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Multi-D Simulations of Ultra-Stripped Supernovae to Shock Breakout

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ABSTRACT

The recent discoveries of many double neutron star systems and their detection as LIGO-Virgo merger events call for a detailed understanding of their origin. Explosions of ultra-stripped stars in binary systems have been shown to play a key role in this context and have also generated interest as a potential explanation for rapidly evolving hydrogen-free transients. Here we present the first attempt to model such explosions based on binary evolution calculations that follow the mass transfer to the companion to obtain a consistent core-envelope structure as needed for reliable predictions of the supernova transient. We simulate the explosion in 2D and 3D, and confirm the modest explosion energies \( \sim 10^{50} \) erg and small kick velocities reported earlier in 2D models based on bare carbon-oxygen cores. The spin-up of the neutron star by asymmetric accretion is small in 3D with no indication of spin-kick alignment. Simulations up to shock breakout show the mixing of sizeable amounts of iron group material into the helium envelope. In view of recent ideas for a mixing-length treatment (MLT) of Rayleigh-Taylor instabilities in supernovae, we perform a detailed analysis of the mixing, which reveals evidence for buoyancy-drag balance, but otherwise does not support the MLT approximation. The mixing may have implications for the spectroscopic signatures of ultra-stripped supernovae that need to be investigated in the future. Our stellar evolution calculation also predicts presupernova mass loss due to an off-centre silicon deflagration flash, which suggests that supernovae from extremely stripped cores may show signs of interactions with circumstellar material.

Key words: supernovae: general – binaries: close – stars: massive – stars: evolution – stars: neutron

1 INTRODUCTION

In recent years, there has been considerable progress in our theoretical understanding of both the explosions of massive stars as core-collapse supernovae and the modelling of their progenitors in close binary systems. In light of the increasing number of discoveries of double neutron star (DNS) systems and their mergers, leading to the high-frequency gravitational wave (GW) bursts detected by LIGO/Virgo (GW170817, Abbott et al. 2017a), it is of utmost importance to understand the formation of DNS systems (Tauris et al. 2017). In particular, the second supernova (SN) explosion is a key ingredient to gain further knowledge of the survival rates of such systems and thereby the expected LIGO-Virgo detection rates.

The second SN also determines the kinematics of the surviving DNS systems (i.e., their resulting systemic runaway velocity) which is important for determining the offset distance from their host galaxies by the time the two NSs merge and produce a short gamma-ray burst (Fong & Berger 2013; Abbott et al. 2017c; Blanchard et al. 2017) and a kilonova (Kasliwal & Nissanke 2014; Soares-Santos et al. 2017; Coulter et al. 2017; Abbott et al. 2017b; Smartt et al. 2017; Drout et al. 2017) following the GW signal.

The vast majority (if not all) of massive stars are born in binary systems (Sana et al. 2012) and it is well known that the outcome of stellar evolution in close binaries differs significantly from that of single stars (Langer 2012). Binary interactions affect, for example, the rotation rate, the amount of envelope material, and the final core mass prior to the core collapse (Brown et al. 2001; Podsiadlowski et al. 2004). In extreme scenarios for the second SN forming

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a close-orbit DNS system, it has been demonstrated (Tauris et al., 2013, 2015) that the progenitor stars are ultra-stripped prior to their core collapse; i.e., these stars basically become almost naked metal cores due to mass transfer via so-called Case BB Roche-lobe overflow (RLO) to the first-born NS of the system. Therefore, to make further progress in our understanding of the formation of close-orbit DNS systems and the LIGO-Virgo GW sources, it is timely and necessary to start modelling such ultra-stripped SN explosions in detail.

On the side of transient observations, the advent of high-cadence surveys (e.g., PanSTARRS, Chambers et al. 2016; PTF, Law et al. 2009; iPTF, Masci et al. 2017; SkyMapper, Keller et al. 2007) has increased the interest in studying the diversity among core-collapse SN events (e.g., Drout et al. 2011; Pejcha & Prieto 2015; Terreran et al. 2017). Especially faster and fainter transients become more accessible by observations. In terms of the explosion parameters, this means that the regime of ultra-stripped SNe with small ejecta masses and explosion energies comes into focus thanks to the new observational capabilities. There are already noteworthy cases like SN 2005ek and SN 2010X (Drout et al. 2013; Tauris et al. 2013; Moriya et al. 2017) that have been identified as candidates for ultra-stripped SNe, and more are likely to follow. It is imperative to put the interpretation of such fast and faint transients on a firmer footing using self-consistent explosion models.

In this paper, we therefore investigate the low-mass end of stripped-envelope progenitors of Type Ib/c SNe, i.e., ultra-stripped SNe, by means of multi-dimensional simulations, extending earlier work by Suwa et al. (2015). Different from the 2D study of Suwa et al. (2015), we conduct simulations both in 3D and 2D, thus further extending the growing list of successful 3D explosion models in the field (Takiwaki et al. 2012, 2014; Melson et al. 2015a,b; Lentz et al. 2015; Müller 2015, 2016; Janka et al. 2016; Müller et al. 2017; Summa et al. 2018; Roberts et al. 2016; Chan et al. 2018; Ott et al. 2017; Kuroda et al. 2018). Moreover, we perform calculations from collapse to shock breakout and apply, for the first time in a 3D SN simulation, a progenitor obtained from binary stellar evolution modelling with a helium envelope rather than exploding bare C/O cores.

Although the 2D study of Suwa et al. (2015) established the basic parameters of ultra-stripped explosions from a theoretical point of view, namely a low explosion energy of \( \sim 10^{50} \) erg, a small nickel mass of \( \lesssim 0.01 M_\odot \), and a small kick velocity, a number of questions about this SN channel still remain open, and our approach allows us to address some of these. On a very basic note, 3D modelling is necessary simply for confirming the 2D results of Suwa et al. (2015) and for modelling the possible spin-up of neutron star (NS) due to asymmetric accretion.

Perhaps more importantly, the connection between the theoretical models of Suwa et al. (2015) and observed fast and faint transients still remains rather tenuous. That the theoretically predicted explosion properties of ultra-stripped models roughly fit the light curves of such a type Ic event like SN 2005ek has been demonstrated by Moriya et al. (2017) using 1D models, but there is only rough agreement between the observationally inferred nickel masses of \( 0.01 \sim 0.05 M_\odot \) and explosion energies of a few \( 10^{50} \) erg (Drout et al. 2011; Moriya et al. 2017) and the theoretical predictions, which may partly be due to degeneracies in the light curve fits. An unambiguous identification of ultra-stripped SNe needs to be based on spectroscopy.

This, however, requires an understanding of the mixing by Rayleigh-Taylor instabilities during the propagation of the shock through the envelope. The extent of the mixing is crucial as even the gross spectral type (Ib vs. Ic) is sensitive to the mixing of radioactive nickel into the helium envelope. This is because gamma-ray energy deposition from the nickel affects the non-thermal excitation of helium, as shown by Dessart et al. (2012, 2015) and Hachinger et al. (2012) in detailed non-LTE radiative transfer simulations and by Piro & Morozova (2014) based on analytic estimates. The question of mixing in Type Ib/c SNe is in fact relevant not only for ultra-stripped progenitors, but for the entire class of hydrogen-free progenitors, and has so far been explored only to a very limited extent even in parameterised 2D models (Hachisu et al. 1991, 1994; Kifonidis et al. 2003) — in contrast to the very extensive body of computational studies on mixing in Type II SNe such as SN 1987A (Arnett et al. 1989; Benz & Thielemann 1990; Müller et al. 1991; Fryxell et al. 1991; Hachisu et al. 1992; Kifonidis et al. 2000, 2003; Hammer et al. 2010; Ellinger et al. 2013; Wongwathanarat et al. 2015) and Cas A (Wongwathanarat et al. 2017). By combining self-consistent multi-dimensional explosion models and a realistic envelope structure from binary evolution, we can now start to address the question of mixing in Type Ib/c SNe more reliably and take a first step towards connecting the multi-D explosion simulations to observations by detailed spectral modelling in the future.

Our paper is organised as follows: In Section 2, we describe the binary progenitor model and the numerical methods and setup used for multi-dimensional simulations of the explosion from collapse to shock breakout. We then discuss the results of the SN simulations in Sections 3 and 4. Section 3 focuses on the explosion properties — i.e., the explosion energy, the composition of the inner ejecta, and the PNS mass, spin, and kick. Mixing instabilities in the envelope are addressed in Section 4, where we analyse the growth conditions for Rayleigh-Taylor-driven mixing and describe the final state of mixing at shock breakout. We also investigate to what extent the non-linear phase of the Rayleigh-Taylor instability can be described by effective 1D models in the vein of mixing-length theory, as recently suggested by Duffell (2016) and Paxton et al. (2018). Section 5 discusses our results in the context of DNS system properties. We conclude with a summary and a discussion of the implications of our results in Section 6.

2 INPUT MODELS AND NUMERICAL METHODS

2.1 Progenitor Model

Tauris et al. (2015) calculated a large grid of progenitor models for ultra-stripped SNe by evolving helium stars of metallicity \( Z = 0.02 \) including mass transfer via Case BB\(^1\) RLO to a NS companion. This phase of binary evolution (e.g., Tauris & van den Heuvel 2006) follows after the high-mass X-ray binary stage which evolves into common-envelope evolution where the hydrogen envelope is ejected via in-spiral of the NS. In the subsequent phase of evolution the exposed core (i.e., the helium star) will initiate RLO to its NS companion if the orbit is not too wide.

In this paper, we consider the model with an initial helium star mass of \( 2.8 M_\odot \) and an initial orbital period of 20 d. As a result of a stellar wind, the mass is reduced to \( \sim 2.5 M_\odot \) by the time the helium star initiates mass transfer while undergoing core carbon burning. The subsequent stage of Case BC RLO reduces the helium star

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\(^1\) Strictly speaking: Case BA, Case BB or Case BC depending on whether the RLO is initiated while the helium star undergoes core helium burning, helium shell burning or has evolved to core carbon burning or beyond.
mass to $1.72 M_\odot$ with a remaining helium envelope of $0.217 M_\odot$ (Figure 1).

Up to the stage of early oxygen burning, the evolution is followed using the binary evolution code BEC of Wellstein et al. (2001), which is based on the single-star code of Langer (1998); for details see Section 2 in Tauris et al. (2015). Progenitor rotation is not explicitly considered in the calculation. Due to tidal coupling one expects the progenitor to spin extremely slowly with a spin period of the order of the final orbital period of 19.4 d. In general, the final orbital periods (and hence the pre-collapse spin periods) for ultra-stripped SN progenitor are expected to vary considerably, however, with a broad distribution down to less than 1 h (Tauris et al. 2015).

At oxygen ignition, the binary has detached again, and the final C/O core mass of the helium star is about $\sim 1.47 M_\odot$. Since the nuclear network in BEC is not well-suited for advanced phases well beyond carbon burning, we then map the model into the Kepler code during neon burning, which is ignited in a shell off-centre in this low-mass core. This is similar to what has been done by Heger et al. (2000) where a mapping was done when a central temperature of $10^6$ K was reached. Kepler has been well-developed to properly treat the advanced burning stages. In particular, silicon burning is treated using a quasi-nuclear equilibrium (QNSE) network, and the iron core past silicon burning uses a nuclear statistical equilibrium (NSE) network (Weaver et al. 1978; Heger & Woosley 2010). The remaining time from mapping (off-centre neon ignition) to core collapse is $\sim 37.9$ yr.

The subsequent evolution to collapse is noteworthy. Other recent works on ultra-stripped SNe (Tauris et al. 2013, 2015; Suwa et al. 2015) have already remarked upon the structural similarities of the progenitor cores to single-star electron-capture supernova (ECSN) progenitors Nomoto (1984, 1987); Jones et al. (2013, 2014); Doherty et al. (2017) and low-mass iron-core progenitors Woosley & Heger (2015), which are essentially due to the small C/O core mass and result in similar explosion dynamics. The small C/O core mass also has other interesting consequences because some of the final core and shell burning episodes occur under strongly degenerate conditions (Woosley & Heger 2015), which can lead to off-centre ignition and very violent flash-like burning that triggers presupernova mass ejection. In the progenitor considered here, off-centre neon burning first ignites at about $0.47 M_\odot$ is accompanied by off-centre oxygen burning as the shell progresses further inward. Various smaller off-centre O and Ne shell burning stages occur further our as well during that phase but have little effect in the over-all progress of the inward burning shell. When the shell reaches about 0.09 $M_\odot$ silicon burning ignites violently causing a sound wave that travels to the surface, steepening into a shock as it runs down the density gradient. This leads to the ejection of most of the helium envelope. Only a small residual envelope of 0.02 $M_\odot$ remains. In this work, we cut the ejected matter and evolve the rest of the star further to collapse, which occurs 78 d later. At this stage, much of the ejected material has already reached radii of $\sim 10^{15}$ cm, and expansion velocities are of order $\sim 1000$ km s$^{-1}$.

Due to the presupernova mass ejection, the observable transient may be very strongly affected by interaction and evolve into a Type Ibn supernova at some stage. It could thus appear considerably brighter than the faint transients that have been predicted for ultra-stripped progenitors (Moriya et al. 2017). Based on the properties of the ejected shell, we still expect to see a distinguishable supernova Type Ib/c-like transient before the supernova stars to interact with the circumstellar material from the pre-collapse mass ejection. With maximum ejecta velocities of 16,000 km s$^{-1}$ in the supernova, we expect strong interaction features to emerge no earlier than about 12 d after the explosion, i.e., after the peak of the light curve judging by the results of Moriya et al. (2017) – especially if we consider that the mass of the supernova ejecta will be even lower than in Moriya et al. (2017) so that the transient should evolve more rapidly. For the first phase of the observable transient, one can therefore justifiably disregard the circumstellar material when discussing mixing, light curves and spectra, although the interaction phase will be of great interest for future work.

### 2.2 Simulating the Neutrino-Driven Explosion

We simulate the collapse, the post-bounce accretion phase, and the initial explosion phase using the neutrino hydrodynamics code CoCoNvT-FMT (Müller & Janka 2015). The hydrodynamics module CoCoNvT (Dimmelmeier et al. 2002; Müller et al. 2010) solves the equations of general relativistic hydrodynamics in spherical polar coordinates in an unsplit finite-volume approach using piecewise parabolic reconstruction (Colella & Woodward 1984) and the HLLC Riemann solver (Mignone & Bodo 2005). The metric equations are solved in the extended conformal flatness approximation (Cordero-CarrIÓN et al. 2009), and a spherically symmetric metric is assumed.

As in previous works (Müller 2015), we employ a mesh-coarsening scheme for variable resolution in the longitudinal direction to avoid strong time step constraints near the grid axis, and we model the inner region with density $\geq 10^{15}$ g cm$^{-3}$ in spherical symmetry, using a mixing-length treatment for proto-neutron star (PNS) convection. In the high-density regime, we use the equation of state of Lattimer & Swesty (1991) with a bulk incompressibility modulus of $K = 220$ MeV.

We conduct two 2D runs (s2.8-2D-a and s2.8-2D-b) and one
2.3 Simulation to Shock Breakout

When the explosion energy is reasonably converged and further energy input by neutrino heating becomes negligible, we map models s2.8-3D and s2.8-2D-b into the Newtonian hydrodynamics code PROMETHEUS (Fryxell et al. 1991; Mül1er et al. 1991). PROMETHEUS is a directionally-split implementation of the piecewise-parabolic method of Colella & Woodward (1984). In 3D, we use an over-set grid Yin-Yang grid consisting of two spherical polar coordinate patches (Kageyama & Sato 2004; Wongwathanarat et al. 2010a) as implemented by Melson et al. (2013a). The initial grid resolution on each patch is 1600 × 56 × 148, corresponding to an angular resolution of 1.6°. The initial radial grid is equally spaced in log \( r \) with a resolution of \( \Delta r / r = 6.9 \times 10^{-3} \).

PROMETHEUS allows for a moving radial grid (Mül1er 1994); in principle an arbitrary grid velocity function \( v \) can be specified. In our models, we choose a grid velocity of the form

\[
\dot{r}_i = (\alpha + \beta i) r_i, \tag{1}
\]

for the radial zone \( i \). This allows us to make the expansion of the grid non-homologous so that the inner boundary can “catch up” with the outward-moving ejecta once the central region becomes sufficiently evacuated. This largely eliminates the need to remove interior grid zones to increase the time step (as in Hammer et al. 2010; Wongwathanarat et al. 2013). Due to the form of Equation (1), the grid retains equal spacing in \( \ln r \).

In the PROMETHEUS runs, we use the equation of state of Timmes & Swesty (2000). Self-gravity is accounted for in the Newtonian approximation; for the CoCoNcT model, the monopole approximation is employed.

3 EVOLUTION DURING THE FIRST SECOND

3.1 Explosion Dynamics

The evolution of the diagnostic explosion energy (defined as in Müller et al. 2017) for all models as well as the maximum, minimum and average shock radii and trajectories of selected mass shells of the 3D model are shown in Figure 2. For the 3D model, we also present snapshots of the specific entropy on 2D slices in Figure 3.

As expected based on the pre-collapse density profile, the ultra-stripped progenitor explodes in a manner very similar to the bare C/O core models of Suwa et al. (2015) and the low-mass single-star progenitors just above the iron core formation limit (Müller et al. 2015a; Melson et al. 2016; Radice et al. 2017). The shock moves outward steadily, and shock expansion accelerates once neutrino-driven convection develops about 80 ms after bounce. Around 150 ms, neutrino-heated material first reaches positive net energy, and the explosion energy grows to \( \sim 10^{50} \text{ erg} \) by 300 ms. At this time accretion onto the PNS has ceased, and the neutrino-driven wind has developed and continues to pump some power into the explosion at a modest rate. The final explosion energy \( E_{\text{expl}} \) is \( 1.12 \times 10^{50} \text{ erg} \), which is somewhat smaller than the value of \( 1.77 \times 10^{50} \text{ erg} \) for model CO145 with a similar C/O core mass of 1.45 \( M_\odot \), a difference which is likely to be ascribed to the somewhat different neutrino transport treatment in our model. Our final baryonic NS mass of 1.42 \( M_\odot \) is also somewhat higher, reflecting considerable structural differences of our progenitor compared to the bare CO-core models of Suwa et al. (2015). Using the fit formula for the NS binding energy \( E_{\text{bind}} \) from Lattimer & Prakash (2001),

\[
E_{\text{bind}} = 0.084 M_\odot c^2 (M_{\text{grav}} / M_\odot)^2, \tag{2}
\]

this translates into a gravitational mass of \( M_{\text{grav}} = 1.28 M_\odot \), assuming a final NS radius of 12 km. This value is in agreement with the typical mass measured for the young, second-formed NS in DNS systems (Tauris et al. 2017).

We see a moderate increase of the explosion energy in 3D compared to 2D. This is in line with the small increase in explosion energy in 3D found by Melson et al. (2015a) due to the faster quenching of accretion by 3D turbulence in the explosion of low-mass iron core progenitors with fast shock propagation. The effect is somewhat larger than in Melson et al. (2015a), especially for model s2.8-2D-a, which also shows the more unsteady growth of explosion energy due to partial outflow quenching that is characteristic for more massive progenitors (Müller 2015).

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\[^2\] This is because \( \frac{d}{dt} (\ln r_{i+1} - \ln r_i) = \alpha + \beta (i + 1) - \alpha - \beta i = \beta \) is independent of \( i \).
Figure 3. 2D slices showing the entropy in units of $k_b$/nucleon in model s2.8-3D at post-bounce times of 86 ms (top left), 133 ms (top right), 190 ms (bottom left), and 401 ms (bottom right). The $x$-axis is aligned with the axis of the spherical polar grid. As the explosion develops and the shock radius grows, a mild bipolar $\ell = 2$ asymmetry in the flow develops. At later times (bottom right), the bipolar deformation becomes weaker, and the inner ejecta display a more unipolar geometry with no apparent alignment with the grid axis. At 401 ms, the transition to the neutrino-driven wind phase is already underway.

Compared to ECSNe (Kitaura et al. 2006; Fischer et al. 2010; Hüdepohl et al. 2010) and the structurally most extreme low-mass iron core progenitors (Müller et al. 2013; Melson et al. 2015a; Müller 2016; Radice et al. 2017), shock propagation is slightly less rapid. At a post-bounce time of 150 ms, the average shock radius is only around 400 km in s2.8-3D compared to almost 1000 km in the 9.6$M_\odot$ model of Melson et al. (2015a). As a result, there is sufficient time of the 3D explosion model to develop a modest level of large-scale asymmetries, resulting in a visible bipolar asymmetry at late times (Figure 3). This is to be compared to the small-scale asymmetries that dominate multi-dimensional ECSN models (Wanajo et al. 2011, 2018) and the low-mass iron core models of Müller et al. (2013); Melson et al. (2015a); Müller (2016). Global asymmetries are, however, less pronounced than in the ultra-stripped models of Suwa et al. (2015) with a ratio of the maximum and minimum shock radius of no more than $\sim 1.25$ around shock revival.

The time-dependent composition of the ejecta is shown in Figure 4. The yields of iron group elements are also similar to the simulations of Suwa et al. (2015). Roughly $10^{-2}M_\odot$ of iron-group material is synthesised. Due to the simple flashing treatment in our code and uncertainties in the electron fraction due to our use of an approximate transport scheme, only the total amount of iron group material can be given with some confidence, and the detailed composition of this ejecta component remains uncertain. Yoshida et al. (2017) found a significant amount of neutron-rich ejecta and the production of the lighter trans-iron elements similar to the case of electron-capture supernovae (Wanajo et al. 2011) and low-mass iron core supernovae (Wanajo et al. 2018), where the rapid expansion of Rayleigh-Taylor plumes after shock revival leads to a freeze-out of the electron fraction below 0.5. Although our models exhibit similar explosion dynamics, simulations with more accurate transport than our FMT scheme or the IDSA approximation...
will be required to better assess the potential for neutron-rich nucleosynthesis. The amount of $^{56}\text{Ni}$ produced in the explosion therefore remains uncertain as well. A lower limit for the nickel mass is provided by the amount of material with $Y_e=0.5$ that undergoes explosive burning to the iron group, which is about $10^{-3}M_\odot$.

Iron group nucleosynthesis has already finished by the end of the 3D neutrino hydrodynamics simulation. At this stage, only a few $10^{-3}M_\odot$ of intermediate-mass elements have been swept up by the shock. During the subsequent evolution the shells still outside the shock will be completely ejected without any fallback. Including the material ahead of the shock, the ejecta will eventually comprise 0.024$M_\odot$ of helium, 0.011$M_\odot$ of oxygen, 0.01$M_\odot$ of neon, and smaller amount of magnesium and carbon. Despite the small mass of the helium envelope, helium thus remains the most abundant element in the ejecta.

### 3.2 Neutron Star Kick and Spin

Following Scheck et al. (2006) and Wongwathanarat et al. (2010b), we evaluate the kick velocity $v_{\text{kick}}$ of the PNS from the momentum of the ejecta using momentum conservation,

$$v_{\text{kick}} = -\frac{1}{M_{\text{PNS}}} \left( \int_{\text{ejecta}} \rho v dV + \int_0^t \oint \mathbf{F}_v \cdot d\mathbf{A} dt \right),$$

where $M_{\text{PNS}}$ is the (gravitational) PNS mass. The momentum of the radiated neutrinos is also included in the budget in the second term in brackets, where $\mathbf{F}_v$ denotes the neutrino energy flux density of all flavours on a spherical shell far away from the PNS (in our case at $r = 500$ km). The vector $\mathbf{n}$ denotes the unit normal vector in the radial direction. Note that this term contains a double integral over the surface normal vector $d\mathbf{A}$ and over time $t$.

For the 3D model, we can also calculate the angular momentum $J_{\text{PNS}}$ of the PNS by integrating the angular momentum flux through a sphere of radius $r_0$ around the central remnant (Wongwathanarat et al. 2010b, 2013),

$$\frac{dJ_{\text{PNS}}}{dt} = \int \alpha \phi \beta r_0^2 \rho_{\text{ejecta}} v \times r d\Omega,$$

where $\alpha$ and $\phi$ are the lapse function and conformal factor. The evolution of $v_{\text{kick}}$, $J_{\text{PNS}}$, and the baryonic PNS mass $M_{\text{PNS}}$ is shown in Figure 5. The kicks have almost asymptoted to their final values for all three models. Even extrapolating the kick by assuming that the gravitational acceleration $a_{\text{PNS}}$ of the PNS by the ejecta continues after the end of the simulations and scales with time $t$ as $t^{-2}$ (based on approximately homologous expansion of the ejecta) does not change the values appreciably.

As expected from the general similarity of the explosion dynamics with the C/O core models of Suwa et al. (2015) of similar mass, we also find small kick velocities with final values ranging from 2.5 to 28 km s$^{-1}$ for the s2.8-2D-b and s2.8-2D-a mod-
els, respectively. Our 3D model results in a kick velocity of about 9.4 km s$^{-1}$. Since more models would be needed to probe stochastic variations, these numbers are only rough order-of-magnitude indicators for the distribution of expected kick velocities. It is already clear that the kicks obtained for this particular progenitor are somewhat larger than for the most extreme single-star ECSN models in the literature (Gessner & Janka 2018). Different from ECSN models, the contribution of anisotropic neutrino emission to the kick is not negligible, though it remains by far subdominant to the gravitational tug on the PNS. In the 3D case, it amounts to about 0.6 km s$^{-1}$. The comparison with the models of Gessner & Janka (2018) should not be over-interpreted, however, since the progenitor used in this study does not exhibit the most extreme core-envelope structure among the models of Tauris et al. (2015). Kicks of just a few km s$^{-1}$ may be generic for the ECSN channel of ultra-stripped progenitors, but this remains to be tested by future work.

Such small kicks for ultra-stripped supernovae of small iron cores are indeed in agreement with earlier theoretical and observational arguments (Tauris et al. 2015, 2017) – see also the proposed relation between NS mass and kick velocity for the second supernova in forming DNS systems (Tauris et al. 2017), as supported by current observational data of NS masses, eccentricities, proper motions and spin-orbit misalignment angles.

The 2D simulations of ultra-stripped supernovae of Suwa et al. (2015) did not allow any statement on the spin-up of the PNS due to asymmetric accretion. Our 3D model shows that the spin-up is very modest; the angular momentum imparted onto the PNS is merely $0.27 \times 10^{46}$ g cm$^{-2}$ s$^{-1}$ (bottom panel of Figure 5). Using the fit formula of Lattimer & Schutz (2005) for the moment of inertia $I$ of cold NSs,

$$I \approx 0.237 M_{\text{grav}} R^2 \left[1 + 4.2 \left( \frac{M_{\text{grav}} \text{ km}}{M_{\odot} R} \right) + 90 \left( \frac{M_{\text{grav}} \text{ km}}{M_{\odot} R} \right)^4 \right],$$

this translates into a spin period of 3 s for $M_{\text{grav}} = 1.28 M_{\odot}$ and assuming $R = 12$ km. The spin-up is thus considerably smaller than for massive progenitors with sustained accretion (Müller et al. 2017). It is also on the low side compared to parameterised 3D explosion models of more massive progenitors (Wongwathanarat et al. 2013), which do not exhibit considerable accretion after shock revival. The small spin-up is a natural result of the explosion dynamics of ultra-stripped models, i.e., a fast explosion with small global asymmetries, the absence of sustained accretion, and the small mass in the gain region that is involved in overturn motions.

This result, however, comes with a large amount of uncertainty and is again only accurate to about an order of magnitude since we can neither explore the stochasticity of the aspherical accretion onto the PNS nor variations in the progenitor structure. Models with more massive metal cores and stronger accretion after shock revival could presumably achieve considerably shorter spin periods. Moreover, the angular momentum of the progenitor core will not be negligible in general. For the shortest final orbital period of 0.035 d considered in Tauris et al. (2015), the initial angular momentum alone is sufficient to explain NS spin periods as short as $\sim 400$ ms. Comparing to observations, there are two DNS systems known where the young NSs are detectable as radio pulsars and their spin periods are 144 ms and 2.77 s. One must keep in mind that their spin periods at birth are smaller, potentially significantly smaller, depending on their true ages and the braking index.

From our simulations, we find no correlation between the direction of the spin axis and the kick velocity vector (Figure 6). This result is in agreement with recent simulations (Tauris et al. 2017) of the post-SN kinematics resulting from the second SN in DNS systems when calibrated to empirical data. A spin-kick alignment, however, has been suggested for isolated radio pulsars (Noutsos et al. 2013, and references therein).

## 4 EVOLUTION TO SHOCK BREAKOUT

### 4.1 Extent of Mixing

As the shock propagates to the stellar surface, mixing driven by the Rayleigh-Taylor instability occurs as the acceleration and subsequent deceleration of the shock at shell interfaces establishes regions where the pressure and density gradients point in opposite directions (Chevalier 1976; Müller et al. 1991; Fryxell et al. 1991). These episodes of acceleration and deceleration are essentially determined by variations in $\rho r^3$ in the density profile through which the blast wave propagates (Sedov 1959), although the overall decrease of the shock velocity $v_{sh}$ is primarily due to the accumulation of ejecta mass $M_{ej}$. Both effects can be well fitted by (Matzner & McKee 1999),

$$v_{sh} = 0.794 \left( \frac{E_{\text{kin}}}{M_{ej}} \right)^{1/2} \left( \frac{M_{ej}}{\rho_0 r^3} \right)^{0.19}. \tag{6}$$

For the ultra-stripped progenitor, this implies two episodes of shock acceleration and deceleration (top panel of Figure 7). The shock velocity peaks at 30,000 km s$^{-1}$ at the base of the almost completely burned C shell and again at 25,000 km s$^{-1}$ at the base of the He envelope. Rayleigh-Taylor instabilities indeed occur in these regions as confirmed by local linear stability analysis, which yields a growth rate of (Bandiera 1984; Benz & Thielemann 1990; Müller et al. 1991)

$$\omega_{RT} = \frac{c_s}{\Gamma} \sqrt{\left( \frac{\partial \ln P}{\partial r} \right)^2 - \Gamma \frac{\partial \ln P}{\partial r} \frac{\partial \ln \rho}{\partial r}}. \tag{7}$$

for the compressible Rayleigh-Taylor instability, where $P$, $\Gamma$, and $c_s$ are the pressure, adiabatic index, and the speed of sound. We evaluate Equation (7) using spherically averaged profiles of model s2.8-3D and show the expected number $\omega_{RT}$ of e-foldings over one characteristic time-scale in the bottom panel of Figure 7. Rayleigh-Taylor instabilities at the two interfaces develop already at $\sim 1$ s and $\sim 30$ s, respectively.
Especially at the inner edge of the O/Ne/Mg/C shell, the nominal growth factors appear small (about 2.5 e-foldings during the first 400 s), but this is merely due to the fact that the instability quickly becomes non-linear due to the asphericities seeded by the neutrino-driven engine, so that the feedback on mixing on the density and pressure profiles reduces the nominal linear growth rate.\(^3\) There is in fact considerable mixing of iron group elements into and through the O/Ne/Mg/C shell as can be seen from the final distribution of the ejecta at shock breakout in velocity space (Figure 8) and as a function of mass coordinate (Figure 9). An appreciable amount of O, Ne, Mg, and of Fe group material makes it far into the He envelope, with somewhat more efficient mixing in the 3D model.

\(^3\) Note that many papers in the literature (e.g. Benz & Thielemann 1990; Müller et al. 1991; Fryxell et al. 1991; Kifonidis et al. 2003; Wongwathanarat et al. 2015) provide growth rates based on 1D models with similar energetics as their multi-D simulations and therefore obtain a larger number of e-foldings.

Merely judging by the mass fractions of iron group elements of \(~0.1\) in the He shell, the mixing of \(^{56}\)Ni appears more than sufficient to make the He visible in the spectra, which requires mass fractions \(\geq 0.01\) (Dessart et al. 2012, 2015). The 3D distribution of the mixed Fe group material precludes any firm conclusions on the impact of the mixing on the spectra at this stage, however: The \(^{56}\)Ni that is mixed into the He shell is concentrated into a few thin plumes (Figure 10) with significantly higher density than the ambient He, and it still needs to be determined whether such a strongly clumped distribution of \(^{56}\)Ni can lead to efficient non-thermal excitation of He in a large fraction of the envelope.

The extent of the mixing falls between the few studies that have addressed stripped-envelope supernovae of Type Ib/c and Ib with consistent (Ellinger et al. 2013) or artificially altered (Kifonidis et al. 2003; Wongwathanarat et al. 2017) envelope structures. Whereas – at the extreme end – Ellinger et al. (2013) found little mixing during the explosion in a Type Ib supernova model due to the lack of a maxing episode at an H/He interface, \(^{56}\)Ni is thoroughly homogenised from the mass cut far into the He shell in a Ib supernova model of Kifonidis et al. (2003) with an artificially truncated envelope (see their figure 19). Our model is less extreme than that of Kifonidis et al. (2003), as one still recognises three fairly distinct layers, i.e., the He shell with C enrichment from the active shell source, the remains of the O/Ne/Mg/C shell, and the inner ejecta that mainly consist of \(^{56}\)Ni from explosive burning, Fe group material and He from neutrino-driven outflows, and O, Ne, Mg, and Si swept up by the shock at early times. More efficient mixing into the He shell is impeded by the development of the reverse shock from the base of the He envelope, which confines all but the fastest

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**Figure 7.** Top panel: Shock velocity (solid line) according to the fit formula of Matzner & McKee (1999) (Equation 6) and variation of \(\rho r^3\) in the progenitor model (dashed line). Middle panel: Spherically averaged composition of model s2.8-3D at the time of mapping into ProASMUS 730 ms after bounce. Bottom panel: Expected e-foldings \(\omega_{RT} t\) of the Rayleigh-Taylor instability per characteristic time-scale at various stages of the explosion from the local growth rate \(\omega_{RT}\). The local growth rate \(\omega_{RT}\) is calculated from spherically averaged profiles of the 3D model according to Equation (7). Rayleigh-Taylor instabilities are triggered by the inner interface of the O/Ne/Mg/C shell and the He envelope. The unstable region below the He envelope extends inwards as the reverse shock propagates deeper into the ejecta.

**Figure 8.** Binned distribution of selected elements in the ejecta as a function of radial velocity at the time of shock breakout in model s2.8-2D-b (top) and model S2.8-3D (bottom).
Multi-D Simulations of Ultra-Stripped Supernovae to Shock Breakout

Figure 9. Spherically averaged mass fractions of selected elements in the ejecta at the time of shock breakout as a function of enclosed mass in model s2.8-2D-b (top) and model S2.8-3D (bottom).

Ni-rich plumes within the O/Ne/Mg/C shell, similar to the situation at the He/H shell interface of some blue supergiant models (Kifonidis et al. 2003; Wongwathanarat et al. 2015). This phenomenon illustrates that it is crucial that mass loss is included appropriately in the evolution of the progenitor and that the structure of the He envelope in stripped-envelope supernovae is modelled consistently. The formation of the reverse shock depends critically on the relatively strong acceleration and deceleration of the shock at the base of the He envelope, which is in turn tied to the expansion of the envelope to a radius of $5 \times 10^{12}$ cm, which could not be obtained by cutting the hydrogen envelope of an “appropriate” hydrogen-rich single-star progenitor model.

Such a variety of outcomes in stripped-envelope models is not unexpected, and it would be premature to make general statements about the effects of envelope stripping on mixing instabilities. There is no reason to expect considerably less variation in mixing than between hydrogen-rich red and blue supergiant progenitors: The development of the Rayleigh-Taylor instability at shell interfaces inside the C/O-core is bound to be very sensitive to variations in the structure and configuration of the interior burning shells (which can be considerable, see Collins et al. 2018; Sukhbold et al. 2017) and the seed asphericities imprinted by the supernova engine. One also expects that the character of the core-envelope interface – and hence of the mixing and the reverse shock associated with it – varies considerably since the radial extent of the He envelope of Type Ib/c supernova progenitor should span about two orders of magnitude (Yoon et al. 2010).

Figure 10. Isosurfaces showing the mixing of iron-group ejecta at the time of shock breakout. The red isosurface denotes an iron group mass fraction of $X_{\text{Fe}} = 0.4$. The transparent yellow surfaces correspond to a helium mass fraction of $X_{\text{He}} = 0.5$, and the outer one coincides with the stellar surface. Only a few small-scale plumes of nickel/iron-rich ejecta have penetrated roughly half way through the helium envelope.

Figure 11. 2D slice showing the logarithm ($\log_{10} \rho$ in g cm$^{-3}$) of the density 1700 s after the onset of the explosion. Most of the Rayleigh-Taylor plumes formed at the inner interface of the O/Ne/Mg/C shell are caught behind the dense shell (white annular structure) formed by the reverse shock at the core-envelope interface. The black curve denotes the $X_{\text{He}} = 0.5$ isocontour for the mass fraction of He.
terms. The di-

Figure 12. Comparison of the root-mean-square fluctuations $\delta v$ of the radial velocity from model s2.8-3D (black) to the estimate from Equation (12) based on a simple buoyancy-drag-model (red) at three selected times.

4.2 Non-Linear Regime of the Rayleigh-Taylor Instability and Assessment of 1D Mixing Models

Paxton et al. (2018) recently suggested that mixing by the Rayleigh-Taylor instability is amenable to a 1D treatment based on an appropriate turbulence model (Duffell 2016). The possibility of an effective 1D treatment has largely remained unexplored during the long history of multi-D simulations of mixing instabilities in core-collapse supernovae. Although Paxton et al. (2018) presented an encouraging comparison of their 1D mixing algorithm in the 

Figure 13. Comparison of the relative root-mean square density contrast $\delta \rho/\rho$ in model s2.8-3D (black) to mixing-length approximations based on the local gradients of the spherically averaged density and pressure. The red and blue curves show the results for the equations including the adiabatic expansion of bubbles (Equation 20) and the estimate from Duffell (2016) (Equation 19), respectively. In formally stable regions where these stability criteria give $\delta \rho/\rho < 0$, the value is set to zero instead.

The mixing length is chosen as

$$\Lambda = C \frac{\delta v}{c_s},$$

where $C$ is an appropriate non-dimensional coefficient. This choice is motivated by the realisation that the mixing length should depend on the distance that Rayleigh-Taylor plumes can traverse within one characteristic time-scale of the system.

For the turbulent velocity fluctuations, Paxton et al. (2018) solve a time-dependent equation for the ratio $\kappa = \delta v^2/c_s^2$,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 (\rho c_v - C \kappa c_s) \frac{\partial \rho}{\partial r} \right) = (A + B \kappa) \max(0, -\frac{\partial P}{\partial r} - D \kappa p c_s r^{-1}),$$

where $A$, $B$, and $D$ are again appropriately chosen non-dimensional coefficients. Paxton et al. (2018) set those non-dimensional coefficients to $A = 10^{-3}$, $B = 2.5$, $C = 0.2$, $D = 2$.

Whereas the validity of Equation (8) can be tested directly by comparing to the actual turbulent fluxes in multi-D simulations (see below), it is less straightforward to pit Equation (10) against a multi-D model without actually solving the time-dependent equations of the turbulence model. In fact, our simulations show that
Figure 14. Turbulent mass fluxes $\mathcal{F}_0$ (left) and $\mathcal{F}_\rho$ (right) for $^{16}\text{O}$ and $^{60}\text{Fe}$ (a tracer for the neutrino-processed iron/nickel-rich ejecta) in model s2.8-3D (black) compared to the generalised mixing-length approximation (red) from Equation (21) at selected times.

the turbulent velocity fluctuations appear to be captured by an even simpler model in the non-linear regime of the Rayleigh-Taylor instability. Judging by the morphology of the plumes in typical supernova simulations, the Rayleigh-Taylor instability typically proceeds far into the second stage, where the plume velocity is determined by the balance between buoyancy and the drag force (Sharp 1984; Zhou 2017a,b). This implies

$$\delta v = \sqrt{\Lambda \frac{\delta \rho}{\rho} \frac{1}{\rho^2} \delta \rho \cdot \delta \rho}, \quad (11)$$

in terms of the density contrast $\delta \rho$, the spherically averaged density $\rho$, the effective acceleration $g_{\text{eff}} = \rho^{-1} \partial P / \partial \rho$, and a length scale $\Lambda$ that encapsulates the drag coefficient and the volume-to-surface ratio of the plumes. Expressing $g_{\text{eff}}$ in terms of $\rho$ and the gradient of the spherically averaged pressure $P$, this becomes

$$\delta v = \sqrt{\Lambda \frac{\delta \rho}{\rho} \frac{1}{\rho^2} \frac{\partial P}{\partial \rho} \cdot \delta \rho}. \quad (12)$$

In Figure 12 we evaluate Equation (12) for three representative times using root-mean-square fluctuations (RMS) from a spherical Favre decomposition for $\delta \rho$ and $\delta \rho$. It can be seen that this simple buoyancy-drag model quite accurately captures the evolution of $\delta v$ for $\Lambda = 0.016\rho_s$. The small value of $\Lambda$ is consistent with the morphology of Rayleigh-Taylor mixing in our model, which is dominated by small-scale plumes. These findings suggest that balance between buoyancy and drag is indeed what determines the evolution of the Rayleigh-Taylor plumes in this particular model.

But what does this finding imply for effective 1D turbulence models for Rayleigh-Taylor mixing and can it be related to Equation (10)? The concept of a balance between effective buoyancy and drag is not included in Equation (10) by construction; indeed one notices that the dominant source and sink terms on the right-hand side cannot balance each other because they are both proportional to $\kappa$ (and the small term proportional to $A$ merely serves to kick off the growth of the instability and is not important in the non-linear regime). The only terms that can lead to a balance condition for the plume velocity are the linear source term and the diffusive term, which is quadratic in $\kappa$. If we match these two terms using dimensional analysis (i.e., replacing radial derivatives with $r^{-1}$), we arrive at the condition

$$C_c r^{-1} \rho c_s^2 \sim \frac{\kappa}{B} \sqrt{\frac{\partial P}{\partial r} \frac{\partial \rho}{\partial r}}, \quad (13)$$

or

$$\kappa \sim \frac{B}{C_c} \sqrt{\frac{\partial P}{\partial r} \frac{r}{\partial r} \rho c_s}, \quad (14)$$

To obtain a form similar to Equation (12), we note that $\partial \rho / \partial r$ is directly related to the density contrast $\delta \rho$ between the plumes and the background flow in the framework of a mixing-length approach (if we discount compressibility effects as in Duffell 2016 and Paxton et al. 2018). Using $\kappa = \delta v^2 / c_s^2$ and discarding non-dimensional coefficients of order unity, we thus obtain:

$$\delta v^2 \sim \frac{r c_s}{\rho} \sqrt{\frac{\partial P}{\partial r} \frac{\partial \rho}{\partial r} \frac{r c_s}{\rho} \sqrt{\frac{\partial \rho}{\partial r} \frac{\partial P}{\partial r} \Lambda \frac{\rho c_s}{\rho} \frac{\partial \rho}{\partial r} \frac{r c_s}{\rho} \sqrt{\frac{\partial \rho}{\partial r} \frac{\partial P}{\partial r} \frac{r}{\partial r} \rho c_s}, \quad (15)$$
or

\[
\delta\nu^{3/2} \sim c_s^{3/2} \sqrt{g_{\text{eff}} r \frac{\delta\rho}{\rho}}.
\]

(16)

The resulting plume velocity \(\delta\nu\) is somewhere in between the equilibrium velocity \(\delta\nu_{\text{eq}}\) from Equation (12) and the sound speed. Considering that the plume velocities are typically of a similar order of magnitude of \(c_s\), anyway, this may not have a major effect. It is probably still advisable to modify the turbulent damping term in Equation (10) to better capture the interplay between buoyancy and drag forces. This could easily be achieved by modifying the source and sink terms to reflect the work expended against the drag force. The sink term then needs to be proportional to \(\delta\nu^3/\lambda\); note that such a cubic dissipation term is also well established in 1D turbulence models for subsonic stellar convection (Kuhfuss 1986; Wuchterl & Feuchtinger 1998). The source term also needs to be modified: While the term \(B_{\text{vort}}\) given in Duffell (2016) correctly reproduces the exponential growth during the initial phase of the Rayleigh-Taylor instability, the instability enters a different regime once elongated plumes form. The growth rate of the kinetic energy is then given by the product of the velocity perturbations \(\delta\nu\) and the force felt by plumes with density contrast \(\delta\rho\),

\[
\left(\frac{\delta v^2}{\partial t}\right)_{\text{source}} = \delta v g_{\text{eff}} \frac{\delta\rho}{\rho} = \sqrt{\kappa c_s \omega_{\text{RT}}} = \kappa c_s \omega_{\text{RT}}.
\]

(17)

Since the quantity \(\rho v\) rather than the turbulent kinetic energy density \(\epsilon\) is evolved in the model of Duffell (2016), the modified evolution equation becomes

\[
\frac{\partial \rho}{\partial t} + \frac{1}{\rho} \frac{\partial}{\partial r} \left( r^2 (\rho v r - C_{\kappa} \rho v \frac{\partial \rho}{\partial r}) \right) = (A + B_k) \frac{r}{\rho c_s} \max(0, -\frac{\partial \rho}{\partial r} - D_e \frac{\partial v}{\partial r}),
\]

(18)

where an appropriate approximation for the effective volume-to-surface ratio \(A\) of the plumes is needed. This would give the desired balance condition provided that local gradients can indeed be used to estimate the density contrast.

We next examine whether local gradients can be used to obtain the density contrast of the Rayleigh-Taylor plumes and the turbulent transport of different species in the spirit of a mixing-length approach. To this end, we first compare (Figure 13) the RMS fluctuations of the density from our 2D model to the assumption that the density contrast \(\delta\rho\) merely depends on the mixing length and the density gradient (cp. Equation (13) in Duffell 2016),

\[
\delta\rho = C A \frac{\partial \ln \rho}{\partial r},
\]

(19)

and also to the more usual estimate from stellar mixing-length theory that takes compressibility effects into account,

\[
\frac{\delta\rho}{\rho} = C A \left( \frac{\partial \ln \rho}{\partial r} - \frac{1}{\Gamma} \frac{\partial \ln T}{\partial r} \right).
\]

(20)

Here, the mixing A is computed according to Equation (9) using the RMS velocity fluctuations from the 3D model. It should be noted that we show \(\sqrt{\delta\nu^2/\rho}\) instead of the density contrast \(\delta\rho/\rho\), in Figure 13, since the former determines the equilibrium velocity of the plumes and is therefore a more appropriate metric for evaluating an effective 1D turbulence model.

Neither the compressible nor the incompressible local approximation fares very well at predicting the density contrast. Even a recalibration of the proportionality factor \(C\) would do little to remedy this. The local estimate for the density contrast becomes especially problematic at later times (middle and bottom panel of Figure 13) when the reverse shock from the C/He interface has formed and propagates deeper into the O/Ne/Mg/C shell. However, Equation (20) for the compressible case at least correctly predicts the sign of the density contrast in most regions and appears to be a sufficiently good approximation to obtain the plume velocity within a factor of a few.

Finally, we compare the mixing-length estimate for the partial mass flux of species \(i\),

\[
\mathcal{F}_i = 4\pi r^2 (\rho) \frac{\partial v}{\partial r} \frac{\partial X_i}{\partial r},
\]

(21)

to that of turbulent partial mass flux of species \(i\) in the 3D simulation,

\[
\mathcal{F}_i = \int r^2 (\rho - \rho_i)(X_i - \langle X_i \rangle)(v_i - \langle v_i \rangle) \frac{\partial \rho}{\partial t} \ d\Omega.
\]

(22)

where \(X_i\) denotes the mass fraction of species \(i\) and angled brackets denote spherical Favre averages. Results for \(^{16}\text{O}\) and the most abundant representative neutron-rich iron group nucleus, \(^{60}\text{Fe}\), are shown in Figure 14.

Again, the agreement between the diffusive approximation and the actual turbulent fluxes is not too convincing. For \(^{16}\text{O}\), there is rough agreement within a factor of two at early times, but this agreement subsequently deteriorates. We even encounter situations where the actual turbulent flux point in the same direction as the gradient of the mass fluxes, i.e., a mass flux, which is anti-diffusive, for example in the region between enclosed masses of 1.44\(M_\odot\) and 1.45\(M_\odot\) during the later phases. That such non-diffusive mixing occurs in both the 3D model and the 2D model was in fact already evident from Figure 9, which showed a considerably higher mass fraction of iron group elements at the base of the He envelope than deeper down within the O/Ne/Mg/C shell. The non-diffusive nature of the mixing is also not unexpected from the morphology of the Rayleigh-Taylor instability in supernova simulations; very often one finds strongly elongated plumes originating from deep in the supernova that have traversed overlying shells without substantial small-scale mixing so that the initial layering is partially inverted rather than erased by diffusive mixing.

Taken together, our analysis of the turbulent fluctuations and fluxes in the 3D model does not provide convincing, positive justification for an effective 1D treatment of Rayleigh-Taylor mixing in core-collapse supernovae. This does not imply, however, that such an approach is invalid as an approximation. The comparison with 3D models of Wongwathanarat et al. (2015) in the work of Paxton et al. (2018) demonstrated that their 1D turbulence model can at least passably mimic the end result at shock breakout for some progenitors. More work is needed to ascertain whether it does so accidentally, or whether there is a deeper reason that allows it to approximately reproduce the gross features of mixing on a global scale even though some of its basic equations may not be very good approximations locally.

5 In other words, volume-weighted spherical averages are used for the density, and density-weighted averages are used for other quantities. The comparison between the 3D results and the mixing-length estimates is not changed substantially by using volume-weighted averages instead.
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5 APPLICATIONS TO OBSERVED DOUBLE NEUTRON STAR SYSTEMS

Based on the detailed pre-SN evolution of the helium star–NS binary using a binary stellar evolution code (Tauris et al. 2015), in this work extended up to collapse using the Kepler code, and on our models of the ensuing supernova explosion, we can calculate the post-SN orbital properties of the resulting binary system.

Originally, the binary system consisted of a 2.8 $M_{\odot}$ helium star–NS binary with an orbital period of 20 d. At the onset of oxygen burning, the helium star has become a detached star with a total mass of 1.72 $M_{\odot}$ (a 1.47 $M_{\odot}$ metal core and an envelope mass of $\sim0.25 M_{\odot}$, of which $\sim0.22 M_{\odot}$ is helium). The orbital period at this point was 19.4 d. It was assumed that the NS companion had an initial mass of 1.35 $M_{\odot}$ and thus we also take this value to be its final mass, since it only accreted 3 $M_{\odot}$ of helium during the accretion phase (Tauris et al. 2015).

In this work (see Section 2.1), we modelled the final stages of nuclear burning using the Kepler code and found a strong off-centre silicon deflagration flash which ejected all but 0.02 $M_{\odot}$ of the helium envelope, about 78 d prior to core collapse. At the time of the SN explosion, the total mass of the ultra-striped star is 1.48 $M_{\odot}$. Depending on the details of the envelope ejection from the silicon flash, we find that the pre-SN orbital period is between 22.8 and 23.1 d. In Section 3.2, we presented the resulting NS kick velocity from our modelling, ranging between 2.5 to 28 km s$^{-1}$, from which we can calculate the post-SN orbital properties of the resulting DNS system. The gravitational mass of the newly formed NS is 1.28 $M_{\odot}$, as mentioned earlier.

In Figure 15, we show the distribution of possible DNS systems following the ultra-striped SN explosion of our star. Similarly to the method outlined in, e.g., Freire & Tauris (2014), we simulate 10,000 SN explosions using Monte Carlo techniques to obtain a random kick direction in each event, assuming an isotropic distribution and a circular pre-SN orbit. The resulting distributions of DNS orbits are seen to cover a fairly large area in the orbital period–eccentricity plane, depending on the kick magnitude ($w = 2.5, 9.4$ or 28 km s$^{-1}$). We note that, by coincidence, we are able to reproduce the properties of the wide-orbit DNS system PSR J1930–1852 (Swiggum et al. 2015). This demonstrates that ultra-striped SN can indeed be responsible for even the widest orbit DNS system known. For a full range of solutions to PSR J1930–1852, see the appendix of Tauris et al. (2017).

6 CONCLUSIONS

In this work, we performed the first detailed modelling of the SN explosion of an ultra-striped star produced from a binary stellar evolution code. This means that different from previous work (Suwa et al. 2015), we actually calculate the mass loss and orbital evolution of the supernova progenitor model up to collapse. Our initial model is a 2.8 $M_{\odot}$ helium star–NS binary with an orbital period of 20 d. At the onset of core collapse, the donor star has been reduced to an ultra-striped star with a total mass of $\sim1.48 M_{\odot}$ as a result of Case BC RLO followed by a silicon deflagration flash after detachment. Hence, in this model the helium envelope is removed in a three-step process: first by the stellar wind mass loss prior to Case BC RLO, then as a result of mass transfer to the NS companion, and finally, after detachment, via a silicon deflagration flash. The result is an ultra-striped SN with a tiny envelope containing only 0.02 $M_{\odot}$ of helium. Interestingly, the almost complete expulsion of the envelope by the silicon flash occurs less than 100 d before explosion, which implies that the ensuing supernova may exhibit interaction with circumstellar material (CSM).

We performed one 3D simulation and two 2D simulations of the SN explosion using the neutrino hydrodynamics code CoCoNuT-FMT. The outcome of these simulations is broadly in agreement with previous work of Suwa et al. (2015) who investigated the explosion of bare C/O cores in 2D. We find rapid shock revival and small explosion energies of order $\sim10^{50}$ erg. The explosion dynamics is reminiscent of explosions of low-mass iron core progenitors in the single star channel (Müller et al. 2012; Melson et al. 2015a; Müller 2016; Radice et al. 2017). It is also similar to ECSNe, though less extreme in the sense that shock expansion is sufficiently slow to allow the development of the low-mode asymmetries that are absent in ECSN models (Wanajo et al. 2011; Gessner & Janka 2018). Our results are also compatible with a modest boost to the explosion energy due 3D turbulence as found by Melson et al. (2015a) for a low-mass iron core model.

By the end of the simulations, accretion onto the PNS has already stopped, and the neutrino-driven wind emerges, so that an we can put an upper limit of 1.28 $M_{\odot}$ on the gravitational NS mass for the current progenitor. We see evidence for small NS kick velocities between 2.5 and 28 km s$^{-1}$ for the ultra-striped SN, in agreement with Suwa et al. (2015). Our results confirm current ideas that ultra-striped core-collapse SNe of small iron cores lead to small kicks (Tauris et al. 2015, 2017). We do not find any evidence for a spin-kick alignment in such SNe, as opposed to some empirical evidence presented for isolated radio pulsars (Noutsos et al. 2013, and references therein).

Based on the calculated pre-SN orbital properties and the donor star mass at the onset of core collapse, the ejecta mass, the kick velocity and the final gravitational mass of the resulting NS, we simulated the post-NS binary systems using Monte Carlo techniques. We are able to reproduce the properties of the widest orbit DNS system known (PSR J1930–1852, Swiggum et al. 2015) which has an orbital period of 45 d. This suggests that ultra-striped SNe are not only relevant for tight-obit DNS systems but even apply for such wide-orbit systems. The total mass of our DNS system...
is $2.63 \, M_\odot$ compared to $2.59 \, M_\odot$ measured for PSR J1930–1852 (Swiggum et al. 2015).

As a preliminary step towards the calculation of observable signatures, we followed the evolution of the 3D SN model and one 2D model up to shock breakout. Mixing driven by the Rayleigh-Taylor instability develops at the base of the O/Ne shell and at the base of the He envelope. Isolated dense plumes of iron group material make it roughly half way through the He shell. The result is a considerable presence of iron group elements (mass fraction $\sim 0.1$) throughout the outer layers of the ejecta.

As it has been recently proposed that Rayleigh-Taylor mixing in supernova envelopes is amenable to a simple mixing-length treatment (MLT) (Duffell 2016; Paxton et al. 2018), we quantitatively investigated turbulent velocity fluctuations and fluxes in our 3D model to check the validity of such an MLT approach. Our simulation suggests that the plume velocities are well described by balance between buoyancy and drag forces, which could be captured in a modified 1D turbulence model. We found, however, that the turbulent fluxes cannot be well approximated by diffusive fluxes. Further work is necessary to determine to what extent an effective 1D treatment of the Rayleigh-Taylor instability is possible and can at least furnish a rough global approximation for the mixing in supernova envelopes.

The investigation presented here is the first attempt to model the evolution leading to the second SN in forming a DNS system by combining detailed binary stellar evolution and state-of-the-art multi-dimensional SN modelling. In the future, we plan to investigate such models for ultra-stripped SNe in tighter systems leading to post-SN DNS systems that will merge within a Hubble time, thus leading to systems similar to those detected by LIGO-Virgo. Furthermore, we will attempt to explode ultra-stripped stars with more massive iron cores to test the hypothesis of a correlation between NS mass and kick velocity (Tauris et al. 2017), and to determine whether one can obtain substantially more energetic explosions with higher nickel mass that are more similar to observationally inferred values (Drouet et al. 2013). Follow-up work is also needed on the observable signatures of ultra-stripped supernovae. It will be necessary to put radiative transfer calculations of such events (Moriya et al. 2017) on a more solid basis by incorporating the results of 3D simulations of mixing instabilities as presented in this work. Moreover, the intriguing possibility of CSM interaction for explosions of ultra-stripped progenitors needs to be investigated further.

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