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ABSTRACT

We used structural equation modeling to evaluate an enhanced version of the Home Numeracy Model proposed by Skwarchuk and colleagues (2014, *Journal of Experimental Child Psychology, 121*, 63-84). Participants were 390 Chilean preschool children and their parents. Children completed numeracy and literacy tasks at the beginning of preschool (mean age: 4 years and 7 months) and approximately 8 months later. Parents reported on the home numeracy activities they engaged in with their children, including formal (i.e., mapping and operational), informal (i.e., parents' number-game knowledge), and home literacy activities (i.e., code-related and meaning-related), as well as on numeracy and literacy attitudes and expectations for children's performance prior to Grade 1. We found that parents with more positive numeracy attitudes and higher academic expectations reported a higher frequency of formal numeracy (mapping and operational) activities. In turn, formal operational activities predicted number line estimation and applied problem-solving skills. In contrast, informal activities (i.e., parents' number-game knowledge) predicted children's non-symbolic arithmetic and non-symbolic number comparison tasks, as well as their applied problem-solving skills. The links between home activities and numeracy outcomes were domain specific: Parents' reports of literacy activities did not predict early numeracy skills. We discuss how our results support the enhanced home numeracy model and thus provide a more complete framework connecting parents' engagement in numeracy activities and children's mathematical outcomes.

Keywords: early numeracy, home numeracy, home environment, numeracy activities, children, early number skills.

Children's early experiences support the development of skills that form the basis of their later academic achievement both in mathematics (Dunst, Hamby, Wilkie, & Dunst, 2017) and reading (Sénéchal, Whissell, & Bidfell, 2017). Children's early learning experiences are many and varied: The focus of the present research is on parent engagement within the home numeracy environment (HNE) and its relation to children's early mathematical skills (Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009; Manolitsis, Georgiou, & Tziraki, 2013; Skwarchuk, Sowinski, & LeFevre, 2014). These skills are predictive of children's later mathematics performance (Duncan et al., 2007; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Watts, Duncan, Siegler, & Davis-Kean, 2014) and of children's performance in other academic domains (Duncan et al., 2007). The goal of the present study was to develop and test a comprehensive conceptual framework that will enhance research on the relation between children's home numeracy environment and their numeracy skills in preschool. To reach this goal, we extended a model to explain the links between the home numeracy environment and children's early numeracy skills to a sample outside the commonly-studied groups in the field. Thus, we tested whether the findings from an extended conceptual framework connecting parents' engagement in numeracy activities and children's numeracy outcomes would replicate in a different culture and an emerging economy, Chile. A test of this enhanced model using a sample from a different context would provide support for a conceptual model to understand home numeracy.

Home Numeracy Activities and Children's Numeracy Skills

Parents engage with their children at home in various activities that promote their academic skills. In terms of literacy, storybook reading and the **meaning-related or indirect activities** involved in these experiences predict the development of vocabulary knowledge (Hindman, Connor, Jewkes, & Morrison, 2008; Sénéchal, 2015; Sénéchal & LeFevre, 2014;

Sénéchal et al., 2017). In contrast, **direct** or **code-related** activities, which involve parental instruction in emergent literacy skills, predict children's early word reading (Hindman et al., 2008; Sénéchal et al., 2017). Skwarchuk et al. (2014) applied a similar distinction between indirect and direct numeracy activities, to understand the links between children's outcomes and home numeracy activities that varied in their didactic focus. Initial application of these distinctions characterized home numeracy activities as **indirect or informal** versus **direct or formal**. *Informal home numeracy activities* correspond to opportunities that incidentally promote children's numeracy skills. Thus, children might learn specific skills, but teaching math is not the goal (although parents could be using informal contexts for deliberate teaching of math). Examples of these activities are playing number-related games at home or assisting while parents cook. *Formal numeracy activities* are those in which parents engage in numeracy with the goal of teaching children some specific aspects of math, such as counting or arithmetic (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Skwarchuk et al., 2014). Subsequent uses of these categories have not always been straightforward and have led to some inconsistencies in the literature (see Elliott & Bachman, 2017, for a review).

Although home numeracy activities are positively related to children's numeracy outcomes in general, there are differences depending on the types of activities, children's age, and the outcomes analyzed (i.e., symbolic or non-symbolic skills; Dunst et al., 2017; Thompson, Napoli, & Purpura, 2017). In terms of the types of activities, parental reports of *informal home numeracy activities* positively relate to some aspects of children's math knowledge in kindergarten, Grade 1, and Grade 2 (LeFevre et al., 2009). Other studies have shown positive links between children's informal numeracy activities measured by the exposure to number board games and their numerical understanding (i.e., counting, number line estimation, numerical magnitude comparison, and numeral identification; Dunbar, Ridha, Cankaya, Jiménez Lira, & LeFevre, 2017; Ramani & Siegler, 2008; Whyte & Bull, 2008). Also, correlations exist between parents' familiarity with games with a numerical component and their kindergarten children's non-symbolic arithmetic skill (Skwarchuk et al., 2014).

On the other hand, *formal numeracy activities* are related to children's symbolic numeracy outcomes (Skwarchuk et al., 2014). Parents' reports of formal numeracy activities have been linked to their preschool children's general numeracy outcomes (Thompson et al., 2017; Zippert & Ramani, 2017) and counting skills (Manolitsis et al., 2013). For kindergarten children, parents' reports are related to their children's applied problem-solving skills (del Río, Susperreguy, Strasser, & Salinas, 2017) and symbolic number knowledge (Skwarchuk et al., 2014); and to arithmetic fluency for children in kindergarten, Grade 1, and Grade 2 (LeFevre et al., 2009). *Formal numeracy activities* vary in terms of complexity, from *basic* formal activities that include teaching children how to count objects, identify numerals, or print numbers to *advanced* formal activities, such as teaching simple sums or doing math mentally. Only advanced formal activities were positively related to children's outcomes in Skwarchuk et al. (2014), suggesting that the complexity of the activities plays a role in promoting numeracy skills. However, labels of basic and advanced are problematic in that they do not necessarily provide sufficient description of the types of activities being done by parents and they have been interpreted differently by researchers (Dunst et al., 2017; Saxe, Guberman, & Gearhart, 1987; Skwarchuk et al., 2014).

Despite the research showing that different home numeracy activities are predictive of various outcomes, some studies have used global measures of numeracy activities (Anders et al., 2012; DeFlorio & Beliakoff, 2015; Niklas & Schneider, 2014). Although a global measure of the

HNE may be informative, it becomes difficult to extract conclusions about the specific activities that matter for children's numerical outcomes with this approach. Given the ample research in both the home numeracy and literacy domains suggesting that the connections between activities and outcomes are specific (Dunst et al., 2017; Skwarchuk et al., 2014; Thompson et al., 2017), we argue that it is useful to make clear distinctions among types of numeracy activities.

Parental Factors Linked to Numeracy Activities

A variety of parental factors are linked to parents' involvement in numeracy activities with their children, such as parental attitudes and expectations. Parents with more positive numeracy attitudes report engaging in more formal numeracy activities with their children (del Río et al., 2017; Missall, Hojnoski, Caskie, & Repasky, 2015; Skwarchuk et al., 2014). Those who have higher expectations for their children's academic skills also report engaging in a higher frequency of formal home numeracy activities with them (del Río et al., 2017).

Demographic characteristics of parents, such as educational attainment or socio-economic status (SES), also relate to differences in the HNE that parents provide to their children. Some studies have found that children from high SES families or those where parents have more education have a higher quality of numeracy interactions than children in lower SES families (Saxe et al., 1987; Tudge & Doucet, 2004; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009). Other studies, however, have not found such links (Manolitsis et al., 2013; Skwarchuk et al., 2014), possibly due to small sample sizes or homogeneity in terms of SES. Finally, differences have begun to emerge across cultures in terms of the relations between parent characteristics and children's numeracy skills, and in some cases, these cultural differences may reflect confounds between parent factors (e.g., education, income) and home experiences (LeFevre et al., 2010; Authors et al.). Thus, there is a need for further research that examines parental factors in samples that are more diverse and within other cultural contexts.

The Home Numeracy Model

The Home Numeracy Model (HNM; Skwarchuk et al., 2014) is a conceptual framework that links the HNE to children's numeracy skills. There are other ways of conceptualizing relations between the home environment and children's skills (e.g., Bradley, Corwyn, McAdoo, & Coll, 2001; Hart, Ganley, & Purpura, 2016). The HNM, however, is a parsimonious framework that includes several factors linked to children's outcomes while distinguishing between formal and informal numeracy activities. Skwarchuk et al. (2014) first tested the HNM in a study with kindergarten children and their parents. The results showed that parents' attitudes toward numeracy predicted their children's numeracy outcomes, and parents' academic expectations positively related to their reports of formal numeracy activities. Furthermore, Skwarchuk et al. found that parents' *advanced formal* numeracy activities predicted children's symbolic number skills, and parents' *informal* numeracy activities predicted children's non-symbolic arithmetic skill. Although the HNM has contributed to the field by integrating several HNE factors previously studied only in isolation, many aspects require further investigation. In addition, previous research has not fully addressed the domain specificity of the links between parental activities and children's outcomes. This raises the question of whether the relations between numeracy activities and children's outcomes are specific to children's numeracy outcomes, or whether they extend to other outcomes in the domain of literacy. Thus, we propose several ways to extend this conceptual framework.

Current Research

The current research used a short-term longitudinal study with two testing times to evaluate a model to explain the relations between the HNE and numeracy outcomes in four-year-old children attending a preschool transition program (i.e., pre-kindergarten). To do so, it extends the HNM (Skwarchuk et al., 2014) in several ways: (a) it tests the model using a different and diverse population, a large Chilean sample of pre-kindergarteners coming from families of various educational backgrounds, allowing for a consideration of the results outside the commonly-used samples for these studies; (b) it statistically evaluates the complete model rather than conducting separate regression analyses; (c) it uses several numeracy outcomes, both symbolic and non-symbolic, to evaluate the nuances of the links between HNE and children's performance; (d) it includes a measure of parental education to account for the differences in home activities due to educational background; and (e) it assesses the domain specificity of the role of the home learning environment on children's outcomes at this age, by including measures of the home literacy environment.

Figure 1 shows the extended Home Numeracy Model. We used this extended model to frame and test five hypotheses in a large sample of Chilean children. We chose the Chilean context due to the need to extend the research on the HNE to countries outside of North America and Europe, and to evaluate the relations between home experiences and children's development in an advancing economy (Strasser, Rolla, & Romero-Contreras, 2016). Thus, this study provides an opportunity for evaluation and replication of the extended model in a more diverse sample than the ones commonly used in the field of home numeracy. This allows for a better understanding of the HNE not only in the commonly-studied samples, but also in other populations. Despite cultural differences in the Chilean context, we expected to find replicable results that account for common home numeracy activities that foster early math skills in different contexts.

The first hypothesis links parental factors (i.e., numeracy attitudes, numeracy expectations, and parental education) and home numeracy activities. Specifically, we expected that parents' numeracy expectations would predict their reports of *formal* numeracy activities (Skwarchuk et al., 2014). We distinguished between two kinds of formal home activities, **mapping** and **operational**, whose labels specify the nature of the activities in which parents engaged with their children to teach some aspects of math. *Mapping activities* relate to the understanding of the different ways of mentally or physically representing numbers, such as connecting digits to quantities and to number words, whereas *operational activities* involve operating on digits and quantities, such as mental arithmetic. Based on prior research, we expected that parents' numeracy expectations would predict both mapping and operational numeracy activities (Skwarchuk et al., 2014). We also proposed that parents with positive math attitudes will report more operational numeracy activities with their children (Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Missall et al., 2015), as they may be more aware and more interested in advanced home numeracy activities. Similarly, we hypothesized that more educated parents would be more familiar with games that have a numerical component (i.e., shared number-game play; Ramani & Siegler, 2008; Skwarchuk et al., 2014) and also create more advanced numeracy interactions for their children. Although some previous studies did not find support for relations between parental education and formal numeracy activities (Manolitsis et al., 2013; Skwarchuk et al., 2014), with our much larger and more diverse sample we anticipated detecting small effects. Parental education, more generally, is a good predictor of parenting activities in a wide variety of contexts (e.g., Zadeh, Farnia, & Ungerleider, 2010), and thus we expected that more educated parents would engage in a higher frequency of operational activities, which focus on more advanced formal interactions.

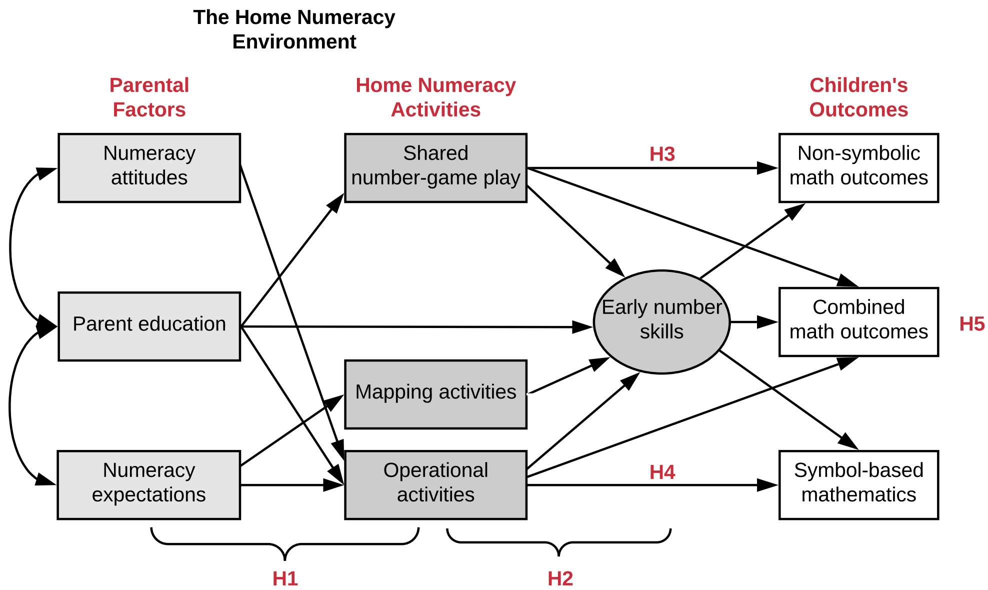

*Figure 1***.** Home numeracy model summarizing relations among parental factors, home numeracy activities, and children's outcomes. H1 captures the relations among the parental factors and parents' activities; H2 captures the relations among home numeracy activities and early number skills; and H3, H4 and H5 refer to the specific hypotheses linking home activities to outcomes.

Hypothesis 2 states that all home numeracy activities (i.e., self-reported mapping and operational activities and shared number-game play) would predict children's early number skills (e.g., counting, digit recognition). This hypothesis is based on prior findings where home numeracy activities predicted early number skills similar to those we are testing, such as verbal counting (Manolitsis et al., 2013) and object counting (Kleemans, Peeters, Segers, & Verhoeven, 2012). We note, however, that this differs from Skwarchuk et al.'s finding that shared game playing was not related to early symbolic number skills (2014). Hypotheses 3 and 4 capture the relations between home numeracy activities and children's mathematical outcomes while accounting for early number skills. Hypothesis 3 states that shared number-game play would predict children's success in non-symbolic tasks, such as object arithmetic and non-symbolic number comparison. The emphasis in these activities with very young children is presumably on representing and understanding quantities, rather than on developing the links between quantities and symbols. Hypothesis 4 is that operational activities would predict symbol-based mathematical outcomes (i.e., symbolic number comparison and number line estimation), either directly or indirectly through early numeracy skills because those early skills provide the foundation for more advanced symbolic knowledge. Hypothesis 5 is that both shared number-game play and operational activities would directly predict applied problem-solving, as this outcome includes both symbolic and non-symbolic math problems, and the total score inextricably links both aspects.

Finally, to confirm that the home numeracy environment is unique and distinct from the home literacy environment and has a domain-specific influence on mathematical skill, we compared the home literacy environment with the home numeracy environment. In line with previous research, we predicted parents would have more positive attitudes towards literacy than numeracy (Cannon & Ginsburg, 2008; Skwarchuk, 2009) and would engage more frequently in literacy than numeracy home activities (Anders et al., 2012; Skwarchuk et al., 2014). To evaluate the domain specificity of the links between the home environment and children's skills, we tested whether home literacy activities predicted math outcomes. We hypothesized that the relations between home activities and performance are domain-specific and thus literacy activities will not predict numeracy outcomes.

METHOD

Participants

This study is part of a large research project investigating factors that predict early math skills prior to the onset of Grade 1. Children (*N* = 419, 48% male, all Spanish monolingual), mean age 4:7 (years:months, range 3:4 – 5:7) and their parents (85% mothers), were recruited from seven schools in the greater urban area of Santiago, Chile. In Chile, children attend a year of kindergarten preparation (i.e., half-day preschool) at the elementary school they will be attending once they enter kindergarten. All children were attending this preschool year. Schools reflected a wide range of socioeconomic groups in Chile, including schools serving low/middle-low, middle, and middle-high/high SES families. The sample also included a variety of parental education levels. Of the 390 parents who completed the home survey, 26% had a high school diploma or lower, 10% had some level of technical training, 23% attended or graduated from community college, and 43% had attended or graduated from university.

Procedure

The *[blinded]* Institutional Ethics Committee, as Project # *[blinded],* approved the procedures for this research in advance. After principals agreed to participate, parents provided written consent for themselves and their children. Children also gave verbal assent prior to the assessments. There are no prescribed common curricula for all schools in Chile, but the Ministry of Education provides general national learning standards. These specify learning goals and the areas that the pre-kindergarten programs should cover, but they do not refer to the frequency or sequence of the learning experiences children should receive, allowing teachers to adapt the activities they choose based on different methods, programs, or contexts (MINEDUC, 2001).

Children completed assessments at two time points (Time 1 and Time 2). We tested children's numeracy, literacy, and other cognitive skills over a two-month period at the beginning of the fall semester of 2016 (Time 1), and then approximately 7-8 months later at the end of the winter term of the same academic year (Time 2). Thus, the children had little school experience at the time of the first assessment. Of the 419 children included at Time 1, 13 did not participate in Time 2, due to changes in school or prolonged absences due to health issues or moving overseas. Parents completed the questionnaires at the beginning of the school year.

Materials

Parental factors. Parents completed a pencil-and-paper Spanish translation of a questionnaire (Skwarchuk et al., 2014), reporting on the frequency of home numeracy and home literacy activities they shared with their children. This translation, done and reviewed by two native speakers of Spanish, and then back-translated to English, was piloted in a prior study. The results indicated that the instructions and questions were culturally appropriate for Chilean parents. Parents answered additional questions about their attitudes towards math and literacy, and the numeracy and literacy skills they expected their child to acquire by Grade 1 (see below). They also provided background and demographic information (i.e., educational attainment).

Parents' attitudes and expectations. To capture parent attitudes towards math and literacy, parents rated their level of agreement/disagreement (1= strongly disagree, 5 = strongly agree) with statements such as "I enjoy math activities" or "I was good at reading and writing in school". Parents indicated their academic expectations by noting the importance (0 = unimportant, 4 = very important) of children achieving academic benchmarks such as "Count to 100" or "Read a few words". Table 1 includes descriptives for numeracy attitudes and expectations, whereas we present the literacy data on parent attitudes and expectations in the supplementary data. We used mean scale scores in descriptive analyses and saved regression factor scores to create two uncorrelated variables for each parent factor, that is, numeracy expectations and numeracy attitudes, and literacy expectations and literacy attitudes.

Table 1

Descriptive Statistics and Rotated Factor Loadings for Parent Numeracy Factors

Numeracy			PCA component loadings	
	M	*SD*	Factor 1	Factor 2
Numeracy Expectations				
Count to 10[a]	3.3	0.9		
Count to 100	2.2	1.1	**.86**	.11
Read printed numbers up to 100	2.0	1.1	**.90**	.05
Solve simple sums (e.g., 2+2)	2.6	1.0	**.77**	.11
Count to 1000	1.4	1.1	**.87**	-.05
Multiply	1.3	1.1	**.81**	-.04
Scale Reliability (Cronbach's α)			.90	
Numeracy Attitudes				
I enjoy math activities	4.1	.87	.11	**.78**
I avoid situations involving math[b]	1.9	1.0	-.05	**.68**
I was good at math in school[c]	3.7	1.2	.04	**.80**
My job is related to math[c]	3.5	1.4	.04	**.70**
Scale Reliability (Cronbach's α)				.70

Notes. (i) PCA converged in 3 iterations (ii) The two-factor structure accounted for 64.4% of the variance in parent home numeracy factors. [a] Item excluded from factor analysis given the lower variability in parental responses. [b] Reverse-scored in the factor analysis. [c] Questions not included in Skwarchuk et al. (2014).

Home activities.

Numeracy - Mapping and operational activities. Following the survey in Skwarchuk et al. (2014), parents rated how frequently they did home numeracy activities with their children. We observed high internal reliability of the ten numeracy-focused questions (Cronbach's α = .86). As shown in Table 2, a principal component analysis (PCA) resulted in two components that accounted for 58.2% of the variance in home numeracy activities. We named the first component (basic formal home numeracy in Skwarchuk et al., 2014) *mapping activities,* as it involved activities related to numerical representation and identification, and the links among the various representations of a number (e.g., "I teach my child to recognize printed numbers" and "I ask about quantities"). We named the second component, called advanced formal home numeracy in Skwarchuk et al. (2014), *operational activities*, as it involved more advanced activities focused on manipulating digits or quantities (e.g., "I help my child learn simple sums" and "I encourage my child to do math in his or her head"). We used mean scale scores in descriptive analyses and saved regression factor scores to create two uncorrelated variables (mapping and operational activities) that we used for correlational analysis and model testing.

Numeracy - Shared number-game play. Parents' number-game knowledge was used to index one aspect of the home numeracy environment, that is, shared number-game play (Skwarchuk et al., 2014). Although parents' self-reports provide one perspective on the frequency of their shared game play (see Table 2), parents' number-game knowledge provides additional insights into the home numeracy environment that is not based on explicit self-evaluations. The rationale for testing game knowledge follows Sénéchal and colleagues' (Sénéchal, LeFevre, Hudson, & Lawson, 1996; Sénéchal, LeFevre, Thomas, & Daley, 1998) widely-used test of parents' knowledge of children's books to capture shared reading. Experimenters gave parents a version of the number-games checklist (Skwarchuk et al., 2014) and asked them to check the game titles they recognize (see supplementary data). We designed this list based on the availability of number games in Chilean stores, and we previously piloted it to ensure its appropriateness for Chileans. The list includes real number-based game titles, real non-number game titles, and made-up game titles (i.e., foils). As in Skwarchuk et al. (2014), we included the non-number game titles to broaden the list, but we did not include these in the scoring. In this study, the median number of foils selected was 0, indicating that parents did not guess when completing the survey. We calculated a corrected number-game score (i.e., real number games selected – foils selected), and we standardized this score (*z*-score) for further analysis. Eight scores were greater than 3 SD above or below the mean. We made these scores less extreme by reducing them to 3 (as per Skwarchuk et al., 2014). Guttman split-half reliability comparing responses on games 1-13 and 13-25 was .70.

Table 2

Descriptive Statistics and Rotated Factor Loadings for Self-Reported Home Numeracy Activities

			PCA loadings	
	M	*SD*	Factor 1	Factor 2
Mapping activities				
I help my child to recite numbers in order	3.0	1.1	**.80**	.21
We sing counting songs	2.3	1.4	**.67**	.14
I teach my child to recognize printed numbers	2.7	1.2	**.70**	.24
I ask about quantities (e.g., how many spoons)	3.0	1.0	**.67**	.32
I encourage the use of fingers to indicate how many	3.0	1.1	**.81**	.19
Scale Reliability (Cronbach's α)			.81	
Operational activities				
I help my child learn simple sums	1.8	1.3	.10	**.78**
I encourage my child to do math in his or her head	1.8	1.4	.16	**.79**
We talk about time with clocks and calendars	1.9	1.4	.30	**.62**
I help my child weigh, measure, and compare quantities	1.9	1.4	.22	**.68**
We play games that involve counting, adding or subtracting	2.4	1.2	.43	**.68**
Scale reliability (Cronbach's α)				.81
Non-numeric activities (not included in factor analysis)				
We sort and classify by color, shape and size	2.7	1.2		
I encourage collecting (e.g., cards, stamps, rocks)	1.2	1.4		
We play board games or cards	1.3	1.2		

Note. Parents responded on a five-point scale to the question, "How often do you do the following activities with your child?" Response options were (0) rarely or never, (1) monthly, (2) weekly, (3) several days a week, and (4) most days per week. (ii) PCA converged in 3 iterations.

Literacy - Code-related and meaning-related activities. We surveyed ten home literacy activities as per Skwarchuk et al. (2014; see supplementary data). Observed internal reliability of the questions was high (Cronbach's alpha = .88). A PCA resulted in two components that accounted for 59.6% of the variance in home literacy activities. Following the commonly used terminology (Hindman et al., 2008; Sénéchal et al., 2017), these two formal home literacy components were named *code-related* and *meaning-related activities*. *Code-related activities* are those focused on the features of print such as early letter skills identification or alphabet knowledge (e.g., "I help my child print words" and "I teach my child to recognize printed letters"). *Meaning-related activities* refer to activities that promote oral language development (e.g., "I ask questions when we read together" and "I introduce new words and their definitions to my child"). We used mean scale scores in descriptive analyses and saved factor scores to create two uncorrelated variables (code-related and meaning-related activities) for correlational analysis. Research on the home literacy environment uses parents' knowledge of children's books (storybook and author checklists, Sénéchal et al., 1996; Sénéchal et al., 1998), the number of books at home, or the frequency of shared storybook reading, to assess informal home literacy. Because children's books are expensive (Strasser, Vergara, & del Río, 2017) and storybook reading is not very common in Chile (Strasser & Lissi, 2009), we did not assess informal home literacy in the current study.

Children's numeracy measures. Because this study is part of a larger project examining factors that predict early math skills before Grade 1, children also completed other tasks, including executive function tasks. We did not include executive function in the models tested in this study, as the pattern of results concerning the relations between the home numeracy environment and children's math outcomes did not change with its inclusion, but for a discussion on the links between executive function and early number skills, see Montoya et al. (2018). Table 3 includes descriptive statistics for the children's tasks,

Early number skills. We chose three tasks to capture different aspects of early number knowledge: verbal counting, number identification, and object counting, all measured at Time 1. In the *verbal counting* task, children counted as high as they could. The experimenter discontinued counting when the child made a mistake in the count sequence. Scoring was the highest number recited. For *number identification* (adapted from Purpura & Ganley, 2014), the experimenter showed children digits on flashcards and asked, “What number is this?” 17 flashcards were used, with the following numbers included: 1, 2, 9, 7, 4, 6, 3, 5, 8, 10, 17, 15, 12, 11, 22, 28, and 23. The score was the total of items identified correctly. For *object counting*, children completed a Give-N-Task adapted from Purpura and Ganley (2014). Children received 20 tokens, and the experimenter asked them, on each of 7 trials, to place a given number of tokens on a sheet of paper (i.e., 3, 4, 6, 8, 10, 11, or 16). Scoring was the number of correctly produced sets. We entered the raw scores into a PCA. The solution accounted for 76% of the variance among the tasks, and the factor loadings were 0.84, 0.90, and 0.88, respectively. We named the resulting factor score **early number skills**, and we used it in correlational analyses. We entered the task raw scores as indicators for the early number skills latent variable in the structural equation models.

Non-symbolic mathematical outcomes.

Object arithmetic. We used the Purpura and Lonigan (2013) addition and subtraction with objects task at Time 2 to assess children’s understanding of the ways quantities can be composed and decomposed. In this task, the experimenter tells the child how many items she is placing in an empty box (e.g., 3), which she then demonstrates. She then tells the child that another quantity (e.g., 1) is being added or subtracted from the box. The experimenter also demonstrates this. The child then has to name the number of items in the box without looking inside. There are 8 trials (5 additions and 3 subtractions), and the score is the total number of correct responses.

Number comparison – non-symbolic. At Time 2, children completed the numeracy screener task (Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013). This two-part paper-and-pencil task includes a symbolic (digits) and non-symbolic (dots) component. In the *non-symbolic version*, children have 2 minutes to cross off the larger quantity of dots (e.g., 7 dots or 3 dots). The experimenter times the children, and the score is the rate of correct responses (i.e., total correct responses/time to complete the task in seconds).

Symbolic mathematical outcomes.

*Number comparison – symbolic***.** In the *symbolic version* of the numeracy screener (Nosworthy et al., 2013), the experimenter gives the children two minutes to quickly and correctly cross off the larger of two digits (e.g., What is larger, 7 or 3?). The score is the rate of correct responses.

Number Line estimation. In the iPad version of the number line task (https://hume.ca/ix/estimationline.html), children see a target number (i.e., 1-9) and a number line ranging from 0 (on the left) to 10 (on the right). The child quickly and accurately has to tap the spot on the number line where the target number belongs. Scoring is the absolute error between the target number and the child's placement of the target. We used mean absolute error across trials in the analyses.

Mathematical outcomes that include both symbolic and non-symbolic knowledge.

Applied problem-solving. Children were given the *Problemas Aplicados* (Applied Problems) subtest of the Batería III Woodcock-Muñoz (Muñoz-Sandoval, Woodcock, McGrew, & Mather, 2005). In this test, the experimenter reads aloud the questions and children follow along with written or pictorial stimuli. Problems become increasingly difficult, progressing from counting to basic arithmetic, and capturing both symbolic and non-symbolic elements. According to the administration procedures, we discontinued testing after six consecutive errors. We used the total correct score in further analyses.

Children's literacy measures.

Receptive vocabulary. The Vocabulario en Imágenes Peabody (TVIP; Dunn, Padilla, Lugo, & Dunn, 1986) is a Spanish adaptation of the PPVT-R test of receptive vocabulary. In this task, the child sees a page with four pictures. The experimenter says a target word and the child points to the picture that best reflects that target word. Following the test's administration procedures, we terminated the test when the child made six incorrect responses in a set of eight words. Scoring is the total of correct words.

Letter-word identification. Children's decoding skills were assessed using the subtest *Identificación de Letras y Palabras* (Letter–Word Identification) of the Batería III Woodcock-Muñoz (Muñoz-Sandoval et al., 2005). Children are shown letters, syllables, and words and asked to identify them. According to the testing protocol, the test stopped after six consecutive errors. Scoring is the total number correct. We excluded one extreme outlier (Score = 54, M = 3.6, SD = 3.2) from further analysis.

Table 3

Descriptive Statistics for Children's Measures

	Time 1				Time 2			
	M	*SD*	Range	*Reliability*	*M*	*SD*	Range	*Reliability*
Early number skills								
Verbal counting[1]	13.2	8.5	0-69	*r* = .53				
Number identification[2]	6.2	4.2	0-17	α = .89				
Give N Task (Object counting) [2]	3.1	2.2	0-7	α = .82				
Outcomes								
Number line[3]					2.4	1.3	.3-6.3	α = .72
Number comparison – Symbolic[4]					0.22	0.10	.00-.48	N.A.
Non-symbolic[4]					0.27	0.07	.01-.47	N.A.
Object arithmetic[2]					5.3	1.5	0-8	α = .53
Applied problem-solving	11.4	3.5	0-27	α = .65	15.1	3.8	5-26	α = .70
Letter-word identification[2]	3.5	2.1	0-13	α = .64	8.2	89	1-73	α = .71
Vocabulary (TVIP)[2]	40.5	13.0	0-70	α = .62	49.1	12.0	4-79	α = .65

Note. For Time 1, $N = 388$ to 415, for Time 2, $N = 403$ to 406.

[1]Highest count. [2]Total correct trials. [3]Mean absolute error. [4]Number correct/second.

We used Cronbach's alpha to calculate reliability in all cases as a measure of internal consistency, but for verbal counting, we used test-retest reliability (between Time 1 and Time 2). We computed reliability for object arithmetic using split-half reliability.

RESULTS

Descriptive Analyses

Parental factors and home activities. Table 4 shows the correlations among the parental factors and home activities. We found numeracy attitudes to be modestly correlated with operational activities and shared number-game play. As expected, numeracy expectations were positively related to both mapping and operational numeracy activities. We also found parents' education to be positively related to shared number–game play, but we did not find significant links with operational activities.

We compared the reported frequencies of home numeracy and home literacy activities, and the parental expectations in both domains. Parents reported more positive attitudes towards literacy (M=4.17, SD = 0.75) than numeracy (M = 3.82, SD = 0.83), $t(387) = -6.51$, $p<.001$, Cohen's d = 0.44. Parents also had significantly higher expectations for literacy (M=2.40, SD = 0.88) than numeracy (M=1.91, SD = 0.93), $t(388) = -15.9$, $p<.001$, Cohen's d = 0.54. However, they reported spending more time doing mapping and operational numeracy activities (M=2.36, SD = 0.82) than code- and meaning-related literacy activities with their children (M=2.04, SD = 0.93), $t(387) = 9.93$, $p<.001$, Cohen's d = 0.36. Correlations were moderate between the frequency of literacy activities and numeracy activities and were higher for mapping than for operational activities. Parents' numeracy and literacy attitudes were weakly but significantly related. The results showed literacy attitudes to be the most highly correlated with meaning-related activities. The highest correlation in Table 4 was between literacy and numeracy expectations, and it suggests that parents who value achievement do not differentiate between numeracy and literacy skills.

Children's Performance. Table 5 includes the correlations among parental factors, home numeracy activities, and children's outcomes. We include the literacy correlations in the supplementary data.

Table 4

Correlations among the Parent Factors and Shared Activities: Numeracy vs. Literacy

	2	3	4	5	6	7	8	9
1. Numeracy attitudes	.12*	.00	.04	-.01	.18**	.11*	-.03	.11*
2. Literacy attitudes		-.02	.00	.16**	.03	.21**	.04	.25**
3. Numeracy expectations			.79**	.15**	.21**	-.03	.29**	.04
4. Literacy expectations				.22**	.20**	-.05*	.33**	.05
5. Mapping activities					.00	.07	.41**	.46**
6. Operational activities						.03	.32**	.32**
7. Shared number-game play							-.00	.02
8. Code-related activities								.00
9. Meaning-related activities								

* p <.05. ** p<.01.

Model Development

We tested the home numeracy models using structural equation modeling in MPlus Version 7 (Muthén & Muthén, 1998-2012). We tested four different models to capture the relations among parental factors, home numeracy activities, and the child's mathematical outcomes, four different models were tested, with each model predicting different numerical outcomes and each aligning with the theoretical model shown in Figure 1. We tested indirect effects from operational activities to all mathematical outcomes (we present the details of indirect effects in the supplementary data).

Table 5

Correlations among Control Measures, Parental Factors, Home Numeracy Activities, and Children's Numeracy Performance

	2	3	4	5	6	7	8	9	10	11	12	13	14
1. Age	-.01	**.19**	.04	-.01	-.02	.10	**.11**	**.37**	**.20**	**.16**	**.24**	-.07	**.20**
2. Gender		**-.12**	-.09	.07	.03	.01	-.02	*-.11*	.06	**.17**	-.02	.00	-.06
3. Parent education			**.20**	**-.14**	-.09	*.12*	**.20**	**.33**	**.17**	*.12*	**.23**	-.05	**.16**
4. Attitudes[a]				.00	-.01	**.18**	*.11*	**.17**	.02	-.01	*.11*	-.04	*.11*
5. Expectations[a]					**.15**	**.21**	-.03	-.06	-.03	-.01	.00	*-.13*	.02
6. Mapping activities[a]						.00	.07	.01	-.06	-.04	*-.12*	-.01	-.04
7. Operational activities[a]							.03	**.15**	*.12*	*.11*	**.25**	**-.17**	.08
8. Shared number-game play								**.15**	**.23**	**.18**	**.23**	-.06	**.19**
9. Early number skills[a] (T1)									**.41**	**.27**	**.57**	**-.29**	**.44**
10. Numb.comp: symbolic (T2)										**.63**	**.54**	**-.30**	**.34**
11. Numb.comp: non-symb. (T2)											**.41**	**-.20**	**.28**
12. Applied problem-solving (T2)												**-.27**	**.42**
13. Number line (T2)													**-.24**
14. Object arithmetic (T2)													

Notes. Italicized numbers indicate $p < .05$, **bolded** numbers indicate $p < .01$ [a] factor/latent scoring described in the methods section.

T1= Time 1 measure; T2= Time 2 measure.

Hypothesis 1: The home numeracy environment. We tested the portion of the model that captures the relations among parental factors and parents' reports of home numeracy activities first because we used it in all subsequent models. As hypothesized, parents with more positive math attitudes reported a higher frequency of formal operational activities. Parents with higher academic expectations also reported more mapping and operational numeracy activities. Higher-educated parents recognized more number games and reported a higher frequency of operational activities. This model fit the data well: $\chi^2(4) = 4.04$, $p = .401$, SRMR = .020, CFT = .999, RMSEA = .005 (90% CI = [0, .077]), thus, we used this structure in each of the outcome models.

Hypothesis 2: Home numeracy activities and early number skills. The proposed model predicted that all home numeracy activities would predict early number skills. As shown in Figure 2, when we entered all home numeracy activities into the model, only the path between operational activities and early number skills was significant. We did not find a significant correlation between mapping activities and early number skills (Table 5), so it is not surprising that this path was not significant. Shared number-game play was positively correlated with early number skills but the path was not statistically significant, replicating the pattern found by Skwarchuk et al. (2014). Thus, in subsequent models, we did not specify the paths between early number skills and both shared number-game play and mapping activities. This decision is supported by Kline's recommendation for model parsimony (Kline, 2016).

Hypothesis 3: Shared number-game play and non-symbolic math outcomes. We tested this hypothesis with two non-symbolic mathematical outcomes, object arithmetic and non-symbolic number comparison. Our findings supported Hypothesis 3 in that shared number-game play uniquely predicted children's performance on these non-symbolic math outcomes (see

Figure 3). As expected, early number skills also predicted these outcomes. Operational activities directly predicted early number skills and related indirectly to both outcomes through early number skills (see Supplementary data). Model fit is strong, thus supporting Hypothesis 3.

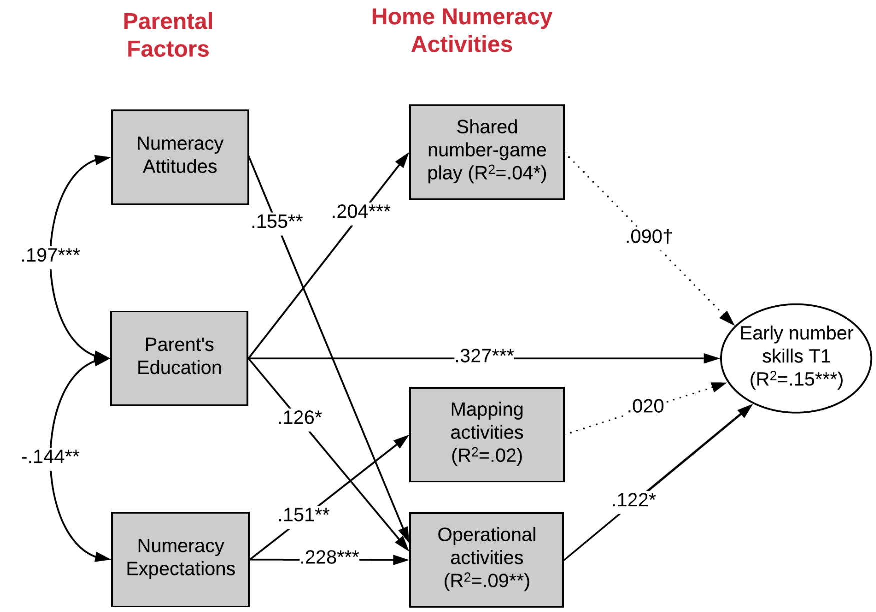

Figure 2. Home numeracy model capturing the relations between the home environment and children's early number skills. Model fit statistics: $\chi^2(21) = 25.89$, $p = .211$, SRMR = .031, CFT = .992, RMSEA = .024 (90% CI = [0, .050]). †$p \leq .10$. *$p \leq .05$. **$p \leq .01$. *** $p \leq .001$.

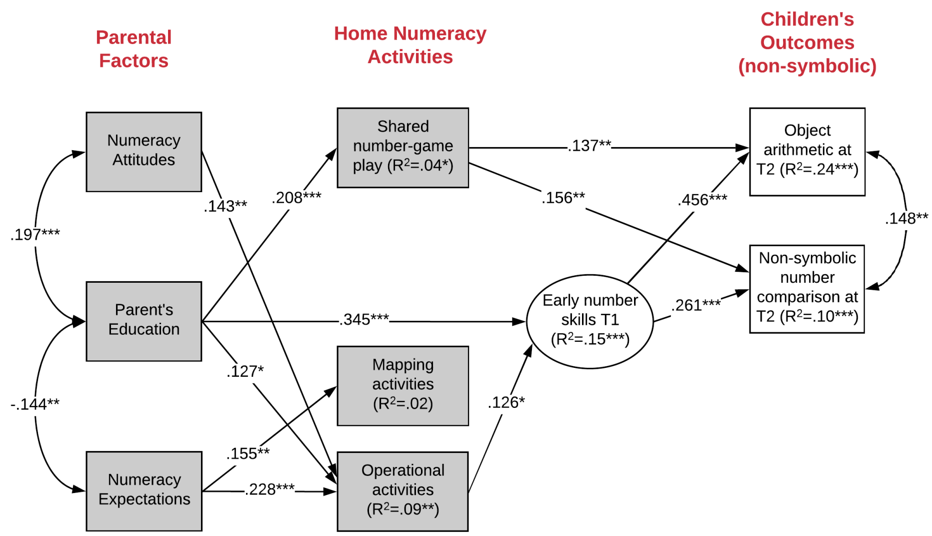

Figure 3. Home numeracy model predicting non-symbolic mathematical outcomes. Model fit

statistics: $\chi^2(35) = 37.65$, $p = .349$, SRMR = .032, CFT = .997, RMSEA = .013 (90% CI = [0, .038]). Numbers on paths are standardized coefficients. $*p \leq .05$. $**p \leq .01$. $***\ p \leq .001$.

Hypothesis 4: Operational activities and symbolic math outcomes. We included two symbolic mathematical outcomes in the model: symbolic number comparison and the number line task (see Figure 4). As expected, operational activities directly predicted the number line task and indirectly predicted symbolic number comparison through early number skills, $\beta = .055$, 90% CI = [.011, .107].

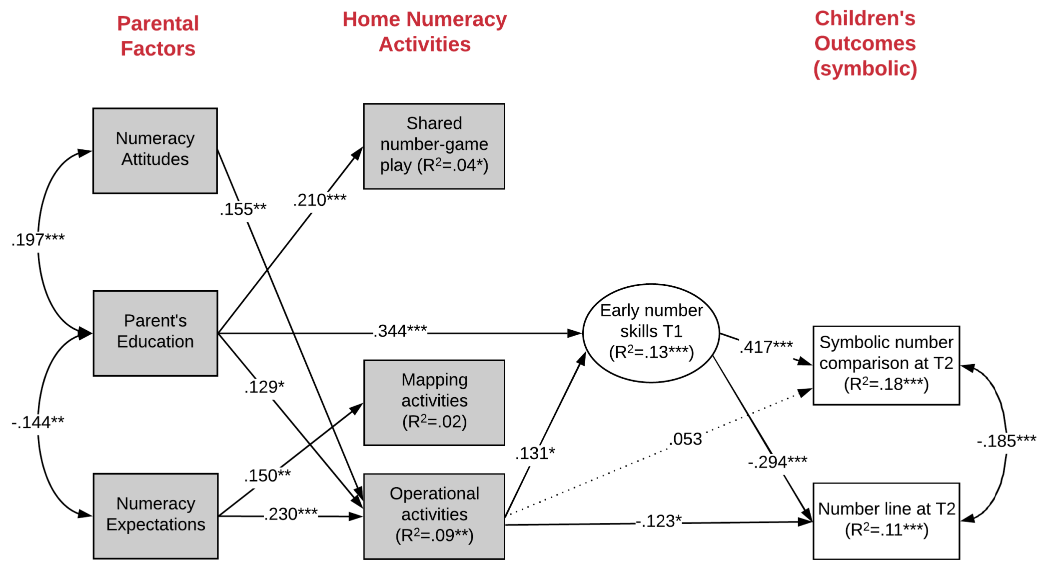

Figure 4. Home numeracy model predicting symbolic mathematical outcomes. Model fit statistics: $\chi^2(32) = 42.12$, $p = .109$, SRMR = .033, CFT = .987, RMSEA = .027 (90% CI = [0, .048]). Numbers on paths are standardized coefficients. $*p \leq .05$. $**p \leq .01$. $***\ p \leq .001$.

Hypothesis 5: Both shared number-game play and operational activities will predict outcomes that involve symbolic and non-symbolic elements. The applied problem-solving task included both symbolic and non-symbolic math problems. Accordingly, we modelled direct paths from shared number-game play and operational activities to this task. Model fit was strong (see Figure 5). As hypothesized, applied problem-solving skills were predicted directly by both shared number-game play and operational activities. Parents' reports of a higher frequency of mapping activities were related to lower problem-solving skills, however.

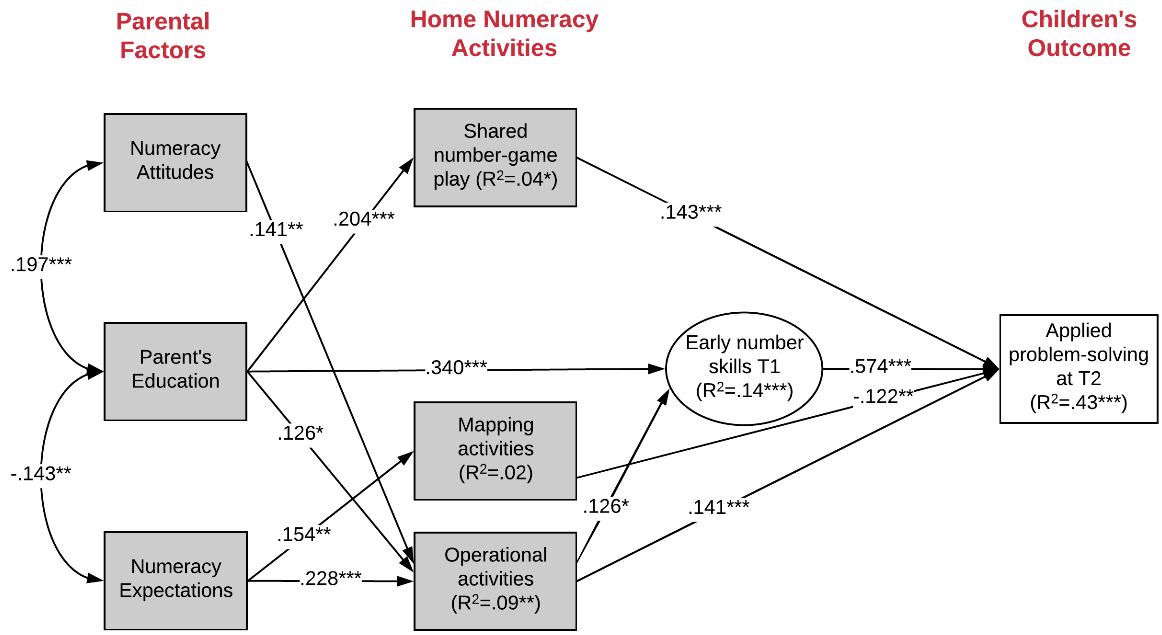

Figure 5. Home numeracy model predicting applied problem-solving skills. Model fit statistics: $\chi^2(28) = 30.29$, $p = .350$, SRMR = .033, CFT = .997, RMSEA = .014 (90% CI = [0, .041]). *$p \leq .05$. **$p \leq .01$. *** $p \leq .001$.

Domain-Specificity of Home Activities

The results showed a moderate correlation between home numeracy and literacy activities, raising the question of whether literacy activities uniquely relate to literacy outcomes and numeracy activities uniquely relate to numeracy outcomes. To test the hypothesis that home activities were domain-specific in their influence, we conducted regressions for a) numeracy outcomes predicted by code- and meaning-related activities and b) literacy outcomes predicted by mapping and operational activities. The results (see supplementary materials) showed that literacy activities did not significantly predict applied problem-solving or number line estimation, and numeracy activities did not predict letter-word identification or vocabulary.

DISCUSSION

In this study, we extended the Home Numeracy Model (HNM) proposed by Skwarchuk et al. (2014) to a large and diverse sample of Chilean preschoolers. We developed a comprehensive conceptual framework and tested a complete model that included several symbolic and non-symbolic outcomes. Our results replicated three main findings of the HNM: (a) positive links from parental factors to home numeracy activities, specifically, from socioeconomic status (i.e., parental education in the present study, income in Skwarchuk et al.) to shared number-game play, and from parental expectations to mapping and operational activities; (b) positive relations between shared number-game play and non-symbolic mathematical outcomes; and (c) positive relations between parents' reports of operational activities to children's symbolic mathematical outcomes. Furthermore, in the present study we identified additional connections between the home numeracy environment and children's outcomes because we included more measures and evaluated children's performance over the course of the school year. First, parents' education and the measure of their attitudes toward math were related to operational activities. Second, operational activities indirectly predicted non-symbolic skills. Thus, our results extended the conceptual framework provided by the HNM by including paths that were not tested in the version proposed by Skwarchuk et al. In particular, the finding that effects of parents' operational activities on children's outcomes was an indirect effect of children's early number skills provides a clear pathway for a continued influence of early numeracy activities on the growth in children's skills over the pre-kindergarten year.

Extensions to the Home Numeracy Model

We found three results that we did not predict based on the HNM. First, we found that parents with more positive numeracy attitudes engaged in a higher frequency of operational activities. This link was not directly tested in Skwarchuk et al. (2014)'s model because they did not assess the same range of numeracy attitudes. Our findings about parents' attitudes are consistent with other research (Blevins-Knabe et al., 2000; del Río et al., 2017; Missall et al., 2015). For example, del Río et al. (2017) found that Chilean parents with lower levels of math anxiety reported a higher frequency of math-related activities with their children. Research with older children (i.e., in grades 1 and 2) has also shown that parents' negative feelings about math are related to children's math learning, such that children of math-anxious parents learn less math over the school year if the parents help the children with their homework (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015). The present results suggest that connections between parents' attitudes towards math and their children's performance may start very early.

Second, we found that parents with more education were more likely to involve their children in operational numeracy activities. Skwarchuk et al. (2014) did not directly test this link because they used income to index SES. Our results were similar to those of del Río et al. (2017), who found that highly-educated Chilean mothers were more likely to involve their children in formal operational numeracy activities. Thus, our results extend the links between education and parents' numeracy activities in the home environment, showing that for Chilean families, more-educated parents provide more frequent experiences through both formal and informal activities compared to less-educated parents (Ramani & Siegler, 2008; Skwarchuk et al., 2014).

Third, we found an unexpected indirect path from operational activities to non-symbolic math skills (Skwarchuk et al., 2014). In the non-symbolic number comparison task used in this study, children compared two sets of up to 9 dots each (Nosworthy et al., 2013). A possible explanation for this indirect link is that children who had a stronger grasp of the cardinality principle (as indexed by the early number skills composite in the model) were more likely to use counting to determine the quantity of dots and make accurate comparisons compared to children who had a weaker understanding of the cardinality principle. Similarly, on the object arithmetic task, children could have relied on counting and knowledge of verbal number words to perform well on the task (Jiménez Lira, Carver, Douglas, & LeFevre, 2017). Thus, this link might explain the strong correlation between early number skills and the non-symbolic mathematical outcomes, while also providing an explanation for the indirect path from operational activities to non-symbolic skills.

Although we replicated the main findings of the HNM, the effect sizes for the links between activities and children's performance are smaller than some other studies. For example, our model only explained a modest amount of the variance in some outcomes (i.e., number line estimation and non-symbolic number comparison). For non-symbolic number comparison, other cognitive factors such as inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013) may explain additional variance. For number line performance, spatial skills are important (Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013), and thus parental activities tailored to supporting spatial skills, such as playing with puzzles (Levine, Ratliff, Huttenlocher, & Cannon, 2012) or building with blocks (Verdine et al., 2014), might also predict performance on the number line. Furthermore, young children may need to achieve a threshold of performance on multiple numeracy skills (e.g., digit recognition, mapping between quantities and digits, relative ordinal positions of numbers) before they can construct a useful strategy for this task (Xu & LeFevre, 2016). Overall, the relatively modest relations between the home numeracy environment and children's outcomes are consistent with other research with Chilean families (Authors, under review), and they might reflect cultural differences in the range of home activities that are important for early academic skills. Future work should explore a wider range of home activities, such as oral numeracy traditions or early childhood educational quality, to fully understand the home numeracy environment for Chilean children.

Domain-Specificity of the Links between Home Environment and Children's Outcomes

Several of our findings are consistent with prior research on links between parental factors and the home environment, suggesting domain-related differences in the home environment. First, parents reported more positive attitudes and expectations for literacy than numeracy (Cannon & Ginsburg, 2008). Second, parents reported engaging with their children more frequently in numeracy activities than in literacy activities (Skwarchuk et al., 2014). These apparently contradictory findings may reflect different emphases: Parents reported higher frequencies of *mapping* activities compared to *operational* activities, in terms of numeracy, whereas they seemed to focus more often on meaning-related activities than on code-related ones for literacy (with the exception of teaching printed letter recognition).

In addition, we found that home literacy activities did not predict children's numeracy outcomes, and that home numeracy activities did not explain children's literacy skills, allowing us to conclude that the relations between home activities and children's performance are domain specific. In contrast, Napoli and Purpura (2018) found that numeracy activities are also related to vocabulary, but their measure of numeracy activities overlapped with literacy activities (i.e., math language). Similarly, Anders et al. (2012) reported that home literacy activities were more predictive of early numeracy skills than were home numeracy activities, but the math outcome used in their research was strongly dependent on language skills. The current research provides a more nuanced assessment of the relations between home activities and numeracy outcomes.

Replications of the Home Numeracy Model in the Chilean Context

The HNM was a good fit for the Chilean home numeracy context and was consistent with some other evidence about the HNE in Chilean families. First, the finding that parents with higher numeracy expectations reported a higher frequency operational activities (Skwarchuk et al., 2014) is consistent with prior research with Chilean kindergarteners (del Río et al., 2017). Second, consistent with Skwarchuk et al. (2014)'s findings, parents with higher expectations also reported higher frequencies of *mapping* activities. In general, mapping activities may be ones that parents recognize as relevant to math learning that occurs in school and hence support their beliefs that such activities at home will foster skills needed for their children's success. Our findings also show that parents with higher education recognized more number books (Skwarchuk et al., 2014). These results suggest that more-educated parents support numeracy skills and/or greater access to relevant resources than less-educated parents, especially given the class differences in access to educational resources among Chilean families (Strasser et al., 2017).

Our study provides further support for the central premise of the HNM, that is, we showed differential links between types of home numeracy activities and children's numeracy outcomes (Skwarchuk et al., 2014). First, our results replicate the finding that parents' reports of operational formal activities predict children's performance on symbolic tasks. Specifically, these activities predicted number line estimation, problem-solving skills, and symbolic number comparison performance (directly and/or indirectly through early number skills), suggesting that parents' engagement in activities in which children mentally manipulate numbers supports children's acquisition of these skills (del Río et al., 2017; Skwarchuk et al., 2014; Zippert & Ramani, 2017). Second, and as hypothesized, we found that parents' shared number-game play was related to children's performance on non-symbolic addition and subtraction, non-symbolic number comparison, and applied problem-solving, as suggested by the HNM (Skwarchuk et al., 2014) and other studies (Niklas & Schneider, 2014). These findings might imply that when parents engage in these activities, their focus might not be on symbolic representations, but on manipulation of quantities (e.g., how many spaces to move in a board game). We also identified a positive correlation between shared number-game play and symbolic number comparison. Based on the HNM's findings, we did not hypothesize this link, and thus we did not test this path in the models. Future research should further explore the relations between shared number-game play and other children's measures, including symbolic aspects. This might be an avenue to explore in more depth using samples from different cultures. Third, mapping activities, which focus on activities that tap different ways of representing numbers, did not positively relate to children's numeracy outcomes (Skwarchuk et al., 2014). On the contrary, parents' reports of more mapping activities related to poorer problem-solving skills in their children. Because parents are relatively accurate at estimating their child's mapping knowledge (Zippert & Ramani, 2017), the observed relation may indicate that parents spend more time on mapping activities if their child is struggling than if their child is doing well (for similar results for reading in Grade 1, see Sénéchal & LeFevre, 2014). These findings support the view that parents are sensitive to their children's specific capabilities and modify the home environment accordingly (Martini & Sénéchal, 2012; Sénéchal, 2015).

In brief, our results replicate and extend the HNM to a more diverse sample coming from a different country, outside the commonly-employed samples in the field. This study provided support for this conceptual framework in a different culture. Thus, despite the cultural differences between countries, we found that there are several activities that foster early math in this Chilean sample, which are similar to ones reported in studies from other countries.

Limitations and Further Directions

Although we measured several aspects of the HNE, we used parents' reports, which have limitations related to aspects such as social desirability and interpretation of the questions. We also focused on the frequency of home activities rather than on the quality of the interactions between parents' and children (Sénéchal et al., 2017). Although our measures distinguished between formal (i.e., mapping and operational) and informal (i.e., shared number-game play) activities, we do not know whether parents use informal contexts to deliberately teach math. Further studies should include supplemental observations (Elliott & Bachman, 2017), such as recordings of parent number talk (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) or diary approaches in which parents are queried in real time about their activities. Intervention studies (Berkowitz et al., 2015; Niklas, Cohrssen, & Tayler, 2016) are critical to support causal relations between specific parent activities and children's performance. Further, instructional and educational factors, such as prior preschool experience, the amount of numeracy activities in preschool or teacher math talk (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006), the quality of preschool settings (UNESCO, UNICEF, Brookings Institution, & World Bank, 2017), and teacher-child interactions (Leyva et al., 2015) are also important for understanding the key contexts for children's early numeracy skills. The literature would also benefit from longitudinal studies to evaluate whether links persist over time, and how home numeracy activities exert a role at different ages. Finally, including additional assessments or direct observations suitable for young children of different backgrounds and countries (e.g., UNESCO et al., 2017) could enhance a comprehensive model of the relations between the home environment and children's outcomes.

Conclusion

The current research expanded the Home Numeracy Model to Chilean children. By testing four models with different numerical outcomes and by using a diverse sample of preschoolers, this study provided additional support for a conceptual framework for how the home numeracy environment relates to the development of children's numeracy skills. It also highlighted that the nature of the numeracy activities in which parents engage with their children is crucial for promoting specific skills, and it offered further evidence of the domain-specificity of the links between the home environment and children's outcomes.

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