Properties of hydraulic jumps over apparent corrugated beds

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ABSTRACT

The characteristics of hydraulic jumps were investigated for three shapes of artificial apparent corrugated beds in a horizontal rectangular flume. Rectangular, triangular, and circular-shaped tire waste corrugated beds were used. Froude number ranged from 2.75 to 4.25. The experimental observations included water surface profiles, bed shear stress, and the hydraulic jump length. Results showed that the shape of the corrugation had relatively insignificant effects on hydraulic jump properties for small Froude numbers. The rectangular, triangular, and circular-shaped corrugated beds reduced the hydraulic jump length by up to 7, 10, and 11%, respectively. The corrugated bed also reduced the tailwater depth by up to 11.5% compared with the smooth bed. The apparent conditions of corrugated bed reduced the hydraulic jump relative length and height by about 0.4 and 0.5, respectively. The circular-shaped tire waste was found to be more effective in reducing the length and depth of the hydraulic jump.

Keywords: Hydraulic structures, Apparent corrugated bed; Bed shear stress; tire waste.

1. Introduction

A hydraulic jump is the rapid and sudden transition from a high-velocity supercritical flow to a subcritical flow. Hydraulic jumps are widely used for energy dissipation downstream of hydraulic structures and are common in rivers and canals, in industrial applications and manufacturing processes. When the jump is formed in a wide rectangular and horizontal channel with a smooth bed, it is called classical jump which has been studied extensively in the literature (e.g. Peterka 1958, Rajaratnam 1967, McCorquodale 1986, Hager 1992).

Many studies investigated the hydraulic jump characteristics in corrugated bed channels. The results indicated that corrugated bed has significantly reduced the tailwater depth ($y_t$), as well as the length of the jump (e.g. Rajaratnam 1968, Hughes and Flack 1984, Alhamid 1994, Ead et al. 2000, Samadi-Boroujeni et al. 2013). Ead & Rajaratnam (2002) found that the shear stress of rough beds is ten times higher than that of smooth beds, and the length of the jumps was about the half. In another study, Ead (2007) found that, for three tested shapes of corrugated beds (sinusoidal, triangular and trapezoidal), the tailwater depth required to form a jump was appreciably smaller than that of smooth beds. The length of the jump for the different corrugated beds was less than half of that of smooth beds. The integrated bed shear stress for corrugated beds was more than 15 times that of smooth beds.

A study by Izadjoo and Shafai-Bajestan (2005) on a trapezoidal-shaped corrugated bed showed that the jump length was more dependent on the wavelength than wave amplitude. The depth and length of the hydraulic jump were reduced by 20% and 50%, respectively, compared with those of the smooth
bed. Samadi-Boroujeni et al. (2013) studied the effect of the triangular corrugated beds on the hydraulic jump characteristics and their results showed that the corrugated bed reduced the jump depth and length by 25% and 54.7%, respectively. The shear stress coefficient for the corrugated bed was about 8.5 times of that of smooth bed.

Abdelhaleem et al. (2012) experimentally studied the effects of three classical corrugated beds with semi-circular, trapezoidal, and triangular shapes on hydraulic jump. The jump length was reduced by 10%, 11%, and 14% for semi-circular, trapezoidal and triangular corrugated beds, respectively. They also found the shear stresses of the semi-circular, trapezoidal and triangular corrugated beds to be about 8, 9 and 11 times of those of smooth bed respectively. Carollo et al. (2007, 2009), who studied hydraulic jumps over both smooth and various rough horizontal beds made up of closely packed crushed gravel particles cemented to the bottom, proposed a generalized solution for the momentum equation for sequent depth ratio of a hydraulic jump over both smooth and rough beds. The jump characteristics were studied for homogeneous and non-homogeneous roughened bed channels, and a jump equation that accounts for bed roughness and non-homogeneity was proposed by Pagliara et al. (2008).

Mohamed-Ali (1991) conducted a series of experiments to study the effect of roughened beds using regularly placed cube blocks that occupied 10% of the bed surface. He found that the length of the jump was reduced by around 27 to 67% for Froude number ranged from 10 to 4, respectively. The more reduction values of the hydraulic jump length were for Froude number less than 6. Mahound (1984) and Abdelsalam et al. (1986) found that the optimum bed roughness intensity of cubic shape is 10% from both the hydraulic and economical point of view, and the best ratio for the height of the cubic roughness (r) was r/y1=0.4 to 0.5 (Ezizah et al. 2012). Also, Ezizah et al. (2012) studied the hydraulic jumps on new roughened beds (U-shape) and concluded that the best roughness intensity was 12.5% and the best relative roughness length is 18. Aboulatta et al. (2011) concluded that the T-shape-roughened bed reduced the jump length and materials more than the cubic block-roughened bed.

The corrugated beds have effectively dissipated energy downstream hydraulic structures, which decreases the cost of stilling basins. The apparent triangular and U-shape corrugated beds were found to be more effective in reducing the jump length and sequent depth. Also, the relative corrugation height had insignificant effects on the jump characteristics (Abdelhaleem et al. 2012, Ead 2007, Ezizah et al. 2012, Shafai-Bajestan & Neisi 2009, Samadi-Boroujeni et al. 2013, Imran & Akib 2013, Izadjoo and Shafai-Bajestan 2005).

From the literature, it is clear that most of the previous studies have focused on the case when the corrugated bed was unapparent, i.e. embedded within the bed of the channel. To our best knowledge, no previous study has investigated the case when the corrugated bed was apparent. The tire-shred
waste corrugated bed was not also found in the literature. Therefore, in this study, the tire-shred waste was used as an apparent corrugated bed beside the rectangular and triangular shapes. Scrap tires are increasingly used in today’s growing market of civil engineering applications. Tire shreds are used for a wide range of applications, including drainage layers beneath roads, vibration damping layers beneath rail lines, landfill leachate collection layers, drainage layers in landfill, drainage aggregate materials for wastewater treatment, and lightweight backfill materials for walls and bridge abutments (Humphrey & Katz 2000, Brophy & Graney 2004, Sheehan et al. 2006, USEPA 2010).

The hydraulic jump characteristics of smooth and three different apparent corrugated beds were analysed and compared to identify the most effective conditions of the channel bed for reducing the jump length and sequent depth. These apparent corrugated shapes were tested for a range of Froude numbers from 2.75 to 4.25. This research is of practical applications to dams and other hydraulic structures, where the jump is used to dissipate the excess kinetic energy.

2. Materials and methods

The experiments were conducted over both smooth and three apparent corrugated beds installed on the flume bed to investigate the hydraulic jump properties. The hydraulic jump was produced in a rectangular flume that was 10 cm wide, 15 cm deep and 9 m long. The side walls of the flume were made of transparent Plexiglas sheets. Water was pumped by a centrifugal pump from a storage tank to the head tank before releasing it into the flume. Water depth was controlled by a control valve installed on the pipe, which was connected to the feeding pump. The point gauge was used to measure both the jump depths and water surface profile along the centreline of the flume. Moreover, the initial depth ($y_1$), sequent depth ($y_2$) and jump length ($L_j$) were measured at the same time.

The corrugated beds were built by using rectangular, triangular, and circular tire waste sheets on the flume bed, as shown in Figs. 1-3. The corrugation created a system of turbulent eddies, which increased the bed shear stress. The wavelength and amplitude of the corrugation beds were 40, and 20 mm respectively (Fig. 2).

To determine the Froude number of the flow, a measuring tank at the end of the flume was used. The time to fill the required volume was recorded, and Eq. (1) was used to calculate the Froude number $F_1$ of the flow:

$$F_1 = \frac{v}{\sqrt{gy_1}} \tag{1}$$

where $v$ = velocity of water, $g$ = gravitational acceleration, and $y_1$ = supercritical depth.

Different values of $F_1$ were considered with a discharge range of 4.95 to 6.8 l/s while the associated Reynolds number was always sufficiently high to guarantee a fully turbulent flow (Table 1). After
determining the Froude number of the flow, the gradient slope was adjusted slowly to create the hydraulic jumps and to control the position of the jumps (Figs. 2 and 3). The jump was adjusted directly at the starting point of the length indicator. This step took time, as the jump slowly moved into its equilibrium position. When the jump reached the starting position of the length indicator and became stable, the depths of the jump were recorded every 5 cm along the flume (Fig. 3).

The next step was to take the measurement of the jump on the corrugated beds. The corrugated beds were slowly and carefully placed on the bottom of the flume at the starting point of the length indicator. The gradient slope was then adjusted by screwing the tail gate until the appropriate jump position is obtained. After that, the hydraulic jump characteristics were measured as described above and these test steps were repeated for all other shapes of corrugated beds.

3. Results and discussion

3.1. Water Surface Profiles

The length of hydraulic jump ($L_j$) and the subcritical sequent depth ($y_2$) at the end of the jump were calculated based on the water surface profile, which was completely horizontal. The water surface profiles for the three corrugated and smooth beds are shown in Fig. 4. The trends of the water surface profiles were similar for all types of corrugated beds, and the fluctuations of jumps height did not show very high magnitude for the lower range of Froude number.

3.2. Sequent Depth Ratio

The length of hydraulic jump ($L_j$) and the sequent depth ($y_2$) over corrugated beds mainly depend on the upstream flow characteristics, such as flow velocity ($V_1$), flow depth ($y_1$), fluid density ($\rho$), as well as viscosity ($\mu$), acceleration of gravity ($g$), bed corrugation amplitude ($t$), and the shape and installation conditions of the corrugated bed ($\zeta$). Thus, $y_2$ and $L_j$ can be written as:

$$y_2 \text{ or } L_j = f(V_1, y_1, g, \rho, \mu, t, \zeta)$$

(2)

Considering $y_1, g, \text{ and } \rho$ as three repeated variables and using the Pi theorem, it can be written in the following form:

$$y_2 / y_1 \text{ or } L_j / y_2 = f(F_1 = V_1 / \sqrt{gy_1}, Re = V_1 y_1 / \mu, t / y_1, \zeta)$$

(3)

where $F_1$ and $Re$ are the Froude number and Reynolds number, respectively, at the upstream side of the jump. For larger Reynolds number, viscous force can be neglected (Rajaratnam 1976, Hager and Bremen 1989), and final expressions of sequent depth and length of the jump can be developed as:

$$y_2 / y_1 \text{ or } L_j / y_2 = f(F_1, t / y_1, \zeta)$$

(4)
The graphical presentation of the sequent depth ratio \((y_2/y_1)\) against the different Froude numbers for relative roughness \((t/y_1=0.67)\) is shown in Fig. 5. Generally, the relative sequent depth ratio of all the apparent corrugated beds was smaller than that of smooth bed. For the apparent triangular corrugated bed, a relationship was established between the sequent depth ratio \((y_2/y_1)\) and the upstream Froude number \((F_1)\) and compared with Abdelhaleem et al. (2012) for the same corrugated bed shape and Froude number. The \((y_2/y_1)\) ratio has the same trend but smaller than that of Abdelhaleem et al. (2012) by about 0.5. The difference between the two results is because of the corrugated bed installation conditions, which in the present study was installed above the bed, while in Abdelhaleem et al. (2012) the triangular apex was in the same level with the bed surface. The results also suggested that the relative roughness had small effects on the sequent depth ratio.

Fig 6 shows the relation between the mean \(y_2/y_1\) values and Froude number as mean values for all shapes, in which the sequent depth ratio was nearly 96% of Froude number. The relationship trend was similar and corresponds to previous findings of Izadjoo & Shafai-Bajestan (2005), Ead (2007), and Abdelhaleem et al. (2010). This relationship can be defined as:

\[
y_2 / y_1 = 0.963F \tag{5}
\]

The difference between the tailwater depth of classical jump \((y_2^*)\) and tailwater depth of corrugated beds \((y_2)\) was estimated using the following Equation:

\[
D = \frac{y_2^* - y_2}{y_2} \tag{6}
\]

where \(D\) is the dimensionless index. From current experiments, the values of \(D\) were around 0.08, 0.09, and 0.115 for rectangular, triangular, and circular corrugated beds, respectively. These results demonstrate that the tailwater depths of hydraulic jumps over the apparent corrugated beds were reduced by up to 11.5% compared to the smooth bed channel. Abdelhaleem et al. (2012) found that the maximum \(D\) value at 0.174 for unapparent (classical) triangular corrugated bed, which indicated that the tailwater depths were reduced for this shape by 17.40% compared to the smooth bed. The difference in the maximum \(D\) values for the triangular shape (5.9%) between the current study and that of Abdelhaleem et al. (2012) is because of the turbulent eddies, which increased the bed shear stress.

### 3.3. Jump Length

The relationship between Froude number \((F_1)\) and dimensionless length of the jump \((L/y_2)\) is shown in Fig. 7. The jump length of the corrugated beds was smaller than that of smooth bed. The reduction of the jump length depended on the Froude number. Abdelhaleem et al. (2012) found that the corrugated beds had a small effect on the jump length when the Froude number was less than three.
As shown in Fig. 7, values of jump relative length \( L_j/y_2 \) in case of an apparent triangular bed were compared with those of Abdelhaleem et al. (2012). The \( L_j/y_2 \) values have the same trend but smaller than those in Abdelhaleem et al. (2012) by about 0.40.

The magnitude of reduction of the jump length is calculated from the following Equation:

\[
L_s = \frac{L_{yj} - L_j}{L_j} \times 100
\]

(7)

where \( L_s \) is the reduction percentage of jump length, and \( L_{yj} \) and \( L_j \) are the jump length of smooth bed and corrugated bed, respectively. The average reduction values of the jump length for circular tire waste, triangular, and rectangular corrugated beds were around 11%, 10%, and 7%, respectively. This result indicates that roughened beds were effective in reducing the length of hydraulic jump; this reduction increases as the Froude number increases.

3.4. Bed Shear Stress

The sequent depth and hydraulic jump length on corrugated beds were smaller than their respective values for the smooth bed, which resulted from the increased bed shear stress caused by the bed corrugations. The interaction forces between the supercritical flow of the liquid and bed corrugations had significant effect in increasing the bed shear stress especially at high values of Froude number and for the apparent corrugated beds. The apparent bed corrugations induced more turbulent intensity, which generated more drag force and bed shear stress, and consequently reduced the sequent depth and hydraulic jump length. In this study, circular shaped tire waste was relatively more efficient than smooth bed and the other tested shapes of corrugated beds to produce more eddies and drag forces against the flow direction for increasing bed shear stress. The integrated bed shear stress and the shear force \( (F_i) \) can be calculated using the momentum Eq. (8):

\[
F_i = (P_2 - P_1) + (M_1 - M_2)
\]

(8)

where \( P_1, P_2, M_1, \) and \( M_2 \) are the integrated pressures and momentum before and after the hydraulic jump. In addition, the shear force index \( (\varepsilon) \) is calculated using Eq. 9 (Rajaratnam 1965):

\[
\varepsilon = \frac{F_i}{0.5 \gamma y_1^2}
\]

(9)

where \( \gamma \) is the kinematic viscosity of water. Figure 8 shows the shear force index variations \( (\varepsilon) \) against different values of Froude number \( F_I \). The shear stresses over the rectangular, triangular, and circular corrugated beds were almost 5-, 7-, and 8-times more than that of smooth bed. The shear stresses over the apparent triangular corrugated beds were almost 4-6 times that of unapparent triangular in Abdelhaleem et al. (2012). The study illustrated that the corrugated bed of circular-shaped tire waste
was the most effective to produce more drag forces, and it consequently generated more bed shear stress.

The relationship between shear force index and Froude number is shown in Fig. 9. The trend of the relationship obtained in the present study is similar to that obtained by other investigators (Ead & Rajaratnam 2002, Izadjoo & Shafai-Bajestan 2005, Ead 2007, Ead & Elsebaie 2009, Abdelhaleem 2012). The relationship between the shear force index and Froude number found in the current study was:

\[ \varepsilon = 0.405 F_1^2 - 0.253 F_1, \quad R^2 = 0.98 \]  

(10)

4. Conclusions

We investigated the characteristics of hydraulic jumps for three shapes of artificial apparent corrugated beds. To our best knowledge, the circular corrugated bed made from tire waste has not been tested in previous studies. The jump height for apparent circular shaped tire waste corrugated beds was consistently slightly lower than that of smooth beds and other apparent corrugated beds of triangular and rectangular shapes.

The apparent rectangular, triangular, and circular corrugated beds reduced the tailwater depth of the jump by about 8%, 9%, and 11.5%, respectively, and the jump length was reduced by up to 7%, 10%, and 11%, respectively, compared to smooth bed surface. The bed shear stresses of apparent rectangular, triangular, and circular-shaped tire waste corrugated beds were almost 5-, 7-, and 8-times the corresponding stress of smooth bed. The shear stresses over the apparent triangular corrugated bed were almost 4-6 times that of unapparent triangular corrugated bed.

The relative sequent depth \( y_2/y_1 \) values in case of apparent triangular corrugated bed have the same trend but smaller than those for unapparent triangular corrugated bed by about 0.5 for Froude number ranged from 2.75 to 4.5. An apparent corrugated bed produced more eddies due to the interaction between supercritical flow liquid and corrugations of channel bed and thus was efficient to produce more drag force against the flow direction, and consequently the jump length and sequent depth were significantly reduced. Circular-shaped tire waste in the corrugated bed produced more turbulent intensity and drag force compared to other tested corrugated beds, which in turn induced more shear stresses at the bottom of the channel bed.

Future studies should investigate the high performance of the tire waste in corrugated bed. These studies may also consider the large Froude numbers with variations in wavelength and amplitude. Further research should also be carried out to confirm the beneficial effects of tire waste on durability issues and thus to encourage the use of this material as corrugated bed material.
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References


**Table 1** The experimental flow conditions and parameters.

<table>
<thead>
<tr>
<th>Types of bed</th>
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<th>Velocity ($m/s$)</th>
<th>Froude No.</th>
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</table>
Figure 1 Different shapes of corrugation (Rectangular, Circular waste tire, and Triangular).
Figure 2 Hydraulic jump on apparent triangular and rectangular corrugated beds.
Figure 3 Formation of hydraulic jump at length indicator (a) on smooth bed, (b) on triangular-shaped corrugated beds, (c) on rectangular-shaped corrugated beds.
Figure 4 Water surface profiles at different Froude number and bed conditions.
Figure 5 Variation of the $y_2/y_1$ against $F_1$ over different bed formations.
Figure 6 Relation between $y_2/y_1$ and $F_1$ over corrugated bed.
Figure 7 Variation of normalized jump length \( \frac{L_j}{y_2} \) with Froude number \( F_1 \).
Figure 8 Variation of shear force index ($\varepsilon$) against different Froude number ($F_1$).
Shear Force Index ($\varepsilon$)

Present Study; $\varepsilon = 0.4051F_1^2 - 0.253F_1$  \( R^2 = 0.98 \)

**Figure 9** Relation between shear force index ($\varepsilon$) and Froude number ($F_1$).