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# Pilot Assignment and Power Allocation for Multipair Massive MIMO DF Relaying Networks

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**Abstract**—This paper considers a multi-pair massive multiple-input multiple-output relaying channel where  $K$  single-antenna sources communicate with  $K$  single-antenna destinations via the assistance of a decode-and-forward relay equipped with massive antenna arrays. The relay station uses maximum-ratio processing to decode the signals transmitted from the sources and precodes the signals before broadcasting to the destinations. To perform maximum-ratio processing, the relay needs to know the channels to the sources and the destination. This is done during the training phase where all sources and destinations send pilot sequences to the relay. We consider a very general case in which the pilot sequences are arbitrary. An exact closed-form expression for the end-to-end spectral efficiency is derived taking into account the imperfect channel estimates with arbitrary power control coefficients. From the spectral efficiency formula, we formulate and propose to solve a max-min fairness power allocation. With this power control scheme, the spectral efficiency improves significantly, compared to the uniform power control. In particular, a simple greedy pilot assignment scheme is proposed to reduce the pilot contamination effect, and hence, improve the system performance under the limitations of training duration.

**Index Terms**—Channel estimation, decode-and-forward, massive MIMO, maximum-ratio processing, multipair relaying channels, pilot assignment, power allocation.

## I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is one of the core technologies in improving the data speed and the reliability of wireless communication systems, especially for the fifth generation mobile networks and beyond [1]–[6]. In massive MIMO, many antennas at the base station can simultaneously serve many users in the same frequency resource. By leveraging the properties of large antenna arrays (i.e. channel hardening and favorable propagation), massive MIMO can reduce substantially the noise effect and interference, and hence, can offer very high energy efficiency and throughput with simple linear processing. As a consequence, there has been a great deal of interest in massive MIMO from the communications engineers in both academia and industry.

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When massive MIMO is combined with relaying technologies, the concept of multipair massive MIMO relaying system comes up, which is one of the most widely adopted ways for extending the coverage area and improving the data rate of the users at the cell edge of cellular networks [7]–[12]. Reference [7] analyzed the spectral and energy efficiency of a multi-pair massive MIMO relay network with ZF processing under the practical RF-chain constraint. In [8], the authors investigated the performance of a multipair massive MIMO full-duplex decode-and-forward (DF) relaying system under the assumption of orthogonal pilot sequences. Moreover, this paper showed the spectral efficiency comparison between maximum-ratio (MR) processing and zero-forcing (ZF) processing. The wireless information and power transfer of multipair massive MIMO DF relaying systems were exploited in [10]. Paper [12] proposed a specific pilot transmission scheme in which pilots are sent in partial overlap with the payload data to reduce pilot contamination effect and the channel estimation overhead. Most of above works have assumed that all pilot sequences transmitted from the sources and destination during the training duration are pairwise orthogonal. This requires that the training duration (in symbols) is at least equal to the total number of the sources and the destinations. However, the training duration is limited by the coherence interval. In many scenarios where the number of sources and destinations is large or/and the coherence interval is short (high mobility environments), we cannot use a long training duration [13], [14]. As a consequence, non-orthogonal pilot sequences are used across the sources and the destinations. With non-orthogonal pilots, pilot contamination appears which can seriously compromise the system performance [15], [16]. Thus, the design of pilot assignment is of particular importance in multipair massive MIMO relaying networks.

Motivated by the aforementioned discussion, in this paper, we consider a multi-pair DF massive MIMO relaying channel where multiple sources transmit their signals to multiple destinations with the aid of a massive-antenna relay station. We consider a very general case where the pilot sequences assigned to the sources and the destinations during the training phase can be arbitrary. The power allocation, a novel pilot assignment scheme, and channel estimation overhead are investigated. The key contributions of our paper are as follows:

- We provide an exact closed-form expression for the spectral efficiency of the considered system model. This closed-form expression is very general which can be used for any pilot sequences and any power control coefficients.

- Based on the closed-form expression of spectral efficiency, we propose a max-min fairness power optimization problem which aims at maximizing the smallest spectral efficiency of all source-destination pairs. This optimization problem can be reformulated in a form of geometric programs and can be solved efficiently using convex optimization tools. The power allocation is implemented over large-scale fading time-scales and can considerably improve the system performance.
- We also examine the effect of pilot sequences on the system spectral efficiency. The obtained insights enable us to design a novel greedy pilot assignment algorithm. Our proposed pilot assignment scheme aims at reducing the effect of pilot contamination when the number of relay antennas is large.

The rest of this paper is structured as follows. In Section II, we describe the system model. In Section III, we provide the results of the spectral efficiency analysis. Max-min fairness power optimization and pilot sequences design are developed in Sections IV and V, respectively. We present some numerical results in Section VI and conclude the paper in Section VII.

**Notations:** We use  $\mathbf{X}^H$  and  $\mathbf{x}^H$  to denote the Hermitian of the matrix  $\mathbf{X}$  and vector  $\mathbf{x}$ , respectively. The notations  $|\cdot|$ ,  $\mathbb{E}\{\cdot\}$ ,  $\text{Var}(\cdot)$ , and  $\|\cdot\|$  are the absolute value, expectation, variance, and norm operators, respectively. We use  $\mathbf{x}_k$  to denote the  $k$ -th column of matrix  $\mathbf{X}$ . In addition,  $\text{Tr}(\cdot)$  and  $\text{cov}(\cdot)$  represent the trace and covariance of a matrix. Finally,  $\mathbf{I}_K$  is a  $K \times K$  identity matrix.

## II. SYSTEM MODEL

We consider a multipair massive MIMO relaying system in which a relay station  $R$  serves a group of  $K$  sources  $S_k$  and  $K$  destinations  $D_k$ , for  $k = 1, \dots, K$ , as illustrated in Fig. 1. The source  $S_k$  wants to transmit signals to the destination  $D_k$ . The relay station has  $M$  antennas, while each source/destination is equipped with a single antenna.<sup>1</sup>

**Channel Model:** Let  $\mathbf{g}_{\text{SR},k}$  and  $\mathbf{g}_{\text{RD},k}$  be the channel vectors from the relay to  $S_k$  and  $D_k$ , respectively. Then  $\mathbf{G}_{\text{SR}} = [\mathbf{g}_{\text{SR},1} \dots \mathbf{g}_{\text{SR},K}] \in \mathbb{C}^{M \times K}$  and  $\mathbf{G}_{\text{RD}} = [\mathbf{g}_{\text{RD},1} \dots \mathbf{g}_{\text{RD},K}] \in \mathbb{C}^{M \times K}$  are the channel matrices from the relay station to the  $K$  sources and the  $K$  destinations, respectively. The channel matrices  $\mathbf{G}_{\text{SR}}$  and  $\mathbf{G}_{\text{RD}}$  include independent small-scale fading (assuming Rayleigh fading) and large-scale fading (path loss and log-normal shadow fading), and can be modeled as

$$\mathbf{G}_{\text{SR}} = \mathbf{H}_{\text{SR}} \mathbf{D}_{\text{SR}}^{1/2},$$

and

$$\mathbf{G}_{\text{RD}} = \mathbf{H}_{\text{RD}} \mathbf{D}_{\text{RD}}^{1/2},$$

<sup>1</sup>In massive MIMO, when  $M$  is large, the channels become favorable, and hence, having  $L$  autonomous single-antenna sources/destinations or having one  $L$ -antenna source/destination represent two cases with similar sum spectral efficiency (see Footnote 2 of [2]). If  $M$  is not very large (and hence, the channel is not very favorable), then the performance of multiple-antenna sources/destinations will be different compared to that of the single-antenna setup. In this case, it is expected that the multiple-antenna source/destination setup will be beneficial when  $K$  is small since it offers more degrees of freedom at the source/destination sides. However, when  $K$  is large, the channel estimation overhead and interference are large, and hence, using multiple antennas at the sources/destinations may not be beneficial.

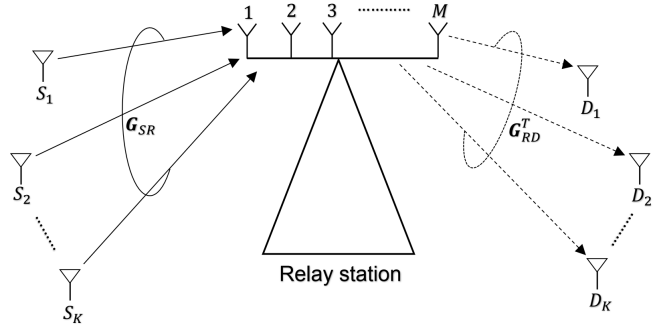


Fig. 1: Multipair massive MIMO half-duplex relaying system model.

where  $\mathbf{H}_{\text{SR}}$  and  $\mathbf{H}_{\text{RD}}$  are small-scale fading matrices with i.i.d.  $\mathcal{CN}(0, 1)$  components, while  $\mathbf{D}_{\text{SR}}$  and  $\mathbf{D}_{\text{RD}}$  are  $K \times K$  diagonal large-scale fading matrices. The  $k$ -th diagonal components of  $\mathbf{D}_{\text{SR}}$  and  $\mathbf{D}_{\text{RD}}$  are denoted by  $\beta_{\text{SR},k}$  and  $\beta_{\text{RD},k}$ , respectively.

### Transmission Model:

All  $K$  communication pairs  $(S_k, D_k)$ ,  $k = 1, \dots, K$ , use the same time-frequency resource. The relay operates in a half-duplex (HD) mode with DF transmission. We assume that there are no direct transmission links from the sources to the destinations due to the large obstacles or heavy shadowing. We further assume that transmission is performed under time-division-duplex (TDD) protocol with perfect channel reciprocity. The imperfect channel state information (CSI) is taken into account. Thus, for each coherence interval, there are two main phases: channel estimation and data transmission phases.

#### A. Channel Estimation Phase

In this phase, the relay estimates the instantaneous channels (small-scale fading coefficients) to all sources and destinations. We assume that the statistical properties of the channels (large-scale fading coefficients) are known at the relay. This assumption is reasonable, since the large-scale fading coefficients change very slowly with time, and hence, are easy to be estimated [3]. Let  $T$  be the length of a coherence interval (in symbols). For each coherence interval, a duration of  $\tau$  symbols is used for the channel estimation at the relay station. More precisely, during this channel estimation phase, all  $K$  sources and  $K$  destinations at the same time send their pilot sequences of  $\tau$  symbols to the relay. Let  $\varphi_{\text{S},k} \in \mathbb{C}^{\tau \times 1}$  and  $\varphi_{\text{D},k} \in \mathbb{C}^{\tau \times 1}$  be the pilot sequences sent from the  $k$ -th source and the  $k$ -th destination, respectively. We assume that  $\{\varphi_{\text{S},1}, \varphi_{\text{S},2}, \dots, \varphi_{\text{S},K}\}$  and  $\{\varphi_{\text{D},1}, \varphi_{\text{D},2}, \dots, \varphi_{\text{D},K}\}$  are unit-norm vectors, i.e.,  $\|\varphi_{\text{S},k}\|^2 = \|\varphi_{\text{D},k}\|^2 = 1$ . The received pilot signal at the relay is an  $M \times \tau$  matrix, given by

$$\mathbf{Y}_{\text{R}} = \sqrt{\tau P_{\text{p}}} \sum_{k=1}^K \mathbf{g}_{\text{SR},k} \varphi_{\text{S},k}^H + \sqrt{\tau P_{\text{p}}} \sum_{k=1}^K \mathbf{g}_{\text{RD},k} \varphi_{\text{D},k}^H + \mathbf{N}_{\text{R}}, \quad (1)$$

where  $\mathbf{N}_{\text{R}}$  is the AWGN noise matrix at the relay which includes i.i.d.  $\mathcal{CN}(0, 1)$  elements and  $P_{\text{p}}$  is the normalized transmit power of each pilot symbol.

In matrix form, (1) can be rewritten as

$$\mathbf{Y}_R = \sqrt{\tau P_p} \mathbf{G}_{SR} \boldsymbol{\Phi}_S^H + \sqrt{\tau P_p} \mathbf{G}_{RD} \boldsymbol{\Phi}_D^H + \mathbf{N}_R, \quad (2)$$

where  $\boldsymbol{\Phi}_S = [\varphi_{S,1} \dots \varphi_{S,K}]$  and  $\boldsymbol{\Phi}_D = [\varphi_{D,1} \dots \varphi_{D,K}]$ .

The relay station wants to estimate  $\mathbf{G}_{SR}$  and  $\mathbf{G}_{RD}$ . To do this, it first projects the received pilot signal  $\mathbf{Y}_R$  onto  $\boldsymbol{\varphi}_{S,k}$  and  $\boldsymbol{\varphi}_{D,k}$  to obtain

$$\begin{aligned} \tilde{\mathbf{y}}_{SR,k} &= \mathbf{Y}_R \boldsymbol{\varphi}_{S,k} = \sqrt{\tau P_p} \sum_{k'=1}^K \mathbf{g}_{SR,k'} \boldsymbol{\varphi}_{S,k'}^H \boldsymbol{\varphi}_{S,k} \\ &+ \sqrt{\tau P_p} \sum_{k'=1}^K \mathbf{g}_{RD,k'} \boldsymbol{\varphi}_{D,k'}^H \boldsymbol{\varphi}_{S,k} + \mathbf{n}_{SR,k}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} \tilde{\mathbf{y}}_{RD,k} &= \mathbf{Y}_R \boldsymbol{\varphi}_{D,k} = \sqrt{\tau P_p} \sum_{k'=1}^K \mathbf{g}_{RD,k'} \boldsymbol{\varphi}_{D,k'}^H \boldsymbol{\varphi}_{D,k} \\ &+ \sqrt{\tau P_p} \sum_{k'=1}^K \mathbf{g}_{SR,k'} \boldsymbol{\varphi}_{S,k'}^H \boldsymbol{\varphi}_{D,k} + \mathbf{n}_{RD,k}. \end{aligned} \quad (4)$$

Then it will use the linear minimum mean-square-error (MMSE) scheme to estimate the channels. As a consequence, the MMSE estimates of the  $k$ -th columns of  $\mathbf{G}_{SR}$  and  $\mathbf{G}_{RD}$ , conditioned on  $\tilde{\mathbf{y}}_{SR,k}$  and  $\tilde{\mathbf{y}}_{RD,k}$ , are given by

$$\hat{\mathbf{g}}_{SR,k} = c_{SR,k} \tilde{\mathbf{y}}_{SR,k}, \quad (5)$$

$$\hat{\mathbf{g}}_{RD,k} = c_{RD,k} \tilde{\mathbf{y}}_{RD,k}, \quad (6)$$

respectively, where

$$c_{SR,k} = \frac{\sqrt{\tau P_p} \beta_{SR,k}}{\tau P_p \sum_{k'=1}^K \left| \boldsymbol{\varphi}_{S,k'}^H \boldsymbol{\varphi}_{S,k} \right|^2 \beta_{SR,k'} + \tau P_p \sum_{k'=1}^K \left| \boldsymbol{\varphi}_{D,k'}^H \boldsymbol{\varphi}_{S,k} \right|^2 \beta_{RD,k'} + 1},$$

and

$$c_{RD,k} = \frac{\sqrt{\tau P_p} \beta_{RD,k}}{\tau P_p \sum_{k'=1}^K \left| \boldsymbol{\varphi}_{D,k'}^H \boldsymbol{\varphi}_{D,k} \right|^2 \beta_{RD,k'} + \tau P_p \sum_{k'=1}^K \left| \boldsymbol{\varphi}_{S,k'}^H \boldsymbol{\varphi}_{D,k} \right|^2 \beta_{SR,k'} + 1}.$$

The detailed derivations of (5) and (6) are provided in Appendix A.

Let  $\boldsymbol{\epsilon}_{SR,k}$  and  $\boldsymbol{\epsilon}_{RD,k}$  be the corresponding channel estimation error vectors, i.e.,

$$\boldsymbol{\epsilon}_{SR,k} = \mathbf{g}_{SR,k} - \hat{\mathbf{g}}_{SR,k}, \quad (7)$$

and

$$\boldsymbol{\epsilon}_{RD,k} = \mathbf{g}_{RD,k} - \hat{\mathbf{g}}_{RD,k}. \quad (8)$$

Then,  $\hat{\mathbf{g}}_{SR,k}$  is independent of  $\boldsymbol{\epsilon}_{SR,k}$ , and  $\hat{\mathbf{g}}_{RD,k}$  is independent of  $\boldsymbol{\epsilon}_{RD,k}$ . This comes from the property of the MMSE estimator together with the fact that  $\hat{\mathbf{g}}_{SR,k}$ ,  $\boldsymbol{\epsilon}_{SR,k}$ ,  $\hat{\mathbf{g}}_{RD,k}$ , and  $\boldsymbol{\epsilon}_{RD,k}$  are Gaussian vectors. In addition, the elements of  $\hat{\mathbf{g}}_{SR,k}$  and  $\boldsymbol{\epsilon}_{SR,k}$  are Gaussian random variables distributed as  $\mathcal{CN}(0, \sigma_{SR,k}^2)$  and  $\mathcal{CN}(0, \varepsilon_{SR,k}^2)$ , respectively, where

$$\sigma_{SR,k}^2 = \sqrt{\tau P_p} \beta_{SR,k} c_{SR,k}, \quad (9)$$

and

$$\varepsilon_{SR,k}^2 = \beta_{SR,k} - \sigma_{SR,k}^2. \quad (10)$$

Similarly, the elements of  $\hat{\mathbf{g}}_{RD,k}$  and  $\boldsymbol{\epsilon}_{RD,k}$  are Gaussian random variables distributed as  $\mathcal{CN}(0, \sigma_{RD,k}^2)$  and  $\mathcal{CN}(0, \varepsilon_{RD,k}^2)$ , respectively, where

$$\sigma_{RD,k}^2 = \sqrt{\tau P_p} \beta_{RD,k} c_{RD,k}, \quad \text{and} \quad \varepsilon_{RD,k}^2 = \beta_{RD,k} - \sigma_{RD,k}^2. \quad (11)$$

## B. Data Transmission Phase

1) *First Hop—Transmission from the  $K$  Users to the Relay:* Signals are transmitted from all  $K$  sources  $S_k$ ,  $k = 1, \dots, K$ , to the relay in the same time-frequency resource. The  $M \times 1$  received signal vector at the relay is

$$\mathbf{y}_R = \mathbf{G}_{SR} \mathbf{P}_S^{1/2} \mathbf{x} + \mathbf{n}_R, \quad (12)$$

where  $\mathbf{x} = [x_1, \dots, x_K]^T$ , with  $\mathbb{E}\{|x_k|^2\} = 1$ , is the symbol vector transmitted from the  $K$  sources,  $\mathbf{n}_R$  is an  $M \times 1$  AWGN noise vector with i.i.d.  $\mathcal{CN}(0, 1)$  elements, and  $\mathbf{P}_S$  is a  $K \times K$  diagonal matrix whose  $(k, k)$ -th diagonal element is  $P_{S,k}$  representing the normalized transmit power of the  $k$ -th user.

After receiving  $\mathbf{y}_R$ , the relay uses the channel estimates to perform the maximum-ratio combining (MRC) technique, i.e., it multiplies  $\mathbf{y}_R$  with the conjugate transpose of the channel estimate matrix:

$$\mathbf{r} = \hat{\mathbf{G}}_{SR}^H \mathbf{y}_R = \sqrt{P_{S,k}} \hat{\mathbf{G}}_{SR}^H \mathbf{G}_{SR} \mathbf{x} + \hat{\mathbf{G}}_{SR}^H \mathbf{n}_R. \quad (13)$$

Then, it will detect the signal transmitted by  $S_k$  from the  $k$ -th element of  $\mathbf{r}$ , for  $k = 1, \dots, K$ . From (13), the  $k$ -th element of  $\mathbf{r}$  is

$$r_k = \sqrt{P_{S,k}} \hat{\mathbf{g}}_{SR,k}^H \mathbf{g}_{SR,k} x_k + \sum_{i \neq k} \sqrt{P_{S,i}} \hat{\mathbf{g}}_{SR,k}^H \mathbf{g}_{SR,i} x_i + \hat{\mathbf{g}}_{SR,k}^H \mathbf{n}_R. \quad (14)$$

2) *Second Hop—Transmission from the Relay to the  $K$  Destinations:* The signals detected in the first hop are precoded and sent to the destinations. We assume that the relay employs the MR precoding scheme. With MR precoder, the precoded signal transmitted from the relay is

$$\mathbf{s} = \sum_{k=1}^K \sqrt{P_{R,k}} \hat{\mathbf{g}}_{RD,k}^* x_k = \hat{\mathbf{G}}_{RD}^* \mathbf{P}_R^{1/2} \mathbf{x}, \quad (15)$$

where  $\mathbf{P}_R$  is a  $K \times K$  diagonal matrix whose  $k$ -th diagonal element is denoted by  $P_{R,k}$  representing the power control coefficient at the relay. The power control coefficients are chosen to meet a total transmit power constraint at the relay:  $\mathbb{E}\{\|\mathbf{s}\|^2\} \leq P_{r,\text{th}}$ . Therefore, we obtain the following constraint on  $P_{R,k}$ :

$$M \sum_{k=1}^K P_{R,k} \sigma_{RD,k}^2 \leq P_{r,\text{th}}. \quad (16)$$

With the transmit signal given in (15), the  $k$ -th destination,  $D_k$ , receives

$$y_{D,k} = \sqrt{P_{R,k}} \mathbf{g}_{RD,k}^T \hat{\mathbf{g}}_{RD,k}^* x_k + \sum_{i \neq k} \sqrt{P_{R,i}} \mathbf{g}_{RD,k}^T \hat{\mathbf{g}}_{RD,i}^* x_i + n_{D,k}. \quad (17)$$

### III. SPECTRAL EFFICIENCY ANALYSIS

In this section, a closed-form expression for the end-to-end (e2e) spectral efficiency of the transmission link  $S_k \rightarrow R \rightarrow D_k$  is derived. We follow similar methodology as in [8] to obtain the spectral efficiency. The e2e spectral efficiency of the transmission link  $S_k \rightarrow R \rightarrow D_k$  is

$$R_k = \min(R_{\text{SR},k}, R_{\text{RD},k}), \quad (18)$$

where  $R_{\text{SR},k}$  is the spectral efficiency of link from  $S_k$  to the relay and  $R_{\text{RD},k}$  is the spectral efficiency of link from the relay to  $D_k$ .

To find  $R_{\text{SR},k}$ , we first rewrite  $r_k$  in (14) as

$$r_k = \sqrt{P_{\text{S},k}} \mathbb{E} \left\{ \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},k} \right\} x_k + \sum_{i \neq k}^K \sqrt{P_{\text{S},i}} \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},i} x_i + \sqrt{P_{\text{S},k}} \left( \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},k} - \mathbb{E} \left\{ \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},k} \right\} \right) x_k + \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{n}_{\text{R}}. \quad (19)$$

Then we treat the sum of the last three terms as the effective noise and use the fact that Gaussian noise is the worst-case noise [8], we obtain an achievable spectral efficiency

$$R_{\text{SR},k} = \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{P_{\text{S},k} \text{NU}_{\text{SR},k}}{P_{\text{S},k} \text{VA}_{\text{SR},k} + \text{IU}_{\text{SR},k} + \text{AN}_{\text{SR},k}} \right), \quad (20)$$

where

$$\text{NU}_{\text{SR},k} = \left| \mathbb{E} \left\{ \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},k} \right\} \right|^2,$$

$$\text{VA}_{\text{SR},k} = \text{Var} \left( \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},k} \right),$$

$$\text{IU}_{\text{SR},k} = \sum_{i \neq k}^K P_{\text{S},i} \mathbb{E} \left\{ \left| \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},i} \right|^2 \right\},$$

and

$$\text{AN}_{\text{SR},k} = \mathbb{E} \left\{ \left| \hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{n}_{\text{R}} \right|^2 \right\}.$$

The pre-log factor in (20) comes from the fact that, for each coherence interval of  $T$  symbols, we spend  $\tau$  symbols for the training phase and we need two hops to transmit signals from source to the destinations.

Similarly, we obtain

$$R_{\text{RD},k} = \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{P_{\text{R},k} \text{NU}_{\text{RD},k}}{P_{\text{R},k} \text{VA}_{\text{RD},k} + \text{IU}_{\text{RD},k} + 1} \right), \quad (21)$$

where

$$\text{NU}_{\text{RD},k} = \left| \mathbb{E} \left\{ \mathbf{g}_{\text{RD},k}^T \hat{\mathbf{g}}_{\text{RD},k}^* \right\} \right|^2,$$

$$\text{VA}_{\text{RD},k} = \text{Var} \left( \mathbf{g}_{\text{RD},k}^T \hat{\mathbf{g}}_{\text{RD},k}^* \right),$$

and

$$\text{IU}_{\text{RD},k} = \sum_{i \neq k}^K P_{\text{R},i} \mathbb{E} \left\{ \left| \mathbf{g}_{\text{RD},k}^T \hat{\mathbf{g}}_{\text{RD},i}^* \right|^2 \right\}.$$

The closed-form expressions of the spectral efficiencies (20) and (21) are provided in the following theorem.

*Theorem 1:* The closed-form expressions of the e2e spectral efficiencies of the transmission links  $S_k \rightarrow R$  and  $R \rightarrow D_k$  are, respectively, given by.

$$R_{\text{SR},k} = \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{M^2 P_{\text{S},k} \sigma_{\text{SR},k}^4}{M P_{\text{S},k} \sigma_{\text{SR},k}^2 \beta_{\text{SR},k} + \text{IU}_{\text{SR},k} + M \sigma_{\text{SR},k}^2} \right), \quad (22)$$

$$R_{\text{RD},k} = \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{M^2 P_{\text{R},k} \sigma_{\text{RD},k}^4}{M P_{\text{R},k} \sigma_{\text{RD},k}^2 \beta_{\text{RD},k} + \text{IU}_{\text{RD},k} + 1} \right), \quad (23)$$

where

$$\text{IU}_{\text{SR},k} = c_{\text{SR},k}^2 M \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \beta_{\text{SR},i} \left( \tau P_{\text{p}} (M+1) \beta_{\text{SR},i} \left| \varphi_{\text{S},i}^H \varphi_{\text{S},k} \right|^2 + \tau P_{\text{p}} \sum_{\substack{k'=1 \\ k' \neq i}}^K \beta_{\text{SR},k'} \left| \varphi_{\text{S},k'}^H \varphi_{\text{S},k} \right|^2 + \tau P_{\text{p}} \sum_{k'=1}^K \beta_{\text{SR},k'} \left| \varphi_{\text{D},k'}^H \varphi_{\text{S},k} \right|^2 + 1 \right), \quad (24)$$

$$\text{IU}_{\text{RD},k} = c_{\text{RD},k}^2 M \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{R},i} \beta_{\text{RD},i} \left( \tau P_{\text{p}} (M+1) \beta_{\text{RD},i} \left| \varphi_{\text{D},i}^H \varphi_{\text{D},k} \right|^2 + \tau P_{\text{p}} \sum_{\substack{k'=1 \\ k' \neq i}}^K \beta_{\text{RD},k'} \left| \varphi_{\text{D},k'}^H \varphi_{\text{D},k} \right|^2 + \tau P_{\text{p}} \sum_{k'=1}^K \beta_{\text{RD},k'} \left| \varphi_{\text{S},k'}^H \varphi_{\text{D},k} \right|^2 + 1 \right). \quad (25)$$

*Proof 1:* See Appendix B.

*Remark 1:* Some important insights obtained from the result in Theorem 1 are:

- Plugging (22) and (23) into (18), we obtain a simple closed-form expression for the e2e spectral efficiency of the link  $S_k \rightarrow R \rightarrow D_k$ . In the special case that all pilot sequences are mutually orthogonal (i.e.  $\varphi_{\text{S},k}^H \varphi_{\text{S},k'} = 0$ ,  $\varphi_{\text{D},k}^H \varphi_{\text{D},k'} = 0$ , and  $\varphi_{\text{S},i}^H \varphi_{\text{D},j} = 0$ , for all  $k \neq k'$  and all  $i, j$ ), the e2e spectral efficiency in Theorem 1 is identical to the one in [8] where the loop interference is neglected.
- If all pilot sequences  $\{\varphi_{\text{S},k}\}$  and  $\{\varphi_{\text{D},k}\}$  are not mutually orthogonal, then the numerators and the denominators of the SINRs in (22) and (23) scale as  $M^2$ , as  $M$  increases. As a result, when the number of base station antennas goes to infinity, the e2e spectral efficiencies are bounded. More precisely, as  $M \rightarrow \infty$ , we have

$$R_{\text{SR},k} \rightarrow \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{P_{\text{S},k} \beta_{\text{SR},k}^2}{\sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \beta_{\text{SR},i}^2 \left| \varphi_{\text{S},i}^H \varphi_{\text{S},k} \right|^2} \right), \quad (26)$$

$$R_{\text{RD},k} \rightarrow \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{P_{\text{R},k} \beta_{\text{RD},k}^2}{\sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{R},i} \beta_{\text{RD},i}^2 \left| \varphi_{\text{D},i}^H \varphi_{\text{D},k} \right|^2} \right). \quad (27)$$

This stems from the pilot contamination effect.

- Assume that  $P_{\text{S},k} = E_{\text{S}}/M$  and  $P_{\text{R},k} = E_{\text{R}}/M^2$  (corresponding to the case where the transmit power at the relay

is proportional to  $1/M$ ) for all  $k$ . Then, as  $M \rightarrow \infty$ , we obtain

$$R_{\text{SR},k} \rightarrow \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{E_S \beta_{\text{SR},k}^2}{\sum_{\substack{i=1 \\ i \neq k}}^K E_S \beta_{\text{SR},i}^2 |\varphi_{\text{S},i}^H \varphi_{\text{S},k}|^2 + \frac{\beta_{\text{SR},k}^2}{\sigma_{\text{SR},k}^2}} \right), \quad (28)$$

$$R_{\text{RD},k} \rightarrow \frac{T-\tau}{2T} \log_2 \left( 1 + \frac{E_R \beta_{\text{RD},k}^2}{\sum_{\substack{i=1 \\ i \neq k}}^K E_R \beta_{\text{RD},i}^2 |\varphi_{\text{D},i}^H \varphi_{\text{D},k}|^2 + \frac{\beta_{\text{RD},k}^2}{\sigma_{\text{RD},k}^2}} \right). \quad (29)$$

The above results imply that, when  $M$  is large, the transmitted powers of the sources and the relay can be made inversely proportional to  $M$  with no reduction in e2e performance.

#### IV. MAX-MIN SPECTRAL EFFICIENCY POWER OPTIMIZATION

In this section, a max-min fairness power optimization problem is proposed to maximize the smallest spectral efficiency of all communication pairs, subject to the constraints on the transmit powers at the users and at the relay. Note that our intention is not to find the optimal results in closed-form. We aim at designing a simple and efficient power allocation algorithm. The power allocation problem can be formulated as follows:

$$\arg \max_{\{P_{\text{S},k}, P_{\text{R},k}\}} \min_{k=1, \dots, K} R_k \quad (30a)$$

$$\text{s.t. } 0 \leq P_{\text{S},k} \leq P_{\text{S,th}}, \quad k = 1, \dots, K, \quad (30b)$$

$$\sum_{k=1}^K P_{\text{R},k} \sigma_{\text{RD},k}^2 \leq \frac{P_{\text{R,th}}}{M}, \quad k = 1, \dots, K. \quad (30c)$$

In problem (30),  $R_k$  is given in (18) which is equal to  $\min(R_{\text{SR},k}, R_{\text{RD},k})$ . Since  $R_{\text{SR},k}$  depends only on  $P_{\text{S},k}$  and  $R_{\text{RD},k}$  depends only on  $P_{\text{R},k}$ , the problem (30) can be decoupled into two sub-problems: i) power allocation optimization at the sources in the first hop; and ii) power allocation optimization at the relay in the second hop.

##### A. Power Allocation Optimization at the Sources in the First Hop

Power allocation optimization at the sources in the first hop is formulated as

$$\arg \max_{\{P_{\text{S},k}\}} \min_{k=1, \dots, K} R_{\text{SR},k} \quad (31a)$$

$$\text{s.t. } 0 \leq P_{\text{S},k} \leq P_{\text{S,th}}, \quad k = 1, \dots, K. \quad (31b)$$

From (22), the optimization problem (31) can be rewritten as

$$\arg \max_{\{P_{\text{S},k}\}} \min_{k=1, \dots, K} \log_2(1 + \gamma_{\text{SR},k}) \quad (32a)$$

$$\text{s.t. } 0 \leq P_{\text{S},k} \leq P_{\text{S,th}}, \quad k = 1, \dots, K, \quad (32b)$$

where

$$\gamma_{\text{SR},k} = \frac{M^2 P_{\text{S},k} \sigma_{\text{SR},k}^4}{M P_{\text{S},k} \sigma_{\text{SR},k}^2 \beta_{\text{SR},k} + M c_{\text{SR},k}^2 \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \eta_{\text{SR},i} + M \sigma_{\text{SR},k}^2}, \quad (33)$$

and

$$\eta_{\text{SR},i} = \tau P_p \beta_{\text{SR},i} \left( (M+1) \beta_{\text{SR},i} |\varphi_{\text{S},i}^H \varphi_{\text{S},k}|^2 + \sum_{\substack{k'=1 \\ k' \neq i}}^K \beta_{\text{SR},k'} |\varphi_{\text{S},k'}^H \varphi_{\text{S},k}|^2 + \sum_{k'=1}^K \beta_{\text{SR},k'} |\varphi_{\text{D},k'}^H \varphi_{\text{S},k}|^2 + \frac{1}{\tau P_p} \right). \quad (34)$$

By introducing a new variable  $t_{\text{SR}}$ , and using the fact that  $\log_2(1+x)$  is an increasing function of  $x$ , problem (32) is equivalent to

$$\arg \max_{\{P_{\text{S},k}\}, t_{\text{SR}}} t_{\text{SR}} \quad (35a)$$

$$\text{s.t. } \gamma_{\text{SR},k} \geq t_{\text{SR}}, \quad k = 1, \dots, K, \quad (35b)$$

$$(32b). \quad (35c)$$

The constraint (35b) can be reformulated to the following posynomial function:  $\frac{1}{\gamma_{\text{SR},k}} \leq \frac{1}{t_{\text{SR}}}$ . Therefore, the problem (35) is rewritten as

$$\arg \max_{\{P_{\text{S},k}\}, t_{\text{SR}}} t_{\text{SR}} \quad (36a)$$

$$\text{s.t. } (32b), \quad (36b)$$

$$\frac{a_{\text{SR},k}}{f_{\text{SR},k}} + \frac{c_{\text{SR},k}^2}{f_{\text{SR},k}} P_{\text{S},k}^{-1} \sum_{\substack{i=1 \\ i \neq k}}^K b_{\text{SR},ki} P_{\text{S},i} + \frac{e_{\text{SR},k}}{f_{\text{SR},k}} P_{\text{S},k}^{-1} \leq \frac{1}{t_{\text{SR}}}, \quad \forall k, \quad (36c)$$

where  $a_{\text{SR},k} = M \sigma_{\text{SR},k}^2 \beta_{\text{SR},k}$ ,  $b_{\text{SR},ki} = M \eta_{\text{SR},ki}$ ,  $e_{\text{SR},k} = M \sigma_{\text{SR},k}^2$ , and  $f_{\text{SR},k} = M^2 \sigma_{\text{SR},k}^4$ .

The problem (36) becomes a geometric program, and hence, it can be solved efficiently by leveraging convex optimization tools [17], [18]. Note that to implement the power allocation at the sources, we need to know only the large-scale fading coefficients which are easy to be estimated because they change very slowly (see Section II-A).

##### B. Power Allocation Optimization at the Relay in the Second Hop

Power allocation optimization at the relay in the second hop can be represented as

$$\arg \max_{\{P_{\text{R},k}\}} \min_{k=1, \dots, K} R_{\text{RD},k} \quad (37a)$$

$$\text{s.t. } \sum_{i=1}^K P_{\text{R},k} \sigma_{\text{RD},k}^2 \leq \frac{P_{\text{R,th}}}{M}, \quad k = 1, \dots, K. \quad (37b)$$

From (23), the optimization problem (37) is equivalent to

$$\arg \max_{\{P_{R,k}\}} \min_{k=1,\dots,K} \left( \log_2 \left( 1 + \frac{M^2 P_{R,k} \sigma_{RD,k}^4}{M P_{R,k} \sigma_{RD,k}^2 \beta_{RD,k} + M c_{RD,k}^2 \sum_{\substack{i=1 \\ i \neq k}}^K P_{R,i} \eta_{RD,i} + 1} \right) \right) \quad (38a)$$

$$\text{s.t.} \quad \sum_{i=1}^K P_{R,k} \sigma_{RD,k}^2 \leq \frac{P_{R,\text{th}}}{M}, \quad k = 1, \dots, K, \quad (38b)$$

where

$$\eta_{RD,i} = \tau P_p \beta_{RD,i} \left( (M+1) \beta_{RD,i} |\varphi_{D,i}^H \varphi_{D,k}|^2 + \sum_{\substack{k'=1 \\ k' \neq i}}^K \beta_{RD,k'} |\varphi_{D,k'}^H \varphi_{D,k}|^2 + \sum_{k'=1}^K \beta_{RD,k'} |\varphi_{S,k'}^H \varphi_{D,k}|^2 + \frac{1}{\tau P_p} \right). \quad (39)$$

Similarly, we can reformulate problem (38) as a geometric program:

$$\arg \max_{\{P_{R,k}\}, t_{RD}} t_{RD} \quad (40a)$$

$$\text{s.t.} \quad \sum_{k=1}^K P_{R,k} \sigma_{RD,k}^2 \leq \frac{P_{R,\text{th}}}{M}, \quad k = 1, \dots, K, \quad (40b)$$

$$\frac{a_{RD,k}}{f_{RD,k}} + \frac{c_{RD,k}^2}{f_{RD,k}} P_{R,k}^{-1} \sum_{\substack{i=1 \\ i \neq k}}^K b_{RD,ki} P_{R,i} + \frac{1}{f_{RD,k}} P_{R,k}^{-1} < \frac{1}{t_{RD}}, \quad \forall k, \quad (40c)$$

where  $a_{RD,k} = M \sigma_{RD,k}^2 \beta_{RD,k}$ ,  $b_{RD,ki} = M \eta_{RD,ki}$ , and  $f_{RD,k} = M^2 \sigma_{RD,k}^4$ . As a result, problem (40) can be also solved efficiently by using convex optimization tools, such as CVX or YALMIP [17], [18].

## V. PILOT SEQUENCES DESIGN

Note that the training duration is defined by  $\tau$  (in symbols) which corresponds to  $\tau$  orthogonal pilot sequences. If we utilize the conventional orthogonal pilot sequence assignment, where all sources and destinations are assigned mutually orthogonal pilot sequences, then the training duration  $\tau$  is at least  $2K$ . Since the length of training duration depends on the coherence interval (i.e. it cannot be larger than the coherence interval), we can use orthogonal pilot assignment only when the coherence interval is long (relative to the number of sources and destinations). Otherwise, the sources and destinations have to use non-orthogonal pilot sequences. With non-orthogonal pilot assignment, pilot contamination appears which affects significantly the system performance. In general, our key objective here is to optimally assign pilot sequences to all sources and destinations to maximize the smallest spectral efficiencies of all communication pairs. Unfortunately this is a very complicated task. Therefore, we propose a simple greedy algorithm, which iteratively refines

the pilot assignment. Our proposed scheme is based on two main observations:

- From the spectral efficiency expressions (22) and (23), the terms of the denominators in the SINRs which scale as  $M^2$  (the same of order as the numerators) are

$$M \sum_{\substack{i=1 \\ i \neq k}}^K P_{S,i} \beta_{SR,i} (M+1) \beta_{SR,i} |\varphi_{S,i}^H \varphi_{S,k}|^2, \quad (41)$$

respectively,

$$M \sum_{\substack{i=1 \\ i \neq k}}^K P_{R,i} \beta_{RD,i} (M+1) \beta_{RD,i} |\varphi_{D,i}^H \varphi_{D,k}|^2. \quad (42)$$

Because of these terms, the spectral efficiency is bounded even when the number of relay antennas goes to infinity.

- The term (41) depends only on the pilot sequences assigned to the sources, while the term (42) depends only on the pilot sequences assigned to the destinations.

From the above observations, our proposed pilot assignment scheme will be implemented at the sources and the destinations independently, and aim at reducing terms (41) and term (42). Therefore, we have a greedy pilot sequences assignment design for sources and destinations as given follows:

### 1) Greedy pilot sequences assignment design for sources:

We first use random pilot assignment scheme to assign pilots to all  $K$  sources, i.e., the  $K$  sources are randomly assigned  $K$  pilot sequences from a set of  $\tau$  orthogonal pilot sequences. Then, we compute  $R_{SR,k}$  for all  $k$ . Finally, we determine the source that has the lowest  $R_{SR,k}$ , say source  $\tilde{k}$ , and change its pilot sequence to the new one so that

$$\sum_{\substack{i=1 \\ i \neq k}}^K M(M+1) \beta_{SR,i}^2 |\varphi_{S,i}^H \varphi_{S,\tilde{k}}|^2$$

is minimized. The algorithm processes repeatedly for a predetermined number of iterations. Note that

$$\begin{aligned} & \sum_{\substack{i=1 \\ i \neq k}}^K M(M+1) \beta_{SR,i}^2 |\varphi_{S,i}^H \varphi_{S,\tilde{k}}|^2 \\ &= \frac{\varphi_{S,\tilde{k}}^H \left( \sum_{\substack{i=1 \\ i \neq k}}^K M(M+1) \beta_{SR,i}^2 \varphi_{S,i} \varphi_{S,i}^H \right) \varphi_{S,\tilde{k}}}{\varphi_{S,\tilde{k}}^H \varphi_{S,\tilde{k}}} \end{aligned} \quad (43)$$

is a Rayleigh quotient. Thus, to minimize  $\sum_{\substack{i=1 \\ i \neq k}}^K M(M+1) \beta_{SR,i}^2 |\varphi_{S,i}^H \varphi_{S,\tilde{k}}|^2$ ,  $\varphi_{S,\tilde{k}}^H$  has to be the eigenvector which corresponds to the smallest eigenvalue of  $\sum_{\substack{i=1 \\ i \neq k}}^K M(M+1) \beta_{SR,i}^2 \varphi_{S,i} \varphi_{S,i}^H$ .

### 2) Greedy pilot sequences assignment for the destinations:

This follows a similar methodology as in the case of pilot sequences assignment for the sources. Finally, our proposed greedy pilot assignment algorithm can be summarized in Algorithm 1. Note that the main part which contributes to the computational complexity of the greedy pilot assignment (Algorithm 1) is finding the eigenvalues and eigenvectors of

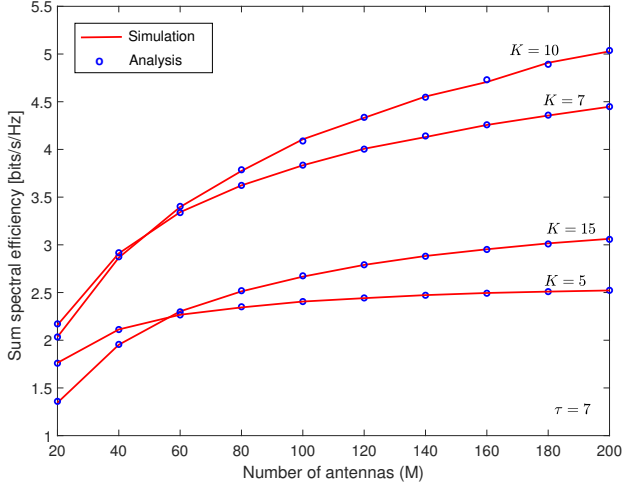


Fig. 2: Analytical and simulation sum spectral efficiency versus  $M$ . Here, we choose  $T = 200$ ,  $\tau = 7$ ,  $P_S = 10$  dB,  $P_R = P_S/M$ ,  $\beta_{SR,k} = \beta_{RD,k} = 1$  for all  $k$ .

the  $\tau \times \tau$  matrices  $\mathbf{A}$  and  $\mathbf{B}$ . The corresponding complexity is  $O(\tau^3)$ .

---

#### Algorithm 1: Greedy pilot assignment

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1. Initialization: choose the number of iterations  $N$ , and set  $n = 1$ .

1.1. If  $K \leq \tau < 2K$ : assign pilots so that  $\{\varphi_{S,1}, \dots, \varphi_{S,K}\}$  are mutually orthogonal and  $\{\varphi_{D,1}, \dots, \varphi_{D,K}\}$  are mutually orthogonal.

1.2. If  $\tau < K$ : Using the random pilot assignment approach to assign pilots to  $K$  sources/destinations.

2. Compute  $R_{SR,k}$  and  $R_{RD,k}$ . Find the source and destination with lowest spectral efficiency:

$$\tilde{k} = \arg \min_k R_{SR,k} \text{ and } \hat{k} = \arg \min_k R_{RD,k}$$

3. Update the pilot sequence for the  $\tilde{k}$ -th source and the  $\hat{k}$ -th destination by choosing eigenvector of

$$\mathbf{A} = \left( \sum_{\substack{i=1 \\ i \neq \tilde{k}}}^K M(M+1)\beta_{SR,i}^2 \varphi_{S,i} \varphi_{S,i}^H \right) \text{ and} \\ \mathbf{B} = \left( \sum_{\substack{i=1 \\ i \neq \hat{k}}}^K M(M+1)\beta_{RD,i}^2 \varphi_{D,i} \varphi_{D,i}^H \right) \text{ corresponding}$$

to the smallest eigenvalue of  $\mathbf{A}$  and  $\mathbf{B}$ , respectively.

4. Set  $n = n + 1$ . Go back to Step 2 and repeat until  $n > N$ .

---

## VI. NUMERICAL RESULTS AND DISCUSSION

In this section, the numerical results are provided to validate our analysis and to evaluate the system performance shown in previous sections, i.e., the correctness of analytical results, the benefits of the proposed max-min power control, and the benefit of the proposed pilot assignment scheme. For all examples, we choose  $T = 200$  symbols.

We first consider a simple case where the large-scale fading coefficients corresponding to all sources and destinations are

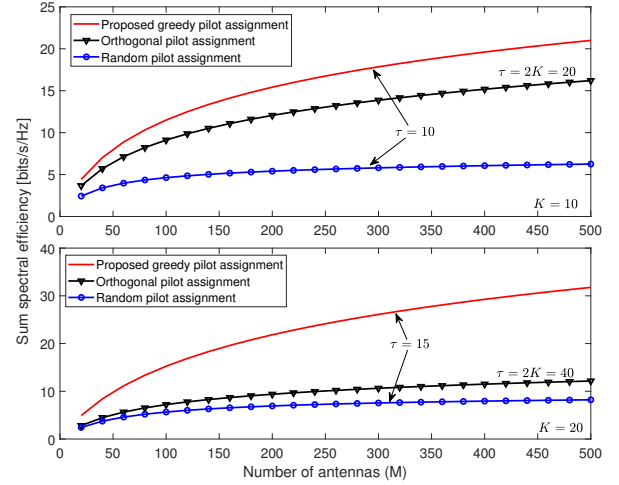


Fig. 3: Sum spectral efficiency versus  $M$  with different pilot assignment schemes. We choose  $P_S = 10$  dB,  $P_R = P_S/M$  and  $\beta_{SR,k} = \beta_{RD,k} = 1, \forall k$ .

equal to 1, i.e.,  $\beta_{SR,k} = \beta_{RD,k} = 1$  for all  $k$ . Furthermore, we choose  $P_{S,k} = P_S$  and  $P_{R,k} = P_R$  for all  $k$ . Figure 2 shows the sum spectral efficiency versus the number of relay antennas with different numbers of sources and destinations, for  $\tau = 7$  symbols,  $P_S = 10$  dB, and  $P_R = P_S/M$ .<sup>2</sup> In this example, the pilot sequences of sources and destinations are selected randomly. We can clearly observe from the figure that the “analysis” curves, given by Theorem 1, match perfectly with the one of “simulation” curves. This corroborates the correctness of our analysis. Moreover, the spectral efficiency increases dramatically when the number of antennas at the relay increases. We can also observe that the spectral efficiency with  $K = 10$  is larger than with  $K = 15$ , particularly for a large number relay antennas. This is due to the fact that we use the random pilot assignment scheme for our simulations, which means that a very large  $K$  will boost the pilot-contamination effect.

Then, we study the benefits of our proposed greedy pilot assignment scheme in improving the spectral efficiency for different  $(K, \tau)$ , see Fig. 3. The “Random pilot assignment” curves correspond to the case where the sources and destinations randomly select pilots from a set of  $\tau$  orthogonal pilot sequences, the “Orthogonal pilot assignment” curves correspond to the case where pilots at sources and destinations are pairwise orthogonal, and the “Proposed greedy pilot assignment” curves correspond to our proposed scheme (Algorithm 1). It is clear that the spectral efficiency of our proposed pilot assignment scheme is much larger than that of other schemes. For example, at  $M = 100$  and  $K = 10$ , our proposed greedy pilot assignment scheme can improve the spectral efficiency by factors of more than 2 and 1.5 compared with the random pilot assignment and orthogonal pilot assignment schemes, respectively. The proposed scheme can provide more spectral efficiency improvement in the case

<sup>2</sup>These are normalized values and therefore expressed in dB.



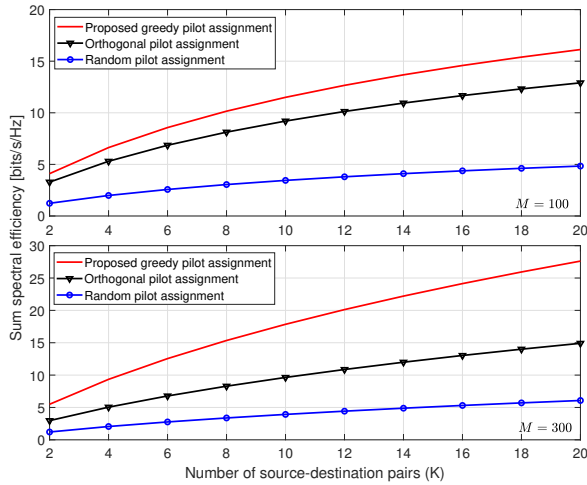


Fig. 4: Sum spectral efficiency versus the number pairs of  $K$ . Here, we choose  $M = 100$  and  $M = 300$ ,  $\tau = 10$ ,  $P_S = P_R = 10$  dB,  $P_R = P_S/M$  and  $\beta_{SR,k} = \beta_{RD,k} = 1, \forall k$ .

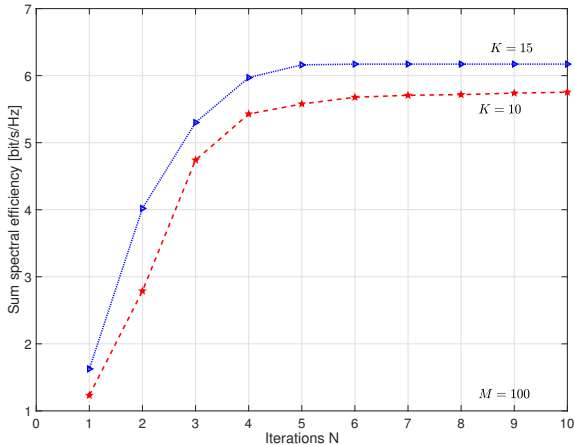


Fig. 5: The convergence of the proposed algorithm versus the number of iterations  $N$  with different  $K$ . Here, we choose  $M = 100$ , and  $\tau = 10$ .

of ( $\tau = 15, K = 20$ ). This is due to the fact that in the case of  $\tau < K$ , the system suffers strong pilot contamination, and hence, our proposed scheme is more beneficial. We further investigate the effect of the number of source-destination pairs on the performance of proposed scheme in Fig. 4. We can see that our proposed scheme outperforms the others, especially when  $K$  is large.

We now study the convergence of our proposed scheme (Algorithm 1) as depicted in Figure 5. The figure illustrates the spectral efficiency versus the number of iterations  $N$  with  $M = 100$  for different  $K$ . We can see that the proposed approach converges very quickly, just after a few iterations.

Finally, we demonstrate the benefits of our max-min power control algorithm. We consider a practical large-scale fading model. More precisely, we use the model in [19] to generate

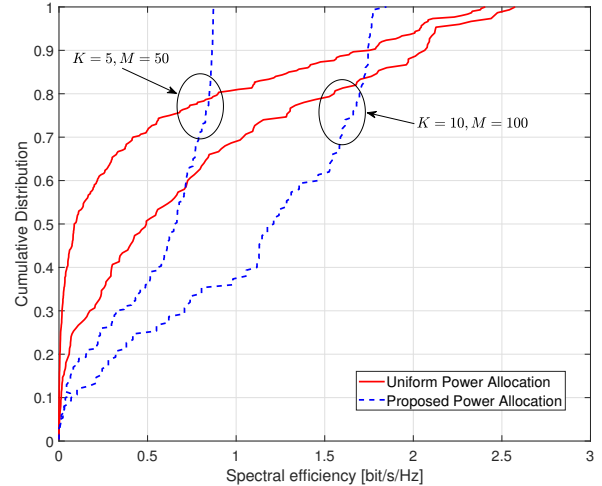


Fig. 6: The cumulative distribution of the spectral efficiency with uniform and proposed power allocation schemes for different  $(M, K)$ . Here, we choose  $T = 200$ , and  $\tau = 5$ .

the large-scale coefficients as follows:

$$\beta_{SR,k} = \frac{z_{SR,k}}{1 + (d_{SR,k}/d_0)^\nu}, \quad (44)$$

where  $z_{SR,k}$  represents the shadowing which is the log-normal random variable,  $\nu$  is the path loss exponent,  $d_{SR,k}$  denotes the distance between source  $k$  and the relay, and  $d_0$  is a reference distance. The same model is used for  $\beta_{RD,k}$ . We assume that all the  $K$  sources and destinations are located uniformly at random inside a disk with radius  $R_d = 500$  (m),  $d_0 = 50$  (m), and  $\nu = 4$ . In addition, we use the notations  $\bar{P}_{S,k}$  and  $\bar{P}_{R,k}$  to denote the transmit powers of the  $k$ -th source and the  $k$ -th diagonal element of the power matrix at the relay, respectively;  $\bar{P}_{S,th}$  and  $\bar{P}_{R,th}$  to indicate the maximum powers of the sources and the relay, respectively, and  $\bar{P}_{Sp}$  to represent the pilot powers sent from the sources. The corresponding normalized transmit SNRs  $P_{S,k}, P_{R,k}, P_{S,th}, P_{R,th}$  and  $P_{Sp}$  can be calculated by dividing these powers by the noise power. With uniform power allocation, we select  $\bar{P}_{S,k} = 0.1$  W for all  $k = 1, \dots, K$ , while with our proposed power allocation scheme, we choose  $\bar{P}_{S,th} = \bar{P}_{R,th} = \bar{P}_{Sp} = 0.1$  W. Figure 6 shows the cumulative distribution of the spectral efficiency for uniform and proposed power allocation, with  $(K = 5, M = 50)$  and  $(K = 10, M = 100)$ . We can see from the figure that our proposed power allocation scheme offers noticeably more spectral efficiency than the uniform power optimization does. More precisely, the proposed power allocation method can increase the 95%-likely spectral efficiency by factors of 6.2 and 7.5 compared to uniform power allocation for the case of  $(K = 5, M = 50)$  and  $(K = 10, M = 100)$ , respectively.

## VII. CONCLUSION

We considered a very general multipair massive MIMO relaying system where the pilot sequences assigned to all sources and destinations during the training phase can be arbitrary. The relay station deployed decode-and-forward with

maximum-ratio combining technique to help the simultaneous transmissions of  $K$  source-destination pairs. An exact and simple closed-form expression of the spectral efficiency was derived. Based on this closed-form expression, a novel power allocation was proposed which was solved efficiently via geometric programs. It was shown that by applying our proposed power allocation scheme, the spectral efficiency can improve dramatically, compared to the uniform power allocation. Moreover, a novel greedy pilot design was proposed. Compared to the conventional random pilot assignment, the proposed greedy pilot assignment increased the spectral efficiency noticeably.

## APPENDIX

### A. Derivations of (5) and (6)

From [20], the MMSE estimate of  $\mathbf{g}_{\text{SR},k}$ , conditioned on  $\tilde{\mathbf{y}}_{\text{SR},k}$ , is given by

$$\begin{aligned} \hat{\mathbf{g}}_{\text{SR},k} &= \mathbb{E}\{\mathbf{g}_{\text{SR},k}\} \\ &+ \text{COV}(\mathbf{g}_{\text{SR},k}, \tilde{\mathbf{y}}_{\text{RS},k}) \text{COV}(\tilde{\mathbf{y}}_{\text{RS},k}, \tilde{\mathbf{y}}_{\text{RS},k})^{-1} (\tilde{\mathbf{y}}_{\text{RS},k} - \mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k}\}) \\ &= \text{COV}(\mathbf{g}_{\text{SR},k}, \tilde{\mathbf{y}}_{\text{RS},k}) \text{COV}(\tilde{\mathbf{y}}_{\text{RS},k}, \tilde{\mathbf{y}}_{\text{RS},k})^{-1} \tilde{\mathbf{y}}_{\text{RS},k}, \end{aligned} \quad (45)$$

where the second equality follows the fact that  $\mathbb{E}\{\mathbf{g}_{\text{SR},k}\} = \mathbf{0}$  and  $\mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k}\} = \mathbf{0}$ . We now compute all terms in (45). We have

$$\begin{aligned} \text{COV}(\mathbf{g}_{\text{SR},k}, \tilde{\mathbf{y}}_{\text{RS},k}) &= \mathbb{E}\left\{(\mathbf{g}_{\text{SR},k} - \mathbb{E}\{\mathbf{g}_{\text{SR},k}\})(\tilde{\mathbf{y}}_{\text{RS},k} - \mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k}\})^H\right\} \\ &= \mathbb{E}\{\mathbf{g}_{\text{SR},k} \tilde{\mathbf{y}}_{\text{RS},k}^H\} \\ &= \sqrt{\tau P_p} \mathbb{E}\{\mathbf{g}_{\text{SR},k} \mathbf{g}_{\text{SR},k}^H\} \\ &= \sqrt{\tau P_p} \beta_{\text{SR},k} \mathbf{I}_M, \end{aligned} \quad (46)$$

and

$$\begin{aligned} \text{COV}(\tilde{\mathbf{y}}_{\text{RS},k}) &= \mathbb{E}\left\{(\tilde{\mathbf{y}}_{\text{RS},k} - \mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k}\})(\tilde{\mathbf{y}}_{\text{RS},k} - \mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k}\})^H\right\} \\ &= \mathbb{E}\{\tilde{\mathbf{y}}_{\text{RS},k} \tilde{\mathbf{y}}_{\text{RS},k}^H\} \\ &= \tau P_p \mathbb{E}\left\{\sum_{k'=1}^K \mathbf{g}_{\text{SR},k'} \boldsymbol{\varphi}_{\text{S},k'}^H \boldsymbol{\varphi}_{\text{S},k} \boldsymbol{\varphi}_{\text{S},k}^H \boldsymbol{\varphi}_{\text{S},k'}^H \mathbf{g}_{\text{SR},k'}^H\right\} \\ &+ \tau P_p \mathbb{E}\left\{\sum_{k'=1}^K \mathbf{g}_{\text{RD},k'} \boldsymbol{\varphi}_{\text{D},k'}^H \boldsymbol{\varphi}_{\text{S},k} \boldsymbol{\varphi}_{\text{S},k}^H \boldsymbol{\varphi}_{\text{D},k'} \mathbf{g}_{\text{RD},k'}^H\right\} \\ &+ \mathbb{E}\left\{\mathbf{N}_R \boldsymbol{\varphi}_{\text{S},k} (\mathbf{N}_R \boldsymbol{\varphi}_{\text{S},k})^H\right\} \\ &= \left(\tau P_p \sum_{k'=1}^K |\boldsymbol{\varphi}_{\text{S},k'}^H \boldsymbol{\varphi}_{\text{S},k}|^2 \beta_{\text{SR},k'} \right. \\ &\quad \left. + \tau P_p \sum_{k'=1}^K |\boldsymbol{\varphi}_{\text{D},k'}^H \boldsymbol{\varphi}_{\text{S},k}|^2 \beta_{\text{RD},k'} + 1\right) \mathbf{I}_M. \end{aligned} \quad (47)$$

The substitution of (46) and (47) into (45) yields (5). Similarly, we can obtain (6).

### B. Proof of Theorem 1

To compute  $R_{\text{SR},k}$  in a closed-form, we need to find  $\text{NU}_{\text{SR},k}$ ,  $\text{VA}_{\text{SR},k}$ ,  $\text{IU}_{\text{SR},k}$ , and  $\text{AN}_{\text{SR},k}$  in close form.

By expressing  $\mathbf{g}_{\text{SR},k}$  as  $\hat{\mathbf{g}}_{\text{SR},k} + \boldsymbol{\epsilon}_{\text{SR},k}$ , and using the fact that  $\hat{\mathbf{g}}_{\text{SR},k}$  is independent of  $\boldsymbol{\epsilon}_{\text{SR},k}$ , we obtain  $\text{NU}_{\text{SR},k} = M^2 \sigma_{\text{SR},k}^4$ , and  $\text{VA}_{\text{SR},k} = M \sigma_{\text{SR},k}^2 \beta_{\text{SR},k}$ . Using (5), the term  $\text{IU}_{\text{SR},k}$  can be computed as follows:

$$\begin{aligned} \text{IU}_{\text{SR},k} &= \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \mathbb{E}\left\{|\hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{g}_{\text{SR},i}|^2\right\} \\ &= c_{\text{SR},k}^2 \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \mathbb{E}\left\{|\tilde{\mathbf{y}}_{\text{RS},k}^H \mathbf{g}_{\text{SR},i}|^2\right\}. \end{aligned} \quad (48)$$

From (48), we now compute

$$\begin{aligned} &\sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \mathbb{E}\left\{|\tilde{\mathbf{y}}_{\text{RS},k}^H \mathbf{g}_{\text{SR},i}|^2\right\} \\ &= \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \mathbb{E}\left\{\tau P_p \sum_{k'=1}^K \sum_{j=1}^K \mathbf{g}_{\text{SR},k'}^H \mathbf{g}_{\text{SR},i} \mathbf{g}_{\text{SR},i}^H \mathbf{g}_{\text{SR},j} (\boldsymbol{\varphi}_{\text{S},k'}^H \boldsymbol{\varphi}_{\text{S},k})^H (\boldsymbol{\varphi}_{\text{S},j}^H \boldsymbol{\varphi}_{\text{S},k}) \right. \\ &\quad \left. + \tau P_p \sum_{k'=1}^K \sum_{j=1}^K \mathbf{g}_{\text{RD},k'}^H \mathbf{g}_{\text{SR},i} \mathbf{g}_{\text{SR},i}^H \mathbf{g}_{\text{RD},j} (\boldsymbol{\varphi}_{\text{D},k'}^H \boldsymbol{\varphi}_{\text{S},k})^H (\boldsymbol{\varphi}_{\text{D},j}^H \boldsymbol{\varphi}_{\text{S},k}) \right. \\ &\quad \left. + \mathbf{n}_{\text{RS},k}^H \mathbf{g}_{\text{SR},i} \mathbf{g}_{\text{SR},i}^H \mathbf{n}_{\text{RS},k}\right\} \\ &= \sum_{\substack{i=1 \\ i \neq k}}^K P_{\text{S},i} \left(\tau P_p M(M+1) \beta_{\text{SR},i}^2 |\boldsymbol{\varphi}_{\text{S},i}^H \boldsymbol{\varphi}_{\text{S},k}|^2 \right. \\ &\quad \left. + \tau P_p M \beta_{\text{SR},i} \sum_{\substack{k'=1 \\ k' \neq i}}^K \beta_{\text{SR},k'} |\boldsymbol{\varphi}_{\text{S},k'}^H \boldsymbol{\varphi}_{\text{S},k}|^2 \right. \\ &\quad \left. + \tau P_p M \beta_{\text{SR},i} \sum_{k'=1}^K \beta_{\text{SR},k'} |\boldsymbol{\varphi}_{\text{D},k'}^H \boldsymbol{\varphi}_{\text{S},k}|^2 + M \beta_{\text{SR},i}\right). \end{aligned} \quad (49)$$

Substituting (49) into (48), we obtain  $\text{IU}_{\text{SR},k}$  as in (24). Finally,  $\text{AN}_{\text{SR},k} = \mathbb{E}\left\{|\hat{\mathbf{g}}_{\text{SR},k}^H \mathbf{n}_R|^2\right\} = \mathbb{E}\left\{|\hat{\mathbf{g}}_{\text{SR},k}|^2\right\} = M \sigma_{\text{SR},k}^2$ . Then, we can obtain (22). Similarly, we can obtain (23).

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