beta_k - \bar{\beta}^2) \sum_{k'\neq k} \sigma_{k'}^2 \sigma_{k'k+1} + M^2 \bar{\beta}_k^2 \sigma_{k+1}^2$$

(50)

Similarly, we obtain

$$T_2 = M^3(\beta_k - \bar{\beta}^2) \sum_{k'\neq k} \sigma_{k'}^2 \sigma_{k'k+1} + M^2 \bar{\beta}_k^2 \sigma_{k+1}^2$$

(51)

$$T_3 = M^3(\beta_k - \bar{\beta}^2) \sum_{k'\neq k} \sigma_{k'}^2 \sigma_{k'k+1} + M^2 \bar{\beta}_k^2 \sigma_{k+1}^2$$

(52)

$$T_4 = M^2 \sigma_k^2 \sigma_{k+1}^2$$

(53)

By using (6), and substituting (50), (51), (52) and (53) into (43), we arrive at the desired result as in (23).

3) Compute $\Sigma_k$ and $\Lambda_k$:

Following a similar methodological approach as in 1) and 2), we obtain $\Sigma_k$ and $\Lambda_k$ given in (24) and (25), respectively.

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