



**QUEEN'S
UNIVERSITY
BELFAST**

Comment on "On the Uniqueness of Stable Marriage Matchings" [Economic Letters 69(1):1-8, 2000]

Consuegra, M. E., Kumar, R., & Narasimhan, G. (2013). Comment on "On the Uniqueness of Stable Marriage Matchings" [Economic Letters 69(1):1-8, 2000]. *Economics Letters*, 121(3), 468.
<https://doi.org/10.1016/j.econlet.2013.09.019>

Published in:
Economics Letters

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
[Link to publication record in Queen's University Belfast Research Portal](#)

Publisher rights

© 2015, Elsevier. Licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International <http://creativecommons.org/licenses/by-nc-nd/4.0/> which permits distribution and reproduction for non-commercial purposes, provided the author and source are cited.

General rights

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.

Open Access

This research has been made openly available by Queen's academics and its Open Research team. We would love to hear how access to this research benefits you. – Share your feedback with us: <http://go.qub.ac.uk/oa-feedback>

Accepted Manuscript

Comment on “On the Uniqueness of Stable Marriage Matchings”
[Economic Letters 69(1):1-8, 2000]

Mario Consuegra, Rajnish Kumar, Giri Narasimhan

PII: S0165-1765(13)00426-6
DOI: <http://dx.doi.org/10.1016/j.econlet.2013.09.019>
Reference: ECOLET 6076

To appear in: *Economics Letters*

Received date: 3 September 2013
Accepted date: 13 September 2013

Please cite this article as: Consuegra, M., Kumar, R., Narasimhan, G., Comment on “On the Uniqueness of Stable Marriage Matchings” [Economic Letters 69(1):1-8, 2000]. *Economics Letters* (2013), <http://dx.doi.org/10.1016/j.econlet.2013.09.019>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



Comment on “On the Uniqueness of Stable Marriage Matchings” [Economic Letters 69(1):1-8, 2000]

Mario Consuegra*

Rajnish Kumar†

Giri Narasimhan*‡

September 3, 2013

Abstract

We prove with the help of a counterexample that *Lemma 6* and *Corollary 7* from Eeckhout [1] are incorrect.

Theorem 1 in Eeckhout [1] provides the following sufficient condition for the existence of unique stable matchings.

Condition 1 *There exists an ordering of the set of females $\mathcal{F} = (X_i)$ and an ordering of the set of males $\mathcal{M} = (x_i)$ such that the preference profile satisfies*

$$\forall X_i \in \mathcal{F} : x_i \succ_{X_i} x_j, \forall j > i, \quad \text{and} \\ \forall x_i \in \mathcal{M} : X_i \succ_{x_i} X_j, \forall j > i$$

It is claimed in Eeckhout [1] that Condition 1 is also necessary for uniqueness of stable matchings when $N \leq 6$. We observe here that although the claim holds for $N = 4$ (*Lemma 5*), it is incorrect for the case $N = 6$ (*Lemma 6*).

Claim 2 *There exists a matching problem $(\mathcal{F}, \mathcal{M}, \succ)$ with $|N| = 6$ where the set of stable marriages \mathcal{S} is a singleton and the preference profile does not satisfy Condition 1.*

Proof. Consider the matching problem $(\mathcal{F}, \mathcal{M}, \succ)$ with the following preference lists:

$$(a, b, c)_A, (b, a, c)_B, (a, b, c)_C; (B, A, C)_a, (A, C, B)_b, (A, C, B)_c$$

The only stable matching μ^* for $(\mathcal{F}, \mathcal{M}, \succ)$ is given by $\mu^*(A, B, C) = (b, a, c)$. This is easily verified using the Gale-Shapley algorithm by observing that the male and female optimal solutions are identical. However, one can confirm that it does not satisfy Condition 1 by considering all possible orderings of \mathcal{F} and \mathcal{M} . An easier way to see that Condition 1 is not satisfied is by observing that none of the females get their top preference in the only stable matching. Therefore, none of the females can be first in the ordering (X_i) from Condition 1. Furthermore, there also exists a ring (A, a, B, b) which, together with Lemma 2 from [1], shows that Condition 1 cannot be satisfied for this example. ■

The proof of *Lemma 6* in Eeckhout [1] argued that if Condition 1 is violated then either μ^* is not an equilibrium or there exists $\mu' \neq \mu^*$ that is also an equilibrium. The author’s statement that if $b \succ_A a$ and/or $B \succ_a A$ and the remaining conditions of Condition 1 hold then $\mu'(A, B, C) = (b, a, c) \in \mathcal{S}$, is not true. Consider, for example, the following preference lists: $(b, a, c)_A, (b, c, a)_B, (a, b, c)_C; (A, B, C)_a, (B, C, A)_b, (A, C, B)_c$. Clearly, we have $b \succ_A a$ and all other conditions of Condition 1 are satisfied. However, $\mu'(A, B, C) = (b, a, c) \notin \mathcal{S}$. Indeed one can easily check that $\{B, b\}$ blocks μ' .

Remark 3 *Since none of females get their most preferred partners in the only stable matching in the example provided above, it is clear that Corollary 7 in Eeckhout [1] is also incorrect.*

Acknowledgments

The work of GN was partly supported by NSF Grant # 1018262. MEC was supported by NSF Graduate Research Fellowship DGE-1038321. We also acknowledge Jan Eeckhout’s comments on an earlier version.

References

[1] Jan Eeckhout. On the uniqueness of stable marriage matchings. *Economics Letters*, 69(1):1–8, 2000.

*School of Computing and Information Sciences, Florida International University, Miami, Florida 33199, USA

†Queen’s University Management School, Queen’s University Belfast, 185 Stranmillis Road, Belfast, UK - BT9 5EE

‡Corresponding Author: giri.narasimhan@fiu.edu