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Boyle, L. M., Marshall, A. H., & Mackay, M. (2021). A framework for developing generalisable discrete event simulation models of hospital emergency departments. *European Journal of Operational Research*. Advance online publication. https://doi.org/10.1016/j.ejor.2021.12.033

Published in:
European Journal of Operational Research

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
Link to publication record in Queen's University Belfast Research Portal

A framework for developing generalisable discrete event simulation models of hospital emergency departments.

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Abstract

Discrete event simulation (DES) is routinely used to model hospital emergency departments (EDs), primarily due to its ability to represent complex patient flow processes and investigate improvement strategies. Despite this, it is clear from published studies that many DES models are not subsequently implemented in hospitals or reused for other sites. This research addresses a gap in the literature by presenting a new data-driven modelling framework 'GE-DES', which outlines an approach to the design and development of generalisable ED models. The nature of the framework means that it is sufficiently flexible (i) for use across multiple EDs, and (ii) for investigating hospital-specific problems through data-driven customisation. The primary aim of GE-DES is to support model reuse and implementation. The framework is demonstrated through application to a case study ED in Australia.

Keywords: Simulation; OR in health services; Emergency departments; Discrete event simulation; Generalisability

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1. Introduction

Hospital emergency departments (EDs) operate under significant pressure worldwide. Overcrowding is a frequent occurrence, caused by a combination of high numbers of patient presentations, insufficient resources within EDs, and long delays in the transfer or discharge of patients due to a lack of available hospital beds and care in the community (Richardson et al. (2009). Frequent breaches of performance targets are reported in a wide range of countries, including Australia, where presentations have been increasing by an average of 3.2% each year between 2014-15 and 2018-19. Discrete event simulation (DES) is a well-established technique for modelling EDs (Salmon et al. (2018)). It is well suited for capturing the complexity of ED processes and for evaluating proposed system improvement strategies. Despite the clear benefits of using DES to support ED decision making, many published models are not implemented within ED units (Mohiuddin et al. (2017)). This has been attributed to (i) insufficient collaboration between modellers and domain experts; (ii) highly detailed models, which represent a specific ED (and cannot be used for other EDs); and (iii) the development of models which will quickly become outdated. Sinreich & Marmor (2004) and Fletcher & Worthington (2009) identified a potential to improve model reuse and implementation through the use of 'generic' models. Momentum towards the development of 'generic' ED DES models has continued in the literature (Salmon et al. (2018)), including the development of a 'generic' conceptual modelling framework for EDs (Furian et al. (2018)).

'Generic' ED DES models have the capacity to investigate 'general problems faced by hospitals' and 'general solutions to improve service delivery' (Fletcher & Worthington (2009)). They can be constructed entirely with theoretical inputs and without reference to site-specific ED data. To the best of our knowledge, there is no defined class of (or modelling framework for) ED DES models which are *sufficiently general for use across multiple EDs*, and which can be customised with hospital-specific data to investigate *unit-specific problems and solutions* to improve service delivery. We address this gap in the literature through the following research contributions:

- the term 'generalisable' is formally defined and presented in the context of ED DES models;
- a new data-driven modelling framework is presented for developing generalisable discrete event simulation models of hospital emergency departments (generalisable ED DES, or 'GE-DES'), which provides structured, domain-specific, guidance for both the conceptual modelling and model-building processes;
- a successful application of GE-DES is demonstrated with a case study in Australia.

GE-DES supports the development of a generalisable DES model of an ED. Then, if the care process and data collection system are sufficiently similar to a different ED, the generalisable model can be used in it also. Because the framework employs a data-driven approach, the generalisable model can be customised to each individual ED unit by using site-specific data to represent it's unique system behaviour. This generalisable model will be suitable for reuse across many EDs. If the generalisable model is not suitable for reuse in a different ED (e.g., if the care process and data collection system differ significantly) it is recommended that the reader use the GE-DES framework to design a different generalisable model structure.

In light of the COVID-19 pandemic, there is an even more pressing need for such frameworks to support model development, reuse, and implementation across multiple healthcare organisations through the rapid development of conceptual models (Currie et al. (2020)). EDs in particular have experienced major changes during the pandemic, such as fluctuations in presentation numbers (Australian Institute of Health & Welfare (2021)) and new procedures to safely handle social distancing and the treatment of COVID-19 patients (The Royal College of Emergency Medicine (2020)). Use of a generalisable model would be advantageous in such situations, where a sudden change in ED activity can be accommodated quickly through re-estimation of the model parameters.

The rest of the paper is structured as follows. Section 2 discusses the theoretical development of generalisable ED models in the literature and

formally defines 'generalisable' ED DES modelling. Section 3 introduces the new GE-DES framework and Section 4 presents a case study application to an Australian ED. Section 5 concludes with the strengths and limitations of GE-DES, as well as opportunities for future work.

2. Generalisable discrete event simulation of emergency departments

2.1. Generalisable versus generic discrete event simulation

Recent reviews of the ED DES literature have categorised models as either 'generic' or 'specific' (Salmon et al. (2018); Mohiuddin et al. (2017); Furian et al. (2018)). However, it is evident that some of these models can be described differently - as 'generalisable', rather than generic or specific. For example, Facchin et al. (2010) and Hoot et al. (2008) have referred to their models as 'generalisable' and Ceglowski et al. (2007) as 'generally applicable'.

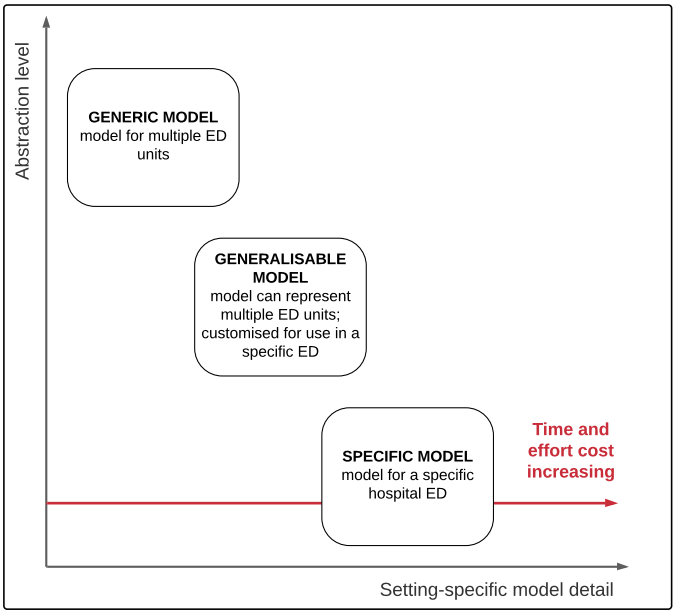

Figure 1: The differences between 'generic', 'generalisable', and 'specific' ED DES models.

We identify a key difference between generic and generalisable models. In relation to generic models, Fletcher & Worthington (2009) have stated, 'generic hospital models suggested a potential for understanding general problems faced by hospitals, and the potential of general solutions to improve service delivery'. Thus, 'generic' models can be constructed using theoretical inputs, without reference to hospital-specific data. By contrast, *we define 'generalisable' models as those that can represent multiple units and, through customisation using real data, be used to investigate site-specific problems.* Figure 1 shows an illustration of the roles of 'generic', 'generalisable' and 'specific' models along the scales of abstraction level and ability to represent setting-specific detail. Generalisable models form a middle ground, where models can be reused across multiple hospitals *and* unit-specific problems can be investigated, as well as giving a balance to the time and effort required for model development.

2.2. Summary of the existing literature on generalisable discrete event simulation models for hospital emergency departments

A subset of models have been identified from recent literature reviews (Salmon et al. (2018); Mohiuddin et al. (2017); Furian et al. (2018)) as generalisable under the definition in Section 2.1 and summarised in Table 1 (and Tables 1 & 2, supplementary material 1). These publications have approached generalisability from a range of perspectives with consequent wide variety in the purpose, detail, and structure of DES models. Several barriers to generalisability are evident in the published studies:

1. differences in the description of generality for each model (table 1);
2. differences and lack of clarity on the generalisable aspects of each model, for example patient flow through stages of ED, or the arrival flow of patients to ED units (Mielczarek (2014));
3. narrow model purpose (table 1) - some models are designed only for high-level forecasting, and are therefore unsuitable for investigating operational aspects of EDs, such as required bed numbers;
4. different software packages utilised in the publications (table 1);

Table 1: Generalisable ED DES model literature review: Background

	Healthcare System	**Description of Generality**	**Model purpose**				**Software**
			Proof of concept	**Service design**	**Forecast/ disaster**	**Understand**	
Miller et al. (2004)	USA	Generic		•		•	EDSim
Sinreich & Marmor (2004)	Israel	Generic		•			Arena
Günal & Pidd (2006)	UK	Generic		•			Micro Saint
Günal & Pidd (2009)	UK	Generic				•	Micro Saint
Virtue et al. (2006)	UK	Generic				•	SIMUL8
Virtue et al. (2011)	UK	Generic	•				SIMUL8
Ceglowski et al. (2007)	Australia	General	•				SIMUL8
Fletcher et al. (2007)	UK	Generic		•			SIMUL8
Hoot et al. (2008)	USA	Generalisable	•	•			C
Facchin et al. (2010)	Italy	Generalised	•				Micro Saint
Mes & Bruens (2012)	Netherlands	Generalised		•			Tecnomatix Plant
Paul & Lin (2012)	USA	Generic	•	•			ProModel
Hurwitz et al. (2014)	USA	No description	•			•	R
Mielczarek (2014)	Poland	No description			•		Not specified
Gul & Guneri (2015)	Turkey	No description			•		AnyLogic
Cimellaro & Piqué (2016)	Italy	No description	•		•		ProModel

5. unrealistic process time assumptions (such as average values) and estimated process times using expert judgement (e.g., triangular distributions), rather than distributions fitted using primary data on length of stay (tables 1 & 2, supplementary material 1);
6. major differences between the approaches for modelling activity progression, arrival processes, and ED resources (tables 1 & 2, supplementary material 1);
7. unclear or unspecified data sources for some models, whilst other studies required specialised data that is not routinely collected, and was therefore substituted with observational data or expert opinion (tables 1 & 2, supplementary material 1).

2.3. Motivation for the theoretical development of a generalisable modelling framework for emergency departments

The models reviewed in Section 2.2 present a wide range of approaches to generalisability, with little guidance for conceptual model development. A unified approach, detailing the steps (including conceptual steps) for model development would be beneficial to both simulation modellers and health care managers. Moreover, this would increase the potential for model implementation and reuse in specific hospital EDs. Table 1 shows that generalisable models have emerged internationally. Notably, similarities exist in the patient flow process descriptions, geographical layout (Salmon et al. (2018)), performance assessment in respect of time-based targets, and grouping of patients by urgency or acuity level. These considerations suggest that it is possible, and appropriate, to design a framework for generalisable simulation modelling, that represents patient flow through EDs internationally.

GE-DES adopts the overarching structure of the Robinson (2008) general conceptual modelling framework for simulation studies. This has since been adapted for stroke systems by Monks et al. (2017), who identified EDs as a key area that would benefit from a domain-specific conceptual modelling framework. A different generic ED DES toolkit by Furian et al. (2018) classifies a range of model component choices from the literature, with the aim of speedy model building. The common approach of these frameworks is

to summarise the existing DES literature into a set of options for conceptual model development.

Although the sub-section headings are similar, the content of GE-DES differs significantly from the above-referenced frameworks. In contrast to providing a summary of options from the literature, GE-DES's *purpose is to guide the conceptual design of a DES model that (i) can be populated entirely with data from emergency medical records, (ii) is sufficiently generalisable for use across many different hospitals and healthcare systems, and (iii) can be used to investigate unit-specific problems by data customisation.* GE-DES also includes a unique 'model building' section, which outlines a combined DES and survival analysis approach, to guide the translation of the conceptual ED DES model into a computer model.

3. The generalisable discrete event simulation for emergency departments (GE-DES) Framework

The GE-DES framework is constructed in four sections: (I) understanding the problem situation; (II) setting the modelling objectives; (III) determining the model content; and (IV) model building (see Fig. 2). Where relevant, the conceptual modelling components in sections (I)-(III) include a link to the preparatory and exploratory data analysis steps of section (IV).

3.1. GE-DES Section I: Understanding the problem situation

Adopting the structure of Monks et al. (2017), four areas of situational knowledge are used to facilitate understanding of the problem situation: (i) the study population, (ii) a process map, (iii) assessment of current system performance, and (iv) exploration of the decision variables for scenario experimentation. The problem situation should be identified through both discussion with domain experts and analysis of hospital data (Robinson (2008)).

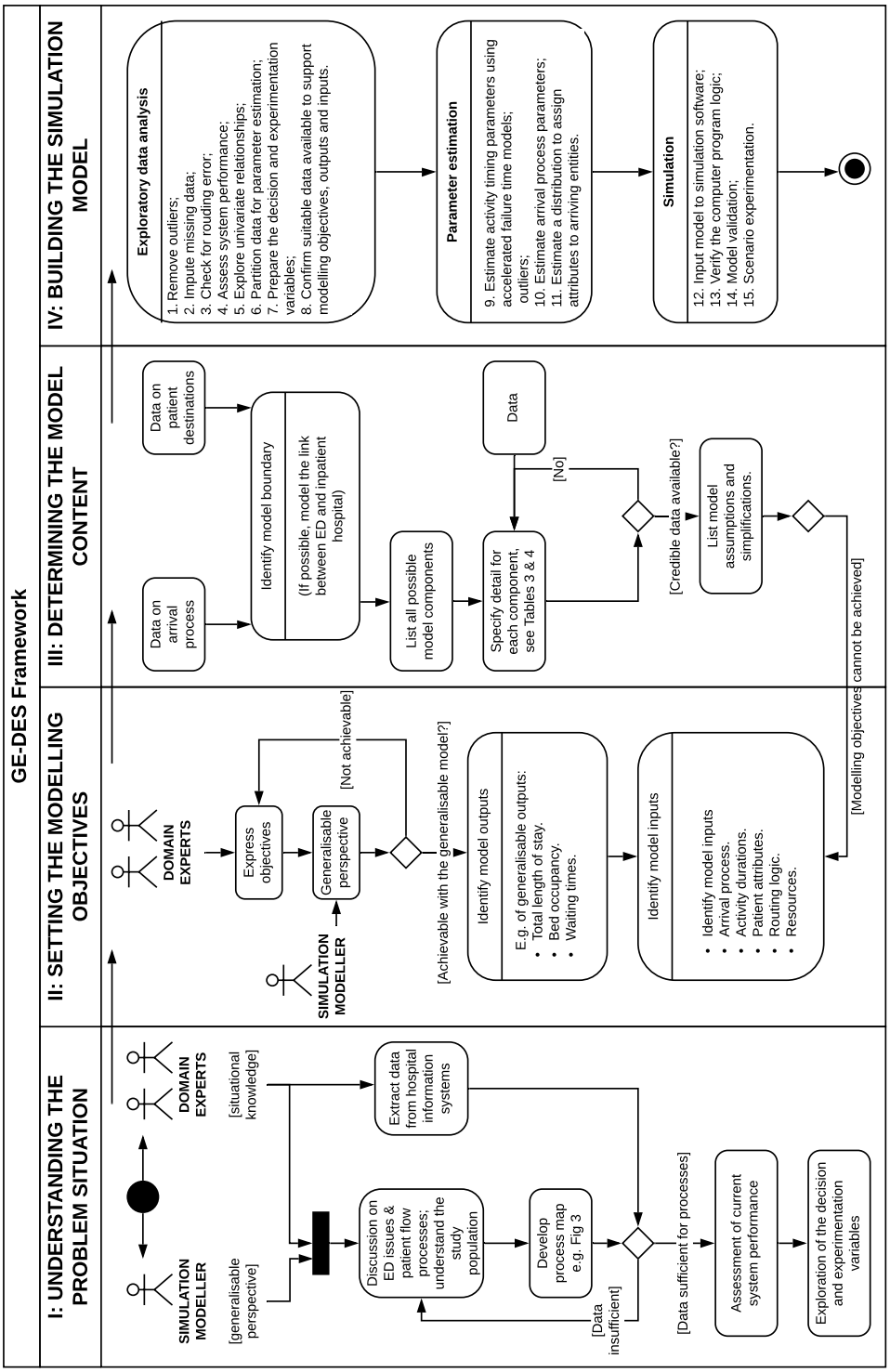

Figure 2: Unified Modeling Language (UML) Activity Diagram for the GE-DES framework.

For example, if the study aims to improve compliance with a four-hour time target, an understanding of the problem situation would be enhanced through exploratory analysis of hospital data, which could help to identify subgroups of patients benefiting most from a targeted improvement strategy. The modelling approach should reflect the questions being asked of the system and at this step the simulation modeller should decide whether DES is the most suitable choice.

3.1.1. The study population

An understanding should be developed of the type of patients seen by the ED (such as adults and/or children, or urgency level of patients) as well as any special services (e.g., direct hospital admission for patients presenting with chest pain, resuscitation area, or long-stay observation area). All information of this type should be recorded in the data as a set of attributes $\boldsymbol{\theta}$ which may inform patient flow pathways $\boldsymbol{\rho}$ and influence ED length of stay (LOS). *(This Section (3.1.1) links to Section 3.4.1-IV Phase 1, Steps 5-6).*

3.1.2. Process map

Fig. 3 provides a map of ED processes that can be used for model development. Patient flow has three ED stages: waiting room time, treatment time, and extended time. These represent the time-stamped data readily available from most hospital information systems, and can be represented using varying levels of detail depending on the objectives of the study. Waiting room time describes the LOS between arrival or triage and the time first seen by an ED doctor. Some hospitals collect the time stamps for one of 'arrival' or 'triage' but not both (as observed in the case study in Section 4). Some patients may not see the ED doctor if they are seen and sent home by another health professional or leave without being seen (LWBS). Treatment time represents LOS between first contact with an ED doctor and treatment completion time. Extended time represents the time between ED treatment completion, and physical departure from ED (by discharge, transfer, or admission). This can be instantaneous, however some patients spend

additional (extended) time in ED - e.g., a period of observation (discharge patients) or waiting for an inpatient bed (admit patients). Patients may leave ED from the treatment time or extended time stages before treatment has completed (LBTC).

At this step in the framework, Fig. 3 should be tailored to reflect the time-stamps available in the data, stimulate a discussion on the different streams of patients, and define the pathways $\boldsymbol{\rho}$ taken by each through the ED. The patient attributes $\boldsymbol{\theta}$ discussed in Section 3.1.1 may help to define any special ED processes to be marked on the map, e.g., the direct admission of cardiac patients to inpatient hospital.

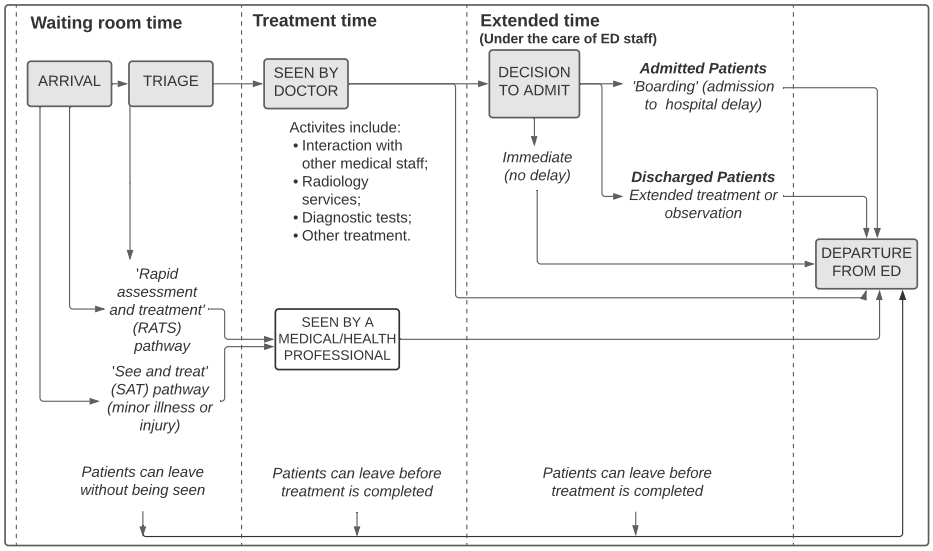

Figure 3: Overview of patient flow through hospital emergency departments. The shaded boxes represent time-stamps that are typically collected in hospitals. Two common alternative emergency department models of care are represented as arrows feeding into the white box.

Fig. 3 shows two additional processes arising from different 'rapid assessment and treatment' (RAT) and 'see and treat' (SAT) ED models of care. In these cases, the RAT/SAT patients should be flagged using an indicator variable in the data. The generalisable model should include distinct rules on which patients can use these pathways, as well as separate queues for the

relevant resources (including any restrictions on the time of day and day of week that they operate). The GE-DES framework can be similarly adapted to other ED processes and models of care. *(This Section (3.1.2) links to Section 3.4.1-IV Phase 1, Step 7).*

3.1.3. Assessment of current system performance

Assessment of current performance should be conducted through both interviews with domain experts and analysis of data. For example, domain experts can give an indication of their understanding of issues in the ED, such as a shortage of beds. Key performance indicators (KPIs) should be employed to assess current ED system performance. LOS is the most frequently used KPI for EDs and is itself used to calculate a significant number of other frequently used KPIs (e.g., arrival to treatment space, or LOS targets (Furian et al. (2018)). At this stage of model development, the LOS data could be split into three sections (process map Fig. 3), and further by pathways $\boldsymbol{\rho}$, allowing a more specific analysis of ED problem areas, including scope for investigating the admit and discharge patients separately. Comprehensive lists of other KPIs (such as queue length) that could be used to assess ED system performance are included in recent review papers (Furian et al. (2018); Mohiuddin et al. (2017); Salmon et al. (2018)). *(This Section (3.1.3) links to Section 3.4.1-IV Phase 1 Step 4).*

3.1.4. Exploration of the variables

Exploration of the variables for analysis should consist of a similar consultation with both domain experts and historic data. Discussions could outline the decision variables $\boldsymbol{\nu}_d$ which can be controlled for within the system (e.g., the number of hospital beds, or capacity of the waiting room) and highlight variables for scenario experimentation $\boldsymbol{\nu}_e$, such as the proportion of ambulance arrivals versus walk-in patients. It is possible to investigate scenarios involving resources, processes, and the environment (Paul et al. (2010)). Resource-related experiments could involve changing the number of ED beds, or increasing the speed at which patients can be admitted to inpatient units. Process scenarios could be investigated, e.g., the introduction

of fast-track beds, or a new pathway for a subgroup of patients e.g., cardiac patients. Environment-related changes could involve altering patient demand. For some experiments it may be important to consider whether new data is required (e.g., to investigate the improvement of a proposed new cardiac pathway, an indicator of which patients were classified as cardiac in the historic data would be needed). However, for many experiments such data is not a constraint (e.g., uncertainty in the demand for ED services next year could be explored using a low, medium, and high percentage increase). This information should inform the choice of variables for inclusion in the model. *(This Section (3.1.4) links to Section 3.4.1-IV Phase 1 Step 7).*

3.2. GE-DES Section II: Setting the modelling objectives

3.2.1. Determining the modelling objectives

An important consideration for generalisable modelling is the relationship between a model's objectives, detail and generality. Günal (2012) indicated that increased model detail would lead to reduced generality. When designing a generalisable model for use in unit-specific ED settings, the study objectives should be defined with an understanding that there is a limitation to the level of detail that it is possible to represent. The detail in generalisable model design is limited by the data which is routinely collected in EDs. If the data available is insufficient to support the modelling objectives, then a more traditional specific ED model could be developed (Günal & Pidd (2010)). For example, a generalisable model could not be used for the objective of investigating ED staff utilisation because most datasets do not reliably record this information.

3.2.2. Identifying the model outputs

Define model outputs $\boldsymbol{\sigma}$ relevant to the modelling objectives. ED DES studies commonly use KPIs as outputs, e.g., waiting time, total LOS, patient throughput, bed occupancy, and proportion of patients who leave without being seen. LOS is the most commonly used model output in ED DES models and is used in the calculation of many other time-based KPIs (Furian et al. (2018); Mohiuddin et al. (2017)) e.g., the number of patients seen

within a given time. A generalisable model is intended to capture commonly used outputs in ED DES, rather than all potential options e.g., resource utilisation could be measured at the level of ED beds, but not X-ray equipment.

3.2.3. Identifying the model inputs

Robinson (2008) defined model inputs as experimental factors, which can be controlled in the simulation. For ED DES models, there are a range of decision and experimentation variables that fall into the following categories: arrival rates, activity durations, activity progressions, and resources (Mohiuddin et al. (2017)). These inputs are outlined in section 3.3.

3.3. GE-DES Section III: Determining the model content

3.3.1. Model scope

Model scope consists of (i) identifying the model boundary, (ii) identifying all of the revelant model components, and (iii) assessing whether to include each of the listed components (Robinson (2008)). GE-DES lists components which have routinely been used in ED simulations, and considers the feasibility of each in a generalisable model setting, combining points (ii) and (iii).

1. **Identify the model boundary**

   Unit- and facility-specific ED case studies tend to ignore what is happening beyond the immediate boundaries of the unit (Günal & Pidd (2010)). In reality, ED operations are heavily influenced by their surrounding environment, both from unscheduled patient arrivals, and from availability of inpatient beds (Richardson et al. (2009)). Studies modelling the inpatient admission process as a simple queue reported the underestimation of boarding times (Bair et al. (2010); Ben-Tovim et al. (2016)). This is because boarding times do not correspond directly to the availability of inpatient beds, and have additional contributors. An alternative approach by Levin et al. (2008, 2011) successfully modelled boarding times using a Cox PH model with covariates which related to various sources of competing bed demand. GE-DES presents an approach for using survival analysis techniques to model boarding

times as a function of factors relating to the inpatient hospital. This approach is flexible, because varying levels of detail can be included as covariates, subject to the availability of suitable data (section 3.4).

2. **Identify the model components and assess which to include** Table 2 contains an illustrative set of examples of common model components identified from the published literature. A recommendation on whether to include each component is made from a generalisable modelling perspective, accounting for the availability of suitable data, as models can be compromised by including components for which there is no credible data Günal (2012).

Component	Decision	Justification
Entities		
Patients	Include	Outcome concerns LOS of patients
Activities		
Triage	Include	Triage category influences LOS
Specific medical tests	Exclude	Lack of credible data
Treatment	Include	Impacts on LOS
Progression		
Waiting room queue	Include	Experimental factor
Queue for inpatient beds	Include	Lack of credible data - can be represented using time distribution instead
Resources		
Medical staff	Exclude	Lack of credible data
ED beds	Include	Bed shortages impact LOS

Table 2: Example components for a generalisable ED simulation model

3.3.2. Model level of detail

This concerns the simulation model depth, and specifically the level of detail required for each model component. Detail should not be included unless there is reliable data to inform the model parameters (Monks et al. (2017)). A generalisable model can be designed by considering ED patient flow as three distinct phases (Fig. 3). In doing so, treatment time is considered as an amalgamated process of diagnostic tests, assessments, treatment, and the time required to arrange this work (i.e. waiting). This approach has been

used successfully in previous generalisable models (Facchin et al. (2010); Hoot et al. (2008)), and is motivated by the type of time-stamped data generally available from hospital information systems. Many DES studies have approached this problem by using expert opinion or time and motion studies (Sinreich & Marmor (2004); Günal & Pidd (2006); Codrington-Virtue et al. (2007)). There are several disadvantages to using this type of data: (i) time and motion studies are liable to suffer from Hawthorne and seasonal variability biases (Abdulwahid et al. (2018)), (ii) unbiased expert opinion is difficult to elicit transparently (Grigore et al. (2016)) and can under- or-over-estimate model inputs (Mohiuddin et al. (2017)), and (iii) the data can become quickly outdated (for example changed clinical practices during the COVID-19 pandemic), so it would be unreasonable to invest in this level of data collection from a generalisable perspective. Furthermore, amalgamating ED processes into three stages avoids a large number of restrictive modelling assumptions; Paul et al. (2010) highlighted, that DES studies which have modelled patient flows as an ordered sequence of treatment procedures have viewed patient flows in an overly simplistic manner. In reality, patients can undergo multiple procedures simultaneously, and repeat stages of a pathway (Bhattacharjee & Ray (2014)).

Tables 3 and 4 show illustrative examples of two theoretical ED problem situations, illustrating how fundamental differences in the problem situation and modelling objectives can lead to differences in the model components, level of detail, assumptions, and simplifications (e.g., consideration of treatment spaces for adult & paediatric vs. adult-only ED). The tables provide suggestions on which level of detail is possible, and appropriate, within generalisable DES model components. Both of these models for *ED A* and *ED B* are generalisable and can be reused to model other EDs with a sufficiently similar problem situation and data collection system. When the same conceptual model structure is applied to two different ED units, their individual system behaviour is captured through estimating parameters from the site-specific data. Section 3.4 shows a detailed Case Study application of the GE-DES framework to an ED.

Table 3: Two examples of generalisable ED models: components, level of detail, assumptions, and simplifications (part 1)

		Emergency Department A	**Emergency Department B**
Problem situation	Population *(section 3.1.1)*	- Adult-only - 5 urgency categories - Special cardiac stream	- Adult and paediatric - 3 urgency categories
	Process *(section 3.1.2)*	- Time stamps: *triage, seen, decision, departure* - Direct admission for cardiac patients - Long stay unit (LSU) for longer ED stay	- Time stamps: *arrival, triage, seen, decision, departure* - Resuscitation room - 'See and treat' stream for minor problems
	Performance *(section 3.1.3)*	Objective: Improve waiting times KPI/$\boldsymbol{\sigma}$= {waiting time T_1}	Objective: Improve access for mental health (MH) patients KPI/$\boldsymbol{\sigma}$={MH patient LOS, MH treatment space utilisation}
	Variables *(section 3.1.4)*	$\boldsymbol{\nu}_d$ ={arrival process, treatment spaces} $\boldsymbol{\nu}_e$ ={diversion of arriving ambulances}	$\boldsymbol{\nu}_d$={arrival process, treatment spaces} $\boldsymbol{\nu}_e$ ={addition of new mental health treatment spaces}
Patients (Entities)	Attributes ($\boldsymbol{\theta}$)	urgency category = {*1-5*}; stream = {*cardiac, admit, discharge (LSU), discharge (no LSU), transfer, left without being seen, died, left before treatment completed*}; age group = {*10 year bands*}; time of day = {*3 staff shifts*}; unit of admission (if admit patient) = *surgery, general, cardiac*}	urgency category = {*1-3*}; stream = {*admit, discharge, see and treat (admit), see and treat (discharge), transfer, resuscitation, left without being seen, died, left before treatment completed*; mental health indicator = {*1,0*}; age group = {*adult, child*}; unit of admission (if admit patient) = {*mental health, general, surgery, women and children's ward*}
	Arrival process	time-varying arrival rate $\lambda_s(h)$ for hour of day h and patient stream s	time-varying arrival rate $\lambda_s(h, d)$ for hour of day h, day of week d and patient stream s
Activities	Triage T_1	Assume patients are triaged immediately (no triage time-stamp)	Capacity unconstrained time distribution for the time between arrival and triage (by urgency category)
	Treatment T_2	Capacity unconstrained time distribution by stream (adjusted for urgency category, age group, time of day)	Capacity unconstrained time distribution by stream (adjusted for urgency category, age group, mental health indicator, time of day)
	Extended time T_3	Capacity unconstrained time distribution by stream (adjusted for urgency category, age group, time of day, unit of admission)	Capacity unconstrained time distribution by stream (adjusted for urgency category, age group, mental health indicator, time of day, unit of admission)

Table 4: Two examples of generalisable ED models: components, level of detail, assumptions, and simplifications (part 2)

		Emergency Department A	**Emergency Department B**
Progression	Pathways $\boldsymbol{\rho}$	- Define the possible pathways through ED for each combination of patient attributes	- Define the possible pathways through ED for each combination of patient attributes
	Queueing logic	- Waiting room capacity limit of 30 patients - Separate queues for each type of resource (e.g., admit/discharge treatment spaces) - Cardiac patients seen immediately - Priority queueing discipline by urgency category	- Assume no limit on waiting room capacity - Separate queues for each type of resource (e.g., for 'see and treat' spaces) - Resuscitation patients seen immediately - Priority queueing discipline by urgency category
	Routing logic	Logic for routing patients through pathways $\boldsymbol{\rho}$, including probability distributions for multiple options and conditions for resource driven pathway choices (e.g., optional use of LSU treatment space for non LSU patient if no other space is available)	Logic for routing patients through pathways $\boldsymbol{\rho}$, including probability distributions for multiple options and conditions for resource driven pathway choices (e.g., 'see and treat' pathway only operational during daytime hours.
Resources	Treatment spaces	- Number of treatment spaces in the ED and LSU - Additional treatment areas (e.g., corridors for surge in attendances) - Rules for which patients can use each space	- Number of resuscitation, adult, paediatric, and 'see and treat' treatment spaces - Number of additional mental health beds
	Staff availability	Modelled by adjusting the time distributions T_i by time of day (captures effect of shift patterns)	Modelled by adjusting the time distributions T_i by time of day (captures effect of shift patterns)
Assumptions and simplifications		- Treatment modelled as an aggregated process - Queue and service time distributions are amalgamated for T_2 and T_3 - Patients triaged immediately upon arrival - Variation in the number and activity of ED staff across different times of the day are captured by using different LOS distributions for each shift	- Treatment modelled as an aggregated process - Queue and service time distributions are amalgamated for T_2 and T_3 - Variation in the number and activity of ED staff across different times of the day are captured by using different LOS distributions for each shift

3.3.3. Model assumptions and simplifications

Simplifications include aggregating model components, replacing components with random variables, and excluding infrequent events (Robinson (2008)). Table 4 lists some examples of necessary modelling assumptions and simplifications for the two example generalisable ED models. At this point in the process of implementing GE-DES, the simulation modeller should confirm whether the modelling objectives (set out in Section 3.2) can be achieved using the model scope, level of detail, assumptions and simplifications set out in Section 3.3. *(Section 3.4.1-IV Phase 1, Step 8).*

3.4. GE-DES Section IV: Model building

GE-DES provides a set of steps to guide transformation of the generalisable conceptual model into a DES model. Model building consists of three phases and 15 steps, which are outlined below.

3.4.1. Phase 1: Preparatory and exploratory data analysis

Let $\boldsymbol{\theta}$ be the vector containing patient and system attributes and T_i represent each time interval (treatment and extended) in Fig. 3. Perform the following data cleaning and exploratory analysis:

1. Examine the data $\boldsymbol{\theta}$ and T_i for outliers and remove any erroneous data.
2. Check for missing values and consider imputation methods.
3. Inspect the time data T_i for rounding error.
4. Compute KPIs from the data and assess current system performance.
5. Use visualisation techniques, Kruskal-Wallis tests (Sweeney et al. (1994)) and Cox PH models (Cox (1972)) to examine the univariate relationship between each time interval i in T_i and each attribute in $\boldsymbol{\theta}$.
6. Where there are significant differences in the distributional shape between one or more levels of each attribute in $\boldsymbol{\theta}$, partition the data T_i into j parts for parameter estimation in phase 2. Note this is likely to be the case for patients in different pathways $\boldsymbol{\rho}$ (e.g., admit versus 'see and treat' patients).
7. Prepare data to support the model decision variables $\boldsymbol{\nu}_d$ and variables for scenario experimentation $\boldsymbol{\nu}_e$.

8. Confirm whether suitable data is available to support the modelling objectives, outputs $\boldsymbol{\sigma}$, and inputs (entities, activities, progression, and resources). If not, revisit steps (I)-(III) of the framework.

3.4.2. Phase 2: Parameter estimation

9. Estimate the activity timing parameters for each time interval using Algorithm 1. The distribution may change across each AFT model to best suit the data. Coxian phase-type regression models are used for all parameter estimation in the case study (Section 4) because they can closely approximate any continuous positive distribution.

Algorithm 1 Parameter estimation for model activities

1: **for each** partition $j \in T_i$
2: Estimate $\lambda(T_i|\theta) = \theta\lambda_0(\theta t)$
3: **end for**
4: **return** β ▷ AFT model regression coefficients

10. Estimate the parameters of a time-varying arrival process $\lambda_s(\tau)$ for patient stream s and time interval τ, using for example, a non-homogeneous Poisson process.
11. Use a distribution to assign attributes to arriving entities. The attributes will contain information for (i) adjusting the activity timings T_i, (ii) routing patients through pathways $\boldsymbol{\rho}$, (iii) the priority queuing discipline, and (iv) rules for resource access.

3.4.3. Phase 3: Simulation

12. Input model structure (entities, activities, progression, and resources) to simulation software and set up the collection of model outputs $\boldsymbol{\sigma}$.
13. Verify the computer program logic, to ensure it performs as expected.
14. Validate multiple replications of the model inputs and outputs against historic data using visualisation techniques, statistical tests, and conversations with domain experts (Banks (2010)).
15. Perform scenario experimentation using the variables for experimentation $\boldsymbol{\nu}_e$ to achieve the modelling objectives.

4. Case study

4.1. Background and study objectives

This case study has applied the GE-DES framework to model the ED of a large teaching hospital in Australia, with the aim of validating the method, demonstrating its uses for scenario experimentation, and ascertaining its limitations. The dataset contains records of all 119,306 patients who attended the ED between January 2012 and August 2013.

4.2. Emergency department layout and patient flow

The ED is composed of five units - resuscitation, paediatrics, treatment, admissions, and extended emergency care unit (EECU) areas, with 5, 6, 13, 21, and 9 beds respectively. At triage, patients are assigned a triage category (TC) from 1 (most urgent) to 5 (least urgent). ED patients are also separated by their likely discharge disposition into two streams - 'expected admission' or 'expected discharge' (Ben-Tovim et al. (2016)).

Together, the five TC scores, two expected disposition streams, and age category of patients determine their path through ED. All TC 1 patients require immediate attention, and are brought directly to the 'resuscitation' area. TC 2-5 patients wait until a treatment space becomes available; paediatric patients queue for the paediatrics unit, 'expected discharge' adults queue for the treatment unit, and 'expected admission' adults queue for the admissions unit. The EECU is used to provide an additional period of care for 'expected discharge' stream patients, e.g., overnight observations.

4.3. Data analysis

4.3.1. Length of stay

The ED data contains four date-time stamps, namely 'triage', 'seen', 'admission' and 'outcome' (Fig. 3), in addition to the following variables: (i) triage category, (ii) outcome destination, (iii) complaint category, and (iv) mode of arrival. The date-time stamps were used to partition LOS into three sections - stage (i) waiting, stage (ii) treatment, and stage (iii) extended care (where applicable). Note that stage (i) was considered to be a model output, and therefore not considered for distribution fitting.

In order to facilitate a detailed understanding of LOS determinants, the data was further partitioned using TC and outcome destination, producing 13 and 10 datasets for stages (ii) and (iii) respectively. For each dataset, the effect of various patient and system attributes on LOS was evaluated using the Coxian phase-type regression model (Boyle et al. (2019)). The procedure for fitting Coxian phase-type regression models, and results are included in the supplementary material (supplementary material 2, section 1). Figures 4 and 5 summarise the covariate effects for stages (ii) and (iii). The red '+' indicates an increased LOS, and the green '-' a decreased LOS over the reference group.

These diagrams highlight several points of interest:

- *Stage (ii) - treatment*: in the majority of cases, patients who presented with a psychiatric illness or substance abuse (PISA) complaint had an increased treatment time over patients with other illnesses. Method of arrival significantly influenced treatment time in many groups - ambulance arrival consistently related to increased treatment duration for discharge patients, however had a varying effect for admit patients, depending on whether they were transferred in. The indicator representative of patient progression to stage (iii) of ED was influential on treatment time in the majority of cases.

- *Stage (iii) - discharge (EECU) patients*: EECU patients admitted between 11pm-7am had a decreased LOS over those admitted between 3pm-11pm, and PISA patients had an increased EECU LOS.

- *Stage (iii) - admit (boarding) patients*: The inpatient division destination was found to be influential on boarding time for triage categories 1-4, but not 5. Patients admitted to the Psychiatric ICU (PICU) generally had longer boarding times than those in the reference group (general medicine). By contrast, patients admitted to the Women and Children's division generally had shorter boarding times than the reference group. Patients admitted to the Surgery division had a mixture of effects in comparison to the reference group. Patients who presented to ED between 3pm-7am had an increased boarding time.

ADULTS TREATMENT STAGE		Admitted								Discharged				
		Triage Category												
Covariate		1	2	3T	3O	4T	4O	5T	5O	1	2	3	4	5
Time of day (ref: 7am-3pm)	3pm-11am		+	-	-		-	-				-	-	-
	11am-7am		+	-	+		+	-			-	-	-	-
Ambulance (ref: other)	Ambulance arrival			-	+	-	+	-			+	+	+	+
Boarding (ref: no boarding)	Boarding patient	+	-	-	-	-		-	-					
EECU (ref: no EECU)	EECU patient									-	-	+	+	+
PISA (ref: other)	PISA patient	+	+	+	+	+				+	+	+	+	

Figure 4: Diagram showing the explanatory variables which were found to be influential on length of treatment time for adult patients. ('3T' represents the transferred-in patients in triage category 3, and '3O' represents the other patients in triage category 3).

ADULTS BOARDING/ EECU STAGE		Admitted					Discharged				
		Triage Category									
Covariate		1	2	3	4	5	1	2	3	4	5
Time of day (ref: 7am-3pm)	3pm-11am	+	+	+	+						
	11am-7am	+	+	+	+		-	-	-	-	-
Ambulance (ref: other)	Ambulance arrival		+								
Inpatient Division (ref: General Medicine)	PICU	+	+	+	+						
	Surgery	-	+	-	-						
	Women & Children	-	-	-	-						
PISA (ref: other)	PISA patient		+	+	+		+	+	+	+	+

Figure 5: Diagram showing the explanatory variables which were found to be influential on length of extended ED care time for adult patients.

4.3.2. Arrival process

The number of patient presentations to ED by hour of day was time varying - attendances increased sharply between 07:00 and 12:00, with a slight dip between 12:00 and 16:00, and fell gradually throughout the night. This distribution was modelled using a non-homogeneous Poisson process (NHPP) with piecewise-constant rate functions, depending on the time of day. The Kolmogorov-Smirnov test was applied to check that the arrivals in each time interval satisfied the conditional-uniform property for using a NHPP. Two independent processes were used to model arrivals to each of the 'expected discharge' and 'expected admit' streams. Details of the arrival process are reported in supplementary material 2, section 3.

4.4. Model logic

The key model logic is summarised in table 12 (supplementary material 2, section 4) for each of the model entities, activities, progression, resources, and assumptions. The logic is determined through analysis of primary hospital data. There are four queues in the triage area for each of admit/discharge and adult/paediatric patients. Each queue operates with a first in first out (FIFO) discipline, within each TC. Once an ED bed becomes available, it is seized by the patient waiting at the top of the relevant queue. The bed remains in use by the patient until their period of treatment has ended. If the patient is to move to a different area of the ED (for example to EECU or admissions area), two actions can occur as a result of the model logic. If there is a bed available in the new area, the patient will seize the new bed, and release the treatment area bed. If no new bed is available, the patient will continue to use the treatment bed at the next step of ED treatment.

4.5. Verification and validation

The case study model was developed using SIMUL8 software. The baseline simulation model was set to run for 367 days including a two-day warm up period, chosen to be larger than the maximum observed patient LOS. Each model run therefore represents hospital operation over a yearly period. The validity of the two day warm-up was confirmed using Welch's method (Welch (1983), see supplementary material 2, section 5).

4.5.1. Verification

Throughout the model development, flow diagrams and documentation were maintained, containing details of patient attributes, SIMUL8 'label' names and values, probability distribution names and parameters, and routing logic. The model was randomly paused during several runs, and each component of the model was checked to ensure consistency with the documentation. LOS and queueing times for patients with each attribute type were checked against the empirical data. The model was further checked by exploring its output with various input parameter adjustments. This is further considered as scenario analysis in section 4.5.2. A sensitivity analysis, to examine the output under changes to the model parameters, is included in supplementary material 2, section 6.

The SIMUL8 graphical user interface was used to visually check model logic, and the 'simulation assistant' feature in SIMUL8 was routinely used to diagnose issues such as potential routing discipline conflicts. Verification was also conducted through discussion with management and clinical staff from the hospital, to ensure that the results obtained from covariate analysis reasonably represented LOS profiles for various patient types.

4.5.2. Validation

Validation was conducted to compare the results from the model with the hospital dataset. The baseline model was run 50 times using different random number streams in SIMUL8. The average treatment time from each run were calculated by TC, for each of the age groups and discharge destinations. Table 5 presents the LOS and queue time results for adult patients. The mean and median LOS from the model closely matched the data, demonstrating that the Coxian phase-type regression models captured LOS to a high level of accuracy. In the majority of cases, the 95% confidence interval (CI) included the mean LOS from the data. Exceptions to this lay mostly within 15 minutes of the CI boundaries, which was the scale utilised for distribution fitting. LOS was overestimated for adult EECU patients, which is likely due to the rescaling of this data to a granularity level of hours. There is also a relatively small number of patients in the EECU group. Fig. 3

in supplementary material 2, section 7, shows that the empirical cumulative distribution function (ECDF) of the LOS data closely matches the ECDF of each simulation replication. Fig. 4 in the same document displays a visual comparison of the average waiting time in the data to the output from multiple simulation runs.

The process, LOS, and queueing aspects of the model were checked regularly with clinical advisors to ensure validity throughout model building Banks (2010). The arrival process was validated by comparing the empirical arrival rates in the data to the arrival rates of each simulation replication. For each hour of the day, it was found that the 95% CIs included the empirical arrival rates from the data. The CIs and plots showing these are included in supplementary material 2, section 7.

Table 5: Length of stay results and 95% confidence interval obtained from the baseline simulation model for adult patients.

Group	Triage Category	Mean LOS Data	Mean LOS Model
Discharge	1	350	345 (342,352)
Non EECU	2	243	245 (243,246)
	3	182	183 (182,184)
	4	130	131 (130,131)
	5	71	70 (70,71)
EECU		906	1012 (1008,1015)
Admit	1	362	380 (370,390)
	2	498	502 (502,509)
	3	510	511 (509,512)
	4	450	430 (427,433)
	5	301	313 (303,322)
Discharge Queue	-	85	80 (78,82)
Admit Queue	-	79	92 (86,97)

4.6. Scenario modelling

4.6.1. Resource-related scenarios

Scenario testing can be categorised as resource-related, process-related, or environment-related (Paul et al. (2010)). The case study model developed using GE-DES is suitable for investigating these three types of scenario

testing. The number of ED beds in the model was increased and decreased by 2, to ensure that the waiting times would drop and climb respectively. The results in Table 6 demonstrate this behaviour, providing further validation of the model logic.

Table 6: Results obtained from scenario analysis on the number of ED beds.

Queue Type	Average queue time (Baseline)	(+ 2 beds)	(- 2 beds)
Admit Adults	92 (86,97)	58 (53,62)	146 (131,160)
Discharge Adults	80 (78,82)	57 (55,60)	110 (106,116)

4.6.2. Environment-related scenarios

Environment-related scenarios relate to experimentation with variables which are external to the ED, such as the daily number of presentations. This was investigated in line with an annually increasing rate in ED presentations of 3.7% in Australia (Australian Institute of Health & Welfare (2021)). This was added as an additional start point to the SIMUL8 model, where it was assumed that the patients would be assigned the same proportion of attribute values as in the ED dataset.

Table 7: Results obtained from scenario analysis of increased presentations.

Queue Type	Average queue time (Baseline)	(+ 3.7%, 1 year)
Admit Adults	92 (86,97)	116 (106,127)
Discharge Adults	80 (78,82)	91 (86,95)

Table 7 shows results from implementation of the scenario using the SIMUL8 model. The average queuing time increased by 29% for admitted adults and 6% for discharged adults. Additional scenarios could look at the impact of projected population increases several years into the future. Further environment-related scenarios could explore potential interventions for the inpatient admission process.

4.6.3. Process-related scenarios

Process-related scenarios correspond to modifying ED processes Paul et al. (2010). The model was utilised to investigate the process of treating psychiatric illness and substance abuse (PISA) patients in ED, as it was found that many patients in this demographic had very long LOS. A hypothetical scenario was developed to investigate the impact of an additional 4-bed unit, dedicated to treating these patients in ED. The routing logic in the SIMUL8 model was altered to direct relevant patients to the PISA beds on a first-come-first-served basis. If no PISA bed was available, the patient occupied one of the regular ED beds. Table 8 shows the results from adding the hypothetical PICU unit in the SIMUL8 model. The average waiting time is reduced for both admit and discharge adults . This is a promising method for improving ED waiting times.

Table 8: Results obtained from scenario analysis on additional unit for PISA patients.

Queue Type	Average queue time (Baseline)	(+ 4 PICU beds)
Admit Adults	92 (86,97)	51 (47,54)
Discharge Adults	80 (78,82)	71 (69,73)

5. Conclusion

This research has a identified an opportunity in the literature to develop a novel methodology for generalisable discrete event simulation (DES) models of hospital emergency departments (EDs). GE-DES has several key advantages: (i) it encourages the use of credible data from hospital information systems, to ensure confidence in data-informed model parameters; (ii) it is flexible, in that the analysis can be customised to suit the contents of a particular dataset; (iii) it provides a step-by-step set of instructions for model building; (iv) the framework is flexible and can be adapted to include the detail of specific ED processes e.g., representation of a unit-specific EECU in the case study; (v) a range of process-related, resource-related, and environment-related scenarios can be investigated using a single DES model; (vi) Coxian phase-type distributions are exploited to closely approximate

skewed length of stay distributions, however the framework has flexibility to accommodate the use of any suitable distribution. A key advantage of GE-DES is that a developed model can easily be updated by re-estimating the parameters from new data. This is particularly beneficial when the ED experiences sudden changes, such as it did during the COVID-19 pandemic. There are some limitations to this work - the framework design is based on 'typical' EDs, and would require modification for use for other models of care. Additionally, model building requires mathematical expertise.

The new framework was successfully applied to model an Australian ED. Although it was developed as a generalisable model, unit-specific processes such as the EECU and split queuing streams were explicitly represented. Comparison of the model output to the data showed that LOS was captured to a high level of accuracy in the simulation. The visual aspect of SIMUL8 was invaluable for communication of results to clinical staff and for model validation. Although the average queueing time was accurately captured in the baseline model, it was sensitive to changes in the scenario testing, particularly for admitted patients. Future work will consider the sensitivity of queuing assumptions. Although there was good communication with hospital staff throughout the project, there were challenges with implementation of the model for long-term use. Future work will involve improvement of the model hand-over to hospital staff. Modelling is an endeavour that can help staff better understand a situation and test out possible changes to the situation through the manipulation of parameters prior to any actual implementation. To maximise the likelihood of success of any change process, the work should be embedded in an improvement, change management, or design-thinking exercise. Modelling is part of the work that occurs in these processes. Such exercises require the training of staff and to be supported by senior staff within the service. A generalisable model will enable more rapid adoption by staff, and incorporate this work as part of their problem-solving and service improvement exercise.

The advantages of the GE-DES framework establish some key implications for both simulation modellers and healthcare managers. Through use of the GE-DES framework, when combined with appropriate quality im-

provement and problem-solving frameworks (e.g., design-thinking), healthcare managers will have a step-by-step guide for ED simulation projects, generating potential for bottleneck identification and performance comparison between multiple hospitals. As a result, health care managers will have a clear means of understanding, modelling, and communicating patient flow-related issues to government and policy makers, in support of applications for increased funding and resources. Future research will consider the application of GE-DES to multiple case studies internationally.

Declaration of interest

Declarations of interest: none.

Funding

The authors wish to thank the Northern Ireland Department for the Economy for funding this research.

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Supplementary material 1 - literature review

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Table 1: Generalisable ED DES model literature review: Activity durations and progression. * P= primary, S= secondary, O= observation or expert opinion, U= unknown or unspecified data source.

	Activity duration						**Activity progression**					
	Detail			**Data**			**Detail**			**Data**		
	Mean/ triangular	Empirical	Distribution	P	O	U	% Probability	Linear	Other/ Unspecified	P	O	U
Miller et al. (2004)						•			•			•
Sinreich & Marmor (2004)	•				•		•				•	
Günal & Pidd (2006)	•			•	•		•			•	•	
Günal & Pidd (2009)						•			•			•
Virtue et al. (2006)		•				•			•			•
Virtue et al. (2011)	•			•	•			•		•		
Ceglowski et al. (2007)		•		•				•		•		
Fletcher et al. (2007)	•				•		•				•	
Hoot et al. (2008)			•	•				•		•		
Facchin et al. (2010)	•					•	•					•
Mes & Bruens (2012)			•			•			•			•
Paul & Lin (2012)				•					•	•	•	
Hurwitz et al. (2014)			•	•	•				•		•	
Mielczarek (2014)		•		•					•	•		
Gul & Guneri (2015)	•		•	•	•		•			•	•	
Cimellaro & Piqué (2016)						•	•			•	•	

Table 2: Generalisable ED DES model literature review: Arrival rates and resources used. * P= primary, S= secondary, O= observation or expert opinion, U= unknown or unspecified data source.

	Arrival rates						**Resources used**					
	Detail			**Data**			**Detail**			**Data**		
	Poisson process	Other	Unspecified	P	S	U	Beds	Staff	None	P	O	U
Miller et al. (2004)			•			•	•	•				•
Sinreich & Marmor (2004)		•		•				•			•	
Günal & Pidd (2006)	•			•			•	•				•
Günal & Pidd (2009)	•			•					•			•
Virtue et al. (2006)		•		•			•	•				•
Virtue et al. (2011)		•		•			•	•		•	•	
Ceglowski et al. (2007)		•		•				•			•	
Fletcher et al. (2007)	•				•			•			•	
Hoot et al. (2008)	•			•			•			•		
Facchin et al. (2010)	•					•	•	•				•
Mes & Bruens (2012)	•					•	•	•				•
Paul & Lin (2012)			•	•			•	•				•
Hurwitz et al. (2014)	•				•		•	•			•	
Mielczarek (2014)	•				•				•			•
Gul & Guneri (2015)			•	•			•	•		•	•	
Cimellaro & Piqué (2016)		•		•			•	•		•	•	

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Supplementary material 2 - model parameters and results

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1. Length of stay parameters

1.1. Procedure for fitting Coxian phase-type regression models to ED length of stay

The total time spent in ED consists of three stages (Manuscript Fig. 1); (i) stage 1 represents the time spent waiting for the treatment to commence with an ED doctor, (ii) stage 2 represents treatment duration, and (iii) stage 3 represents any additional time spent in the ED (as either a boarding inpatient, or for extended observation prior to discharge). Total LOS was therefore sub-divided into these three stages of waiting, treatment, and extended time. Stage 1 (waiting) was considered to be an output of the DES model. Stages 2 and 3 were modelled using Coxian phase-type regression models, fitted using the method of maximum likelihood. The following steps were used to fit the Coxian phase-type regression models to LOS:

1. Fit a Coxian phase-type distribution to the overall LOS in each of the admit and discharge patient groups. Cluster patients by their expected exit phase of the model into LOS groups, then analyse the primary complaint reason for patients in each of the clusters, to identify attributes of patients with consistently longer LOS.
2. Sub-divide patients into admit and discharge groups. Where appropriate, further sub-divide by triage category and age group. Utilise plots of the LOS distribution by age and triage category to assist in determining whether this sub-division is applicable.
3. Rescale ED LOS to an appropriate level of granularity. Although ED data is often recorded in minutes, this can often be inaccurate. In a busy ED, medical staff may be juggling many tasks, resulting in some errors in the recording of time-stamps. Previous ED research has recommended 15 minute intervals as an appropriate scale for modelling LOS (Ben-Tovim et al. (2016)). Stage 3 time for discharge patients was rescaled to hours.
4. For each dataset from step 2, perform an an analysis of the factors which are influential on LOS in ED stages 2 and 3. An exponential accelerated failure time (AFT) model can be used to identify influential

variables via forward, backward, or stepwise selection. The candidate set of explanatory factors in the case study ED data consist of: time of day, arrival method, triage category, age group, and inpatient division (for admit patients). Define a binary variable for each patient, to indicate whether they progressed to ED stage 3. This should be considered as an explanatory variable at ED stage 2, as it may affect the rate at which patients move through this treatment stage. Attributes of patients with consistently protracted LOS (identified in step 1) should be considered as covariates in the analysis. This step should result in the identification of a suitable exponential AFT model for each dataset.

5. The exponential AFT model is mathematically equivalent to a one-phase Coxian regression model. Coxian phase-type regression models of increasing order should be fitted until no improvement is observed in the BIC value of the model. BIC is used to maintain parsimony with respect to the number of model parameters. In the case where no explanatory variables are significant, the model Coxian phase-type regression model reduces to a standard Coxian phase-type distribution.

1.2. Distribution parameters

Tables 2-9 contain the following distribution parameters:

- Table 2 - stage 2 (treatment) for discharge adult patients;
- Table 3 - stage 2 (treatment) for discharge paediatric patients;
- Table 4 - stage 3 (extended care) for all discharge patients;
- Tables 5 and 6 - stage 2 (treatment) for admit adult patients;
- Table 7 - stage 2 (treatment) for admit paediatric patients;
- Table 8 - stage 3 (boarding) for admit adult patients;
- Table 9 - stage 3 (boarding) for admit paediatric patients.

Table 1: Coxian phase-type regression models fitted to stage 2 (treatment) of ED LoS for discharge adult patients. (Extended emergency care unit is abbreviated to EECU, psychiatric illness and substance abuse to PISA, time of day to TDAY, and ambulance to AMB.)

Triage category	No. of observations	No. of phases	Fitted estimates		Log-likelihood	BIC
			Rates	Covariates		
1	230	3	$\hat{\mu}_1 = 1.73 \times 10^{-08}$, $\hat{\mu}_2 = 0.0157$, $\hat{\mu}_3 = 0.1326$, $\hat{\lambda}_1 = 0.1326$, $\hat{\lambda}_2 = 0.1169$	$\hat{\beta}_{EECU} = -0.4010$, $\hat{\beta}_{PICU} = 0.3162$	-865.5	1769.1
2	3850	3	$\hat{\mu}_1 = 0.0061$, $\hat{\mu}_2 = 3.17 \times 10^{-53}$, $\hat{\mu}_3 = 0.1350$, $\hat{\lambda}_1 = 0.3035$, $\hat{\lambda}_2 = 0.3082$	$\hat{\beta}_{AMB} = 0.2513$, $\hat{\beta}_{TDAY1} = -0.0593$ $\hat{\beta}_{TDAY2} = 0.1730$, $\hat{\beta}_{PICU} = 0.3671$, $\hat{\beta}_{EECU} = -0.1947$	-13675.6	27458.4
3	18338	4	$\hat{\mu}_1 = 0.0063$, $\hat{\mu}_2 = 0.0690$, $\hat{\mu}_3 = 0.0080$, $\hat{\mu}_4 = 0.1380$, $\hat{\lambda}_1 = 0.4968$, $\hat{\lambda}_2 = 0.4400$, $\hat{\lambda}_3 = 0.5345$	$\hat{\beta}_{AMB} = 0.3142$, $\hat{\beta}_{TDAY1} = -0.1601$, $\hat{\beta}_{TDAY2} = -0.0059$, $\hat{\beta}_{EECU} = 0.1075$, $\hat{\beta}_{PICU} = 0.1047$	-61961.1	124049.8
4	20207	3	$\hat{\mu}_1 = 0.0121$, $\hat{\mu}_2 = 0.1597$, $\hat{\mu}_3 = 0.0491$, $\hat{\lambda}_1 = 0.4365$, $\hat{\lambda}_2 = 0.0014$	$\hat{\beta}_{AMB} = 0.5238$, $\hat{\beta}_{TDAY1} = -0.1507$, $\hat{\beta}_{TDAY2} = -0.0395$, $\hat{\beta}_{EECU} = 0.3813$, $\hat{\beta}_{PICU} = 0.0849$	-62723.0	125545.1
5	4198	3	$\hat{\mu}_1 = 0.0740$, $\hat{\mu}_2 = 0.2600$, $\hat{\mu}_3 = 0.1775$, $\hat{\lambda}_1 = 1.7486$, $\hat{\lambda}_2 = 0.1104$	$\hat{\beta}_{AMB} = 0.7964$, $\hat{\beta}_{TDAY1} = -0.1004$, $\hat{\beta}_{TDAY2} = -0.0886$, $\hat{\beta}_{EECU} = 0.9606$	-10648.6	21372.2

Table 2: Coxian phase-type regression models fitted to stage 2 (treatment) of ED LoS for discharge paediatric patients. (Time of day is abbreviated to TDAY, and ambulance to AMB.)

Triage category	N	No. of phases	Fitted estimates: Rates	Fitted estimates: Covariates	Log-likelihood	BIC
1	32	N/A	[Use empirical distribution]			
2	781	3	$\hat{\mu}_1 = 0.0040$, $\hat{\mu}_2 = 0.0093$, $\hat{\mu}_3 = 0.1697$, $\hat{\lambda}_1 = 0.3445$, $\hat{\lambda}_2 = 0.3393$		-2526.0	5085.4
3	9432	4	$\hat{\mu}_1 = 0.0083$, $\hat{\mu}_2 = 1.47 \times 10^{-73}$, $\hat{\mu}_3 = 0.9141$, $\hat{\mu}_4 = 0.2444$, $\hat{\lambda}_1 = 2.2776$, $\hat{\lambda}_2 = 0.1976$, $\hat{\lambda}_3 = 1.0538$	$\hat{\beta}_{AMB} = 0.2090$, $\hat{\beta}_{TDAY1} = -0.2033$, $\hat{\beta}_{TDAY2} = -0.1405$	-27502.5	55096.5
4	8890	2	$\hat{\mu}_1 = 0.0103$, $\hat{\mu}_2 = 0.1993$, $\hat{\lambda}_1 = 0.7920$	$\hat{\beta}_{AMB} = 0.4155$, $\hat{\beta}_{TDAY1} - 0.1650$, $\hat{\beta}_{TDAY2} = -0.0950$	-23922.7	47907.2
5	924	2	$\hat{\mu}_1 = 0.0375$, $\hat{\mu}_2 = 0.2225$, $\hat{\lambda}_1 = 1.2938$	$\hat{\beta}_{TDAY1} = -0.2991$, $\hat{\beta}_{TDAY2} = -0.2964$	-2268.1	4570.4

Table 3: Discharge stage 3, extended emergency care unit. (Time of day is abbreviated to TDAY, and psychiatric illness and substance abuse to PISA.)

Triage category	N	No. of phases	Fitted estimates		
			Rates	Covariates	
All	3412	8	$\hat{\mu}_1 = 0.0015$	$\hat{\beta}_{TIMEDAY2}$	-0.3707
			$\hat{\mu}_2 = 0.0045$	$\hat{\beta}_{PISA}$	0.2031
			$\hat{\mu}_3 = 0.0715$		
			$\hat{\mu}_4 = 0.0008$		
			$\hat{\mu}_5 = 0.1335$		
			$\hat{\mu}_6 = 6.07 \times 10^{-14}$		
			$\hat{\mu}_7 = 4.71 \times 10^{-06}$		
			$\hat{\mu}_8 = 0.4967$		
			$\hat{\lambda}_1 = 0.8498$		
			$\hat{\lambda}_2 = 1.6035$		
			$\hat{\lambda}_3 = 0.9538$		
			$\hat{\lambda}_4 = 0.9224$		
			$\hat{\lambda}_5 = 0.8579$		
			$\hat{\lambda}_6 = 0.1565$		
			$\hat{\lambda}_7 = 0.5139$		

Table 4: Coxian phase-type regression models fitted to stage 2 (treatment) of ED LoS for admit adult patients in triage categories 1-3. (Boarding is abbreviated to BOARD, psychiatric illness and substance abuse to PISA, time of day to TDAY, and ambulance to AMB.)

Triage category	N	No. of phases	Fitted estimates		Log-likelihood	BIC
			Rates	Covariates		
1	1438	3	$\hat{\mu}_1 = 0.0006$, $\hat{\mu}_2 = 0.3072$, $\hat{\mu}_3 = 0.0260$, $\hat{\lambda}_1 = 0.3118$, $\hat{\lambda}_2 = 0.0052$	$\hat{\beta}_{BOARD} = 0.5614$, $\hat{\beta}_{PISA} = 0.3846$	-4631.0	9312.9
2	8072	5	$\hat{\mu}_1 = 9.46 \times 10^{-05}$, $\hat{\mu}_2 = 0.2223$, $\hat{\mu}_3 = 0.0120$, $\hat{\mu}_4 = 7.81 \times 10^{-15}$, $\hat{\mu}_5 = 0.1452$, $\hat{\lambda}_1 = 16.6495$, $\hat{\lambda}_2 = 32.3953$, $\hat{\lambda}_3 = 0.3746$, $\hat{\lambda}_4 = 0.3729$	$\hat{\beta}_{TDAY1} = 0.0322$, $\hat{\beta}_{TDAY2} = 0.1376$, $\hat{\beta}_{BOARD} = -0.2629$, $\hat{\beta}_{PISA} = 0.3302$	-27120.9	54358.8
3 Transfers	5028	3	$\hat{\mu}_1 = 0.0846$, $\hat{\mu}_2 = 0.0453$, $\hat{\mu}_3 = 0.2335$, $\hat{\lambda}_1 = 7.36 \times 10^{-05}$, $\hat{\lambda}_2 = 0.5797$	$\hat{\beta}_{AMB} = -0.2831$, $\hat{\beta}_{TDAY1} = -0.1338$, $\hat{\beta}_{TDAY2} = -0.2452$, $\hat{\beta}_{BOARD} = -0.4959$, $\hat{\beta}_{PISA} = 0.3249$	-15936.6	31958.4
3 Other	11081	4	$\hat{\mu}_1 = 0.1221$, $\hat{\mu}_2 = 0.0071$, $\hat{\mu}_3 = 6.05 \times 10^{-20}$, $\hat{\mu}_4 = 0.1424$, $\hat{\lambda}_1 = 9.7966$, $\hat{\lambda}_2 = 0.2912$, $\hat{\lambda}_3 = 0.2783$	$\hat{\beta}_{AMB} = 0.0465$, $\hat{\beta}_{TDAY1} = -0.0149$, $\hat{\beta}_{TDAY2} = 0.0576$, $\hat{\beta}_{BOARD} = -0.0213$ $\hat{\beta}_{PISA} = 0.0837$	- 38299.9	76711.5

Table 5: Coxian phase-type regression models fitted to stage 2 (treatment) of ED LoS for admit adult patients in triage categories 4 and 5. (Boarding is abbreviated to BOARD, psychiatric illness and substance abuse to PISA, time of day to TDAY, and ambulance to AMB.)

Triage category	N	No. of phases	Fitted estimates		Log-likelihood	BIC
			Rates	Covariates		
4 Transfers	2513	3	$\hat{\mu}_1 = 0.1061$, $\hat{\mu}_2 = 58.8083$, $\hat{\mu}_3 = 0.2117$, $\hat{\lambda}_1 = 6.42 \times 10^{-06}$, $\hat{\lambda}_2 = 0.5502$	$\hat{\beta}_{AMB} = -0.3110$, $\hat{\beta}_{BOARD} = -0.4148$, $\hat{\beta}_{PISA} = 0.3595$	-7710.5	15483.6
4 Other	4396	4	$\hat{\mu}_1 = 0.0196$, $\hat{\mu}_2 = 1.59 \times 10^{-05}$, $\hat{\mu}_3 = 1.78 \times 10^{-07}$, $\hat{\mu}_4 = 0.4233$, $\hat{\lambda}_1 = 0.1169$, $\hat{\lambda}_2 = 0.4223$, $\hat{\lambda}_3 = 0.4233$	$\hat{\beta}_{AMB} = 0.0725$, $\hat{\beta}_{TDAY1} = -0.0472$, $\hat{\beta}_{TDAY2} = 0.0874$	-15144.5	30372.9
5 Transfers	220	1	$\hat{\mu}_1 = 0.1370$	$\hat{\beta}_{AMB} = -0.4020$, $\hat{\beta}_{TDAY1} = -0.2635$, $\hat{\beta}_{TDAY2} = -0.7158$, $\hat{\beta}_{BOARD} = -0.6264$,	-580.4	1187.8
5 Other	195	1	$\hat{\mu}_1 = 0.0755$	$\hat{\beta}_{BOARD} = -0.8222$	-658.4	1327.3

Table 6: Coxian phase-type regression models fitted to stage 2 (treatment) of ED LoS for admit paediatric patients. (Time of day is abbreviated to TDAY, and ambulance to AMB.)

Triage category	N	No. of phases	Fitted estimates: Rates	Fitted estimates: Covariates	Log-likelihood	BIC
1	139	5	$\hat{\mu}_1 = 2.16 \times 10^{-254}$, $\hat{\mu}_2 = 4.46 \times 10^{-08}$, $\hat{\mu}_3 = 2.02 \times 10^{-11}$, $\hat{\mu}_4 = 1.1434$, $\hat{\mu}_5 = 12.9629$, $\hat{\lambda}_1 = 0.1401$, $\hat{\lambda}_2 = 1.4234$, $\hat{\lambda}_3 = 1.4234$, $\hat{\lambda}_4 = 6.11 \times 10^{-13}$		-388.2	820.8
2	1079	4	$\hat{\mu}_1 = 0.0187$, $\hat{\mu}_2 = 5.87 \times 10^{-79}$, $\hat{\mu}_3 = 0.7205$, $\hat{\mu}_4 = 0.1031$, $\hat{\lambda}_1 = 0.3542$, $\hat{\lambda}_2 = 0.3666$, $\hat{\lambda}_3 = 0.0761$		-3084.3	62175
3	3263	5	$\hat{\mu}_1 = 0.0002$, $\hat{\mu}_2 = 0.5585$, $\hat{\mu}_3 = 2.97 \times 10^{-13}$, $\hat{\mu}_4 = 0.3134$, $\hat{\mu}_5 = 0.2610$, $\hat{\lambda}_1 = 16.2138$, $\hat{\lambda}_2 = 14.7024$, $\hat{\lambda}_3 = 0.1400$, $\hat{\lambda}_4 = 0.8166$	$\hat{\beta}_{AMB} = -0.3237$, $\hat{\beta}_{TDAY1} = -0.2221$, $\hat{\beta}_{TDAY2} = -0.1749$	-9980.9	20058.9
4	916	2	$\hat{\mu}_1 = 0.0611$, $\hat{\mu}_2 = 0.1614$, $\hat{\lambda}_1 = 0.1003$	$\hat{\beta}_{TDAY1} = -0.1995$, $\hat{\beta}_{TDAY2} = -0.0131$	-3122.6	6279.3
5	62		[Use empirical distribution]			

Table 7: Coxian phase-type regression models fitted to stage 3 (treatment) of ED LoS for admit adult patients (Inpatient division destination is abbreviated to DIV, time of day to TDAY, psychiatric illness and substance abuse to PISA, ambulance to AMB, and overcrowding to O).

Triage category	No. of observations	No. of phases	Fitted estimates		Log-likelihood	BIC
			Rates	Covariates		
1	1001	3	$\hat{\mu}_1 = 0.0559$, $\hat{\mu}_2 = 0.2670$, $\hat{\mu}_3 = 0.0591$, $\hat{\lambda}_1 = 0.1288$, $\hat{\lambda}_2 = 0.0851$	$\hat{\beta}_{TDAY1} = 0.1960$, $\hat{\beta}_{TDAY2} = 0.3713$, $\hat{\beta}_{DIV1} = 1.6091$, $\hat{\beta}_{DIV2} = -0.1095$, $\hat{\beta}_{DIV3} = -1.0685$, $\hat{\beta}_{O} = 0.0263$	-3850.5	7777.0
2	7558	3	$\hat{\mu}_1 = 0.0086$, $\hat{\mu}_2 = 2.99 \times 10^{-28}$, $\hat{\mu}_3 = 0.0793$, $\hat{\lambda}_1 = 0.8069$, $\hat{\lambda}_2 = 0.9302$	$\hat{\beta}_{AMB} = 0.0624$, $\hat{\beta}_{TDAY1} = 0.3053$, $\hat{\beta}_{TDAY2} = 0.2337$, $\hat{\beta}_{DIV1} = 1.2349$, $\hat{\beta}_{DIV2} = 0.0163$, $\hat{\beta}_{DIV3} = -0.8286$, $\hat{\beta}_{PICU} = 0.3771$, $\hat{\beta}_{O} = 0.0286$	-29640.5	59397.1
3	15346	3	$\hat{\mu}_1 = 1.46 \times 10^{-08}$, $\hat{\mu}_2 = 0.1391$, $\hat{\mu}_3 = 0.0404$, $\hat{\lambda}_1 = 0.1785$, $\hat{\lambda}_2 = 0.0394$	$\hat{\beta}_{TDAY1} = 0.1925$, $\hat{\beta}_{TDAY2} = 0.2097$, $\hat{\beta}_{DIV1} = 1.2734$, $\hat{\beta}_{DIV2} = -0.0196$, $\hat{\beta}_{DIV3} = -0.6833$, $\hat{\beta}_{PICU} = 0.3179$ $\hat{\beta}_{O} = 0.0283$	-59952.0	120019.7
4	6450	3	$\hat{\mu}_1 = 1.40 \times 10^{-09}$, $\hat{\mu}_2 = 0.1792$, $\hat{\mu}_3 = 0.0430$, $\hat{\lambda}_1 = 0.2248$, $\hat{\lambda}_2 = 0.0456$	$\hat{\beta}_{TDAY0} = -0.1281$, $\hat{\beta}_{TDAY2} = 0.1088$, $\hat{\beta}_{DIV1} = 1.3248$, $\hat{\beta}_{DIV2} = -0.1295$, $\hat{\beta}_{DIV3} = -0.7031$, $\hat{\beta}_{PICU} = 0.3501$, $\hat{\beta}_{O} = 0.0262$	-24459.1	49023.5
5	314	4	$\hat{\mu}_1 = 0.0037$, $\hat{\mu}_2 = 0.0341$, $\hat{\mu}_3 = 0.2361$, $\hat{\mu}_4 = 0.0251$, $\hat{\lambda}_1 = 0.2787$, $\hat{\lambda}_2 = 0.2482$, $\hat{\lambda}_3 = 0.0463$		-1020.3	2080.8

Table 8: Coxian phase-type regression models fitted to stage 3 (treatment) of ED LoS for admit paediatric patients. (Time of day is abbreviated to TDAY.)

Triage category	N	No. of phases	Fitted estimates: Rates	Fitted estimates: Covariates	Log-likelihood	BIC
1	105	3	$\hat{\mu}_1 = 2.59 \times 10^{-11}$, $\hat{\mu}_2 = 0.2941$, $\hat{\mu}_3 = 0.0096$, $\hat{\lambda}_1 = 0.3016$, $\hat{\lambda}_2 = 0.0075$		-305.2	633.7
2	992	3	$\hat{\mu}_1 = 2.77 \times 10^{-10}$, $\hat{\mu}_2 = 0.3397$, $\hat{\mu}_3 = 0.0175$, $\hat{\lambda}_1 = 0.3488$, $\hat{\lambda}_2 = 0.0091$		-2680.9	5396.3
3	3044	3	$\hat{\mu}_1 = 6.88 \times 10^{-11}$, $\hat{\mu}_2 = 0.3394$, $\hat{\mu}_3 = 0.0204$, $\hat{\lambda}_1 = 0.3509$, $\hat{\lambda}_2 = 0.0115$	$\hat{\beta}_{TDAY1} = -0.1835$, $\hat{\beta}_{TDAY2} = -0.0748$	-8084.8	16225.7
4	915	4	$\hat{\mu}_1 = 1.27 \times 10^{-46}$, $\hat{\mu}_2 = 8.88 \times 10^{-10}$, $\hat{\mu}_3 = 1.3014$, $\hat{\mu}_4 = 0.0380$, $\hat{\lambda}_1 = 0.2652$, $\hat{\lambda}_2 = 1.3462$, $\hat{\lambda}_3 = 0.0448$		-2235.4	4518.5
5	55	1	$\hat{\mu}_1 = 0.1530$		-158.2	320.4

2. Arrival process parameters

Table 9: Non-homogeneous Poisson arrival process for patients to ED (discharge stream)

Time interval	**Discharge stream** Model parameter: expected time between arrivals (mins)
00:00-00:59	22.614
01:00-01:59	26.370
02:00-02:59	30.848
03:00-03:59	36.856
04:00-04:59	41.033
05:00-05:59	43.321
06:00-06:14	47.524
0615-06:29	43.823
06:30-06:44	42.633
06:45-06:59	36.757
07:00-07:59	25.066
08:00-08:14	17.197
08:15-08:29	16.057
08:30-08:44	14.433
08:45-08:59	12.351
09:00-09:29	10.317
09:30-09:59	9.535
10:00-10:59	8.711
11:00-11:59	8.439
12:00-12:59	8.319
13:00-13:59	8.589
14:00-14:59	9.096
15:00-15:29	9.652
15:30-15:59	9.553
16:00-16:59	9.430
17:00-17:59	9.477
18:00-18:59	8.834
19:00-19:59	8.980
20:00-20:59	10.122
21:00-21:59	11.558
22:00-22:59	12.995
23:00-23:59	17.719

Table 10: Non-homogeneous Poisson arrival process for patients to ED (admit stream)

Time interval	**Admit stream** Model parameter: expected time between arrivals (mins)
00:00-00:59	34.973
01:00-01:29	39.249
01:30-02:59	40.125
02:00-02:29	44.705
02:30-02:59	48.967
03:00-03:59	55.511
04:00-04:59	50.746
05:00-05:14	48.869
05:15-05:29	49.976
05:30-05:44	59.953
05:45-05:59	55.783
06:00-06:59	56.490
07:00-07:14	48.032
07:15-07:29	54.229
07:30-07:44	39.527
07:45-07:59	31.689
08:00-08:59	27.784
09:00-09:59	19.411
10:00-10:59	16.418
11:00-11:59	15.534
12:00-12:59	15.338
13:00-13:59	14.089
14:00-14:59	15.324
15:00-15:59	16.456
16:00-16:59	15.901
17:00-17:59	16.503
18:00-18:59	16.374
19:00-19:59	16.308
20:00-20:59	18.759
21:00-21:59	21.247
22:00-22:29	22.646
22:30-22:59	26.401
23:00-23:59	31.370

3. Model logic

Table 11: Case study model logic

		Case Study Emergency Department
Problem situation	Population	Adult and paediatric patients; 5 urgency categories
	Process	- Time stamps: *triage, seen, decision, departure* - Resuscitation area; extended emergency care unit (EECU)
	Performance	Objective: Improve waiting times and access for mental health (MH) patients. KPI: $\boldsymbol{\sigma}$= {waiting time T_1}
	Variables	$\boldsymbol{\nu}_3$ ={arrival process, treatment spaces, treatment time T_2, addition of new mental health treatment spaces}
Patients (Entities)	Attributes ($\boldsymbol{\theta}$)	urgency category = {*1-5*}; stream = {*admit, discharge (no EECU), discharge (EECU), transfer left without being seen, died, left before treatment completed*}; age group = {*adults, children*}; time of day = {*3 staff shifts (7am-3pm, 3pm-11pm, 11pm-7am)*}; arrival mode = {*ambulance, other*} admission (if admit patient) = *surgery, mental health, general medicine, women's and children*} *Complaint* = {*psychiatric illness or substance abuse (PISA), other*}
	Arrival process	- time-varying arrival rate $\lambda_s(h)$ for hour of day h and patient stream s - attributes $\boldsymbol{\theta}$ assigned at arrival using empirical probability distribution
Activities	Triage T_1	Assume patients are triaged immediately (no triage time-stamp)
	Treatment T_2	Capacity unconstrained time distribution by stream (adjusted for attributes)
	Extended time T_3	Capacity unconstrained time distribution by stream (adjusted for attributes)
Progression	Pathways $\boldsymbol{\rho}$	Define pathways for each combination of patient pathways. Some have only one option e.g. resuscitation area for urgency category 1 patients, whereas others have multiple options e.g. treatment space or EECU space for discharge patients
	Queueing logic	- Assume no limit on waiting room capacity; separate queues for each type of resource (admit/discharge/paediatric/EECU treatment spaces) - Resuscitation patients seen immediately, otherwise priority queueing discipline by urgency category
	Routing logic	Logic for routing patients through pathways $\boldsymbol{\rho}$, including probability distributions for multiple options and conditions for resource driven pathway choices (e.g. choice of treatment area space)
Resources	Treatment spaces	Number of resuscitation, adult, paediatric, EECU treatment spaces; rules for who can use each space
	Staff availability	Modelled by adjusting the time distributions T_i by time of day (captures effect of shift patterns)
Assumptions and simplifications		Treatment modelled as an aggregated process; queue and service times are amalgamated for T_2 and T_3; patients triaged immediately upon arrival; variation in the number and activity of ED staff across different times of the day by using different LOS distributions for each shift; patient progression is linear through T_1 and T_2, with outcome assigned using an empirical probability distribution at arrival to the model.

4. Welch's method for assessing the simulation warm-up period

Welch's method (Welch (1983)) was used to check the validity of a two day warm-up period using the output σ 'average waiting time'. The following procedure was used. Let n be the number of replications and r be the simulation run length. For each observation $i = 1, \ldots, m$, the average over all replications $j = 1, \ldots, r$ is calculated using $\bar{\sigma}_i = \sum_{j=1}^{n} \frac{\sigma_{ij}}{n}$. Then the averages $\bar{\sigma}_i$ are smoothed using a moving average $\bar{\sigma}_i(w)$ with window w, where $w \leq \frac{r}{4}$. By visual inspection, the warm-up period is chosen to be the time after which $\bar{\sigma}_i(w)$ appears to be converged. Fig. 1 shows the replication-averaged model output where the 'average waiting time' is initially zero, then increases to fluctuate around a value of 55 after approximately one day of simulation time. From inspection of the graph, it is clear to see that the model has reached a steady state by the second day of simulation time, and therefore two days was deemed to be an appropriate warm-up period.

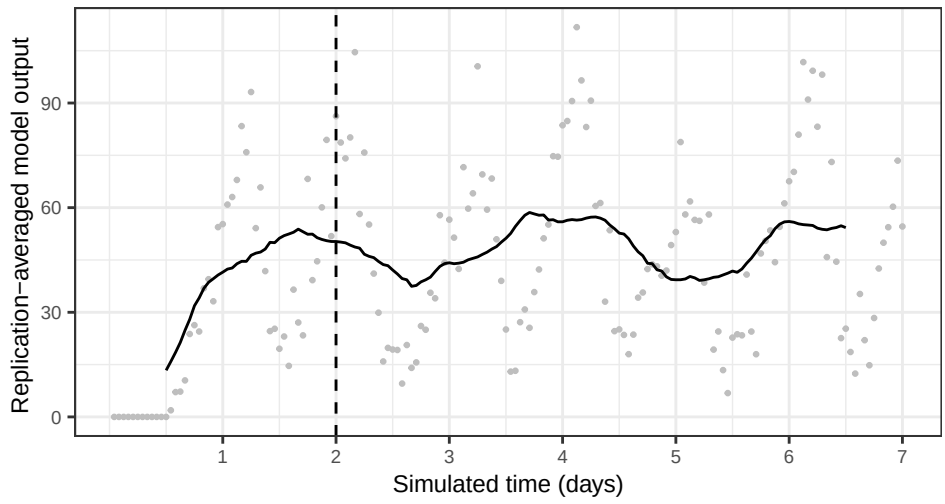

Figure 1: Plot of Welch's Method. The grey points represent replication-averaged model output 'average waiting time', the solid black line shows a moving average through the grey points, and the dashed black line marks 2 days of elapsed simulation time.

5. Sensitivity analysis

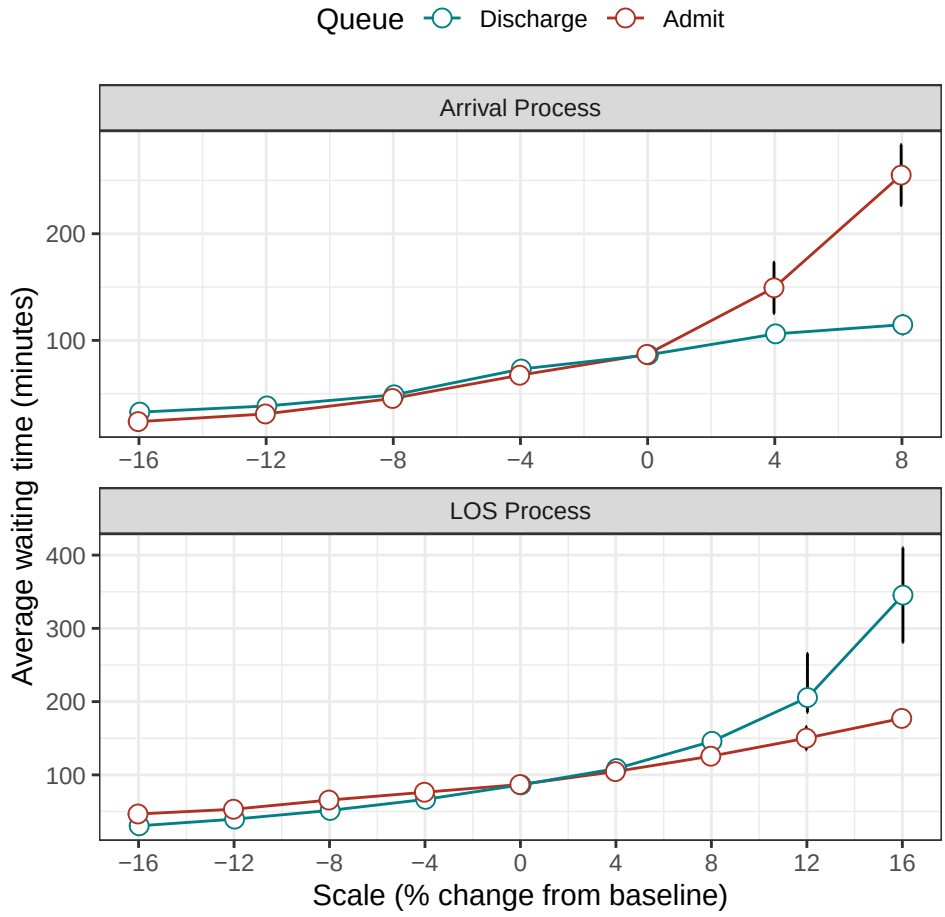

Figure 2: Sensitivity analysis showing how the model output changes as the arrival process and treatment time parameters are scaled using a percentage change from the model baseline.

Fig. 2 shows the change in average model output (average waiting time in the admit and discharge queues) with 95% confidence intervals as the arrival process and the treatment time parameters are scaled using a percentage change from the model baseline. Note that only the treatment time (and not the extended time) parameters were scaled. The graphs show that the

average waiting time increases and decreases in line with an increased or decreased percentage scale from baseline respectively in both queues.

The top graph shows that the average waiting time in the admit queue grows more rapidly than in the discharged queue as the arrival rates increase. This is because the admit patients generally have a longer length of stay than the discharged patients. When the arrival rates are scaled to higher than an 8% increase over the baseline model, the average waiting time grows to an unreasonable size. In this scenario, it is likely that the ED increase the number of treatment spaces (including trolleys, chairs) and would divert patients to other hospitals.

The bottom graph shows that the average waiting time in the discharge queue grows more rapidly than in the admit queue as the treatment time increases. This is due to the method of scaling the treatment times. Admit patients generally have a shorter LOS than discharge patients in the treatment stage of the model and a longer LOS in the extended care stage of the model, as many of them have to wait for an inpatient bed to become available. When the treatment times are scaled to higher than a 16% increase over the baseline model, the average waiting time grows to an unreasonable size in the discharge queue.

6. Validation

6.1. Length of stay

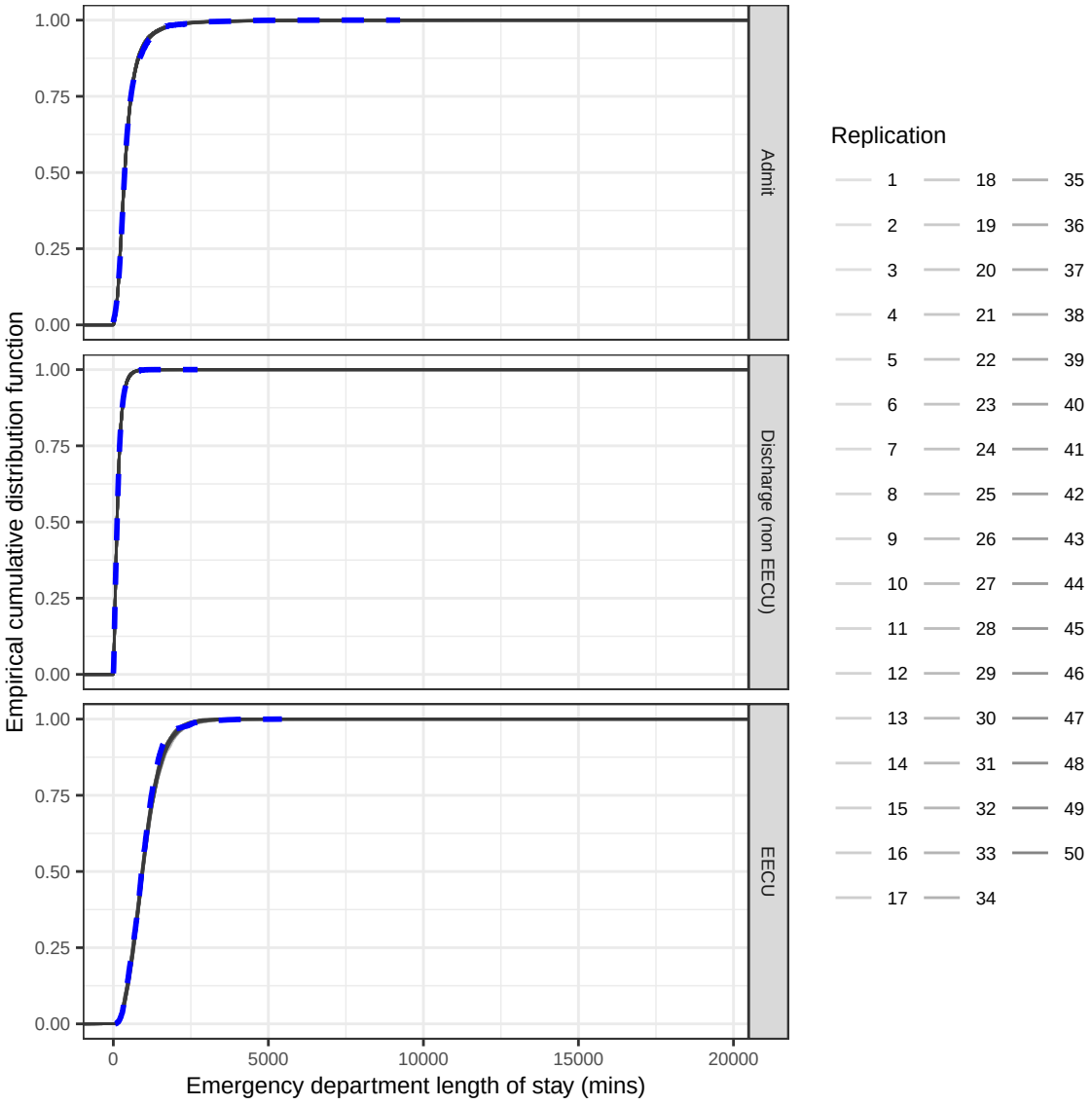

Figure 3: Comparison of the empirical cumulative distribution function of length of stay from the data (blue line) to each replication from the simulation (grey lines) for each of the admit, discharge, and EECU adult patients.

6.2. Average waiting time

Fig. 4 displays a visual comparison of the average waiting time in the data to the output from multiple simulation runs. The graph shows that the replication-averaged model output (black line) fluctuates across the day as in the data (blue line), but with smaller fluctuations. The output from 10 individual runs of the model (grey lines) show that the model output does fluctuate across the day to the same extent as in the data (blue line).

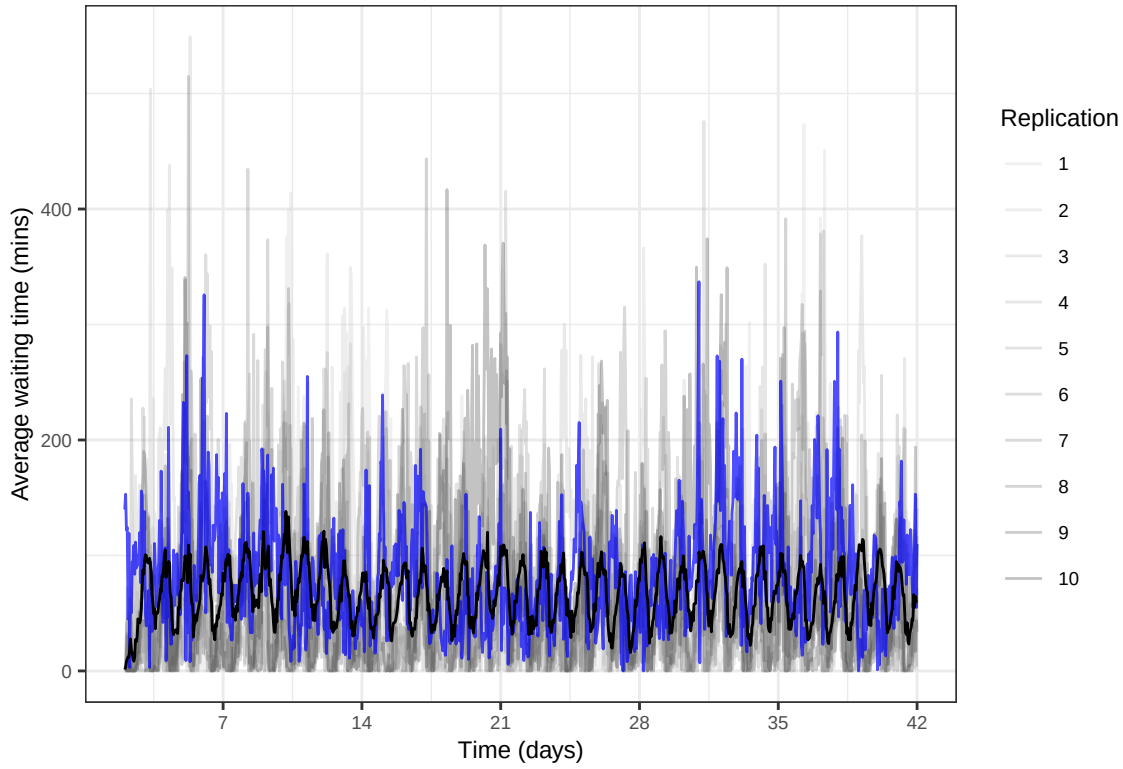

Figure 4: Comparison of the hourly average waiting time from the data (blue line) to each replication from the simulation (grey lines) and the replication-averaged waiting time (black line) for all patients.

6.3. Arrival process

Table 12 contains (i) the empirical arrival rates, (ii) 95% confidence intervals (CI) for the simulated arrival rates, and (iii) p-values from the

Kolmogorov-Smirnov test for the assumption of the conditional-uniform (CU KS test). In all cases, there is insufficient evidence to reject the null hypothesis that the arrivals in each interval follow a Poisson distribution and satisfy the conditional-uniform assumption). The table shows that each of the empirical arrival rates from the data lie within the simulation output 95% CIs. Figs. 5 and 6 show this comparison of the empirical arrival rate from the data (red points) to the empirical arrival rates from each simulation replication (black boxplots). In all cases, the data estimate falls close to the median of the simulation output.

Table 12: Validation of the non-homogeneous Poisson arrival process for patients to ED

	Discharge stream			**Admit stream**		
Time interval	Empirical arrival rate	Model output CI	CU KS test p-value	Empirical arrival rate	Expected time between arrivals (mins)	CU KS test p-value
00:00-00:59	0.0445	(0.0438,0.0451)	0.55	0.0282	(0.0277,0.0287)	0.74
01:00-01:59	0.0379	(0.0373,0.0386)	0.39	0.0254	(0.0248,0.0260)	0.78
02:00-02:59	0.0327	(0.0321,0.0333)	0.79	0.0214	(0.0209,0.0219)	0.67
03:00-03:59	0.0272	(0.0267,0.0277)	0.71	0.0181	(0.0176,0.0185)	0.79
04:00-04:59	0.0244	(0.0239,0.0249)	0.88	0.0197	(0.0193, 0.0202)	0.67
05:00-05:59	0.0233	(0.0228,0.0238)	0.81	0.0188	(0.0183,0.0193)	0.81
06:00-06:59	0.0237	(0.0232,0.0243)	0.78	0.0178	(0.0174,0.0182)	0.84
07:00-07:59	0.0400	(0.0393,0.0406)	0.36	0.0239	(0.0234,0.0244)	0.48
08:00-08:59	0.0680	(0.0671,0.0690)	0.30	0.0360	(0.0355,0.0366)	0.44
09:00-09:59	0.1005	(0.0994,0.1016)	0.15	0.0515	(0.0507,0.0523)	0.39
10:00-10:59	0.1149	(0.1136,0.1162)	0.29	0.0607	(0.0600,0.0613)	0.38
11:00-11:59	0.1190	(0.1180,0.1201)	0.08	0.0645	(0.0637,0.0652)	0.43
12:00-12:59	0.1202	(0.1191,0.1213)	0.17	0.0646	(0.0637,0.0656)	0.37
13:00-13:59	0.1167	(0.1155,0.1179)	0.19	0.0711	(0.0701,0.0722)	0.24
14:00-14:59	0.1099	(0.1090,0.1109)	0.09	0.0654	(0.0645,0.0664)	0.32
15:00-15:59	0.1041	(0.1029,0.1054)	0.16	0.0607	(0.0599,0.0615)	0.29
16:00-16:59	0.1059	(0.1047,0.1070)	0.20	0.0626	(0.0619,0.0634)	0.25
17:00-17:59	0.1056	(0.1044,0.1067)	0.26	0.0604	(0.0595,0.0613)	0.34
18:00-18:59	0.1136	(0.1125,0.1147)	0.20	0.0611	(0.0604,0.0618)	0.43
19:00-19:59	0.1115	(0.1106,0.1124)	0.17	0.0612	(0.0606,0.0619)	0.37
20:00-20:59	0.0988	(0.0979,0.0998)	0.27	0.0533	(0.0525,0.0541)	0.55
21:00-21:59	0.0866	(0.0855,0.0876)	0.33	0.0468	(0.0462,0.0475)	0.44
22:00-22:59	0.0762	(0.0755,0.0770)	0.26	0.0413	(0.0407,0.0419)	0.44
23:00-23:59	0.0560	(0.0552,0.0568)	0.47	0.0317	(0.0311,0.0324)	0.50

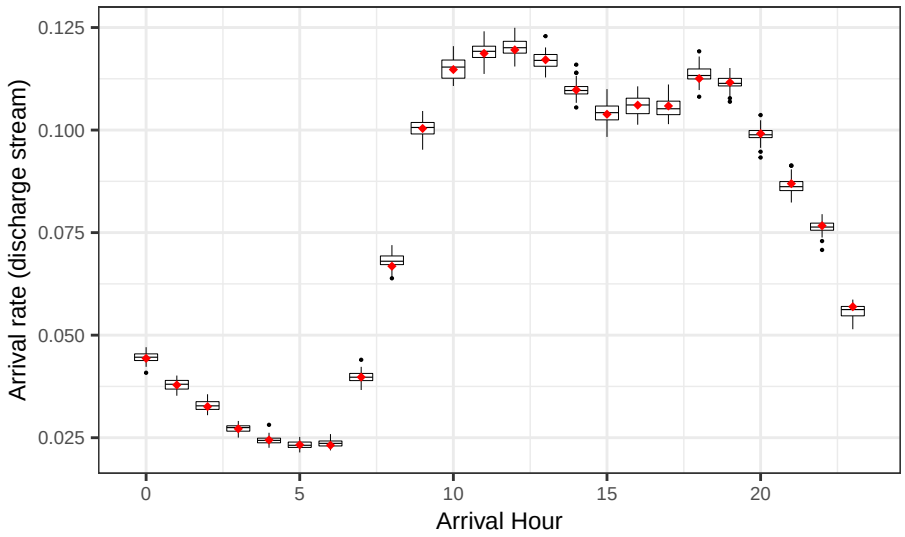

Figure 5: Comparison of the empirical arrival rate from the data (red points) to the empirical arrival rates from the simulation (black boxplots) for discharge patients across each hour of the day.

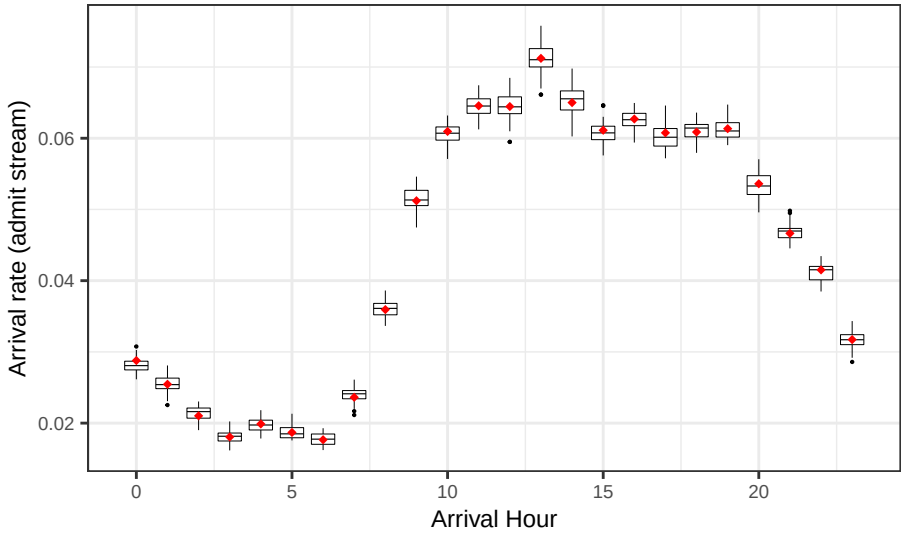

Figure 6: Comparison of the empirical arrival rate from the data (red points) to the empirical arrival rates from the simulation (black boxplots) for admit patients across each hour of the day.

References

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