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A Direct Comparison of Two Measures of Ordinal Knowledge Among 8-Year-Olds

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Supplementary Materials: Materials [see Index of Supplementary Materials]

Abstract

Children’s knowledge of the ordinal relations among number symbols is related to their mathematical learning. Ordinal knowledge has been measured using judgment (i.e., decide whether a sequence of three digits is in order) and ordering tasks (i.e., order three digits from smallest to largest). However, the question remains whether performance on these two ordinal tasks tap into similar cognitive processes. Canadian children (N = 87; Age M = 8.7 years, Grade 3) completed symbolic number tasks (i.e., number comparison, ordering, and order judgment) and measures of arithmetic fluency (i.e., addition and subtraction) and working memory (i.e., digit span backward). For both ordinal tasks, there was a reverse distance effect for ordered sequences such that children responded faster to adjacent than to non-adjacent sequences (e.g., 2 3 4 vs. 4 7 9) and a canonical distance effect for unordered sequences such that children responded faster to non-adjacent than to adjacent sequences (e.g., 4 2 3 vs. 4 9 7). Working memory and number comparison each predicted unique variance in the ordinal measures (ordering, order judgment, and a latent ordinal factor based on the two measures). Furthermore, ordinal skills superseded the role of number comparison as the key predictor of arithmetic, controlling for children’s gender and working memory skills. In summary, although both ordering and order judgment tasks index ordinal knowledge, a latent factor that excludes task-specific error may be a better index than either task separately.

Keywords

ordering, order judgments, arithmetic, cognitive processes, measurement

Knowledge of number order – the ability to determine the relative relations among numbers (e.g., 2 comes after 1 and before 3) – is essential for numerical processing (Lyons et al., 2016; Sury & Rubinsten, 2012). Order judgment tasks are the most widely used measure of ordinal knowledge both for children (e.g., Hutchison et al., 2022; Lyons & Ansari, 2015; Morsanyi et al., 2020) and adults (e.g., Goffin & Ansari, 2016; Lyons & Beilock, 2009; Vogel et al., 2021; Vos et al., 2017). Typically, participants are presented with three numbers and asked to decide whether these digits are in order by making a binary decision (e.g., 1 2 3 is in order whereas 1 3 2 is not). However, for young children, this task can be quite challenging and thus other measures have been used. In particular, ordinal knowledge has been assessed by an ordering
task, in which children are shown three digits and asked to order them from smallest to largest (Chan et al., 2022; Knudsen et al., 2015; Xu & LeFevre, 2016, 2021). Researchers have assumed that both of these measures tap into the same ordinal construct, but none have directly tested this assumption. Thus, the goal of the present study was to determine if two commonly used ordinal tasks, order judgment and ordering, show similar patterns of cognitive processing, and similar relations to other numeracy and arithmetic tasks.

Patterns of Responses in Order Processing Tasks

Two effects are observed in ordinal tasks: the canonical distance effect and the reverse distance effect. The canonical distance effect is so named because in number comparison tasks, which have been used extensively in the field (i.e., which is larger 2 or 6? Schneider et al., 2017), response times and errors decrease with increasing distances between digit pairs (e.g., 2 3 vs. 2 8; Moyer & Landauer, 1967). This canonical distance effect has been observed in children’s number comparison performance beginning at age 5 (De Smedt et al., 2009; Hawes et al., 2019). In a sample of nearly 400 adults, Vogel et al. (2021) found that all participants exhibited a canonical distance effect, although the magnitude of this effect varied among individuals.

In ordinal tasks, a canonical distance effect is observed where people are faster and less error-prone to judge ordered sequences with larger inter-element differences (e.g., 3 6 9 vs. 3 5 7) and for all unordered sequences, including those with adjacent numbers (e.g., 3 2 4; Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons & Beilock, 2013; Turconi et al., 2006; Vogel et al., 2021). However, people are faster and less error-prone when identifying adjacent ordered sequences (e.g., 2 3 4) compared to non-adjacent ordered sequences (e.g., 1 3 5; Dubinkina et al., 2021; Lyons & Beilock, 2009). This pattern was labelled the “reverse distance effect” by Lyons and Beilock (2009) and is assumed to be a central feature of order processing (Lyons & Ansari, 2015).

Reverse distance effects in order judgments depend on the frequency of adjacent sequences in the stimulus set (Vos et al., 2021) and on participants’ solution strategies (Dubinkina et al., 2021). Some ordered adjacent sequences (e.g., 1 2 3 and 2 3 4) are highly familiar and can be easily retrieved from memory (Caplan, 2015; Goffin & Ansari, 2016; LeFevre & Bisanz, 1986; Sella, Sasanguie, et al., 2020; Serra & Nairne, 2000; Vos et al., 2017). In support of a familiarity explanation, the reverse distance effect is found for ascending but not for descending adjacent sequences among adults, presumably because ascending sequences are more familiar and therefore easier to retrieve (Dubinkina et al., 2021; Turconi et al., 2006; Vos et al., 2017). Thus, the reverse distance effect may be largely driven by a few very familiar sequences (Sella, Sasanguie, et al., 2020). On all other sequences, people may need to perform sequential comparisons to make order decisions, resulting in canonical distance effects (Lyons & Ansari, 2015; Lyons & Beilock, 2013). Together, the canonical and reverse distance effects observed in the order judgment task support the view that cardinal and ordinal processing of symbolic numerals are both involved, depending on the task demands and the extent to which adjacent sequences are present in the stimulus set (Dubinkina et al., 2021; Muñez et al., 2022; Vogel et al., 2021; Vos et al., 2017).

The Development of Order Processing

For young children, order processing is closely tied to knowledge of the familiar counting sequence and to number comparison (Gilmore & Batchelor, 2021; Sella & Lucangeli, 2020). For example, Sella and colleagues have shown that when young children (i.e., 3.5 to 6-year-olds) are asked to order adjacent triplets on a number line (e.g., 2 3 4), their performance is closely related to their number comparison performance (Sella et al., 2018, 2019; Sella & Lucangeli, 2020). When asked to order non-adjacent sequences, however, kindergarten and Grade 1 children (i.e., 5 to 6-year-olds) performed below chance (Hutchison et al., 2022), suggesting they have a strict definition of order that is tied to their knowledge of the counting sequence. As children get older (i.e., 7- to 12-year-olds), they become quite accurate at judging both adjacent and non-adjacent sequences; however, sequence type continues to influence mathematical understanding, with performance on ordered adjacent sequences predicting more unique variance in arithmetic than performance on any other type of sequence (Lyons & Ansari, 2015). This finding suggests that the relation between order judgments and arithmetic is driven by fluent recognition of familiar adjacent sequences (Dubinkina et al., 2021).

In contrast, on sequences with non-adjacent digits, people use magnitude comparison mechanisms for assessing relative order instead of retrieval-based processing (Dubinkina et al., 2021; Lyons et al., 2016). For children, performance
on ordinal tasks is consistently related to performance on number comparison tasks, where the explicit requirement is to use magnitude-based decisions. Comparison performance has been found to predict unique variance in both concurrent ordering performance for children in Grade 1 and Grade 2 (i.e., 6- to 8-year-olds; Xu & LeFevre, 2021) and the improvement in ordering performance for children from Grade 1 to Grade 2 (Finke et al., 2021). These results support the view that ordinal processing requires knowledge of the mappings between symbols and magnitude and use of that knowledge to make comparisons (Morsanyi et al., 2020). Similarly, adults reported using a sequential comparison strategy on unordered sequences (e.g., for 2 3 1: 2 < 3, 3 > 1, so sequence is unordered), versus memory retrieval on judgments of ordered adjacent sequences (Dubinkina et al., 2021).orrantia et al. (2019) found that performance on comparison and order judgments predicted mostly shared variance for arithmetic performance for adults, supporting the view that these basic number skills draw on a common process.

In addition to domain-specific number knowledge about cardinal and ordinal associations, domain-general abilities may also influence children’s performance on order processing tasks. In particular, working memory may be involved when people make decisions about unfamiliar sequences (i.e., non-adjacent sequences and unordered sequences). In support of this view, performance on order judgment and other ordinal tasks is correlated with working memory skills for children (Attout & Majerus, 2015; Finke et al., 2021; Morsanyi et al., 2020; Purpura & Ganley, 2014) and adults (Douglas et al., 2020; Lyons & Beilock, 2009). Specifically, people may need to hold the results of the number comparisons in working memory to use for the decision process. If the sequence is shown for a brief time, they may also need to retain the sequence of digits in working memory. In summary, the literature suggests that order judgment performance is related to familiarity with previously learned sequences, knowledge of relative magnitude, and working memory (Devlin et al., 2022; Dubinkina et al., 2021; Finke et al., 2021; Muñez et al., 2022; Vogel et al., 2021; Vos et al., 2021).

**Relations Among Cardinal, Ordinal, and Arithmetic Processing**

Both cardinal and ordinal processing of numbers are assumed to influence the development of children’s numerical representations (Lyons et al., 2016; Sury & Rubinsten, 2012). However, children’s ordinal skills develop more slowly than their cardinal skills (Attout et al., 2014; Colomé & Noël, 2012; Knudsen et al., 2015). In a large cross-sectional study, Lyons et al. (2014) found that number comparison was a better predictor of arithmetic than order judgments for children in Grade 1 (age 7) and Grade 2 (age 8). Order judgments became increasingly more predictive of arithmetic for children in Grades 3 through 6 (ages 9 to 12). Moreover, for children in Grade 1 (6 to 7 years of age), individual differences in arithmetic were correlated with comparison performance, but not with order judgments, suggesting that these two tasks may draw on different underlying processes at this age (Vogel et al., 2015). Similarly, Xu and LeFevre (2021) found that for children in Grade 1 (6 to 7 years of age), performance on comparison, not ordering, predicted the development of arithmetic skills from fall to spring. In the same study, however, Xu and LeFevre also found that for children in Grade 2 (7 to 8 years of age), ordering, but not comparison, predicted the development of arithmetic skills. Moreover, beginning in Grade 2, order judgments fully mediated the relations between number comparison and arithmetic performance (Sasanguie & Vos, 2018; Sommerauer et al., 2020). Overall, there is substantial evidence that ordinal skills supersede the role of number comparison as the key predictor of arithmetic from Grade 2 onward.

**The Current Study**

The goal of the present study was to examine whether ordering and order judgment tasks share the same underlying cognitive processes for children in Grade 3. Grade 3 was chosen because the dominant role of ordinal skills in predicting arithmetic skills becomes stable by Grade 3 (Lyons et al., 2014). Our overarching hypothesis was that the two tasks would show similar patterns of performance and similar relations with number comparison and arithmetic performance. More specifically, we first hypothesized that for both ordinal tasks (i.e., ordering and judgment) there would be a reverse distance effect for ordered sequences and a canonical distance effect for unordered sequences. Second, we hypothesized that number comparison skills and working memory (measured with a digit span backward task) would predict unique variance in performance on both ordinal tasks (Purpura & Ganley, 2014). Third, we hypothesized that ordinal knowledge would supersede the role of number comparison as the key predictor of arithmetic, consistent with other studies of
children in a similar age range (i.e., 7 to 9 years; Sasanguie & Vos, 2018; Sommerauer et al., 2020; Xu & LeFevre, 2021). Specifically, we tested this hypothesis using a latent factor extracted based on ordering and order judgment tasks to reduce error variance in measurement.

Method

Participants

Ethics approval from Carleton university and the school division was first obtained, followed by principal and teacher approval. Letters were then sent home inviting children to participate. Eighty-seven children were recruited from three public schools in a medium-sized Canadian city ( \( M_{\text{age}} = 8.7 \) years, \( SD = 0.3 \). range from 8:1 to 9:2 months; 43 boys) in Ontario, Canada. The schools were selected to draw from neighbourhoods with higher immigrant populations because the data are part of a larger Language Learning and Mathematics Achievement study. Fifty-one children spoke English as their first language and 36 children spoke another first language (i.e., Chinese, Arabic, Hindi, Russian, Turkish, Spanish, Gujarati, Dutch, Japanese, Tigrinya, Hebrew, Urdu, Malayalam, Somali, or Cambodian). For all children the language of instruction at school was English.

Information about parents’ highest education level was available for 81 mothers and 80 fathers. For mothers/fathers, 2.5%/1.3% had less than high school, 4.9%/6.3% had a high-school diploma, 24.7%/17.5% had a community college diploma or degree, 28.4%/30.0% had an university degree, and 28.4%/30.0% had a post-graduate degree. The median level of education for both mothers and fathers was an undergraduate degree.

Procedure

All children were tested individually by trained experimenters in a quiet space at school during three testing sessions within three months. The number comparison and ordering tasks were completed during the first session, the working memory and order judgment tasks were completed during the second session, and the arithmetic task was completed during the third session. The order of tasks was consistent across all children. Children received stickers as compensation for their participation.

Measures

Data were collected as part of a larger project. Description of additional measures and stimuli for the tasks used in the present paper are included in Open Science Framework (OSF; see Supplementary Materials). The reliability for each of the measures in the present study are included in Table 1.

Working Memory

The digit span backward task assessed children’s working memory capacity (Wechsler, 2014). To ensure that speed, volume, and pronunciation were consistent across participants, all trials were presented using pre-recorded audio files. Participants heard a string of numbers and had to repeat them in the reverse order. The task started with a demonstration trial, followed by two practice trials with feedback. Then, children started with two trials of span length two. Each subsequent pair of trials increased by one item. The maximum possible span length was eight. Scoring was the total number of correct trials. The task was discontinued if children were unable to recall both trials for a given span.

Number Comparison

Children completed a number comparison task using an iPad application called “Bigger Number”\(^1\). Children were presented with side-by-side single-digit numbers (i.e., digits 1 to 9) on an iPad screen and were instructed to tap the

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1) https://apps.apple.com/app/id1317619133
numerically bigger number as quickly as possible. A total of 26 trials were presented in a random order. On 13 trials, the
distance between two numbers was small (i.e., 1, 2, or 3); on the other 13 trials, the distance between the two numbers
was large (i.e., 4, 5, 6, or 7). On each trial, a fixation cross appeared in the center of the screen for 500 ms, followed by
a blank screen for 500 ms, followed by the presentation of the two numbers. If the child did not respond within 3 s, the
next trial was presented. Children were each given two practice trials to make sure they understood the task. Response
time (in seconds) and accuracy were recorded automatically by the app.

Ordering Task
Children completed a number ordering task using an iPad application called “Number Ordering”2. For each trial, a
fixation cross appeared in the center of the screen for 500 ms, followed by a blank screen for 500 ms, followed by three
numbers presented in separate boxes (see an example in Figure 1). They were asked to tap the numbers in order from
smallest to largest as quickly and accurately as possible. To ensure that they understood the instructions, children were
given three practice trials with feedback. After the practice trials, 24 sequences (with numbers ranging from 1 to 9) were
presented to children. Twelve of these sequences had adjacent numbers and 12 sequences had non-adjacent numbers.
Twelve of the sequences were presented in order. If children did not provide a response within 7 s the app advanced to
the next trial. Response time (in seconds) and accuracy were recorded automatically by the app.

Figure 1
Examples From the Ordering (Left Panel) and Order Judgment (Right Panel) Tasks

Order Judgment Task
Children completed an order judgment task using an iPad application called “Order Judgment”3. This app was created
based on the paper-and-pencil judgment tasks frequently used in the literature (Hutchison et al., 2022; Lyons et al.,
2014). For each trial, a fixation cross appeared in the center of the screen for 500 ms, followed by a blank screen for 500
ms, followed by the stimulus (see an example in Figure 1). Children were asked to decide whether sequences were in an
ascending order or not, as fast as possible. If all the numbers were in ascending order (e.g., 2 3 4 or 2 5 7), they were
instructed to touch the green check mark; if the numbers were not in ascending order (e.g., 2 3 1 or 5 7 2), they were
instructed to touch the red cross. Children completed the same practice trials as in the number ordering task, followed
by 24 experimental trials. The same stimuli from the ordering task were used for this order judgment task.

Arithmetic Fluency
Children completed addition and subtraction questions using paper-and-pencil as quickly and accurately as possible
(Chan & Wong, 2020). They were presented with one page of single-digit addition problems and one page of single-digit

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3) https://apps.apple.com/app/id1438433960
subtraction problems (3 columns of 20 items for a total of 60 items per page). They were given one minute to complete as many problems as possible without skipping any questions. Scoring was based on the total number of questions answered correctly for each page.

**Data Analysis**

For the speeded tasks (i.e., number comparison, ordering, and order judgment), adjusted response times were calculated to account for both response times and accuracy in a single score. For each participant, an adjusted RT was calculated using this formula:

\[
RT_{adj} = RT_{correct} + (PE \times \frac{SD_{RT}}{SD_{PE}})
\]

where \(RT_{correct}\) is the mean response time on the correct trials and percentage error (PE) is weighted by the ratio of the standard deviations of correct RTs and percentage of error (Vandierendonck, 2017). Vandierendonck refers to these as *linear integrated speed accuracy scores*.

**Results**

**Descriptive Statistics**

Descriptive statistics and correlations are shown in Table 1. Except for one child who did not contribute order judgment data due to a technical error, all children contributed data for all tasks. Outliers were defined as values with \(z\)-scores greater than |3.29| from the mean for the sample (Field, 2013). One outlier was found for each of the following measures: number comparison (\(z = 3.91\)), order judgment (\(z = 3.84\)), arithmetic fluency (\(z = 3.69\)). Sensitivity analyses with and without these outliers showed similar patterns of results, and thus all data were included in the final analyses.

As shown in Table 1, all mathematics measures were correlated with each other. Digit span backward was significantly correlated with the ordinal and arithmetic tasks. The two ordinal tasks were strongly correlated, \(r(85) = .57, p < .001\), sharing approximately one-third of the total variance with each other. Moreover, we used the Fisher \(r\)-to-\(z\) transformation to compare the strength of the correlations between the ordinal tasks and the other measures (i.e., digit span backward, number comparison and arithmetic). The strength of the correlations between ordering and the other measures did not significantly differ from the strength of the correlations between order judgment and other measures, \(p_s > .05\). These results show that performance on the ordering and order judgment tasks had similar correlations with other measures.

Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>M</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Skew</th>
<th>Internal Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Digit Span Backward(^a)</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
<td>4.2</td>
<td>1.7</td>
<td>2.0</td>
<td>9.0</td>
<td>0.92</td>
<td>.78</td>
</tr>
<tr>
<td>2. Comparison (s)(^b)</td>
<td>-.08</td>
<td>–</td>
<td></td>
<td></td>
<td>1.0</td>
<td>0.2</td>
<td>0.7</td>
<td>1.9</td>
<td>0.95</td>
<td>.92</td>
</tr>
<tr>
<td>3. Ordering (s)(^b)</td>
<td>-.26*</td>
<td>.59***</td>
<td>–</td>
<td></td>
<td>2.9</td>
<td>0.7</td>
<td>1.7</td>
<td>5.1</td>
<td>0.76</td>
<td>.87</td>
</tr>
<tr>
<td>4. Judgment (s)(^b)</td>
<td>-.22*</td>
<td>.64***</td>
<td>.57***</td>
<td>–</td>
<td>2.9</td>
<td>0.8</td>
<td>1.4</td>
<td>5.9</td>
<td>0.79</td>
<td>.92</td>
</tr>
<tr>
<td>5. Arithmetic(^c)</td>
<td>.35***</td>
<td>-.30**</td>
<td>-.49***</td>
<td>-.52***</td>
<td>28.5</td>
<td>15.4</td>
<td>2.0</td>
<td>81.0</td>
<td>1.12</td>
<td>.88</td>
</tr>
</tbody>
</table>

\(^a\)Total correct. Reliability (Cronbach’s alpha) was calculated based on performance on all individual trials. \(^b\)Adjusted RT in seconds. Reliabilities were calculated using the sequence types of the adjusted RTs. \(^c\)Sum score of number correct on addition and subtraction. Reliability was calculated based on the subset scores of addition and subtraction.

\(p < .05\). \(^*\)\(p < .01\). \(^***\)\(p < .001\).
There were also no significant differences between ordering and order judgment for percent error, $t(85) = 0.99$, $p = .326$, $d = .11$, or RT on correct trials, $t(85) = 0.94$, $p = .348$, $d = .10$. When considering both error and RT together in the adjusted RTs, children performed equally well on both tasks, $t(85) = 0.48$, $p = .634$, $d = .05$ (see Table 2). These results support the view that the two tasks rely on the same response code.

Table 2

<table>
<thead>
<tr>
<th>Sequence Type</th>
<th>RT Correct</th>
<th>Percent Error (%)</th>
<th>Adjusted RT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ordering</td>
<td>Judgment</td>
<td>Ordering</td>
</tr>
<tr>
<td>Ordered Adjacent</td>
<td>M (SD)</td>
<td>M (SD)</td>
<td>M (SD)</td>
</tr>
<tr>
<td>Ordered Non-Adjacent</td>
<td>2.48 (0.65)</td>
<td>2.33 (0.73)</td>
<td>10.9 (13.9)</td>
</tr>
<tr>
<td>Unordered Adjacent</td>
<td>2.69 (0.69)</td>
<td>2.52 (0.73)</td>
<td>5.9 (9.2)</td>
</tr>
<tr>
<td>Unordered Non-Adjacent</td>
<td>2.80 (0.73)</td>
<td>2.90 (0.75)</td>
<td>18.5 (18.3)</td>
</tr>
<tr>
<td>Overall M (SD)</td>
<td>2.48 (0.65)</td>
<td>2.33 (0.73)</td>
<td>11.3 (9.1)</td>
</tr>
</tbody>
</table>

With one exception, there were no significant gender differences across the measures ($ps > .05$). Boys had higher scores on the arithmetic task than girls (32.5 vs. 24.7), $t(80.62) = 2.44$, $p = .017$, $d = .52$. Thus, we controlled for gender in analyses of arithmetic.

**Canonical Distance Effect on Number Comparison**

On the number comparison task, the error rate across all trials was 4.2% ($SD = 4.6\%$), and the RT on the correct responses was 1.00 s ($SD = 0.19$ s). Adjusted RTs for number pairs with a small distance and number pairs with a large distance were compared. The canonical distance effect was found, such that adjusted RT was greater for number pairs with a small distance than for number pairs with a large distance (1.12 s vs. 0.95 s), $t(86) = 13.86$, $p < .001$, $d = 1.49$.

**Reverse and Canonical Distance Effect on Ordinal Tasks**

Adjusted RTs were analyzed in a 2(task: ordering, judgment) x 2(type: non-adjacent, adjacent) x 2(order: unordered, ordered) repeated measures ANOVA (see Table 3). Means across all factors are shown in Table 2. The main effect of order was significant: Children were faster to respond to ordered sequences than to unordered sequences (2.64 s vs. 3.01 s). The main effects of task and type were not significant.

Table 3

<table>
<thead>
<tr>
<th>Variable</th>
<th>$F(1, 83)$</th>
<th>$MSE$</th>
<th>$p$</th>
<th>$\eta_p^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task (Ordering vs. Judgment)</td>
<td>0.83</td>
<td>0.96</td>
<td>.364</td>
<td>.010</td>
</tr>
<tr>
<td>Type (Non-adjacent vs. Adjacent)</td>
<td>0.98</td>
<td>0.19</td>
<td>.324</td>
<td>.012</td>
</tr>
<tr>
<td>Order (Unordered vs. Ordered)</td>
<td>74.55</td>
<td>0.31</td>
<td>&lt; .001</td>
<td>.473</td>
</tr>
<tr>
<td><strong>Interaction Effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task x Type</td>
<td>0.57</td>
<td>0.19</td>
<td>.453</td>
<td>.007</td>
</tr>
<tr>
<td>Task x Order</td>
<td>5.76</td>
<td>0.35</td>
<td>.019</td>
<td>.065</td>
</tr>
<tr>
<td>Type x Order</td>
<td>16.18</td>
<td>0.25</td>
<td>&lt; .001</td>
<td>.163</td>
</tr>
<tr>
<td>Task x Type x Order</td>
<td>0.53</td>
<td>0.23</td>
<td>.470</td>
<td>.006</td>
</tr>
</tbody>
</table>
The two-way interactions between task and order and between type and order were significant. Simple effect analyses were used to evaluate interactions using Bonferroni adjustments. For the interaction between task and order, on the ordered sequences, children responded faster on the order judgment than on the ordering task (2.55 s vs. 2.73 s, $SE = .09$, $p = .047, \eta^2_p = .046$), but for the unordered sequences, no difference was found between the two tasks (3.03 s vs. 2.99 s, $SE = .09, p = .640, \eta^2_p = .003$). Children can quickly identify ordered sequences, thus, the difference between the two tasks likely reflects response demands, with the order judgment task requiring a single tap and the ordering task requiring three taps. For the interaction between type and order, on the ordered sequences, children responded faster to the adjacent than to non-adjacent sequences (2.55 s vs. 2.74 s, $SE = .05, p < .001, \eta^2_p = .148$), showing a reverse distance effect. In contrast, for the unordered sequences, children responded more slowly to the adjacent than non-adjacent sequences (3.07 s vs. 2.95 s, $SE = .05, p = .022, \eta^2_p = .062$), showing a canonical distance effect. Furthermore, as shown in Figure 2, there was no significant three-way interaction, indicating that the patterns of the reverse and canonical distance effects were not significantly different for the ordering and order judgment tasks.

*Figure 2*

*Non-Significant Three-Way Interaction Between Task, Type, and Order*

Note. Error bars show 95% inferential confidence intervals (Jarmasz & Hollands, 2009).

**Unique Contribution of Working Memory and Number Comparison to Variance in Ordinal Tasks**

Multiple regression analyses were conducted to examine the unique contribution of working memory and number comparison to each of the ordinal measures. Specifically, we considered three ordinal measures (all using adjusted RT): ordering, order judgment, and a latent factor extracted based on the first two. The latent factor, which accounted for 78% of the shared variance between the two measures, was created using principal component analysis. As shown in Table 4, for all three models, performance on the working memory and comparison tasks each predicted unique variance in the ordinal tasks, suggesting that cardinal processing and working memory are likely involved when children perform ordinal tasks.
Table 4

Multiple Regression Predicting Performance on Ordering and Order Judgment Tasks

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>β</th>
<th>t</th>
<th>p</th>
<th>Unique $r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ordering Task</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>-0.10</td>
<td>0.04</td>
<td>-0.22</td>
<td>-2.55</td>
<td>.013</td>
<td>.046</td>
</tr>
<tr>
<td>Number Comparison</td>
<td>2.06</td>
<td>0.30</td>
<td>0.58</td>
<td>6.79</td>
<td>&lt; .001</td>
<td>.329</td>
</tr>
<tr>
<td>Total $R^2$</td>
<td></td>
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<td><strong>Order Judgment Task</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>-0.08</td>
<td>0.04</td>
<td>-0.17</td>
<td>-2.05</td>
<td>.044</td>
<td>.028</td>
</tr>
<tr>
<td>Number Comparison</td>
<td>2.37</td>
<td>0.31</td>
<td>0.63</td>
<td>7.64</td>
<td>&lt; .001</td>
<td>.393</td>
</tr>
<tr>
<td>Total $R^2$</td>
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<td>.441</td>
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<tr>
<td><strong>Latent Ordinal Factor</strong></td>
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</tr>
<tr>
<td>Digit Span Backward</td>
<td>-0.13</td>
<td>0.06</td>
<td>-0.22</td>
<td>-2.98</td>
<td>.004</td>
<td>.049</td>
</tr>
<tr>
<td>Number Comparison</td>
<td>3.25</td>
<td>0.36</td>
<td>0.68</td>
<td>9.12</td>
<td>&lt; .001</td>
<td>.462</td>
</tr>
<tr>
<td>Total $R^2$</td>
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<td></td>
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<td>.539</td>
</tr>
</tbody>
</table>

Note. Unique $r^2$ represents the squared semi-partial correlations within that specific model tested.

Relations Among Cardinal, Ordinal, and Arithmetic Tasks

Hierarchical linear regression analyses were conducted to examine whether the comparison and ordinal measures account for variability in arithmetic fluency. Block 1 included gender, digit span backward, and number comparison. Block 2 included one of the ordinal measures (i.e., ordering, order judgment, or a latent ordinal factor). The patterns of results were the same across the three models, and the best model was the one when a latent ordinal factor was included. Thus, we interpret the results of the model that involved the latent factor (see results for the other models in the Appendix).

Table 5

Hierarchical Linear Regression Showing Gender, Working Memory, Number Comparison, and the Ordinal Latent Factor Predicting Arithmetic Performance

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>SE</th>
<th>β</th>
<th>t</th>
<th>p</th>
<th>Unique $r^2$</th>
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</thead>
<tbody>
<tr>
<td><strong>Block 1</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Gender</td>
<td>-7.04</td>
<td>2.96</td>
<td>-0.23</td>
<td>-2.38</td>
<td>.020</td>
<td>.052</td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>2.90</td>
<td>0.89</td>
<td>0.31</td>
<td>3.24</td>
<td>.002</td>
<td>.096</td>
</tr>
<tr>
<td>Number Comparison</td>
<td>-19.88</td>
<td>7.10</td>
<td>-0.27</td>
<td>-2.80</td>
<td>.006</td>
<td>.072</td>
</tr>
<tr>
<td>Total $R^2$</td>
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</tr>
<tr>
<td>Gender</td>
<td>-5.32</td>
<td>2.67</td>
<td>-0.17</td>
<td>-1.99</td>
<td>.050</td>
<td>.029</td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>1.71</td>
<td>0.84</td>
<td>0.19</td>
<td>2.04</td>
<td>.045</td>
<td>.031</td>
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<td>Number Comparison</td>
<td>9.78</td>
<td>9.02</td>
<td>0.13</td>
<td>1.08</td>
<td>.282</td>
<td>.009</td>
</tr>
<tr>
<td>Ordinal Factor</td>
<td>-9.15</td>
<td>1.98</td>
<td>-0.59</td>
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<td>&lt; .001</td>
<td>.158</td>
</tr>
<tr>
<td>Total $R^2$</td>
<td></td>
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<td>.405</td>
</tr>
</tbody>
</table>

Note. Unique $r^2$ represents the squared semi-partial correlations within that specific model tested.

In Block 1, children’s gender, digit span backward, and number comparison performance together predicted approximately 25% of the variance in arithmetic performance (see Table 5). In Block 2, performance on the ordinal factor, rather than number comparison, predicted unique variance in arithmetic. These results were consistent with patterns in previous research where ordinal skills superseded the role of number comparison and predicted unique variance in arithmetic (Sasanguie & Vos, 2018; Sommerauer et al., 2020; Xu & LeFevre, 2021). The amount of unique variance...
explained by the ordinal latent factor was higher (~16%) than either of the ordering or order judgment tasks entered separately (~10%; see Table A1 and A2 in the Appendix). Additionally, to further evaluate the models, we compared the Akaike’s Information Criteria (AIC) values for each model where lower values indicate better models. The AIC value was lower for the model when the ordinal latent factor was used (681.83) than for the models when either ordering (696.96) or order judgment (689.58) was entered separately. Thus, the best model to predict arithmetic performance was the one that included the latent ordinal factor score.

**Discussion**

In the present study, we examined whether ordering and order judgment tasks share the same underlying cognitive processes for children in Grade 3. We found expected patterns of responses (i.e., reverse distance effects on ordered sequences and canonical distance effects on unordered sequences) for both tasks. The reverse and canonical distance effects observed in the order judgment task are used to index ordinal and cardinal processing, respectively (e.g., Goffin & Ansari, 2016; Lyons & Ansari, 2015; Nosworthy et al., 2013; Turconi et al., 2006; Vogel et al., 2021). Therefore, the results of the present study, as expected, show that both the ordering and order judgment tasks involve cardinal and ordinal processing of symbolic numerals.

With respect to the cognitive predictors of ordinal processing, we found that working memory and number comparison skills predicted ordinal skills for both tasks. Consistent with previous research using the order judgment task (Attout & Majerus, 2015; Finke et al., 2021; Lyons & Beilock, 2009; Morsanyi et al., 2020), we found a link between working memory and ordinal processing, regardless of different task demands (i.e., touching the screen three times for the ordering task versus touching the screen once for the order judgment task). When asked to make order judgments on unfamiliar number sequences, participants need to hold the results of the sequential comparisons in working memory to use for the decision process. Beyond working memory, children’s number comparison skills also predicted unique variance in their ordinal skills, suggesting that children rely on a magnitude-based mechanism to make ordinal decisions, at least some of the time (e.g., on unordered sequences; Bourassa, 2014; Dubinkina et al., 2021; Vos et al., 2021). In summary, the contributions of working memory and cardinal processing to predicting variance in the ordering and order judgment tasks were highly similar, suggesting that these tasks likely tap into similar cognitive processes.

Importantly, ordinal knowledge superseded the role of number comparison as the key predictor of arithmetic, consistent with other studies of children of a similar age (Sasanguie & Vos, 2018; Sella, Lucangeli, et al., 2020; Sommerauer et al., 2020; Xu & LeFevre, 2021). After sufficient instruction and practice with basic arithmetic, children are expected to retrieve basic addition facts (i.e., 5 + 8 = 13). However, learning arithmetic requires children to access specific associations between operands and answers that need to be managed or differentially activated depending on the problem characteristics (Ashcraft, 1992). For example, to solve 5 + 8, children can rely on counting up from the bigger number, or recognize that 8 is close to 10, decompose 5 to 2 + 3, and solve 8 + 2 = 10, 10 + 3 = 13. Similarly, a solution for 16 − 9 may involve recognizing that 9 is almost 10, and then solving 16 − 10 + 1. These arithmetic solutions rely on children’s knowledge of cardinal and ordinal associations. Thus, the strong correlations between ordinal and arithmetic performance may occur because both types of skills rely heavily on accessing symbol-symbol associations from an integrated network (Goffin & Ansari, 2016; Lyons & Beilock, 2013; Xu et al., 2019).

The order judgment task is the most widely used measure of ordinal processing for children. However, in an order judgment task, children aged 4 to 7 years of age did not accept the ordered non-adjacent sequences as “in order” (Hutchison et al., 2022). Alternatively, the ordering task, which clarifies the key phrase “in ascending order” as “from smallest to largest,” is more sensitive to capturing ordinal knowledge for younger children (Xu & LeFevre, 2021). Task performance on both ordering and order judgment tasks for the 8-year-old children in the present study showed excellent reliability and construct validity. Thus, ordering and order judgment tasks can be used to reliably index ordinal knowledge. When feasible, researchers can consider using both tasks for a more precise measure of ordinal processing.
Future Directions and Limitations

The present study examined the concurrent relations among different types of basic numerical skills and arithmetic performance for children with three years of formal schooling, with a focus on validating measures of ordinal processing of symbolic numbers. Although concurrent studies can provide valuable insights into the relations among different levels of numerical knowledge, longitudinal research is needed to capture the development of children’s ordinal skills during the emerging stages (Hutchison et al., 2022; Xu & LeFevre, 2021). Therefore, one limitation of this study is that we did not examine the validity of the ordering and order judgment tasks over time.

Another limitation is that we did not counterbalance the order in which children completed the two ordinal tasks. All children completed the ordering task during the first testing session and the order judgment task during the second testing session. There were several days between testing sessions, attenuating possible practice effects that could have influenced the results. However, if the tasks are administered within the same session, researchers should consider counterbalancing the order of the presentation.

We note that the two ordinal tasks showed highly similar response times despite the ordering task requiring more action than the order judgment task (i.e., three taps to respond versus one tap to respond). We speculate that children decide how to respond before making the actual physical response, however, future research could provide more definitive insights into these similar response times by adjusting the app set up to capture the timing of the first tap (and subsequent taps).

In the present research, verbal working memory was a predictor of ordinal processing, consistent with previous research (Attout & Majerus, 2015; Lyons & Beilock, 2009; Morsanyi et al., 2020; Purpura & Ganley, 2014). In contrast, Finke et al. (2021) found that visuospatial working memory was a unique predictor of the development of order judgments from Grade 1 to Grade 2, potentially indicating a role of visuospatial manipulation in ordinal processing. The present results suggest that working memory processes more generally, are important in ordinal processing. Thus, researchers should evaluate both verbal and visuospatial working memory as a source of individual differences in future studies of ordinal skills.

Conclusion

In mathematics research, different tasks are often used, claiming to tap into the same underlying construct. However, rarely are both tasks administered to the same group of students to quantify their construct validity. In the present study, we show that ordering and order judgment tasks involve similar cognitive processes for children in Grade 3. Despite differences in task demands, performance on ordinal tasks is related to children’s familiarity with number sequences, knowledge of relative magnitude, and working memory skill. Moreover, ordinal skills replaced number comparison as the key predictor of arithmetic for children in Grade 3 (Lyons et al., 2014; cf. Vanbinst et al., 2016). Thus, when evaluating existing studies, readers can be confident that the two measures are tapping into similar relations with other mathematical tasks. Notably, when performance on the two ordinal tasks was used to create a latent factor, more variance in arithmetic performance was explained than when only a single measure was used. Overall, we conclude that both ordering and order judgment tasks are valid indices of ordinal knowledge, but a latent factor that excludes task-specific error may be a better index than either task separately.

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Competing Interests: The authors have declared that no competing interests exist.

Ethics Statement: We confirm that this work, which obtained ethical approval from the Carleton University Research Ethics Board, has been carried out in accordance with their ethical principles and standards.
Supplementary Materials

The Supplementary Materials contain descriptions of additional measures and stimuli for the tasks used in the present paper (for access see Index of Supplementary Materials below).

Index of Supplementary Materials


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Vanbinst, K., Ansari, D., Ghesquière, P., & De Smedt, B. (2016). Symbolic numerical magnitude processing is as important to arithmetic as phonological awareness is to reading. *PLOS ONE, 11*(3), Article e0151045. https://doi.org/10.1371/journal.pone.0151045


### Appendix

#### Table A.1

Hierarchical Linear Regression Showing Working Memory, Number Comparison and Ordering Tasks Predicting Arithmetic Performance ($n = 87$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>$B$</th>
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<th>$\beta$</th>
<th>$t$</th>
<th>$p$</th>
<th>Unique $r^2$</th>
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<td><strong>Block 1</strong></td>
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<td></td>
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<tr>
<td>Gender</td>
<td>-6.90</td>
<td>2.93</td>
<td>-.23</td>
<td>-2.35</td>
<td>.021</td>
<td>.050</td>
</tr>
<tr>
<td>Digit Span Backward</td>
<td>2.84</td>
<td>0.90</td>
<td>.33</td>
<td>3.33</td>
<td>.001</td>
<td>.094</td>
</tr>
<tr>
<td>Number Comparison</td>
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<td>-.27</td>
<td>-2.77</td>
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<td>.072</td>
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Note. Unique $r^2$ represents the squared semi-partial correlations within that specific model tested.

* $p < .05$. ** $p < .01$. *** $p < .001$.

#### Table A.2

Hierarchical Linear Regression Showing Working Memory, Number Comparison and Order Judgment Tasks Predicting Arithmetic Performance ($n = 86$)

<table>
<thead>
<tr>
<th>Variable</th>
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<td>Gender</td>
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<td>-.23</td>
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<td>.052</td>
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<td>3.24</td>
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<td>7.10</td>
<td>-.27</td>
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<td>.072</td>
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<tr>
<td>$R^2$</td>
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<td>.248</td>
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<td><strong>Total $R^2$</strong></td>
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<td>.349</td>
</tr>
</tbody>
</table>

Note. Unique $r^2$ represents the squared semi-partial correlations within that specific model tested.

* $p < .05$. ** $p < .01$. *** $p < .001$. 

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