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Comment on "On the Uniqueness of Stable Marriage Matchings" [Economic Letters 69(1):1-8, 2000]

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Abstract

We prove with the help of a counterexample that *Lemma 6* and *Corollary 7* from Eeckhout [1] are incorrect.

Theorem 1 in Eeckhout [1] provides the following sufficient condition for the existence of unique stable matchings.

Condition 1 *There exists an ordering of the set of females $\mathcal{F} = (X_i)$ and an ordering of the set of males $\mathcal{M} = (x_i)$ such that the preference profile satisfies*

$$\forall X_i \in \mathcal{F} : x_i \succ_{X_i} x_j, \forall j > i, \quad and$$
$$\forall x_i \in \mathcal{M} : X_i \succ_{x_i} X_j, \forall j > i$$

It is claimed in Eeckhout [1] that Condition 1 is also necessary for uniqueness of stable matchings when $N \leq 6$. We observe here that although the claim holds for $N = 4$ (*Lemma 5*), it is incorrect for the case $N = 6$ (*Lemma 6*).

Claim 2 *There exists a matching problem $(\mathcal{F}, \mathcal{M}, \succ)$ with $|N| = 6$ where the set of stable marriages $\mathcal{S}$ is a singleton and the preference profile does not satisfy Condition 1.*

Proof. Consider the matching problem $(\mathcal{F}, \mathcal{M}, \succ)$ with the following preference lists:

$$(a, b, c)_A, (b, a, c)_B, (a, b, c)_C (B, A, C)_a, (A, C, B)_b, (A, C, B)_c$$

The only stable matching μ^* for $(\mathcal{F}, \mathcal{M}, \succ)$ is given by $\mu^*(A, B, C) = (b, a, c)$. This is easily verified using the Gale-Shapley algorithm by observing that the male and female optimal solutions are identical. However, one can confirm that it does not satisfy Condition 1 by considering all possible orderings of $\mathcal{F}$ and $\mathcal{M}$. An easier way to see that Condition 1 is not satisfied is by observing that none of the females get their top preference in the only stable matching. Therefore, none of the females can be first in the ordering (X_i) from Condition 1. Furthermore, there also exists a ring (A, a, B, b) which, together with Lemma 2 from [1], shows that Condition 1 cannot be satisfied for this example. ■

The proof of *Lemma 6* in Eeckhout [1] argued that if Condition 1 is violated then either μ^* is not an equilibrium or there exists $\mu' \neq \mu^*$ that is also an equilibrium. The author's statement that if $b \succ_A a$ and/or $B \succ_a A$ and the remaining conditions of Condition 1 hold then $\mu'(A, B, C) = (b, a, c) \in \mathcal{S}$, is not true. Consider, for example, the following preference lists: $(b, a, c)_A, (b, c, a)_B, (a, b, c)_C; (A, B, C)_a, (B, C, A)_b, (A, C, B)_c$. Clearly, we have $b \succ_A a$ and all other conditions of Condition 1 are satisfied. However, $\mu'(A, B, C) = (b, a, c) \notin \mathcal{S}$. Indeed one can easliy check that $\{B, b\}$ blocks μ'.

Remark 3 *Since none of females get their most preferred partners in the only stable matching in the example provided above, it is clear that* Corollary 7 *in Eeckhout [1] is also incorrect.*

Acknowledgments

References

[1] Jan Eeckhout. On the uniqueness of stable marriage matchings. *Economics Letters*, 69(1):1–8, 2000.

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