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ORIGINAL RESEARCH PAPER

An efficient waveform diversity based on variational mode decomposition of coded beat-frequency shifted signals algorithm for multiple-input multiple-output millimetre-wave imaging

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Abstract
Waveforms used in multiple-input multiple-output (MIMO) systems must be considered in such a way that individual transceiver (T-R) pairs can be separated from the composite received signal. An algorithm called variational mode decomposition of coded beat-frequency shifted signals (VMD-CBFSS) is presented for waveform diversity. First, transmitters are divided into several groups. A small shift is applied to the carrier frequency of the transmitters of different groups. Also, signals within each group are encoded by assigning a binary phase in a time block. On the receiver side, by using a multiresolution analysis technique, the combined signal is decomposed into its main frequency components and then the composite signals of each group are decoded to retrieve each T-R pair. VMD-CBFSS provides an efficient solution in terms of overall transmitting time and sampling rate in addition to robustness against channel noise. Moreover, the algorithm is employed in a near-field MIMO millimetre-wave imaging system, and results are presented in the form of numerical simulations.

1 | INTRODUCTION

Unlike phased-array radars [1], which use the scaled version of a single waveform in each element of the antenna array, multiple-input multiple-output (MIMO) radars [2] use arbitrary signals in each channel. The optimal signals for a MIMO radar are orthogonal to each other [3, 4]. Therefore, designing and optimising waveforms to achieve waveform diversity (meaning transmitting orthogonal waveforms from different antennas) is an important issue in MIMO radars [5]. This is important because the waveforms must be considered in such a way that individual transceiver (T-R) pairs can be separated from the composite received signal.

In a class of methods, waveforms are selected based on information theory, estimation theory, or optimization of the covariance matrix of waveforms based on the Cramer-Rao band (CRB) [6–11]. In the information theory-based method, the target impulse response is considered to model target scattering behaviour [6, 7]. In the estimation theory method, the problem of robust waveform design for MIMO radar is considered based on mutual information and minimum mean-square error estimation for target identification and classification [8, 9]. In the third method, MIMO radar waveform optimization is performed by considering several design criteria, including minimising the trace, determinant, and the largest eigenvalue of the CRB matrix [10, 11]. However, the limitation on the number of orthogonal signals and their cross-correlation property may be considered an open problem in these waveform diversity methods. Methods based on the concept of circulating codes [12] make it possible to transmit time-shifted copies of the same waveform. A popular and practical class of waveform diversity is implemented by various multiple access techniques. Although time-division [13] is the simplest technique for implementing orthogonal waveforms, in each time slot, only one transmitter signal can be transmitted and the receivers can receive only the signal corresponding to a single transmitter. Such a structure greatly increases the overall transmitting time. In frequency-division [14] there is no overlap in the transmitted frequencies; As a result, each transmitter occupies a specific bandwidth in the spectral domain. Receiving the waveforms of different transmitters simultaneously requires a large extended bandwidth. This combined bandwidth may be wider than what can be

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sampled by current analogue-to-digital converters (ADCs). Of course, if the whole bandwidth in the frequency-division technique is equated to the bandwidth in the time-division one, this leads to a lower range resolution because each transmitter uses only a portion of the whole bandwidth. In the code-division technique [15], all transmitters transmit at the same central frequency and bandwidth, and there is no need to increase the sampling rate. However, applying this technique in the decoding step requires the waveform in each time block to be transmitted repeatedly (with different codes) to the number of transmitters. Besides, the number of transmitters must be a power of 2.

Owing to significant advances in semiconductor technology, highly integrated circuits are available in millimetre-wave (mm-wave) frequencies in the range of 30–300 GHz [16] at a moderate cost. This suggests that mm-wave radar realization is becoming more viable for more cost-sensitive applications. mm-wave radars are used in a wide range of applications such as aerial imaging, security scanning, medical diagnostics and non-destructive testing [17, 18]. Most orthogonal waveform generation techniques have been developed for pulse radars and as such are not necessarily applicable to continuous-wave types that benefit from less hardware complexity. Frequency-modulated continuous-wave (FMCW) chirp signal-based radars are popular in industrial and automotive applications. A major advantage of the FMCW signal is that the beat signal bandwidth is much less than the instantaneous bandwidth, which in terms of sampling leads to the much-desired simplification of the acquisition ADC.

An algorithm called the variational mode decomposition of coded beat-frequency shifted signals (VMD-CBFSS) is proposed to design effective multiple waveforms in the transmitter and retrieve individual T-R pairs from the noisy composite received signal in a MIMO structure. To make the best use of time and bandwidth, transmitters are first divided into several non-overlapping groups, each of which includes several transmitters (the number of transmitters in different groups does not have to be equal). The same carrier frequency is assigned to the FMCW chirp signals of all transmitters in each group, each of which has a small frequency shift (much smaller than the carrier frequency) relative to the carrier frequency of adjacent group transmitters. Before transmitting, signals within each group are encoded in a time block by assigning binary phases. On the receiver side, first, using a multiresolution analysis technique, the combined signal is decomposed into its main frequency components, and then the composite signal of each group is decoded based on the coding used on the transmitter-side to retrieve the T-R pair. We will employ this algorithm in a specific application for a near-field (NF) mm-wave MIMO imaging system. In the literature, such a system is usually implemented using the time-division technique [17, 19–22], which can pose a significant challenge in real-time applications owing to the increased data acquisition time, as detailed earlier. The proposed algorithm is not limited to such a system; it can be used for waveform diversity in any MIMO application.

The rest of this work is organised as follows: In Section 2, the system model is presented. Section 3 presents the proposed VMD-CBFSS algorithm. In Section 4, we describe the image reconstruction process, whereas in Section 5, we present the application of the VMD-CBFSS algorithm in the context of MIMO-based mm-wave imaging radar. Finally, Section 6 presents the concluding remarks.

Throughout the work, superscript * represents the complex conjugate. Symbols *, ||||^2, Θ[,] and |.| stand for the dot product, squared $L^2$-norm, convolution, exponent of the next higher power of 2 and absolute value, respectively. $j$ is the imaginary unit.

2 | SYSTEM MODEL

We consider the system based on mm-wave radar sensors that use FMCW chirp signals. Consider an FMCW signal whose instantaneous frequency changes linearly with time as:

$$s_g(t) = e^{j2\pi(f_0 t + 0.5Kr^2)}, \quad 0 < t < T_p, \quad (1)$$

where $f_0$ indicates the carrier frequency at $t = 0$, $T_p$ is chirp pulse duration, and $K$ indicates the range chirp (FM Rate). Accordingly, the signal bandwidth is equal to $B = KT_p$. This is a raw signal that can be transmitted by a single transmitter. We will use this signal form as a reference signal in the receiver. We assume that radar measurements are obtained by continuously moving a MIMO array along the x-axis and that the spatial sampling interval is $d_x$ (Figure 1). The MIMO array is assumed to consist of a uniform linear array (ULA) including $M$ transmitter antenna elements with spacing $d_t$ and a ULA including $N$ receiver antenna elements with spacing $d_r$. It is assumed that these two arrays are located relative to each other with an offset $d_o$. According to the effective phase centre principle, under the far-field (FF) assumption, a multistatic array topology with $M + N$ physical elements can be considered a monostatic virtual array with $M \times N$ elements (equal to the number of T-R channels) [23]. In particular, if $d_t = Nd_r$ is selected, a virtual ULA with spacing $d_r/2$ is provided as a result. Because this system is to be used for NF imaging, we need a more accurate model for image reconstruction. Details of this adaptation are given in Section 4.

3 | PROPOSED ALGORITHM FOR WAVEFORM DIVERSITY

According to the description in Section 2, with $M + N$ physical antennas, we have $M \times N$ T-R channels. The main purpose here is to design the transmitted waveforms so that in each receiving antenna ($R_{x_i}$), the signal corresponding to each transmitting antenna ($T_{x_i}$) can be retrieved from the incoming combined signal, where $i = 1, 2, ..., M$ and $\ell = 1, 2, ..., N$. Owing to the disadvantages listed in Section 1, this design is not based on conventional methods; instead, it leverages a new algorithm that considers both time and frequency efficiencies.
First, the transmitters are divided into $L$ non-overlapping groups such that $\sum_{l=0}^{L-1} P_l = M$, where $P_l$ is the number of transmitters within the $l$-th group, and $l = 0, 1, ..., L - 1$ and $P_l \in \{2^n | n = 0, 1, 2, ...\}$. For simplicity, but without loss of generality, assume that only adjacent transmitters are placed in the same group (Figure 2). The same carrier frequency is assigned to the FMCW chirp signals of all transmitters belonging to the $l$-th group, that is, $s_{l,p}(t)$, where $p = 1, 2, ..., P_l$. This carrier frequency has a small shift $f_\Delta$ relative to the carrier frequency of the previous group transmitters, where $f_\Delta = f_\nu$. On the other hand, before transmitting, the signals within the $l$-th group are encoded by assigning binary phases in a time block of length $P_l T_p$. According to this description, transmitted signal $s_{l,p}(t)$ can be written mathematically in the form:

$$s_{l,p}(t) = e^{i 2\pi (f_\nu t + l f_\Delta - \Phi_{l,p})}, \quad 0 < t < P_l T_p,$$

where $\Phi_{l,p}$ represents the binary phase encoded by Walsh-Hadamard codes [24, 25] (a string of values 0 [+1] and 1 [-1]) of length $P_l$. Figure 3, for instance, shows a sequence of four distinct codes (denoted by $C_1$, $C_2$, $C_3$ and $C_4$) used to encode signals in group 0 in a time block (assuming four transmitters for group 0). The chirp phase for each transmitter is determined by the binary value of the code. Because all four transmitters belong to the same group, they all transmit identical frequency chirps. Mathematically, a set of binary codes $\{C_m\}$ is called orthogonal if they have the following properties over a period of time $\Gamma$ [26]:

$$\begin{align*}
C_m \cdot C_{m'} &= 0, \quad m \neq m' \\
C_m \cdot C_m &= C, \quad \text{for all } m
\end{align*}$$

where $C$ is a constant and $m$, $m' = 1, ..., \Gamma$. The first and second formulae in Equation (3) compute the cross-correlation and autocorrelation of the code string, respectively. The most important feature of Walsh-Hadamard codes is that the generated strings are orthogonal to each other. For example, it can be easily investigated that Equation (3) holds for the codes used in Figure 3. The cross-correlation of this type of code is equal to 0, whereas the autocorrelation is equal to the length of the code.

The signal backscattered from the target received by $Rx_\ell$ and induced by $s_{l,p}(t)$ is a time-delayed and scaled version of it mixed with a reference waveform:

$$s_{\ell, l, p}(t) = s_R(t) \cdot s_{\ell, l, p}^* \left( t - \tau_\ell, l, p \right)$$

$$= \sigma_{\ell, l, p} e^{i \left( \tau_\ell, l, p + l f_\Delta \right)} \left( t - (0.5 K_{\ell, l, p} + l f_\Delta) \right) + \Phi_{l,p},$$

Figure 1: System geometry with one-dimensional scanning by a multiple-input multiple-output array
where $f_{\Delta} = K \Delta f$ is known as the beat frequency, which is the sum of it and the shift $\Delta$ that forms the desired frequency. In real systems, a noise term $n_{\Delta, l, p}(t)$ is added to this equation. Notations $\tau$ and $\sigma$ represent the round-trip delay of the echo and the combination of target reflectivity $g$ and the round-trip amplitude decay off the target, respectively, which can be calculated according to Figure 1:

$$
\begin{align*}
\tau &= (R_T + R_R)/c, \\
\sigma &= g/(R_TR_R), \\
R_T &= \sqrt{(x - x_T)^2 + (y - y_T)^2 + z_0^2}, \\
R_R &= \sqrt{(x_R - x)^2 + (y_R - y)^2 + z_0^2},
\end{align*}
$$

where $c$ is the speed of light. The parameters of this equation are generally a function of $i', l$ and $p$, which for simplicity are written in the presented form.

Remark 1 The last terms related to $\tau$ in Equation (4) are known as residual video phase (RVP). This is an unwanted artefact arising from the dechirping process. Because it can be compensated, it is usually ignored [17, 22]. As an idea to compensate for this here, a phase shift of $\pi/2/\Delta f$ can be added to the transmitter-side signal. Then, at the receiver side, a phase of $\pi/K$ times the square of the desired frequency is considered to compensate for the RVP.

When transmitting simultaneously, what is received by each of receiver antenna is the sum of all the waveforms transmitted by $M$ transmitters, which can be expressed mathematically as:

$$s_i(t) = \sum_{l=0}^{L-1} \sum_{p=1}^{P} s_{i, l, p}(t).$$
In continuation of this section, the proposed mechanism for retrieving $M$ signals corresponding to $M$ transmitters from within the received composite signal will be explained.

Here, we use the variational mode decomposition (VMD) technique [27, 28], which provides a multiresolution analysis, to retrieve signals corresponding to the $L$ groups. VMD is able to decompose a multicomponent signal $x(t)$ into $N_g$ band-limited sub-signals $u_n$ called intrinsic mode functions (IMFs) in a completely non-recursive manner [27, 28]:

$$x(t) = \sum_{n=1}^{N_g} u_n(t). \quad (7)$$

According to the properties of $u_n$ [27, 28], because each mode is compacted around its central frequency $f_n$, VMD simultaneously calculates the mode waveforms and their number of groups, that is subsignals multiplied by the estimated central frequency [27, 28]:

$$\min_{\{u_n, \{f_n\}\}} \left\{ \sum_{n=1}^{N_g} \|dB(t)/dt\|^2 \right\}, \quad \text{s.t. } \sum_{n=1}^{N_g} u_n = x, \quad (8)$$

where $B(t)$ is the base frequency spectrum. To calculate it, the analytic signal of the mode function $u_n$ is first obtained using the Hilbert transform. Then, the analytical signal of each $u_n$ is multiplied by the estimated central frequency [27, 28]:

$$B(t) = [(\delta(t) + j/(\pi t)) \otimes u_n(t)]e^{-j2\pi f_n t}. \quad (9)$$

Equation (8) can be solved by introducing a quadratic penalty and Lagrangian multipliers, the details of which can be found in Zhang et al. [27] and Wu et al. [28]. In the VMD technique, how to determine the number of IMFs, that is $N_g$, is important for correct analysis [28]. Fortunately, we have no problem with this here, because $N_g$ in our case is equal to the number of groups, that is $L$, which is a known value.

Remark 2 Multiresolution analyses such as the VMD technique have advantages over conventional frequency analyses such as Fourier transform and time-frequency analyses such as wavelet transform. Although frequency and time-frequency analyses provide useful information, in many situations it is necessary to separate signal components in time and frequency and examine them individually. For example, for the application discussed here, we ideally need such information to be available on the same time scale as the original data, because to reconstruct images from reflected data, it is important to retrieve all parameters of amplitude, frequency and phase of the signal correctly. Multiresolution analysis accomplishes this. In fact, the proposed mechanism provides a way to avoid the need for time-frequency analysis while allowing it to work directly in the time domain. This is where the superiority of the proposed technique over conventional band-pass filtering becomes apparent: because mathematically, passing a signal through a band-pass filter bank is equivalent to applying a wavelet transform [29]. On the other hand, the band-pass filter approach is inadequate when studying non-stationary signals [30] because the frequency content of such signals varies over time, while a filter bank is limited by assumptions regarding frequency, bandwidth and filter design type. In addition, because the VMD technique is efficiently optimised by using an alternating direction method of multipliers approach, it is more robust to noise and sampling rate issues than conventional techniques [31].

Finally, after the composite signal of each of the $L$ groups is retrieved using the VMD technique, we need to decode them according to the type of encoding used when transmitting. For example, the decoding corresponding with each transmitter in Figure 3 can be implemented as:

$$T_{x1} : (C_1 + C_2 + C_3 + C_4)/4,$$
$$T_{x2} : (C_1 - C_2 + C_3 - C_4)/4,$$
$$T_{x3} : (C_1 + C_2 - C_3 - C_4)/4,$$
$$T_{x4} : (C_1 - C_2 - C_3 + C_4)/4. \quad (10)$$

For more details on Walsh-Hadamard encoding and decoding with different code lengths, see Samanta et al. [24] Santra et al. [25] and Ibrahim and Mansour [32].

These operations are performed separately for each $Rx_i$. The steps for retrieving the signals of each channel are shown schematically in Figure 4. The example provided in Section 5 also illustrates various parts of the proposed algorithm.

The steps for implementing the proposed VMD-CBFSS method on the transmitter and receiver side can be found in Algorithm 1. Step 3 in the receiver side is performed to change the frequency of signals from the desired frequency to the beat frequency. Related explanations are provided in Section 4.

Remark 3 The computational complexity of the VMD technique depends on initialising the centre frequency of each mode and the recursive fast Fourier transform [33]. Therefore, considering the major multiplications to retrieve the main frequency components of the complex composite signals received by each receiving antenna, the required number of operations is around $2LN \log \kappa$, where $\kappa$ is the length of the transform domain signal. Also, $P_t$ matrix multiplication operations with complexity $N_p$ are required to decode signals belonging to the $l$-th group, where $N_p = 2(L/f_p)T_p$ is the number of points sampled in duration $T_p$. Thus, in total, the computational complexity for retrieving the signals by the proposed algorithm is approximately equal to $O(2L \log \kappa + MN_p)$. 


FIGURE 4  Steps for retrieving the signals of each channel in the proposed algorithm

4   IMAGE RECONSTRUCTION

Now that the signals of each T-R pair have been retrieved by the technique described in Section 3, to change the frequency of the signals from the desired frequency to the beat frequency, it is sufficient to multiply the complex signals retrieved in the $l$-th group by $e^{-j2\pi f_{\Delta} t}$. Then, according to Equations (4) and (5) and Remark 1, the received beat signal can be written in the wavenumber field as:

$$s(x_T, y_T, x_R, y_R, k) = ge^{j(R_TR_R)}(R_TR_R),$$  \hspace{1cm} (11)$$

where $k = 2\pi f / c$ is the wavenumber of the corresponding to instantaneous frequency $f = f_0 + kt$. Based on this, raw three-dimensional (3D) data of size $MN \times N_x \times N_p$ captured by each T-R pair over the $xy$-domain can be formed to reconstruct the image, where $N_x$ is the total number of measurement points along the $x$-axis.

ALGORITHM 1 Steps of implementing VMD-CBFSS algorithm on the transmitter and receiver side

**Transmitter Side**

**Input:** $M$, $N$, $L$, $f_0$, $f_\Delta$, $T_p$ and $\phi_{1,p}$ encoded by Walsh-Hadamard codes, where $f_\Delta = f_0$, $l = 0, 1, ..., L-1$ and $p = 1, 2, ..., P$

**Output:** $s_{l,p}(t)$

1. Divide the transmitters into $L$ non-overlapping groups such that  \[ \sum_{l=0}^{L-1} P_l = M, \text{ where } P_l \in \{2^n | n = 0, 1, 2, \ldots \}. \]
2. Assign the same carrier frequency $f_0 - 1f_{\Delta}$ to the FMCW chirp signals of all transmitters belonging to the $l$-th group, that is $s_{l,p}(t)$.
3. Encode the signals within the $l$-th group by assigning $\phi_{l,p}$ in a time block of length $P_lT_p$.

**Receiver Side**

**Input:** $s_{l'}(t) = \sum_{l=0}^{L-1} \sum_{p=1}^{P} s_{l',1,p}(t) + n_{l',1,p}(t), f_{\Delta}$ and $L$, where $l' = 1, 2, ..., N$

**Output:** The retrieved signal associated with channel $Tx_l-Rx_{l'}$, where $l = 1, 2, ..., M$ for $l' = 1$ to $N$ do

1. According to Equations (7)-(9), apply the VMD technique to $s_{l}(t)$ to retrieve the composite signal of each $L$ group.
2. Decode the composite signal of the $l$-th group according to the type of encoding used on the transmitter side.
3. Multiply the complex signals retrieved in the $l$-th group by $e^{-j2\pi f_{\Delta} t}$.

As mentioned in Section 2, because the system is to be used for NF imaging, we need a more accurate system model for image reconstruction than that provided in Section 2. Suppose $(x', y', 0)$ is the position of the phase centre corresponding to the transmitter element at $(x_T, y_T, 0)$ and the receiver element
Table 1 Simulation parameters

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<th>M</th>
<th>N</th>
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<th>d_2</th>
<th>d_m</th>
<th>x_0</th>
<th>D_x^R</th>
<th>D_y^R</th>
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<td>95 kHz</td>
<td>3</td>
<td>4</td>
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</tbody>
</table>

Figure 5 Real part of the noisy composite signal sent from 12 transmitters and received by Rx_1 in a time block.

at (x_R, y_R, 0). By using multistatic-to-monostatic conversion [34], the multistatic data set in Equation (11) can be converted to an effective monostatic version as:

\[
\tilde{s}(x', y', k) = s(x_T, y_T, x_R, y_R, k) \frac{s_R(x', y', k)}{s_R(x_T, y_T, x_R, y_R, k)}
\]

where \(s_R(x', y', k)\) and \(s_R(x_T, y_T, x_R, y_R, k)\) correspond to the monostatic and multistatic reference signals, respectively [22].

Finally, 2D target reflectivity can be reconstructed as [19]:

\[
g(x, y) = \text{IFFT}_{2D}\left[S(k_x, k_y, k)k_xe^{-jk_z}\right]dk,
\]

where \(S(k_x, k_y, k)\) denotes the Fourier transform of \(s(x, y, k)\). The presented system and proposed algorithm are not limited to 2D reconstruction. Therefore, a 3D imaging system can be achieved only by modifying the image reconstruction step and using 3D inverse fast Fourier transform [22].

5 | SIMULATION RESULTS

This section presents the numerical results (in MATLAB) of the proposed algorithm and the introduced system in the presence of white Gaussian noise. The simulation parameters are given in Table 1, where \(\lambda\) is the wavelength calculated at the centre frequency of 79 GHz. All results (except Figure 10) are provided for a signal-to-noise ratio (SNR) of 15 dB. According to the parameters in Table 1, targets with distances less than approximately 39 m from the array are located in the NF [35].

Figure 5 shows the real part of the composite signal sent from 12 transmitters and received by Rx_1 at scanning point \(x = 0\) in a time block. Figure 6 shows the beat spectrum of the received composite signal based on conventional Fourier analysis in both noiseless and noisy cases. As expected, three peaks (corresponding to three groups) are observed. Although these peaks were expected to be observed at intervals of
approximately 95 kHz (equal to $f_\Delta$), Figure 6(b) indicates that the distance between the second and third peaks in the noisy case is significantly greater than expected. Figure 7 shows the magnitude scalogram of the continuous wavelet transform of the corresponding signal using a Morse mother wavelet [36]. At the same peak frequencies observed in Figure 6, bright areas are observed. Also, this figure shows the blue peaks at the end of each chirp and the beginning of the next chirp. Although some information about frequency and phase can be extracted from Figures 6 and 7, they are still insufficient to retrieve channel signals, because especially in our case, the signal amplitude information is important to image reconstruction. Figure 8 shows the result of the signals retrieved in each group using the proposed algorithm. Figure 8(a), which shows the smoothest signal, is equivalent to the corresponding signal received in group 0. Figure 8(b) and (c) also shows the signal corresponding to groups 1 and 2, respectively, which were sent in frequency shifts of one and two times $f_\Delta$. The retrieved signals in terms of amplitude, frequency and phase are in line with the true signal (except towards the ends of the chirp periods). The momentary heterogeneities, the effect of which is also seen in Figure 7, occur owing to sudden changes in the phase of the encoded chirps during the time block. Figure 9 shows, as only two instances, the retrieved signal associated with the second and 10th channels obtained after decoding the signals of Figure 8. Besides, to evaluate the recovery performance of noise-impregnated signals in the proposed algorithm, we consider the mean-squared error (MSE) criterion (in dB) as:

$$MSE = 10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^{n} |Y_i - \hat{Y}_i|^2 \right)$$  \hspace{1cm} (14)
where \( \mathbf{Y} \) and \( \mathbf{\hat{Y}} \) are the vectors of actual values and retrieved values, respectively, and \( n \) is the total number of experiments. Figure 10 shows the MSE performance of the proposed algorithm versus SNR over 1000 independent runs. MSE is obtained in two cases. In the first case, the error between the retrieved signal and the noise-free signal is calculated, which is indicated in the figure by a solid line. In the second case, the error between the retrieved signal and the true noisy signal is calculated, which is indicated by a dashed line. The results of case 1 show less error, which means that the signal retrieved by the proposed algorithm is more similar to the noise-free signal than the noisy signal. Owing to the uniform presence of Gaussian noise at all frequencies, the frequency analysis of the VMD technique, in addition to separating the main components, also inherently acts as a noise removal filter. This advantage can also be seen in Figure 9. In addition to the MSE associated with each group, Figure 10 shows the average MSE corresponding to all T-R pairs. Obviously, the MSEs of pairs should be less than group MSEs, which is also clear in the figure.

Figure 11 shows the image obtained from the target profile in Figure 1 by the mechanism described in Section 4 and with the parameters of Table 1. According to the imaging system and target aperture sizes, the wavelength and the target range, and based on Equations (6) and (7) in [37], the Nyquist sampling constraints are satisfied and the simulated system has a resolution of about 2.1 mm (0.55\( \lambda \)) in both the \( x \)- and \( y \)-axis.

In another numerical example, we consider a 3D NF imaging scenario. Suppose nine point scatterers are arranged at a distance of \( z_0 = 3m \) (Figure 12). Four of these scatterers

**Figure 8** Comparison of real part of signals retrieved using the proposed algorithm and corresponding true signals: (a) group 0, (b) group 1, and (c) group 2.

**Figure 9** Real part of retrieved signal associated with: (a) \( Tx_2-Rx_1 \); (b) \( Tx_{10}-Rx_1 \).

**Figure 10** Shows the MSE performance of the proposed algorithm versus SNR over 1000 independent runs.
located in the corners of the square are placed at $z = 2.9\text{m}$, the central element at $z = 3\text{m}$, and the four elements remain at $z = 3.1\text{m}$. According to Figure 12, $D_x^r$, $D_y^r$ and $D_z^r$ are equal to 1000, 1000 and 200 mm, respectively. The other parameters are similar to the values presented in Table 1. Figure 13 shows the normalized isosurface corresponding to the point scatterers reconstructed by the proposed algorithm. All nine point targets have been reconstructed correctly. The accuracy of the reconstructed targets along the $y$-axis is somewhat lower than the $x$-axis (especially in the case of the four points closer to the array, marked in blue). This can be better seen in Figure 13(b). The reason for this is the approximation resulting from the use of a multistatic-to-monostatic conversion in the NF. Because physical scanning is performed along the $x$-axis, we have obtained good accuracy along the $x$-axis, whereas the aperture along the $y$-axis is covered by a virtual linear array. Artefacts created along the $y$-axis (vertical axis) in Figure 11 are also attributed to this approximation. As we get closer to the array, this problem becomes more acute as we move farther from the FF conditions and the efficiency of Equation (12) decreases. However, Equation (12) is still an effective and acceptable approximation for the NF scenario. Figure 14 shows the isosurface of the normalized reconstructed image when we did not use multistatic-to-monostatic conversion. By comparing

**FIGURE 10** Mean-squared error performance of proposed algorithm versus signal-to-noise ratio

**FIGURE 11** The reconstructed image

**FIGURE 12** Location of nine point targets used in a three-dimensional (3D) and near-field scenario; (a) 3D view, (b) 2D view on $x-y$ plane, (c) 2D view on $x-z$ plane
Figure 13(a) with Figure 14, the importance of the multistatic-to-monostatic conversion is evident. Figure 15 compares the reconstructed images in the proposed algorithm and Yanik et al. [22], which uses time-division multiplexing to achieve transmit waveform orthogonality. These images are obtained from the superposition of all values corresponding to the

**FIGURE 13**  Isosurfaces of normalized reconstructed image; (a) three-dimensional (3D) view, (b) 2D view on the x-y plane, (c) 2D view on the x-z plane
reconstructed 3D data in the depth direction (z-axis). Because the image reconstruction step is the same in both methods, the results are similar, as expected. However, if the images are zoomed in the presence of noise in Figure 15(b) provides better quality than in Figure 15(a). This is the effect of applying the VMD technique in the proposed algorithm. More noteworthy is the time comparisons between the proposed algorithm and Yanik et al. [22]. In general, two time consumption factors can be considered here. One is data acquisition time and the other is processing time. The image reconstruction time is the same in both the proposed method and the method of Yanik et al. [22]. However, a significant advantage of the proposed method becomes evident when we consider the data acquisition time. In time division-based methods such as Yanik et al. [22], only one transmitter can transmit at a time interval. The total data acquisition time in the method of Yanik et al. [22] is equal to 0.173 s, whereas this time in the proposed algorithm is equal to 0.058 (approximately one-third of the method of Yanik et al. [22]).

Figure 16(a) and (b) shows a comparison in terms of overall transmitting time versus the $M$ and $N_x$ parameters, respectively, when the VMD-CBFSS algorithm, and time- and code-division techniques are employed. In Figure 16(a), $P_{\text{max}}$ is assumed to be equal to $M/4$, in which $P_{\text{max}} = \max\{P_0, \ldots, P_{L-1}\}$. In Figure 16(b), $M$ and $P_{\text{max}}$ are considered to be equal to 16 and 4, respectively. The other parameters are as shown in Table 1. Increasing the $M$ and $N_x$ parameters can increase the imaging system aperture size or decrease the spatial sampling steps, both of which improve cross-range resolutions. Figure 16 shows that as these two parameters increase, the performance superiority of the proposed algorithm becomes more evident in terms of data acquisition time. Also, Figure 17 shows a comparison in terms of the sampling rate versus $M$, when the VMD-CBFSS algorithm and frequency-division technique are employed. Figure 17 demonstrates that increasing the number of transmitted signals in frequency division-based techniques can lead to a large increase in sampling rate (possibly unattainable with current ADCs), whereas the increase in sampling rate in the proposed algorithm is a factor of $f_{\Delta}$, which is a small value.

6 | CONCLUSIONS

A waveform diversity algorithm based on encoding beat-frequency shifted signals and VMD was introduced for use in MIMO systems. It was employed in a specific application (an NF MIMO mm-wave imaging system). Owing to the joint use of multiresolution analysis and
coding, the VMD-CBFSS gives desirable flexibility to the designer on the transmitter side and benefits from efficiency at the overall transmitting time and sampling rate. The overall transmitting time in the conventional time-division and code-division techniques is equal to $N_x M T_p$, whereas in the proposed algorithm this time is equal to $N_x P_{\text{max}} T_p$. Although the overall transmission time in the proposed mechanism is much shorter than in the time-division technique, it requires extra processing on the receiver side. However, because this processing can be done in parallel and independently for the data received from all receiver elements, and with the availability of current powerful processors, such a mechanism is more efficient for real-time applications. Another limitation of the code-division mechanism is that the number of transmitters must be a power of 2, whereas in the proposed algorithm, the number of transmitters can be an odd number if required. This facilitates the design flexibility on the transmitter side. In the frequency-division technique, the bandwidth and consequently the sampling rate are expanded to $M$ times, whereas in the proposed algorithm, the sampling rate requires only a minor extension of $(P_{\text{max}} - 1) f_{\Delta}$.

**Figure 15** Reconstructed images; (a) by using the proposed algorithm; (b) by using Yanik et al. [22]
FIGURE 16 Comparison of overall transmitting time when the variational mode decomposition of coded beat-frequency shifted signals algorithm and time- and code-division techniques are employed: (a) versus $M$; (b) versus $N_x$.

FIGURE 17 Comparison of sampling rate when the variational mode decomposition of coded beat-frequency shifted signals algorithm and frequency-division technique are employed.
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CONFLICT OF INTEREST

None.

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