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Case study

The modified nurse-saul (MNS) maturity function for improved strength estimates at elevated curing temperatures

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**Abstract**

Curing temperature affects significantly the compressive strength development of mortar mixtures. Higher curing temperatures accelerate the cement hydration and thus also the early age compressive strength development. However, the age conversion factors in maturity functions, especially that of the Nurse-Saul function, are not sufficient to account for this acceleration and thus an additional “acceleration” factor is needed. The “acceleration” compresses a certain percentage of hydration or strength development into a smaller time interval. The strength development rate was increased because of the “compression” of the hydration. The “acceleration” factor was not equal to the “compression” factor. The reaction at the higher temperature was therefore less efficient in contributing to the compressive strength than the reaction at the lower temperature. A relationship between concrete strength and the Nurse-Saul maturity index combined with an “acceleration” and a “temperature efficiency” factors are used in an iterative procedure for predicting/estimating the strength development for other than the standard 20 °C curing temperature.

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**1. Introduction**

The development of maturity methods, in around 1950, was the result of the need to estimate the effects of steam curing treatments on concrete strength development. Maturity methods aim to account for the combined effect of temperature and time on concrete strength development [1]. The temperature history during the curing period can be used to compute a single number that can be indicative of the concrete strength. Saul [2] called this single factor “maturity”:

\[ M = \sum_{0}^{t} (T - T_0) \Delta t \]

where: \( M \) is the maturity, °C-hours,
\( T \) is the average temperature (20 °C for standard curing) over the time interval \( \Delta t \),
\( T_0 \) is the datum temperature, taken as −11 °C in this work.

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The Nurse-Saul function described above can be used to convert a given temperature-time curing history to an equivalent age of curing at a reference temperature as follows [3]:

\[ t_e = \frac{\sum (T - T_0)}{(T_r - T_0)} \Delta t \]  

(2)

where: \( t_e \) is the equivalent age at the reference temperature, hours,
\( T_r \) is the reference temperature, °C.

Equivalent age is the duration of the curing period at the reference temperature that would result in the same maturity as the curing period at other temperatures. The equivalent age concept, originally introduced by Rastrup [3,4], can be written as:

\[ t_e = \sum \beta \Delta t \]  

(3)

where:

\[ \beta = \frac{(T - T_0)}{(T_r - T_0)} \]  

(4)

The “age conversion factor” \( \beta \) can be used to convert a curing interval \( \Delta t \) to the equivalent curing interval at the standard reference temperature.

The above rely on Saul’s principle or “maturity rule”, being valid, i.e.: “Concrete of the same mix at the same maturity (reckoned in temperature-time) has approximately the same strength whatever combination of temperature and time go to make up maturity” [2]. However, this principle was shown to be only valid provided the concrete temperature did not reach: (a) 50 °C within the first 2 h, or (b) about 100 °C within the first 6 h after the start of mixing. Later studies [5,6] also confirmed that at the same value of low maturity, a high curing temperature resulted in greater strength than a low curing temperature, and conversely at later maturities, it resulted in lower strength. This “crossover” effect was first reported in 1956 by McIntosh [5] and indicated that Saul’s maturity rule was not always valid. For it to be valid there should have been a single strength-maturity curve. It has been suggested that the “crossover effect” was due to the fact that a higher initial temperature resulted in more than a proportional increase in the initial rate of hydration [7]. Therefore, during the early stages of curing, when there is rapid strength development, the strength of concrete cured at the high temperature is greater than that of concrete cured at a lower temperature despite having the same maturity.

Saul’s introduction of the maturity rule led to an outgrowth of studies dealing not only with accelerated curing but also (a) estimation of in-place strength based on strength development data obtained under standard laboratory conditions [8–13], and (b) later age prediction based on early age strengths [14–16]. Numerous maturity functions [4,12,17] have been proposed to account for the deficiencies in the Nurse-Saul maturity function. Freiesleben Hansen and Pedersen’s expression for equivalent age [17], which is based on the Arrhenius equation, is one of the most commonly used:

\[ t_e = \sum_{0}^{-\frac{T_r - T_0}{R E}} e^{\frac{E}{R}} \Delta t \]  

(5)

where: \( t_e \) = equivalent age at the reference temperature, hours
\( T_r \) = average temperature of concrete (°C) during time interval \( \Delta t \),
\( T_r \) = reference temperature, °C,
\( E \) = apparent activation energy, J/gmol, and,
\( R \) = universal gas constant, 8.3144 J/gmol·K.

where:

\[ \beta = e^{\frac{E}{R}} \]  

(6)

The exponential function in the above equations is the age conversion factor and is expressed in terms of the absolute temperature. Apparent activation energies can be determined using “equivalent” mortar specimens [18] and the results applied to the concrete under investigation. Values for activation energies reported in the literature range from 33,500 J/mol to 63,600 J/mol [19–27]. Freiesleben Hansen and Pedersen’s expression appears to give, because of the use of activation energies, more accurate estimate of the acceleration of the cement hydration and the strength development at higher curing temperatures. However, there have been reports that indicate that reliable estimates at early ages [19,22] are for only the first few days. Overestimates of compressive strengths beyond the first few days appear to be due to this method not attempting to account for the “detrimental” effect of high early age curing temperatures on the ultimate/limiting strength of concretes [19,22,25,28]. Functions that describe a decreasing apparent activation energy with increasing relative strength, degree of hydration or maturity, have been reported as “giving some indication of the retarding effect” [29,30]. However, Schindler [31] questioned whether the activation energy actually changes with degree of hydration and suggested that it might change as a function of temperature only.

Functions described above are used for calculating a maturity index (temperature-time factor or equivalent age) based on the temperature history of the concrete. The Three Parameter Equation (TPE) [32] is one of several functions that have been
proposed to relate concrete strength to the maturity index (strength-maturity relationship):

\[ S = S_\infty e^{-(\delta)^\alpha} \]  

(7)

where: \( S \) = strength at maturity M, MPa,
\( S_\infty \) = limiting strength, MPa,
M = maturity index, °C-hours,
\( \alpha \) = characteristic time constant, °C-hours,
\( \delta \) = shape parameter.

Changing the value of the time constant preserves the same general shape of the curve while shifting it to the left or right [3]. Changing the value of the shape parameter alters the shape of the curve in such a way that when \( \alpha \) increases then the curve has a more pronounced “S” shape. Combinations of \( \tau \) (shifting of the curve to the left for higher curing temperatures) and \( \alpha \) (allowing for different ultimate strengths) can be used to get a good fit of the regression curve through all curing temperatures. Whether \( \alpha \) should remain constant or it should change with temperature has been queried [24]. Eq. (7) can also be expressed in a form of strength-time relationship in which the maturity component is replaced with a time component, i.e. an equivalent age (Eq. (5)). The true potential of this equation is however in its differentiated form:

\[ \frac{dS}{dt} = \frac{\alpha \tau^\alpha (T - T_0)}{M^{\alpha+1}} S_\infty e^{-(\delta)^\alpha} \]  

(8)

Where \( M = t(T - T_0) \). Combining with Eq. (5):

\[ \frac{1}{S} \frac{dS}{dt} = \frac{\alpha \tau^\alpha (T - T_0)}{M^{\alpha+1}} \]  

(9)

Differentiating Eq. (5) with respect to “maturity” rather than “time”:

\[ \frac{1}{S} \frac{dS}{dM} = \frac{\alpha \tau^\alpha}{M^{\alpha+1}} \]  

(10)

Eq. (8) can be programmed as an iterative procedure, for changing or non-isothermal curing temperatures, without relying on values of ultimate/limiting strength of concrete. If Saul’s principle was valid then:

\[ \frac{1}{S} \left( \frac{dS}{dM} \right)_r = \frac{1}{S} \frac{dS}{dM} \]  

(11)

Operations need to be determined that can transform \( \frac{1}{S} \left( \frac{dS}{dM} \right)_r \) to \( \frac{1}{S} \frac{dS}{dM} \) for curing temperatures other than the reference. The transformation required may provide a better understanding of the hydration kinetics and also enable the development of an accurate method for predicting strength development under non-isothermal curing conditions. The applicability of Eq. (8) in predicting the effects of temperature on the strength development of mortar mixtures has been investigated.

2. Research significance

The early-age strength development of concretes is greatly enhanced by high curing temperatures, such as those used for steam curing of precast concrete elements, or in structural elements as a result of the hydration being an exothermic reaction. In order for contractors to be in a position to take advantage of these enhanced strengths, e.g., for increased production in precast concrete factories or for fast track construction for in-situ concrete construction, there needs to be a method to predict relatively accurately the strength development for these high early age curing temperatures. There is therefore a need to increase our understanding of the effect of temperature on the cement hydration, i.e. the transformation of the cement hydration with temperature changes.

3. Materials and experimental procedures

The concrete mixtures investigated had design/characteristic 28-day cube compressive strengths of 30, 45 and 60 MPa and thus they are referred to as PC30, PC45 and PC60. A margin of approximately 10 MPa has been allowed which gave 28-day mean compressive strengths of 37, 54, and 69 MPa. Mortars that have the same water-cement ratio as well as coarse aggregate-binder ratio to concretes are considered to be “equivalent” to the concretes under investigation and can be used for determining the activation energies (ASTM C1074, 2011). Proportion of superplasticising admixture (SPA) used in the mortar mixture was kept the same (as a percentage of solids by weight of cement) as was used in the concrete. The equivalent mortar mixtures had 32-day compressive strengths of 37, 55 and 63 MPa. The mixture proportions for the concretes and their equivalent mortars are shown in Table 1.
Table 1
Concrete and mortar mix proportions.

<table>
<thead>
<tr>
<th>Material ID</th>
<th>PC30</th>
<th>Equivalent mortar</th>
<th>PC45</th>
<th>Equivalent mortar</th>
<th>PC60</th>
<th>Equivalent mortar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland Cement [kg/m³]</td>
<td>300</td>
<td>435</td>
<td>365</td>
<td>483</td>
<td>375</td>
<td>506</td>
</tr>
<tr>
<td>Gravel [kg/m³]</td>
<td>1107&lt;sup&gt;a&lt;/sup&gt;</td>
<td>–</td>
<td>1230&lt;sup&gt;b&lt;/sup&gt;</td>
<td>–</td>
<td>1257&lt;sup&gt;c&lt;/sup&gt;</td>
<td>–</td>
</tr>
<tr>
<td>Sand [kg/m³]</td>
<td>817&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1614&lt;sup&gt;b&lt;/sup&gt;</td>
<td>612&lt;sup&gt;c&lt;/sup&gt;</td>
<td>1621&lt;sup&gt;d&lt;/sup&gt;</td>
<td>662&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1707&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>SPA dosage [%]</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Free water [kg/m³]</td>
<td>180</td>
<td>261</td>
<td>185</td>
<td>245</td>
<td>150</td>
<td>202</td>
</tr>
<tr>
<td>Total water [kg/m³]</td>
<td>205</td>
<td>302</td>
<td>199</td>
<td>255</td>
<td>171</td>
<td>246</td>
</tr>
<tr>
<td>Free w/b [-]</td>
<td>0.68</td>
<td>0.62</td>
<td>0.55</td>
<td>0.53</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td>Total w/b [-]</td>
<td>0.60</td>
<td>0.60</td>
<td>0.51</td>
<td>0.51</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

28-day (concrete) and 32-day (mortar) strength [MPa]

<table>
<thead>
<tr>
<th>Material ID</th>
<th>28-day (concrete)</th>
<th>32-day (mortar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC30</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>PC45</td>
<td>54</td>
<td>55</td>
</tr>
<tr>
<td>PC60</td>
<td>69</td>
<td>63</td>
</tr>
</tbody>
</table>

<sup>a</sup> water absorption = 0.36%.
<sup>b</sup> water absorption = 2.55%.
<sup>c</sup> water absorption = 0.79%.
<sup>d</sup> water absorption = 0.63%.

3. Materials

"Portland cement used was from one batch of CEM I 52.5 (conforming to BS EN 197-1:2000 and BS EN 197-1:2011 [33,34]) which was supplied by Castle Cement Ltd. The chemical composition of the cement is shown in Table 2. The coarse aggregate was crushed granite with a maximum aggregate size of 20 mm combined with a very fine sand (81% of the particles passed through the 600 μm sieve). Oven-dried aggregate was used and batch weights allowed for the aggregate water absorption. The superplasticizer used was a polycarboxylate polymer Structuro 111X provided by Fosroc Ltd.

3.2. Mixing, casting, curing and testing of mortar specimens

All mortar specimens were prepared in accordance with ASTM C1074-11 [18]. A 0.02 m³ capacity horizontal pan mixer was used and the materials (after they had been weighed) were placed in it; first the cement followed by the sand and finally the superplasticizer mixed with the water. The materials were mixed for 3 min and after which the mortar was placed in steel 50 mm cube moulds and compacted with the use of a vibrating table. Subsequently they were wrapped in polythene sheet before placing in water tanks for curing at 20, 30, 40 and 50 °C. One set of cubes was wrapped in damp hessian and stored in a cooler incubator whose temperature was set at 10 °C. The specimens were demoulded only prior to them being tested in compression. Three cubes were tested in accordance to BS EN 12390-3:2009 [35] at each testing age and the first testing age was chosen to approximately correspond to a compressive strength of 4 MPa. For practical purposes this was aimed to be at either 3, 6, 12, or 24 h after casting as subsequent tests needed to be carried out at twice the age of the previous test.

4. Results and discussion

The effect of high curing temperature was to accelerate the cement hydration enabling the mortar mixtures to achieve higher strengths at earlier ages. Fig. 1 however also shows that high early age curing temperatures have a “detrimental” effect on the later age strengths. Regression curves were based on Eq. (5) and the shape parameter α was set as unity for all curing temperatures. There is no general agreement as to whether “α” should increase or decrease with increasing curing.

Table 2
Chemical composition of cement used.

<table>
<thead>
<tr>
<th>Chemical constituent</th>
<th>CEM I 52.5</th>
<th>Portland Cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>21.07</td>
<td></td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>5.00</td>
<td></td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>2.92</td>
<td></td>
</tr>
<tr>
<td>CaO</td>
<td>64.40</td>
<td></td>
</tr>
<tr>
<td>MgO</td>
<td>2.07</td>
<td></td>
</tr>
<tr>
<td>SO₃</td>
<td>2.65</td>
<td></td>
</tr>
<tr>
<td>K₂O</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>Na₂O</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Cl</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Insoluble</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>Free Lime</td>
<td>1.70</td>
<td></td>
</tr>
<tr>
<td>LOI</td>
<td>1.19</td>
<td></td>
</tr>
</tbody>
</table>
Regression analysis carried out, with "α" being left a variable, on the available strength data, did not result in any consistent relationship with curing temperature.

The strength development has been plotted against maturity in Fig. 2 which shows that at the same value of low maturity, a high curing temperature resulted in greater strength than a low curing temperature and conversely at later maturities, resulted in lower strengths. The "crossover" effect [5] was apparent indicating that Saul’s maturity rule [1] is not valid; there is no single strength-maturity relationship.

Fig. 1. Compressive strength versus age (hours) after casting for the mortar mixtures.

Fig. 2. Compressive strength versus maturity (°C-hours) after casting for the mortar mixtures.
Chemical kinetics is the part of physical chemistry that studies reaction rates. Kinetics of cement hydration should therefore be investigating the rate of reaction. The strength development was converted to rate of strength development with maturity (dS/dM) rather than time (dS/dt). The effect of temperature on the rate of cement hydration should be determined in the “maturity” rather than “time” domain as this is required for Eq. (8). The maximum rates of strength development, i.e. the peak of the dS/dM curves shown in Fig. 3, did not only occur at earlier maturity values with higher curing temperature but they were also numerically higher. The effect of temperature on the reaction was to accelerate it and thus “compress” a certain maturity interval into a smaller one, e.g., the reaction up till the peak for PC30 takes place at maturities of 675.8 and 323.3 °C-hours, for 20 and 50 °C curing temperatures respectively (see Table 3 for values of other mixes and curing temperatures). The same rates of reaction are also plotted in Fig. 4 but using a logarithmic scale for the maturity axis which accentuates differences at early ages/maturities.

Taking logarithms of both sides of Eq. (8) results in:

\[
\log\left(\frac{1}{dS} \frac{dM}{dM}\right) = \log(\alpha r^a) - (a + 1) \log(M)
\]

indicating that the plots should be straight lines with a gradient of \(-(a+1)\), which therefore is minus 2, as shown in Fig. 5, since the shape parameter \(\alpha\) used for the regression analysis of strength versus time as well as maturity was set as unity. All the lines in Fig. 5 would have overlapped had Saul’s maturity rule been valid. It is therefore necessary to consider how the rate of reaction is modified with temperature.

The rate of the reaction (in terms of maturity index rather than time) is plotted against maturity in Fig. 6. The two hydration curves do not coincide; the age conversion factor, \(\beta = \frac{(T-T_0)}{(T_0-T_0)} = \frac{(50+11)}{(20+11)} = 1.97\), implied by Saul’s maturity rule, is not sufficient to bring the 50 and 20 °C reaction curves to overlap for any of the three mixes. An “acceleration” factor, in addition to the inherent age conversion factor, is therefore needed so that the 20 °C strength-maturity relationship can be applied for curing at 50 °C. The determination of the “acceleration” factor requires first the ratio of the maturity at the maximum dS/dM at 20 °C to the maturity at the maximum dS/dM at 50 °C, i.e.:

\[
\text{Ratio of Maturities at peak dS/dM} = \frac{M_{r(20\degree C)}}{M}
\]

where:
- \(M_r\) = the maturity at peak dS/dM at the reference temperature of 20 °C, °C-hours,
- \(M\) = the maturity at peak dS/dM at any other curing temperature, °C-hours.

![Fig. 3. Rate of compressive strength gain with respect to maturity(dS/dM) versus maturity (°C-hours) for the mortar mixtures plotted on linear x-axis.](image-url)
Table 3
Parameters required for the transformation of strength from the reference temperature to other curing temperatures for the investigated mixes.

<table>
<thead>
<tr>
<th>Mix ID</th>
<th>Parameter</th>
<th>Curing temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>PC30</td>
<td>$S_n$ (MPa)</td>
<td>39.15</td>
</tr>
<tr>
<td></td>
<td>$T^*$ (hours)</td>
<td>43.51</td>
</tr>
<tr>
<td></td>
<td>$\tau^*$ (°C hours)</td>
<td>1349</td>
</tr>
<tr>
<td></td>
<td>($dS/dM)_{max}$ (MPa per °C hours)</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>Time at ($dS/dM)_{max}$ (hours)</td>
<td>21.80</td>
</tr>
<tr>
<td></td>
<td>Age conversion factor, $\beta$ (actual)</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>M at ($dS/dM)_{max}$ (°C hours)</td>
<td>675.8</td>
</tr>
<tr>
<td></td>
<td>Ratio of ($dS/dM)_{max}$</td>
<td>1.000</td>
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<tr>
<td></td>
<td>Ratio of Maturities at ($dS/dM)_{max}$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Gradient c1 (Ratio of Maturities)</td>
<td>1.089</td>
</tr>
<tr>
<td></td>
<td>Gradient c2 (Ratio of ($dS/dM)_{max}$)</td>
<td>0.269</td>
</tr>
<tr>
<td></td>
<td>Acceleration factor</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Compression factor</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Temperature efficiency (%)</td>
<td>100</td>
</tr>
<tr>
<td>PC45</td>
<td>$S_n$ (MPa)</td>
<td>61.55</td>
</tr>
<tr>
<td></td>
<td>$T^*$ (hours)</td>
<td>33.43</td>
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<td></td>
<td>$\tau^*$ (°C hours)</td>
<td>1036</td>
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<td>($dS/dM)_{max}$ (MPa per °C hours)</td>
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<td>Time at ($dS/dM)_{max}$ (hours)</td>
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<td>Age conversion factor, $\beta$ (actual)</td>
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<td></td>
<td>M at ($dS/dM)_{max}$ (°C hours)</td>
<td>519.3</td>
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<td>Ratio of ($dS/dM)_{max}$</td>
<td>1.000</td>
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<td>Ratio of Maturities at ($dS/dM)_{max}$</td>
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<tr>
<td></td>
<td>Gradient c1 (Ratio of Maturities)</td>
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<td></td>
<td>Gradient c2 (Ratio of ($dS/dM)_{max}$)</td>
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<td>Acceleration factor</td>
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<td>Compression factor</td>
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<td>Temperature efficiency (%)</td>
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<td>PC60</td>
<td>$S_n$ (MPa)</td>
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<tr>
<td></td>
<td>$T^*$ (hours)</td>
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<td></td>
<td>$\tau^*$ (°C hours)</td>
<td>1247</td>
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<tr>
<td></td>
<td>($dS/dM)_{max}$ (MPa per °C hours)</td>
<td>0.028</td>
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<td></td>
<td>Time at ($dS/dM)_{max}$ (hours)</td>
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<td>Age conversion factor, $\beta$ (actual)</td>
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<td>M at ($dS/dM)_{max}$ (°C hours)</td>
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</tr>
<tr>
<td></td>
<td>Ratio of ($dS/dM)_{max}$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Ratio of Maturities at ($dS/dM)_{max}$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Gradient c1 (Ratio of Maturities)</td>
<td>0.895</td>
</tr>
<tr>
<td></td>
<td>Gradient c2 (Ratio of ($dS/dM)_{max}$)</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>Acceleration factor</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Compression factor</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Temperature efficiency (%)</td>
<td>100</td>
</tr>
</tbody>
</table>

* Characteristic time constant for the strength-age relationship (Eq. (7)).
* Characteristic time constant for the strength-maturity relationship (Eq. (7)).

The “ratios of maturities at maximum $dS/dM$” are then plotted against $(T + 11)/(T_{20} + 11)$ on a log – log scale, as shown in Fig. 7. The gradient of the straight line $C_1$ can then be used in the following equation so that a relationship is obtained between the “acceleration” factor and the age conversion factor, i.e. the ratio $(T + 11)/(T_{20} + 11)$, inherent in the Nurse-Saul maturity function:

$$Accelerator Factor (AF) = \left(\frac{T + 11}{31}\right)^{C_1}$$ (14)

where:
- AF = “acceleration” factor
- $C_1$ = the gradient of the straight line relating the ratios of maturities at peak $dS/dM$ and age conversion factor

The effect of this “acceleration” is to compress a certain maturity interval into a smaller one. As a result of the “compression” of the hydration the numerical value of $dS/dM$ is increased. If the reaction at the higher temperature was as efficient as at the lower temperature then the “acceleration” factor would be equal to the “compression” factor. The Nurse-Saul function and indeed all other maturity functions consider that the temperature efficiency is 100% irrespective of the curing temperature. The predicted rates of strength development at the higher curing temperature of 50 °C and assuming 100% temperature efficiency (TEF) are shown as dashed lines in Fig. 6(a), (b) and (c) for PC30, PC45 and PC60 respectively. It
was found that these were consistently higher than those estimated from regression analysis of actual strength versus maturity data. The actual “compression” factor (CF) therefore needed to be determined. The first step was to determine the ratio of \( \frac{dS_{\text{am}}}{dM_{\text{am}}} \) for each curing temperature with that at the reference temperature of 20 °C. The ratio of \( \frac{dS_{\text{am}}}{dM_{\text{am}}} \) is then plotted versus the age conversion factor, i.e. the ratio \( (T + II)/(20 + II) \), on a log – log scale as shown in Fig. 7. The gradient of
The regression line $C_2$ is then used to express the “compression” factor (CF) as a function of the age conversion factor:

$$\text{Compression Factor (CF)} = \left( \frac{T + 11}{T_{20} + 11} \right)^{C_2} \tag{15}$$

where:

- CF = “compression” factor

Fig. 6. “Acceleration” and “temperature efficiency” factors used to transform the $20 \degree C$ rate of compressive strength gain ($dS/dM$) to that at $50 \degree C$.

Fig. 7. Ratio of: (a) maturities at peak $dS/dM$, and, (b) ($dS/dM$)$_{\text{max}}$, versus $(T + 11)/(T_{20} + 11)$. 

C_2= the gradient of the straight line relating the ratios of maximum ds/dM and age conversion factor
The temperature efficiency (η) factor is then the ratio of the “compression”/“acceleration” factors. Thus, for 50 °C:

a) PC30 has a Temperature efficiency factor = \eta = \frac{1.419}{2.090} = 67.9%.

b) PC45 has a Temperature efficiency factor = \eta = \frac{1.200}{1.506} = 79.7%.

and,

c) PC60 has a Temperature efficiency factor = \eta = \frac{1.440}{1.833} = 78.6%.

The effect of the “acceleration” and the temperature efficiency factors can be seen in Fig. 8 in terms of the 1/S.ds/dM relationship with maturity. The predicted and the actual are difficult to distinguish as they overlap. It may at first appear surprising that when the predicted ds/dM at 50 °C and for 100% temperature efficiency for all three concretes are converted to 1/S.ds/dM they again overlap the 67.4%, 79.6% and 78.6% temperature efficiencies for PC30, PC45 and PC60 respectively and their actual relationships. This is because strength and rate of strength development are interrelated, and both are needed for the relationship with maturity.

The compression factor (CF) and temperature efficiency factor (η) are both needed to set up the procedure for estimating the strength development for either elevated curing temperatures or also for non-isothermal curing conditions using a spreadsheet. This can be set up as an iterative procedure which is required to account for the effect of temperature at different maturities. Fig. 9 shows the spreadsheet that has been developed for this purpose. The first column (A) requires the temperature history of the concretes/mortars (in this case it is isothermal), the second (B) and third (C) is the time (in hours and days). The maturity increment is then calculated in the fourth column (D) as is the maturity shown in the fifth column (E). The “accelerated” maturity makes use of the acceleration factor (AF) previously determined. The maturity increment is multiplied by the acceleration factor (AF), calculated in column (F), and then the maturity calculated in the previous cell is added to it. The “stretch factor” is obtained by dividing the “accelerated maturity” (column F) with maturity (column E). The compression factor (column H) is calculated from Eq. (17). The temperature efficiency factor (column I) is the (Compression Factor)/(Stretch Factor). Strength at the accelerated maturity is then calculated in column (J) using Eq. (7) with constants for 20 °C shown in Table 3. The maturity M used is the accelerated maturity in column (F). The accelerated (\frac{ds}{dm})_\text{accelerated} is then calculated (Column K) based on Eq. (10) noting that the accelerated maturity needs to be used:

$$
\left(1 \frac{ds}{dm}\right)_\text{accelerated} = \frac{a \tau^b}{M_\text{accelerated}}
$$

![Fig. 8](image-url) "Acceleration" and “temperature efficiency” factors used to transform the 20 °C relationship between 1/S.ds/dM and maturity to that of specimens cured at 50 °C.
The above has been calculated based on a temperature efficiency of 100% but the strength increment for a certain maturity increment, i.e. \( \frac{dS}{dM} \), is affected by both the acceleration factor as well as the temperature efficiency factor (\( \eta \)). Thus:

\[
\frac{1}{S_{\text{accelerated}}} \left( \frac{dS}{dM} \right)_{\text{shifted}} = \left( \frac{1}{S_d}\frac{dS}{dM} \right)_{\text{accelerated}} \times \left( \frac{\Delta M_{\text{accelerated}}}{\Delta M} \right) \times \text{Temp Eff (TEF)}
\]  

(17)

Fig. 9. Spreadsheet developed for the transformation of strength based on the Modified Nurse-Saul (MNS) maturity method.

![Fig. 9](image)

Fig. 10. Strength development estimates obtained from the Modified Nurse-Saul (MNS) function.

![Fig. 10](image)
The above calculation results in a shift upwards of the accelerated \( \frac{dS}{dM}_{\text{accelerated}} \) to that of the \( \frac{1}{\text{accelerated}} \frac{dS}{dM}_{\text{shifted}} \). The shifted relationship is shown in column (L).

The \( \frac{dS}{dM}_{\text{modified}} \) is shown in column (M) and the cumulative strength, i.e. the strength development, is shown in column (L).

The 20 °C strength development versus age relationship has been used with the above procedure to estimate the strength development at curing temperatures of 30, 40 and 50 °C and these are shown in Fig. 10. Fig. 11 shows the estimated/actual strength ratio using the Modified Nurse–Saul (MNS) method. Previous work [36], for a PC mortar with nominal cube compressive 28-day strength of 50 MPa indicated that the percentage error in the strength estimates was 20% over-estimate by the Nurse–Saul function and up to 60% by the Rastrup function for the age of 12 h. All maturity functions, i.e. Nurse–Saul, Weaver–Sadgrove, Rastrup, the Arrhenius, and the Dutch weighted maturity method tended to still over-estimate the strength by 40% even after 28 days. The strength estimates from the Modified Nurse–Saul (MNS) function, see Fig. 11, are more accurate than those obtained from other maturity functions; the under-estimation of the strength is less than 20% at 12 h and the over-estimation is below 10% at later ages. The incorporation of the detrimental effect of high early age temperatures on the long-term strengths results in not only improved early age strength estimates but also improved strength estimates for later ages. Other maturity functions only satisfactorily estimate the strengths up to 72 h beyond which the estimates deviate from the actual as a result of the long-term detrimental effect to strength of early age elevated curing temperatures [36–38].

5. Conclusions

The effect of high early curing temperature is to accelerate the cement hydration, i.e. it starts earlier, and thus early age strengths are higher. This “acceleration” factor can be obtained by plotting the ratio of maturities at peak dS/dM versus the ratio of relative “temperatures + datum”, i.e. \( (T+11)/(T_{20}+11) \).

The effect of the “acceleration” is to compress a specific reaction interval in a smaller duration and therefore the magnitude of the rate of strength development needs to account for this. It has up till now been assumed that the efficiency of the reaction is unchanged at higher curing temperatures. However, the reaction was found to be less efficient at higher temperatures and therefore a “temperature efficiency” factor \( \eta(T) \) is proposed to be incorporated into maturity functions. This “temperature efficiency” factor can be obtained by plotting dS/dM at maximum values versus the ratio of relative “temperatures + datum”, i.e. \( (T+11)/(T_{20}+11) \).

It is suggested that strength-maturity functions require not only an age conversion factor, in the form of an activation energy or an “acceleration” factor, but also a temperature efficiency factor as outlined above. Both are mixture specific but can be incorporated into the Nurse–Saul function to improve the strength predictions for curing temperatures other than the reference one. The Modified Nurse–Saul (MNS) function, i.e. incorporating “acceleration” and “temperature efficiency”
factors, has been set up as an iterative procedure using a spreadsheet and the strength estimates have been shown to be more accurate than those from other maturity functions not only for the early age but also long term strengths. Validation of the Modified Nurse-Saul function is needed for non-isothermal curing conditions.

Conflicts of interest statement

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest, or non-financial interest in the subject matter or materials discussed in this manuscript.

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