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Experimental Validation of an Integral Sliding Mode-Based LQG for the Pitch Control of a UAV-mimicking Platform

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Abstract. In this paper, an enhanced Integral Sliding Mode-based Linear Quadratic Gaussian (ISM-LQG) controller has been proposed and verified in real-time on a Twin Rotor multi-input-multi-output MIMO System (TRMS). A TRMS serves as a suitable laboratory-based platform to evaluate the performance of control algorithms for complex Unmanned Aerial Vehicle (UAV) systems such as rotocraft. In the proposed scheme, an ISM enhancement to an LQG has been introduced, which attempts to overcome modelling inaccuracies and uncertainties. The novelty of the proposed control law lies in hybridizing a robust control approach with an optimal control law to achieve improved performance. Experimental results on the TRMS demonstrate that the ISM-LQG strategy significantly improves the tracking performance of the TRMS pitch and hence confirm the applicability and efficiency of the proposed scheme.

Keywords

Experimental set-up, Integral Sliding Mode control, Linear Quadrature Gaussian, optimal dynamic control, Twin rotor MIMO system.

1. Introduction

In recent years, research and development in the field of Unmanned Aerial Vehicles (UAVs) has gained significant attention in the engineering community. They are extensively being used for reconnaissance, surveillance and scientific data acquisition in various applications. The reduction in cost has also added hobbyists to its growing number of users. Some of the UAV types currently in use include fixed-wing aircraft, tandem rotors, twin-rotor systems and quad-rotor hover crafts. This substantial interest may be attributed to various distinguishing features offered by these crafts including their ability to fly unmanned while invisible to radar. Other factors that have stipulated the developments in UAVs include their lightweight structure and multi-purpose deployment in multi-dimensional applications. Due to these advantages, the field of UAVs is expected to expand incredibly in the near future. However, the control of UAVs poses many challenges due to their highly nonlinear behaviour, dynamic operational environment and other constraints due to their small size, weight, and power.

A summary of the most promising control strategies for different types of UAVs can be found in \cite{1, 2, 3, 4}. In the work introduced in \cite{6}, the author...
employed soft computing techniques to control a quad-rotor vehicle.

Adaptive schemes are also being explored for UAVs control [7]. Additionally, optimal control techniques, which have been around for many decades, such as the Linear Quadratic Gaussian (LQG), are also being developed for autopilot and missile trajectory controller applications. Among the related works reported in the literature is [8], which uses a multi-variable LQG controller which takes into account noise and disturbance. A Proportional Linear Quadratic Regulator (P-LQR) controller for the longitudinal control of a UAV was presented in [9]. A comparison between the LQR and $H_{\infty}$ algorithms for UAV control can be found in [10].

One of the major challenges faced by the researchers is the high complexity and implementation cost of actual UAV devices. Therefore, in order to facilitate the development of robust and sophisticated control algorithms, laboratory platforms have been developed for many of these vehicles. Despite several reported attempts to conduct experiments on these platforms, design and realization of sophisticated control algorithms on the setups still remain challenging. In this paper, we are focussing on the LQG control of the Twin-Rotor MIMO System (TRMS) developed by Feedback Instruments Ltd., UK. The TRMS resembles a conventional helicopter. However, the angle of attack of the rotors is fixed and consequently, the aerodynamic forces are controlled by varying the speed of the DC motors. The TRMS setup serves as a good platform for the implementation purpose and in most cases, the algorithms developed for the TRMS can be easily extended to UAVs.

A plethora of literature on developing control algorithms for the TRMS has been presented over the last two decades. Most of the reported studies are based on Proportional-Integral-Derivative (PID) control law or its variants. The PID has become a benchmark control algorithm as it offers a simple structure, model free and easier implementation [11]. Moreover, if the parameters of the system are precisely known, the controller gains can be designed analytically [12]. In [13], the authors proposed a sigmoid-based variable coefficient PID control law for the TRMS, where the PID coefficients were dynamically updated within a predefined range. In [14], Cajo and Agila presented a comparison between linear and nonlinear PID implementations and derived a set of nonlinear segmented observers for the required states corresponding to all degrees of freedom. A PID-based fuzzy sliding mode control algorithm was implemented in [15] in order to reduce the tracking errors and chattering in the control input, which is usually present in the sliding mode strategy [16]. Another interesting piece of work is reported in [17], where a mathematical model was derived for the TRMS. This model was then used to implement self-tuning control algorithms based on the PID controller using the Takahashi modification of Ziegler-Nichols and the pole placement method with two degrees of freedom. Other noticeable recently reported works can be found in [18], [19], [20], [21] and [22].

In this paper, we examine the use of an enhanced LQG scheme for the pitch control of the TRMS. The LQG needs an accurate and precise model of the system. However, modelling in case of UAVs poses a major challenge due to their highly nonlinear and oscillatory nature as well as their exposure to environmental disturbances [23]. Moreover, due to loading and unloading, the dynamics is affected significantly. In such a scenario, the use of a standard LQG may not be very effective. Hence, we propose an Integral Sliding Mode Controller (ISMC) enhancement to the LQG controller, which can produce promising results in the face of adverse conditions. As a result, the effect of modelling uncertainty can be effectively overcome. This work combines robustness and ease of implementation of the LQG controller with the powerfulness of the ISMC to improve the performance of LQ-based controllers as demonstrated in [24].

2. TRMS System and Modelling

The TRMS supplied by Feedback Instruments Ltd. has two degrees of freedom and ships with a data acquisition and control apparatus that allows for real-time integration with numerous computer based simulation packages including MATLAB/Simulink. The system is depicted in Fig. 1.

For the LQG controller design, a discrete state space model of the TRMS can be described by:

$$\begin{align*}
\bar{x}[k+1] &= A\bar{x}[k] + B\bar{u}[k] + \bar{W}[k], \\
\bar{y}[k] &= C\bar{x}[k] + D\bar{u}[k] + \bar{V}[k],
\end{align*}$$

(1)
where matrix $A$ is the system state matrix, $B$ is the input matrix, $C$ is the output matrix, and $D$ is the coupling matrix representing the direct influence of changes in the inputs on the outputs. The vectors $\vec{W}[k]$ and $\vec{V}[k]$ respectively represent the white Gaussian process and measurement noises with the corresponding covariance matrices denoted by $W$ and $V$.

The matrices $A$, $B$, $C$, $D$ of the TRMS state space model were derived using Matlab/Simulink system identification toolbox employing input-output data (obtained experimentally from TRMS).

3. The LQG Controller

The LQG controller is one of the most common optimal control methods. It is basically a combination of a Kalman filter used to estimate the states of the system and the LQR. In this section, we first describe the LQR optimal controller. The LQR controller assumes perfect knowledge of the system states. Following LQR, we show the complete LQG solution with experimental results.

The optimal control for the system described by the state space model in Eq. (1) minimizes the quadratic cost function using infinite time horizon i.e.:

$$J = \sum_{k=0}^{N} \vec{x}[k]^T Q \vec{x}[k] + \vec{u}[k]^T R \vec{u}[k],$$

where $Q$ is the process weight covariance matrix and $R$ is the control weight covariance matrix. There is no analytical way to determine these covariance matrices. Thus, they must be determined experimentally through a tedious trial and error procedure. A trade-off exists between the two; choosing a large $Q$ penalizes the transients of the state vector $\vec{x}[k]$ and choosing a large $R$ penalizes the effect of the control action $\vec{u}[k]$.

In general, the optimal LQR solution is the row vector $\vec{u}(\cdot)$, which is used to determine the required control signal:

$$\vec{u}[k] = r[k] - \vec{\kappa}_{LQR}\vec{x}[k],$$

where $r[k]$ is the pitch demand at time step $k$. The LQR gain vector is given by:

$$\vec{\kappa}_{LQR} = (R + B^T PB)^{-1}(B^T PA).$$

This leads to the control signal given by:

$$U_{LQR}[k] = n_u r[k] - \vec{\kappa}_{LQR}\vec{x}[k].$$

The LQR does not compare the demand to the output of the system since it is a full state feedback controller. Instead, it compares the demand to the scalar $\vec{k}_{LQR}\vec{x}[k]$ resulting in the output being different from the demand, which is not the desired goal. In order to compensate for this, the reference as well as the estimated states have to be scaled. This is achieved by means of the pre-compensator scaling vector $\vec{n}$ defined as:

$$\vec{n} = \begin{bmatrix} n_x \\ n_u \end{bmatrix} = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where the scalar $n_v$ is used to apply a steady state value to the system input to remove any steady state error and $n_x$ produces the modified states $\vec{\tau}[k]$, i.e.:

$$\vec{\tau}[k] = \vec{n}_x^T \vec{x}[k].$$

The LQG controller was implemented in real-time on the TRMS using MATLAB/Simulink. The pitch angle demand was set to a smoothed square wave. Figure 2 shows the results of LQG-based control law. The covariances of the LQR and Kalman filter are tuned so as to obtain the best performance with the lowest possible oscillation levels. Kalman filter has two design parameteric matrices i.e $V$ is the measurement noise covariance matrix and $W$ is the process noise covariance matrix. It is quite evident from the figure that, even with the best tuning (which may not always be possible in practice as the covariances depend on the TRMS environment) the LQG does not perform very well. The behaviour is very oscillatory and suffers from significant overshoot. The high overshoot in the response may be attributed to various reasons including oscillatory nature of the TRMS. Additional factors include marginal stability of the system (which makes it very susceptible to the outside disturbances) and the existence of uncertainties in the system. Moreover, the tuning of the noise covariances for the LQR and Kalman filter, imperfections in the modelling of the
The input and output of the TRMS system (a), control signal (b), and Kalman filter tracking (c) using the LQG controller with $Q = \text{diag}(60, 60, 60, 60)$ matrix with diagonal elements equal 60, $R$ is $1 \times 1$ matrix having a value $= 10$, $W$ is $1 \times 1$ matrix $= 1000$, and $V$ is $1 \times 1$ matrix $= 1$.

TRMS, and the slow settling time of the device compared to the desired reference value are all adversely affecting the performance.

The matrices $V$ and $W$ of Kalman filter need to be appropriately tuned to get the desired filtering characteristics i.e., estimation of all the unavailable states and removal of the noise from the system output. It is reported in [30] that increasing the value of $W$ reduces transient response but at the cost of increased steady-state uncertainty. Large value of $W$ also improves learning from measurement data since $W$ acts as a forgetting factor in the learning process. $V$ has opposite effect as that of $W$. Both $V$ and $W$ should be chosen carefully to improve performance of Kalman filter subsequently resulting in the improved LQG performance.

For reducing steady-state error, LQG with integral action has also been investigated in [31] and [32]. However, it may lead to more oscillatory behaviour and excessive overshoot. In order to reduce the oscillation and achieve better tracking performance, the present work utilizes an Integral Sliding Mode (ISM) correction term with a certain weight as explained in the next section.

### 4. The Integral Sliding Mode Correction

The ISM Controller (ISMC) was first introduced in [33]. Unlike conventional sliding mode control, ISMC has a motion equation of the same order as that of the system itself. ISMC introduces compensation for matched disturbances right from the beginning. In addition, ISMC leads to less chattering compared to the conventional sliding mode. However, under no circumstances, the ISMC can account for and compensate for any unmatched disturbances. A good description of the control algorithm can be found in [24], [34] and [35]. In [36], the authors used the ISMC to enhance the robustness of the LQG for linear stochastic systems with uncertainties. In the work proposed by Philips [37], sliding mode control for TRMS has been studied in detail.

The overall controller proposed in the present work is conceptualized by the model shown in Fig. 3. First, we define the ISMC correction term as:

$$s[k] = \dot{e}[k] + K_r e[k] + K_i \int_0^k e[\xi] \, d\xi - e[0] - K_r e[0],$$  

where $e[k]$ is the error in the pitch angle, $\dot{e}[k]$ represents the error in the pitch velocity. $K_r$ and $K_i$ define
the desired behaviour of the control scheme once the sliding motion is achieved. Note that the discrete time integration in [9] was achieved using the forward Euler integration method.

The ISMC control law can be written as follows:

\[ U_{\text{ISMC}}[k] = \alpha \frac{s[k]}{\|s[k]\| + \delta}. \]  

(10)

In order to improve the performance of the LQG controller, we combine LQG control input, \( U_{\text{LQG}} \) (given in Eq. (8)) with ISMC control law given in Eq. (10) i.e. \( U[k] = U_{\text{LQG}} + U_{\text{ISMC}} \), to get the following equation as a final ISMC-LQG control law:

\[ U[k] = n_r[k] - \tilde{K}_{\text{LQR}} \hat{x}[k] + \alpha \frac{s[k]}{\|s[k]\| + \delta}, \]  

(11)

where \( \alpha > 0 \) is used to scale the effect of the ISMC correction. It should be chosen large enough to reduce the effect of uncertainty and to achieve the desired robustness. The scalar \( \delta \) is then used to control the level of chattering present in the ISMC; a large value of \( \delta \) less chattering [38] and [39].

5. Results and Discussion

In order to assess the performance of the proposed ISMC–LQG algorithm for the pitch angle control of TRMS different types of desired angles were considered, namely a filtered square wave and a multi-step input. A sampling time of 1 ms was used for all the experiments. The results are discussed in the following subsections.

5.1. Filtered Square Wave Input

In the first part of the experiment, the desired reference signal was chosen to be a square wave with a very low frequency of \( f = 0.04 \text{ Hz} \). Good tracking results (see Fig. 4a) are produced using LQG plus ISMC control.

In the second part of the experiment, the desired reference signal was chosen to be a square wave with a lower frequency of \( f = 0.03 \text{ Hz} \). In order to reduce the frequency range of the input, a low pass filter was used to smooth the edges of the input signal as shown in Fig. 5. The main control parameters for this experiment were chosen as \( \alpha = 0.3, K_i = 5, K_v = 1000, Q = 50, R = 100, V = 1, \) and \( W = 1000 \), leading to a reasonable tracking performance superior to that of the LQG and LQG with an additional integral action.

Since our choice of the control parameters plays an important role in determining the performance of the controller, it is essential to tune these variables to achieve the best performance. For instance, fixing the ISMC weight at \( \alpha \) and varying the values of \( K_v \) and \( K_i \), we can achieve different behaviours. We learned that increasing the weight \( K_v \) leads to a lower settling time but increases the chattering.

5.2. Multistep Input

In this part of the experiment, the reference was modified to a multi-step signal with a frequency of \( f = 0.05 \text{ Hz} \) and a step size of 0.2 V, 0.3 V, 0.4 V and 0.5 V. The proposed algorithm clearly outperforms the conventional LQG as well as LQG with an additional integral action in terms of reduced oscillations and overshoot in step response as illustrated in Fig. 6.
However, the downside of ISMC is the chatter and aggressive nature which may shorten actuator life if not taken into consideration. To reduce chattering and to make the control signal less aggressive, the parameter $\delta$ in Eq. (11) should be increased. Figure 7 reveals that when $\alpha$ is increased, the performance becomes better.

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**Fig. 5:** Experimental results for a square wave demand using LQG, LQG with an integral action and ISMC (a) along with the corresponding control signals (b) and (c) with $f = 0.03$ Hz, $\alpha = 0.3$, $K_i = 1$, $K_r = 1500$, $Q = 50$, $R = 100$, $V = 1$, and $W = 1000$.

**Fig. 6:** ISMC-LQG experimental results for a multi-step input (a) and the corresponding control inputs for LQG and Integral-LQG (b) and for the LQG+ISMC (c). The main control parameters were chosen as $\alpha = 0.3$, $\delta = 100$, $K_i = 1$, $K_r = 1500$, $Q = 150$, $R = 100$, $V = 1$, and $W = 1000$. 

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6. Conclusion

In this paper, the experimental realization of an ISMC based LQG controller for the pitch control of the TRMS was presented. The controller uses a Kalman filter to estimate the states of the system in order to calculate the optimal LQR gains. It was observed that the presence of modeling imperfections and uncertainties degrades the tracking performance of the LQG. An ISMC enhancement was applied to the LQG to improve its real-time tracking performance. Experimental results were presented to show the resilience and robustness of the proposed control scheme using different reference functions.

References


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