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ORIGINAL - SURVEY OR EXPOSITION

Rajiv Banker's lasting impact on data envelopment analysis

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Abstract

This paper provides a comprehensive analysis of Professor Rajiv Banker's significant impact on the field of Data Envelopment Analysis (DEA). Through an extensive review of his scholarly contributions, we explore three major clusters within DEA research: (1) Returns-to-Scale (RTS) and Most Productive Scale Size (MPSS), (2) Statistical Inference in DEA, and (3) Contextual Analysis. Banker's pioneering research has significantly advanced DEA methodologies, addressing fundamental challenges related to scale efficiency, statistical robustness, and the influence of contextual variables on performance. His work has bridged theoretical developments and practical applications, influencing diverse fields such as economics, finance, and management science. By examining citation trends and bibliometric data, we trace the evolution and enduring relevance of his contributions, highlighting key papers that have shaped the trajectory of DEA research. This paper also discusses the evolution of DEA models and approaches, including the integration of stochastic elements and second-stage analyses. In recognising Banker's lifetime dedication to DEA, we celebrate his lasting legacy and his transformative influence on both the academic community and practical implementations of DEA worldwide.

Keywords Rajiv Banker · Data envelopment analysis · Returns-to-scale · Most productive scale size · Statistical inference in DEA · Contextual analysis

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1 Introduction

Data Envelopment Analysis (DEA) serves as a foundational methodology in the field of efficiency and performance evaluation across diverse fields, including operational research, management science, economics and finance, healthcare, and education. Initially conceptualised by Charnes et al. (1978), DEA offers a powerful framework for assessing the relative efficiency of decision-making units (DMUs) that convert multiple inputs into multiple outputs. At its core, DEA operates on the principle of comparing the efficiency of DMUs based on their input–output relationships, without requiring explicit assumptions about the underlying production function. This non-parametric approach differentiates DEA from traditional econometric methods, making it particularly well-suited for situations where the functional form of the production process is unknown or difficult to specify. The versatility of DEA lies in its ability to accommodate multiple inputs and outputs simultaneously, enabling comprehensive efficiency evaluations across various dimensions. By identifying efficient DMUs that serve as benchmarks, DEA facilitates performance improvement by highlighting areas where inefficient units can potentially enhance their productivity.

Over the years, DEA has evolved from its initial formulations to encompass a wide array of methodological advancements and applications. This paper explores the enduring legacy of Banker in the field of DEA, highlighting his lifetime contributions and their impact on shaping the methodology and its applications. By delving into Banker's pioneering research and unwavering dedication to advancing DEA, we seek to provide a comprehensive overview of his significant influence on this influential field of study.

Banker's seminal contributions to DEA methodology are exemplified in his numerous publications, which have garnered widespread recognition and impact. His 1984 paper, "Some models for estimating technical and scale inefficiencies in data envelopment analysis", cited over 27,000 times according to Google Scholar (at the time of writing this paper), introduced robust models for estimating technical and scale inefficiencies, refining DEA's theoretical underpinnings. His 1989 paper, "Sensitivity, precision, and linear aggregation of signals for performance evaluation", cited over 1300 times according to Google Scholar (at the time of writing this paper), further enhanced DEA's rigour and applicability. He continued to build on this foundation with his 1993 paper, "Maximum likelihood, consistency, and data envelopment analysis: a statistical foundation", which introduced statistical foundations for the technique. Additionally, his work on advanced DEA techniques for evaluating non-financial performance measures, as reflected in his paper "An empirical investigation of an incentive plan that includes nonfinancial performance measures", expanded the scope of DEA beyond traditional financial metrics. These advancements have been instrumental in enhancing DEA's practical applications across various industries, including banking, healthcare, education, and agriculture. These highly influential papers demonstrate the significant impact of Banker's work on the field of DEA.

Beyond his methodological advancements, Banker played a pivotal role in disseminating DEA knowledge and fostering a vibrant DEA community. His co-authored textbook, "Management Accounting", remains a cornerstone of DEA education, introducing the technique to generations of aspiring analysts and researchers.

This paper provides a comprehensive tribute to Banker's lifelong contributions to DEA, exploring his pioneering research and unwavering commitment to disseminating the subject. It begins with a review of the paradigm-shifting process in the context of returns-to-scale (RTS) and most productive scale size (MPSS) in DEA (Sect. 2).

Section 3 focuses on Banker's contributions to the statistical foundation of DEA, building the hypothesis test framework and Monte Carlo simulation framework in DEA. Additionally, recent theoretical developments and potential paradigm shifts are discussed. In Sect. 4, Banker's contribution to DEA-based contextual analysis is presented. The authorship network based on Banker's academic career is generated in Sect. 5. Then, a comprehensive bibliometric analysis of Banker's scholarly contributions at both macro and micro levels is provided in Sect. 6. Finally, Sect. 7 concludes the paper.

2 Returns-to-scale and most productive scale size

Following the initial CCR-DEA model developed by Charnes et al. (1978), numerous studies have discussed the economic concept of RTS through different approaches. The bulk of the discussion is confined to qualitative characterisations of the RTS. For instance, whether the RTS is identified as increasing, decreasing, or constant. Let α denote the ratio, the proportional increase in input and proportional increase in output are represented by x, y, respectively. Thus, $\alpha = y/x$. In the conventional production theory, the types of RTS are defined based on the value of the ratio between the proportional increase in output and the proportional increase in input in a single output scenario. Specifically, when the proportional increase in inputs is larger than the proportional increase in outputs ($\alpha = y/x < 1$), RTS is decreasing. Conversely, if the proportional increase in outputs is larger than the proportional increase in inputs ($\alpha = y/x > 1$), RTS is increasing. The RTS is considers to be constant if $\alpha = y/x = 1$. The concept of RTS has been extended to multiple-output scenarios by the work of Banker (1984), Banker et al. (1984), and Banker and Thrall (1992). By investigating the RTS condition and the MPSS point of a DMU, the DEA model supports the decision-making process in policy formulation, business strategy development, and resource allocation. In general, the analysis outcomes determine macro-level strategies to maximise the productivity of a DMU. For instance, Sueyoshi and Goto (2012, 2013) evaluated the RTS conditions of the operational performance for coal-fired power plants and fossil fuel power plants in the US market. For DMUs exhibiting increasing RTS, an increase in operational size is recommended to reach the MPSS point and maximise productivity. Conversely, DMUs under decreasing RTS can further reduce their operational size to enhance productivity. In the development of RTS identification theory, studies have proposed approaches to identify RTS conditions in more complex production processes and multi-period scenarios (See Sect. 2.3.1. for details). The concept of RTS has also been extended to Damage to Scale (DTS), offering additional support for sustainability-related decision-making processes (See Sect. 2.3.2. for details).

Since the identification of RTS is highly related to the determination of MPSS, we first review the original papers on RTS and MPSS by Banker in Sects. 2.1 and 2.2. Recent progress in the identification of RTS and MPSS is discussed in Sect. 2.3. The development of the theory at the early stage by Banker is demonstrated in Fig. 1.

2.1 RTS and MPSS identification by Banker

Banker et al. (1984) indicated that CCR efficiency could be decomposed into technical efficiency and scale efficiency. For the first time in the literature, Banker (1984) extended the concept of MPSS, suggested by Frisch (1964), into DEA theory. To distinguish the concept

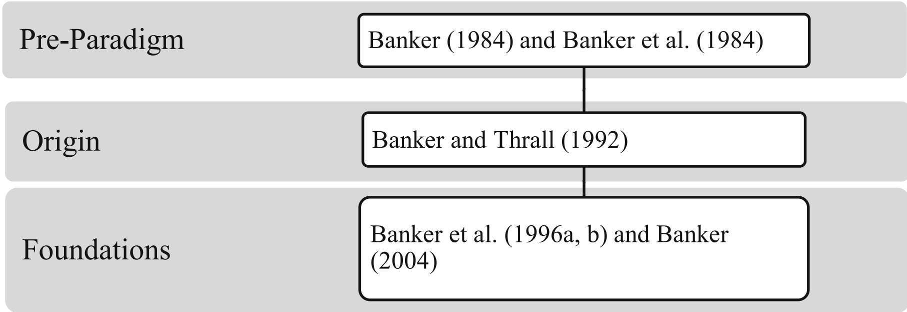

Fig. 1 Development phases of RTS and MPSS identification in DEA by Banker

of MPSS from the concept of minimum cost mix, Banker (1984) defined MPSS under DEA theory as a unit point on the efficiency frontier that maximises the average productivity for its given input–output mix and after which decreasing RTS set in. The model to investigate the MPSS point is also proposed in this paper. Additionally, Banker (1984) suggested a novel approach to identify the unique type of RTS based on the CCR model. By extending the work of Banker (1984), the model proposed by Banker et al. (1984) can be applied under the VRS assumption. Both models proposed by Banker (1984) and Banker et al. (1984) are based on the assumption that the Production Possibility Set (PPS) estimated by the DEA model only exists as a unique supporting hyperplane. However, the literature suggests that in empirical applications, the proposed models by Banker (1984) and Banker et al. (1984) cause an issue where multiple hyperplanes might occur (e.g., in Fig. 2, both points B and C are MPSS points), as seen in Charnes et al. (1986) and Seiford and Thrall (1990). We consider this period as the pre-paradigm stage (Kuhn, 1970), as the proposed approach has apparent pitfalls.

Banker and Thrall (1992) developed a rigorous framework to address the pitfalls identified in Banker (1984) and Banker et al. (1984). The proposed model allows for the possibility of multiple hyperplanes. Banker and Thrall (1992) argued that the issues can be solved by identifying the bounds of the slopes of the multiple supporting hyperplanes for points located at the boundary of the convex PPS. They further proposed a corresponding model to identify the bounds of the slope and an approach to investigate RTS and MPSS in the case of a unique hyperplane. Additionally, Banker and Thrall (1992) also suggested two general standards for RTS and MPSS investigation that have been widely applied in subsequent research:

(1) The concept of RTS is well-defined only for the points located on the boundary of the PPS. For interior points, RTS cannot be investigated since productivity changes due to RTS are confounded with productivity changes due to inefficiency elimination.
(2) The concept of MPSS is directly related to RTS. Specifically, the ray from the origin point (known as (0,0) in Fig. 2) to the boundary point intersects the PPS at two points. If the boundary point is the first of these two points, it indicates increasing RTS. Conversely, if the boundary point is farther from the origin, it indicates decreasing RTS.

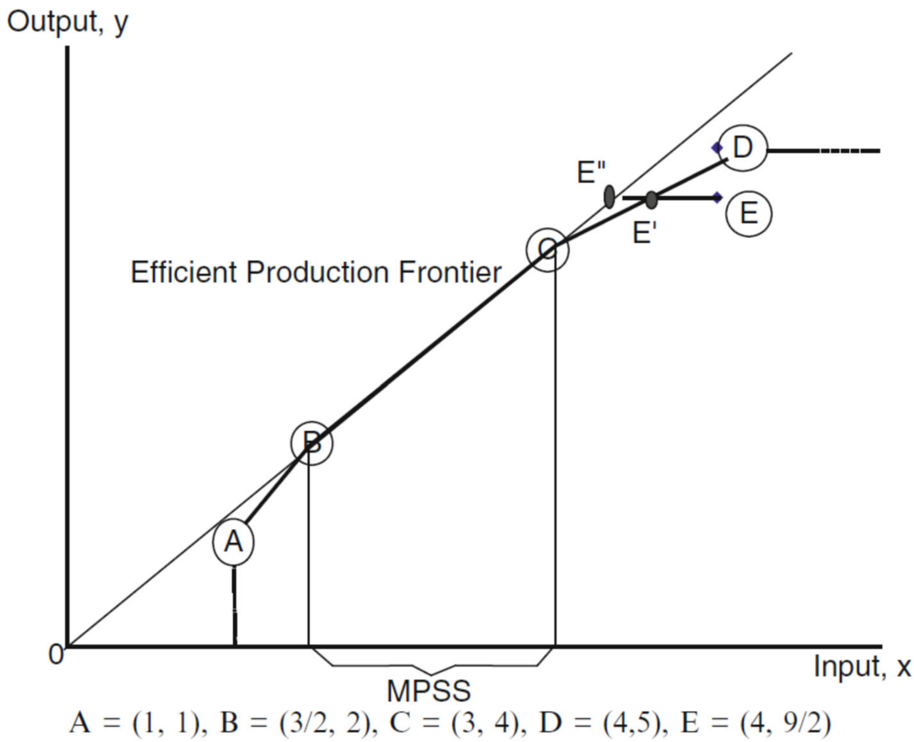

Fig. 2 PPS with two MPSS points (Banker et al., 2011)

2.2 Development of theory in RTS and MPSS identification by Banker

Numerous extended studies have been developed based on the two standards suggested by Banker and Thrall (1992). These studies primarily focus on enhancing the computation performance of the proposed model. For example, Färe et al. (1985) proposed a novel approach to identify RTS based on a group of ratios of radial measures generated from model pairs that differ in the satisfaction of convexity and sub-convexity conditions. Chang and Guh (1991) argued that Banker's (1984) and Banker et al.'s (1984) approach is invalid due to the implicit assumption of linear production functions. Specifically, they contended that Corollary 1 (which states that if the aggregate intensity weight is larger than 1, the DMU under evaluation indicates increasing RTS) suggested by Banker (1984) is invalid in the CCR model. A counterexample is presented in the Appendix of Chang and Guh (1991). In response, Banker et al. (1996b) pointed out that the model proposed by Fare et al. (1985) is equivalent to Banker and Thrall's (1992) model. By proving the equivalent relationship between the two approaches, Banker et al. (1996b) eliminated some of the assumptions underlying the theorems in Fare et al. (1985). Specifically, they argued that the assumption regarding disposability could be eliminated by assigning zero coefficients to the slack variables in the objectives of the BCC and CCR model. Banker et al. (1996a) enhanced the computational convenience of Banker and Thrall's (1992) approach by providing simpler forms for implementing the Banker–Thrall theorems. The suggested approach negates the need to investigate all alternative optima when using the BCC model.

The proposed simplified model is further extended to the CCR model by Banker et al. (1996b).

The framework proposed by Banker (1984), Banker et al. (1984), and Banker and Thrall (1992) defines RTS and MPSS within the context of DEA theory. Banker et al. (1996a, b) established the connection between the two paths to solve the RTS identification problem and simplified the computation process of the proposed model. These papers lay the foundation for RTS and MPSS identification in DEA theory. Additionally, Banker et al. (2004a, b) discussed the identification of RTS and MPSS through various models, including the input-oriented CCR and BCC models, as well as the output-oriented multiplicative model.

2.3 Revolution and progress

Several studies have been published based on the works of Banker et al. (1984, 1996a, b, 2004a, b). Recently, the literature on RTS and MPSS can be divided into two groups. The first group aims to address the research gap in Banker's work, where the RTS and MPSS identification approach is limited to conventional black-box CRS and VRS technology. The literature extends to non-radial models and further proposes more complicated production structures such as network and dynamic structures. The second group extends the definition of RTS and MPSS to related economic concepts regarding scale characterisations. Building on Banker's work, this group of literature defines economic concepts such as various marginal rates and scale elasticity and identifies them within the context of DEA theory. We review the literature regarding these two groups in this section.

2.3.1 Revolution and progress in modelling

We classify the publications regarding modelling development into three groups as shown in Fig. 3.

One of the extended models to identify RTS in a non-oriented case is Wu and An's (2013) Slack-Based Measure (SBM)-based approach. Førsund et al. (2007, 2009) suggested dealing with RTS measurement in non-radial DEA models through Strong Complementary Slackness Conditions from optimisation theory. Krivonozhko et al. (2014) proposed and substantiated an approach to decrease computational complexity in Førsund et al. (2007, 2009). Fukuyama (2000, 2003) extended Banker's literature into the Direction Distance Function (DDF) model to identify RTS and MPSS. Kerstens and Eeckaut (1999) identified RTS through the free disposal hull (FDH) model, which is linearised by Podinovski (2004) to gain a computational advantage. Soleimani-Damaneh et al. (2006) and Soleimani-Damaneh and Reshadi (2007) also suggested a polynomial-time algorithm to improve computational

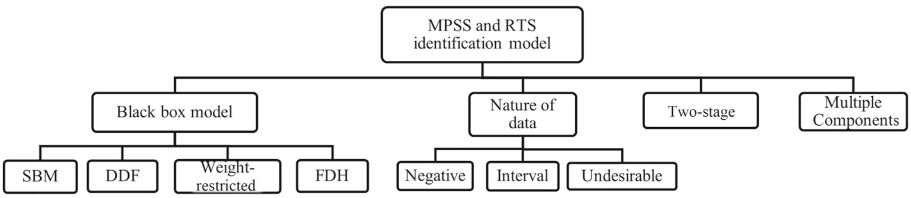

Fig. 3 Division of model development based on model structure and data types

advantage. Literature to identify MPSS and RTS in weight-restricted DEA models includes the studies by Lotfi et al. (2007), Korhonen et al. (2011), and Tone (2001).

Golany and Yu (1997) suggested a new approach to estimate the RTS in BCC model precisely by comparing the outcome of the input-oriented model and the output-oriented model. Specifically, they tested the existence of solutions in the four projection regions defined by the neighbourhood of the DMU under evaluation. The four projection regions represent the case of Decreasing Returns to Scale (DRS), Increasing Returns to Scale (IRS) CRS and technical inefficiency, respectively. This approach solves the issue in Banker and Thrall (1992) that the model is applicable only for technically efficient DMUs and fails to provide unambiguous detection of RTS. This approach was extended by Taleb et al. (2022) as a non-oriented approach to identify RTS through an integrated bi-objective data envelopment model while considering undesirable outputs. Some studies also suggest approaches to deal with negative data (Emrouznejad et al. 2010; Sahoo et al., 2016), ratio data (Olesen et al., 2022; Emrouznejad and Amin, 2009), and interval data (Hatami-Marbini et al., 2014).

The aforementioned models are based on conventional black-box structures. Theoretical extensions have been introduced to account for more complex production processes. Lozano and Villa (2010) and Singh and Ranjan (2018) proposed a model to identify the RTS condition and MPSS point in parallel production systems. Since there are no intermediates in the parallel system, the proposed approach is similar to models based on the black-box system. To account for the internal production process, Lozano (2011) proposed the theoretical foundation for identifying technical, cost, and allocative efficiency, as well as local RTS, in a two-stage system. Zhang and Yang (2015) suggested an approach for identifying the RTS of a two-stage production process through a modified two-stage envelopment model. Assani et al. (2018) identified the MPSS point for a two-stage DEA model. However, there is still a significant research gap in RTS identification for more complex network production structures, such as parallel, series, and general systems. Most network RTS identification approaches are based on the two-stage model, which describes the production process where all inputs are transformed into intermediates and subsequently used to produce outputs at the next stage. However, as noted by Khezrimotlagh and Zhu (2023), the input–output PPS in a two-stage system is generated by integrating the input-intermediate PPS and intermediate-output PPS. Consequently, the input–output PPS in a two-stage system may include exterior projection points, and the aforementioned models require further investigation to address the issue.

The theory of the RTS and MPSS identification under the cross-sectional analysis has also been developed based on the work of Banker. Sueyoshi and Sekitani (2005) suggested a model to address the RTS identification problem in a dynamic production system. For a given time period t, the carryover from period $t - 1$ is treated as an input while the carryover to period t is treated as an output. Based on the above standard, the MPSS and RTS for period t can be identified using a similar approach suggested by Sueyoshi (1999). Sueyoshi's (1999) proposed approach builds on the cost-efficiency scheme model of Banker et al. (1984). Consequently, a multi-stage computation process is needed to identify all the supported hyperplanes. In response, Soleimani-Damaneh (2013) suggested a novel approach to identify RTS in dynamic systems using the envelopment form of the CRS model. The discussion regarding the identification of the MPSS point in Malmquist index analysis can be found in Podinovski et al. (2017). The model suggested by Assani et al. (2018) is capable of identifying the MPSS of multi-period parallel production processes. Kontolaimou and Tsekouras (2010) addressed the measurement issues caused by the identification of MPSS in meta-frontier models. They theoretically defined the MPSS point in a meta-frontier model as the point that is technically efficient across all technologies.

Podinovski (2022) extended the concepts of CRS and VRS to Multiple Component CRS (MCRS) and Multiple Component VRS (MVRS) and suggested a corresponding approach to identify the Scale Elasticity and RTS. Specifically, Podinovski (2022) defined multiple component technologies as production technologies where each process uses its specific inputs and an unknown portion of shared inputs to produce its specific outputs and an unknown portion of shared outputs. The main difference between the two Multiple Component technologies lies in their assumptions: the MVRS model treats each component process as a separate convex technology, whereas the MCRS model additionally assumes that each process is scalable. Building on the work of Banker and Thrall (1992) and Podinovski (2017), Podinovski (2022) stated that RTS situations can be characterised based on the corresponding one-sided scale elasticities. A DMU under evaluation is characterised by increasing RTS if its corresponding right-hand scale elasticities are smaller than 1. Similarly, the DMU under evaluation exhibits decreasing RTS if its left-hand scale elasticities are smaller than 1. A DMU exhibits CRS if and only if it is located at the MPPS point of the current technology. Papaioannou and Podinovski (2023) further extended the work of Podinovski (2022) to scenarios with restricted allocations of shared inputs and outputs.

2.3.2 Revolution and progress in scale characterisations

Rather than focusing solely on performance evaluation, the conventional DEA methodology can be extended to address several other tasks important for decision and policymaking. Most of the literature in this group extends the concepts of RTS and MPSS within the DEA model. In this section, we review recent studies that propose and define new scale characterisations. Figure 4 illustrates the division of the new concepts.

Early DEA theory primarily targeted identifying the specific types of RTS for a given DMU. However, in the theory of economics of efficient production, the term RTS is quantified as the specific value of RTS, known as Scale Elasticity (Fukuyama, 2000). Banker et al. (1984) and Banker and Thrall (1992) suggested a simple non-parametric approach for calculating the scale elasticity in terms of the dual optimal solutions to the CCR and BCC models. Førsund et al. (2007, 2009) stated that the starting point of the DEA approach is the specification of a piecewise-linear production technology, rather than a general production technology. Specifically, the assumption of smoothness and continuous differentiability of the production frontier does not hold in DEA theory because the production frontier generated by the DEA model is piecewise linear. Taking this difference into account, they categorised approaches for calculating scale elasticity into two groups: (1) the indirect approach and (2) the direct approach.

The indirect approach estimates scale and substitution properties through the non-parametric frontier functions in DEA, producing a simple formula for scale elasticity in terms of the dual optimal solutions to the VRS model (Podinovski et al., 2009). For instance,

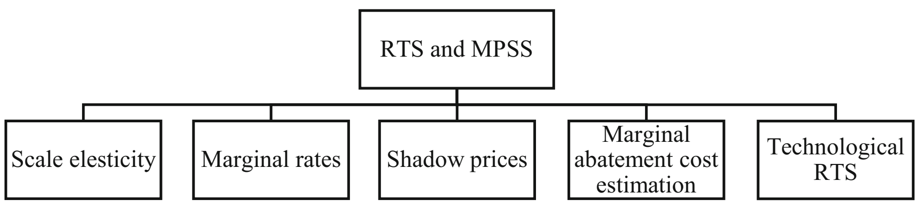

Fig. 4 Division of scale characterisations

Førsund and Hjalmarsson (2004) calculated the value of scale elasticity for radial projections of inefficient points to the frontier based on efficiency scores and shadow values on the convexity constraint. A similar approach was applied by Førsund et al. (2007) to estimate the scale elasticity of inefficient units and their corresponding projections on the frontier. Specifically, they calculated the maximum and minimum value of the shadow price on the convexity constraint and defined the bound of scale elasticity values accordingly. Podinovski et al. (2009) simplified the formula suggested by Førsund and Hjalmarsson (2004) and stated that one-sided elasticities could be viewed as directional derivatives of the optimal value function in a modified envelopment DEA model. This approach extends the scale elasticity investigation to the most general case, dealing with multiple optimal solutions, degeneracy, and non-full dimensional facets.

The direct approach, on the other hand, investigates scale and substitution properties through parametric neoclassical production functions. For instance, Krivonozhko et al. (2004) argued that Banker's approach is limited to investigating partial derivatives (marginal rate of substitution, marginal rate of transformation, marginal product) of those DMUs that are interior to the production frontier. They suggested a parametric approach to calculate partial derivatives at any point on the frontier. Førsund et al. (2007, 2009) directly evaluated numerical scale elasticity on the DEA surface along intersections with planes, applying the parametric optimisation approach to construct the boundary of the VRS production technology. The scale elasticity value is then calculated based on the corresponding production technology boundary. Following the work of Podinovski et al. (2009), Podinovski and Førsund (2010) first calculated a class of mixed partial elasticity measures in both VRS and CRS technology. They proposed two approaches for elasticity measurement: one addressing the instability problem discussed in Cooper et al. (2000) through specially constructed dual linear programmes, and the other based on the standard multiplier model, which increases computational burden but offers interpretative advantages. Zelenyuk (2013) calculated the scale elasticity for multiple inputs and outputs technology based on the DDF model, demonstrating equivalence to input-oriented and output-oriented scale elasticity measures. The DDF-based elasticity measure and the profit function-based elasticity measure also indicated a dual relationship. Kao and Hwang (2011) and Sahoo et al. (2014) decomposed scaly elasticity in a two-stage multiplier DEA model.

Studies have also investigated the productive potential of efficiently operating units in response to particular changes in their input and output profiles by introducing marginal characterizations. Banker and Maindiratta (1986) first calculated the rates of substitution transformation. Bessent et al. (1988) argued that Banker and Maindiratta's (1986) approach failed to address the issue caused by facet interfaces. For the DMU that has a mix of resources or outputs which are different from any frontier point, they suggested a constrained facet analysis approach to consider the lower bound of efficiency for organisational units. The aforementioned approaches aim to estimate the marginal rates of substitution in a scenario with infinitesimal, or small finite, changes in one or more variables. Asmild et al. (2006) argued that the change might be larger in realistic applications. Therefore, they proposed models to estimate the impact of larger non-marginal changes, scalar changes, and additive changes in DEA by extending Rosen et al.'s (1998) approach. Charnes et al. (1996) developed an empirical robust efficient production function and measured the marginal trade-offs of the efficient production function. Cooper et al. (2000) analysed marginal trade-offs through intermediate variables. More recent studies, such as Podinovski and Førsund (2010), estimated the marginal rates of response exhibited by a single input (output) with respect to output (input) under standard VRS and CRS technologies. Podinovski et al. (2016) subsequently suggested a more general framework for calculating the marginal rates of substitution in non-parametric production frontiers.

The concept of shadow price has been discussed in numerous studies. From a computation and efficiency analysis perspective, shadow price stems from using the Lagrangian technique to solve optimisation problems with constraints (Førsund, 2018). Recent literature has targeted extending the concept of shadow price of the undesirable output from an economic perspective, aiming to calculate the marginal abatement cost of a particular undesirable output. For instance, Leleu (2013) referred to the work of Färe et al. (1993, 1994) and calculated the shadow pricing of undesirable outputs while considering the implications of weak disposability. The proposed hybrid model considers the trade-off between modelling the jointness of good and bad output and the expectation of getting the shadow price of bad output. A literature review on calculating the price of undesirable output can be found in Zhou et al. (2014). Kao and Hwang (2019) investigated the shadow price of undesirable output in both the multiplier scheme model and the envelopment scheme model, achieving the highest possible measured efficiency score. Podinovski (2019) estimated the marginal characteristics of non-parametric production frontiers in the presence of undesirable outputs. Wang et al. (2016) calculated the potential gains from carbon emissions abatement cost savings through a special dynamic DEA model. Lee and Wang (2019) measured the marginal abatement cost estimation of air pollutants under Nash equilibrium. Wu et al. (2023) investigated different abatement options to calculate the marginal abatement cost of CO_2 emissions and introduced the concept of expository derivation to calculate the marginal gains caused by energy switching.

Beyond individual DMUs, the literature has also contributed to measuring scale characterisations at the group level. Banker (1993, 1996) suggested a hypothesis to investigate the type of RTS for a given technology. As discussed in Banker (1984) and Banker et al. (1984), deviations from the MPSS point determine the scale inefficiency within the extent of operations. Consequently, Banker (1993, 1996) suggested using no scale inefficiency as the null hypothesis, indicating that the production technology satisfies CRS. The starting point in Banker (1993, 1996) is to calculate inefficiency scores and the corresponding scale efficiency. The null hypothesis assumes no scale inefficiency, meaning the sample data could be rationalised by a production set exhibiting constant RTS. Based on the distribution of true inefficiency (exponentially distributed, half-normally distributed, unknown distribution), the test statistics were calculated using different formulas. Banker (1996) also developed a framework to test the null hypothesis of non-DRS against the alternative of DRS and the null hypothesis of non-IRS against the alternative of IRS. This framework is similar to the CRS test. The first step is to estimate inefficiency under IRS (DRS) by setting the sum of intensity weight to be less (or greater) than 1. The estimated inefficiency is then compared to true efficiency using the corresponding test statistic. Simar and Wilson (2002) extended Banker's (1993, 1996) work by introducing bootstrap estimation procedures, providing appropriate critical values for the test statistics. However, Alirezaee et al. (2018) argued that the statistical assumptions underlying Banker (1993, 1996) are difficult to satisfy, and decision-makers may overlook certain factors or pre-judge the behaviour of the technology in realistic applications. In response, Alirezaee et al. (2018) introduced the concept of technological RTS and suggested a non-statistical approach to identify technological RTS (TRTS) based on the Angles method. Specifically, a measure was introduced to calculate the gap between the constant TRTS hyperplane and the variable TRTS in both the increasing and the decreasing sections of the frontier.

3 Advancements in the statistical foundations of DEA: contributions, frameworks, and recent developments

Advancements in the statistical foundations of DEA, pioneered by Banker, can be categorised into two groups. The first focuses on hypothesis testing based on the DEA model, while the second addresses the foundation and structure of simulation test design in DEA. In summary, the original work by Banker on the statistical foundations of DEA establishes the research paradigm for hypothesis tests and simulations tests under the DEA framework. Banker's hypothesis tests provide the foundation for constructing a wide range of formal statistical tests for DEA models, including technical change, the impact of contextual variables, types of RTS, and input separability. On the other hand, simulation tests under the DEA framework suggest a general approach for selecting suitable analysis models. These simulation tests are also widely applied to assess the performance of nonparametric analysis models. More recent studies have extended Banker's approach to testing the statistical foundations of other nonparametric analysis models. The statistical inference developed by Banker also provides the theoretical foundation for central limit theory (under the DEA model), stochastic DEA analysis, and nonparametric quantile frontier estimation. These models offer more accurate estimations of shadow prices and greater robustness in selecting directional vectors. Furthermore, studies propose the partial frontier approach based on the DEA model's statistical inference. Another group of studies suggests the quantile regression approach, which is considered an extension of Banker et al. (1991). These models provide a more suitable approach for addressing random noise, heteroscedasticity and outliers.

We review the original literature by Banker in Sect. 3.1. In Sect. 3.2, we present the theoretical extension by Banker based on the discussion in Sect. 3.1. Recent progress in simulation tests, hypothesis tests, and statistical inference of DEA and its extensions are reviewed in Sect. 3.3. We also present some more recent extended models, based on statistical theory, to address data noise in Sect. 3.4.

3.1 The origin of statistical foundations of DEA by Banker

The revolution process of the statistical foundation of DEA can be divided into three phases: Pre-Paradigm, Origin and Development. This section reviews the literature that represents these three phases by Banker. The process is visualised in Fig. 5.

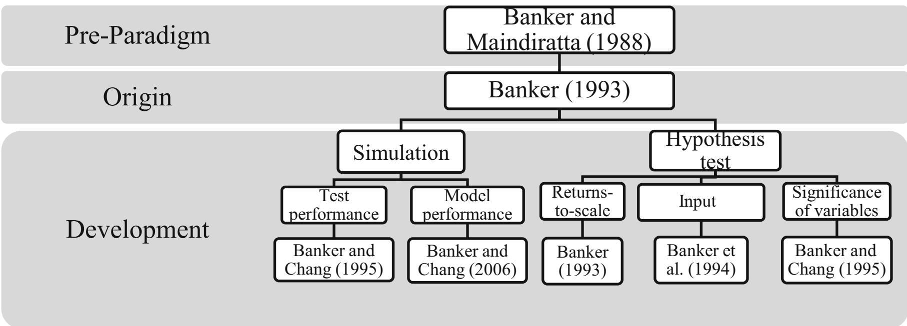

Fig. 5 Revolution phases of Statistics in DEA by Banker

The DEA model is a non-parametric approach grounded in the general axioms of production theory, such as monotonicity, convexity, and homogeneity (Banker et al., 1984). In the 1980s, research on statistics in DEA was in what Kuhn (1970) described as the pre-paradigm stage. Early DEA literature largely ignored discussions of the statistical inference of the generated efficient frontier. Even the work of Banker and Maindiratta (1988) merely established a link between the DEA model and Varian's (1984) algebraic test for consistency with the weak axiom of cost minimisation, measuring whether the technical, allocative, or scale efficiencies of DMUs were low. Thus, during this period, DEA was considered a non-statistical approach (Banker, 1996; Gong and Sickles, 1992; Schmidt, 1985). However, Schmidt (1985) criticised this non-statistical measurement approach, expressing skepticism about its lack of a corresponding measurement foundation. He argued that, because the non-statistical, non-parametric method does not account for error, the literature should be approached cautiously, particularly when the data under evaluation are assumed to be free from statistical noise.

In response to Schmidt's critique, Banker (1993) discussed and established the statistical basis for DEA. Banker (1993) identified four postulates regarding the production set and the probability density function of the "true inefficiency". These postulates include monotonicity, convexity, envelopment, and the likelihood of efficient performance. These four postulates form the foundation for simulation-based testing studies (Banker, 1993, 1996; Banker and Chang, 1995; Banker et al., 1994).

By solving the BCC models to estimate the inefficiency scores, the estimated efficiency scores are compared to the "true inefficiency" generated based on these postulates. The findings suggest that the BCC production frontier aligns with a monotone decreasing probability density function, generated based on the deviation of the outputs under evaluation from the efficient output levels. Additionally, the estimated inefficiency maximises the likelihood function if the inefficiency probability density function exhibits a monotone decreasing trend.

Banker (1993) also proposed three hypotheses to test differences in inefficiency between two different types of DMUs, assessing whether one group is more efficient than another. These three tests can accommodate different kinds of distributions (exponentially distributed, half-normally distributed, unknown distribution) of inefficiency. The proposed approach lays the theoretical foundation for future literature, establishing a formal statistical foundation for DEA inefficiency estimators and suggesting a general framework for statistical hypothesis testing regarding differences in inefficiency distributions in DEA.

3.2 Development of statistical-based extension approaches in DEA by Banker

Based on the work of Banker (1993), two main statistical-based extension approaches have been developed in the field of DEA. The first approach focuses on further developing the statistical foundation of DEA and proposing adjusted hypothesis testing approaches to address potential issues caused by statistical bias. The second approach builds on the simulation approach used by Banker (1993) and aims to test the performance of specific models. This section reviews these two approaches and the related literature by Banker.

3.2.1 Foundations of simulation in DEA by Banker

Banker (1993) laid the groundwork for the simulation-based testing framework in DEA. The simulation testing procedure involves comparing the efficiency score estimated by DEA with the "true efficiency" generated by a production technology that satisfies the four postulates defined by Banker (1993). The critical factors in these simulations are the characteristics of the production technology, the distribution of inefficiency, and the sample size (Banker, 1996).

In almost all of Banker's simulation studies, the designs are aimed at evaluating the performance of a model or test by comparing the estimated efficiency scores to the "true efficiency" through Monte Carlo simulations. The "true inefficiency" is typically generated using a Cobb–Douglas production function in a single-output scenario, with the values of "true efficiencies" distributed in a specific pattern. For instance, Banker and Chang (1995) assessed the performance of asymptotic DEA tests for differences in inefficiency, as presented in Banker (1993). These tests' performance was compared with conventional parametric corrected ordinary least squares (COLS) tests and the Welch and Mann–Whitney tests. The results indicated that the asymptotic DEA tests outperformed COLS-based tests and Mann–Whitney tests. The proposed simulation framework is also utilised to evaluate the accuracy of efficiency estimation in the sup-efficiency DEA model (Banker and Chang, 2006). The findings show that the super-efficiency model is outperformed by the conventional DEA model in identifying data contaminated with outliers and removing the corresponding outliers.

3.2.2 Foundations of hypothesis testing in DEA by Banker

Since the publication of Banker (1993), the literature on the statistical foundation of DEA has evolved significantly. A critical characteristic of production frontiers is the presence of increasing or decreasing RTS. As reviewed in Sect. 2.3, Banker (1996) developed a framework to test RTS at the technology level.

Another hypothesis proposed by Banker et al. (1994) aimed to test the independence of estimations and inputs. Specifically, the separability of inputs—the assumption that inputs can be optimised independently without being substitutable for others—is a common assumption in the DEA model. However, in practical applications, two inputs (such as labour and capital) might be substitutes, and the assumption regarding substitutability or separability is not tested in conventional DEA models. To test this hypothesis over an entire dataset, Banker et al. (1994) provided a connection between the conventional DEA model and Shephard's (1970) distance function. The perfect separable inefficiency is then estimated as the reciprocal of Shephard's (1970) distance measure. They stated that if inputs are separable, in a group of DMUs with I inputs, the input inefficiency estimated by a particular input is unique. Specifically, let $\theta_{ij}(i = 1, \ldots, I)$ be the input inefficiency of DMU_j generated by only input j; we have $\theta_{ij} \neq \theta_{(i+1)j}, \forall i, j$. Therefore, the null hypothesis of input separability can be treated as the situation where $\theta_{ij} \neq \theta_{(i+1)j}, \forall i, j$. Similar to Banker (1993), Banker (1994) also suggested using three different test statistics based on the distribution of true inefficiency.

Banker and Chang's (1995) study was the first to propose a specification testing approach (referred to as "variable selection" in recent studies) to test the significance of variables at the margin in characterising the production process. The null hypothesis posits that extra variables do not affect production correspondence. Specifically, to address the dimensionality issue, the inefficiency score without extra variables θ_o should be larger than the generated inefficiency score θ_o^{extra} generated by the model that includes the extra variables. Three different test statistics were also suggested based on the true efficiency distribution.

3.3 Progress in the statistical foundations of DEA and its extensions

Banker's work has established the research paradigm for statistical testing and simulation testing in DEA. Section 3.3.1 reviews progress in simulation design within DEA, highlighting how Banker's simulation framework has been applied to evaluate the performance of models designed to assess more complex production processes. Section 3.3.2 examines extensions in the statistical testing under the DEA framework. In addition to broadening the scope of statistical testing, studies propose methods to address data noise during both the testing and data-generating stages. Section 3.3.3 discusses recent developments in the proof of statistical inference for extended DEA models. The section also presents some notable stochastic DEA models, which are grounded in the statistical axioms established by Banker.

3.3.1 Progress in simulation design in DEA

Several simulation tests have been designed since Banker (1993), and most of them share the same motivation and aim to examine the satisfaction of a DEA model. The simulation design is similar to Banker's (1993) work, which compares estimated DEA inefficiency to the generated "true inefficiency". As illustrated in Fig. 6, most theoretical extensions target the production process by introducing more factors, such as multiple outputs, undesirable outputs, nondiscretionary variables, or proposing a more complex network production environment.

As a single-output simulation experiment, Bardhan et al. (1998) and Cooper and Tone (1997) compare the performance of DEA, DEA-regression combinations, and stochastic frontier regression approaches under different sample sizes. Ruggiero (1996, 1998) designed the simulation experiments to test the performance of DEA models that consider non-discretionary variables. Giraleas et al. (2012) compared the performance of DEA, COLS, and Stochastic Frontier Analysis (SFA) when generating the frontier based on panel data. The performance of conventional DEA, SFA, and the stochastic DEA model (Resti, 2000; Simar, 2007), and the accuracy of technical efficiency and scale efficiency measured by the DEA model (Perelman and Santín, 2009) are also compared by using Banker's (1993) simulation framework. More examples can be found in Khezrimotlagh (2022), which reviews the current literature on existing simulation experiments.

Banker's (1993) simulation test is based on a conventional Cobb–Douglas production function in a single-output case. However, in realistic applications, DEA models are likely to deal with multiple-output scenarios. Therefore, some studies aim to build simulation experiments by using the production function with multiple inputs and multiple outputs. To generate the "true inefficiency" in multiple-output production technology, the extended Cobb–Douglas production function is applied (Wang et al., 2020). Another production technology based on the quadratic frontier was proposed by Simar (2007). The two-input and two-output translog production function was proposed and applied by Perelman and Santín (2009). Fernandez

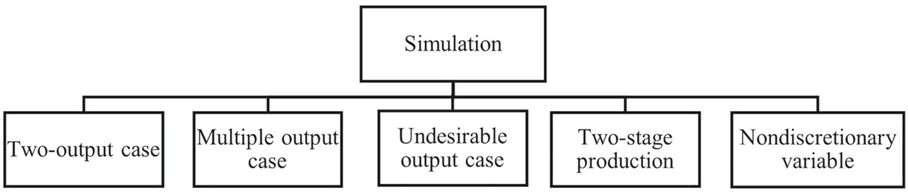

Fig. 6 Recent progress in simulation design in DEA

et al. (2002) used two different production functions to estimate desirable and undesirable outputs separately. This approach was further developed by Chen and Delmas (2012) to generate a multiple-output production function while considering the weak disposability of the undesirable outputs.

To account for the internal production process, the "true inefficiency" is generated by a network production frontier (Michali et al., 2023). Khezrimotlagh (2022) also suggested a general approach for constructing the network production frontier to evaluate the performance of the network DEA model. The core of this approach is to generate the "true inefficiency" and original output level, ensuring that the value of "true inefficiency" and the original output level satisfy a specific distribution. The original output level serves as the output projection or output benchmark for the corresponding DMU. Therefore, the output volume of the DMU is calculated by multiplying the original output level by the corresponding "true efficiency", which is the reciprocal of "true inefficiency". In most cases, the tested models are radial DEA models that are input (output) oriented. A general approach for handling non-radial DEA was proposed by Khezrimotlagh (2022). However, its applicability needs further discussion since it assumes input (output) inefficiency as an aggregation rather than a group of non-radial slacks. The generated non-radial "true inefficiency" is the same as radial "true inefficiency" as it represents the proportional, radial contraction of inputs or expansion of outputs rather than input excesses and/or output shortfalls between benchmarks (Halická and Trnovská, 2021). For instance, in cases where the performance of SBM needs to be tested, the framework suggested by Khezrimotlagh (2022) fails to provide information regarding the "true inefficiency" of a specific variable but instead gives the aggregate "true inefficiency". Moreover, assuming the same level of inefficiency for inputs and outputs might lead to contradictions in realistic applications. Furthermore, there is a research gap in testing the performance of the network non-radial DEA model (i.e., network SBM model) by using the simulation approach suggested by Banker.

3.3.2 Progress and revolution in hypothesis tests in DEA

Banker (1993) provided a formal statistical basis for DEA estimation techniques, which paved the way for the development of various hypothesis tests. As illustrated in Fig. 7, beyond expanding the research scope to test group performance of DMUs or extending the theory proposed by Banker et al. (1996a, b), several studies have also focused on incorporating independent noise terms and addressing issues like missing data or panel data (hypothesis tests for productivity change and allocative efficiency).

In recent research, new estimation and statistical tests have been developed to handle longitudinal and cross-sectional data. The purpose of these studies is to investigate the time-period shifting of the production frontier generated by the DEA model from a statistical perspective. Banker et al. (2004a, b) developed statistical tests for the null hypothesis of no allocative inefficiency by introducing a single aggregate cost variable to represent aggregate

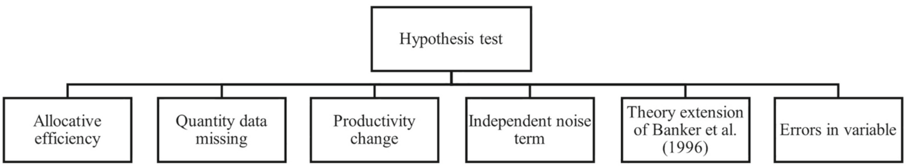

Fig. 7 Recent progress in hypothesis tests in DEA

technical and allocative inefficiency. This approach allows for the analysis of efficiency trends over time and consider the differences across subgroups within a panel dataset. In scenarios involving asymmetric information, output quantity data may not be available, but the monetary value of individual outputs and input quantity information might be. Banker et al. (2007) extended the work of Banker et al. (2004a, b) to create a framework for testing output-based allocative inefficiency while accounting for missing output quantity data. Banker et al. (2005) proposed three null hypotheses to test efficiency changes and applied these tests to the public accounting industry as an empirical case. Compared with the conventional Malmquist index approach, their proposed approach provides statistical properties of the components of productivity change and derives statistical tests based on these estimators.

The hypothesis tests proposed by Banker et al. (1996a, b) have been developed and applied in various areas. For instance, Alperovych et al. (2015) applied the hypothesis test suggested by Banker et al. (1996a, b) to test the RTS assumption related to firm performance. Banker et al. (2010a, b) used Banker et al.'s (1993) approach to examine changes in productivity within the Korean banking system following a crisis. Simar and Wilson (2002) extended Banker et al. (1996a, b)'s work to test hypotheses regarding RTS in non-parametric models of technical efficiency. Pastor et al. (2002) proposed a test for Nested Radial DEA Models by extending the variable selection hypothesis tests suggested by Banker et al. (1996a, b). This hypothesis test aims to analyse the marginal role of a given variable concerning efficiency measured by a DEA model. Monte Carlo simulation proved the performance of the proposed test to be superior to that of Banker et al. (1996a, b). Sueyoshi and Aoki (2001) proposed a non-parametric statistical approach to test productivity changes based on the output of DEA window analysis and suggested the corresponding Kruskal–Wallis rank test. Pastor et al. (1999) extended Banker's (1993) hypothesis framework and proposed a statistical test for detecting influential observations. Kuosmanen et al. (2007) suggested a hypothesis test to detect errors-in-variables, as mentioned in Varian (1985).

In the literature mentioned above, the data-generating process typically treats stochastic inefficiency as the sole cause of deviations of actual output from the production frontier. However, several studies have also suggested considering the influence of an independent noise term in the data-generating process. Banker et al. (2002) is the first to propose the estimation of an extremal function incorporating both a one-sided efficiency term and a two-sided noise term. This function is applied to evaluate the adequacy of parametric functional forms through four statistical tests: Kolmogorov–Smirnov test, the Rank-regression test, the Wilcoxon rank-sum test, and Theil's distribution-free test. Banker and Natarajan (2008) treated the error term as an integration of three distinct components: a linear function of multiple, possibly correlated, contextual variables. Their paper also established a general framework using DEA at the first stage and OLS or ML at the second stage to estimate individual inefficiency conditional on the value of the composed error term. Banker et al. (2010a, b) introduced a two-sided random noise term into the data-generating process and proposed five hypotheses to compare the performance of different groups of DMUs. The proposed approach was shown to perform better than Banker (1993) when noise levels were significant, as demonstrated through Monte Carlo simulation. Further, Simar and Wilson (2007) argued that previous literature failed to describe a coherent data-generating process. In response, they proposed bootstrap procedures that permit valid inference and improve statistical efficiency in second-stage regression.

3.3.3 Developments in statistical inference of DEA and its application

Ever since Banker et al. (1993), the literature has explored statistical inference in DEA and its extensions to more general cases. Grosskopf (1996) surveyed the statistical inference of the DEA model and the FDH model. This paper also reviewed nonparametric regularity tests, sensitivity analysis, two-stage analysis with regression, and nonparametric statistical tests. Similar surveys can be found in Simar and Wilson (2000b, 2007). In more recent work, Daouia and Gijbels (2011) defined and proved the statistical inference of nonparametric (DEA, FDH) partial frontiers.

The statistical inference testing approach proposed by Banker et al. (1993) has also been extended to multi-period analysis. Odeck (2009) applied a bootstrapping method to ensure the statistical inference of the Malmquist index measure. Tortosa-Ausina et al. (2008) examined the sensitivity of the Malmquist index measure measured. Kneip et al. (2021) tested the statistical inference of productivity change (both individual and mean productivity changes for a given technology) measured by Malmquist indices. Pham et al. (2024) extended Kneip et al.'s (2021) work by proposing a novel Malmquist Productivity Index. This new index reflects the relative importance of individuals, and its statistical inference is also proven.

Extending the work of Banker et al. (1993), Korostelev et al. (1995a, b) discussed the consistency and convergence speed of DEA and FDH estimators. This approach is further extended to Banker and Maindiratta's (1988) multiplicative DEA model by Kneip et al. (1998). Tsionas and Philippas (2023) applied Bayesian techniques to test the global sensitivity of the DEA model. Alongside the development of the DEA model, the framework by Banker et al. (1993) has also been applied to test the statistical inference of extended DEA models, including the DDF model (Simar et al., 2012), the Russell measure (Badunenko and Mozharovskyi, 2020), the general technique and allocative measures (Simar and Wilson, 2020; Simar et al., 2024a), and the Hicks–Moorsteen productivity indices (Simar et al., 2024b).

Beyond the direct extensions of Banker et al. (1993), the statistical underpinnings of DEA by Banker et al. (1993) provide the theoretical foundations for the central limit theory (Kneip et al., 2015, 2016; Nguyen et al., 2022; Simar and Zelenyuk, 2020). Moreover, the statistical axioms established by Banker (1993) have been used to suggest stochastic models for addressing data noise. By considering the data noise while building the production frontier, these stochastic models are able to minimise the risk of underestimation and improve the precision of efficiency estimation (Tsionas, 2021). According to Banker et al. (1996a, b), conventional DEA models ignore the data noise and the axioms on the distribution of deviations from a best practice frontier. On the other hand, the SFA approach has been criticised for its restrictive functional form assumptions, particularly in the context of joint production (Kuosmanen and Johnson, 2017). Several models have been developed to improve robustness against data errors and outliers and to incorporate probabilistic settings in SFA theory, drawing on the statistical axioms established by Banker (1993) and Simar and Wilson (2000a, b). Olesen and Petersen (2016) categorised stochastic methodologies into three directions:

1. Treating estimated inefficiencies as random deviations.
2. Developing models to account for either measurement errors or specification errors.
3. Developing models to generate random Production Possibility Sets based on random variations in datasets.

The existing Stochastic DEA models can be divided into three groups based on these directions:

- Group 1: Extends the first direction and includes the bootstrapping DEA model (Aggelopoulos and Georgopoulos, 2017; Bobde and Tanaka, 2018; Boubaker et al., 2023; Dia et al., 2022; Du et al., 2018; Kang et al., 2024; Michali et al., 2023; Staat, 2002; Simar and Wilson, 1998, 1999, 2000a, b; Moradi-Motlagh and Emrouznejad, 2022).
- Group 2: Extends both the first and second directions and includes the semi-parametric DEA model (Assaf and Gillen, 2012; Johnson and McGinnis, 2008; Jradi and Ruggiero, 2019; Kuosmanen and Kortelainen, 2012; Simar and Wilson, 2007).
- Group 3: Extends the second and third directions and includes Chance Constrained DEA models (Amirteimoori et al., 2023; Cooper et al., 1996, 1998, 2002; Lin and Lu, 2023; Mitropoulos et al., 2015; Shiraz et al., 2020; Talluri et al., 2006).

A more detailed review of stochastic DEA can be found in Olesen and Petersen (2016).

3.4 Progress and advances in nonparametric quantile frontier estimation

The aforementioned models in Sect. 3.3.3 address the inherent uncertainty in data and enhance robustness. It is important to note that these models are based on statistical axioms rather than directly addressing stochastic noise in the observed data. However, due to the application of the minimal extrapolation principle that minimises the deviation to the production function in the dataset, several studies (Esteve et al., 2020; Tsionas, 2022) argue that DEA suffers from an overfitting problem. To address these deficiencies, newly proposed approaches focus on estimating quantile frontiers instead of full frontiers that envelop all observations. These models, known as nonparametric quantile frontier estimation, provide more accurate estimations of shadow prices and are more robust in selecting directional vectors. Additionally, these models can address various types of data noise, such as random noise, heteroscedasticity, and outliers (Dai et al., 2022; Kuosmanen and Zhou, 2021; Liao et al., 2024). According to Dai et al. (2022), existing nonparametric quantile frontier estimation methods can be categorised into two groups: (1) Partial frontier and (2) Quantile regression. The partial frontier approach generates a frontier estimator that fits a subset of observations (Dai et al., 2022). In contrast, the quantile regression approach constructs the frontier by applying an asymmetric norm to the full sample of observations (Dai et al., 2022; Koenker and Bassett, 1978). In this section, we review several representative studies on nonparametric quantile frontier estimation.

3.4.1 Partial frontier approach

The partial frontier approach generates a frontier that fits a subset of the observations. The algorithm for generating such a frontier follows the conventional DEA analysis paradigm, which maximises the distance between the DMU under evaluation and its associated projection point. Additionally, inspired by the research paradigm established by Banker (1993), the statistical theory underlying the partial frontier approach has been rigorously proven.

Cazals et al. (2002) initially suggested the order-α approach to generate the FDH frontier while addressing outliers. Building on the work of Cazals et al. (2002), Aragon et al. (2005) proposed the FDH quantile frontier approach of order-m. In subsequent studies, Daouia and Simar (2005) examined the asymptotic properties of these methods and proved the convergence of the order-m and order-α approaches. Further extending this line of research, Daouia and Simar (2007) incorporated the effects of environmental variables on efficiency. Wheelock and Wilson (2008) introduced the concept of root-n to ensure model consistency and convergence, with the added benefit of mitigating the curse of dimensionality. More recently, Atwood and Shaik (2020) proposed a quantile DEA model in both envelopment and multiplier form. The statistical properties of this model are validated using Monto Carlo simulation and nCm subsampling. A similar approach has been applied to context-dependent DEA by Seiford and Zhu (2003), which involves an algorithm that calculates different levels of efficient frontiers by iteratively excluding efficient DMUs from the reference set. However, the statistical inference for context-dependent DEA has not been directly proven. Carvalho and Marques (2014) explored the economies of scope and scale for computing economies of vertical integration through the partial frontier approach.

In general, the aforementioned approaches can be considered extensions of traditional DEA models. They address issues caused by data noise by excluding outliers, with the determination of outliers grounded in statistical theory. Inspired by Banker (1993), the statistical inference for most of the presented partial frontier approaches has been established. However, these methods are still based on the conventional black-box production structure. Furthermore, like conventional DEA approaches, the shadow prices of variables remain non-unique. Future research could focus on extending these approaches to network models or proposing common-weight measures.

3.4.2 Quantile regression approach

The main difference between the quantile regression approach and the partial frontier approach lies in their strategies for inefficiency (noise) estimation. While the partial frontier approach focuses on inefficiency estimation strategies in DEA, the quantile regression approach typically fits a frontier estimator to all observations, considering both inefficiency and noise. Further, Dai et al. (2022) pointed out that the existing quantile regression approaches can be viewed as extensions of the stochastic DEA model proposed by Banker et al. (1991). This section reviews several notable quantile regression approaches.

Inspired by Banker and Maindiratta (1992), who applied Afriat inequalities in maximum-likelihood estimation by introducing non-Gaussian error terms, Kuosmanen (2008) proposed the convex nonparametric least squares (CNLS) model. Later, Kuosmanen and Johnson (2010) established a closer connection between the CNLS and DEA models. Specifically, Kuosmanen and Johnson (2010) demonstrated that DEA could be formulated as a least squares regression where the error term is always negative. Based on this proof, they suggested the Corrected Concave Nonparametric Least Squares (C^2NLS) model, which was tested and found to outperform the conventional DEA model and the COLS model through Monte Carlo simulation. Further, the Stochastic Nonparametric Envelopment of Data (StoNED) approach was introduced as a more general framework that combines conventional DEA and SFA, as developed by Kuosmanen (2006) and Kuosmanen and Kortelainen (2012). Kuosmanen and Johnson (2017) extended the CNLS model (Kuosmanen, 2008) by incorporating constraints for the DDF measure.

While the aforementioned quantile regression approaches are nonlinear, in more recent research, Wang et al. (2014) introduced a convex quantile regression (CQR) model to estimate the production frontier. This model uses linear programming and can be easily solved with standard algorithms (Gurobi, GAMS, CPLEX, MOSEK). However, it should be noted that the CQR model by Wang et al. (2014) fails to generate a unique production frontier. To address this, Kuosmanen et al. (2015) proposed a quadratic formulation as a modified objective function for the CQR model. Jradi and Ruggiero (2019) integrated the quantile regression approach into the DEA model as a stochastic measure. Kuosmanen and Zhou (2021) argued that the conventional DEA approach may overestimate the marginal abatement cost. To address this, they combined Wang et al.'s (2014) CQR model with the DDF measure proposed by Kuosmanen and Johnson (2017) to develop a convex quantile regression approach. This approach explicitly accounts for both noise and inefficiency to mitigate issues caused by high sensitivity to noise data in DEA. Consequently, the proposed approach calculates the marginal abatement cost while free from the data noise issues in conventional DEA models. In more recent research, Dai et al. (2023) summarised existing Quantile regression approaches and proposed a general model of shape-constrained nonparametric functions. Moving beyond the convex models used in CQR (linear programming) and convex expectile regression (quadric programming), Dai et al. (2023) introduced a parameter to represent partial order, generating general nonconvex CQR and CER models. In addition, España et al. (2024) used the DEA model to shape constraints and estimate production functions through additive models based on regression splines. Liao et al. (2024) proposed a nonparametric convex regression approach to address overfitting and outliers.

In general, when generating the production function, some researchers leverage the DEA model's linear-based approach to propose shape constraints that ensure the generated production frontier satisfies the monotonicity and concavity properties of production theory. However, the generated frontier is a quantile frontier rather than one that envelops all observations. The proposed models address random noise, heteroscedasticity, and outlier issues in conventional DEA techniques. The aforementioned approaches are all extensions of Banker et al. (1991), with the DEA frontier serving as a special case of these models.

4 Contextual analysis in DEA

Most early research on evaluating contextual variables relied on the stochastic frontier framework, as seen in studies like Ruggiero and Vitaliano (1999) and Rosko (2001). These approaches estimated DEA efficiency scores in the first stage and then treated these scores as dependent variables in a regression-based analysis in the second stage. However, the two-stage approach lacks a robust statistical foundation and failed to articulate a data-generating process consistent with the two-stage DEA (Banker et al., 2019). Consequently, several studies question the ability of the two-stage approach to effectively investigate the influence of contextual variables (Førsund, 1999; Grosskopf, 1996).

In response to these critiques, Banker and Natarajan (2008) first applied Monto Carlo simulation to compare the performance of the DEA-based two-stage approach (using OLS, maximum likelihood, and Tobit estimation in the second stage) with one-stage and two-stage parametric approaches. The simulation results indicate that DEA-based procedures outperform the parametric methods. This study also established the statistical foundation of the two-stage DEA-based approach and proposed a framework to estimate contextual variables while considering two-sided random noise. Banker et al. (2019) further compared the performance of DEA+OLS with DEA+Tobit regression and Simar and Wilson's (2007) approach within a noisy production environment. In a noise-free environment, simulation results demonstrate that the DEA+OLS approach performs best, particularly when evaluating large sample data (defined as 400 DMUs in the simulation tests). The same result is observed in noisy environments, where the DEA+OLS approach outperforms others. It should be noted that while the performance of the DEA+OLS approach is similar to that of DEA+Tobit regression, the theoretical foundation for DEA+Tobit regression has not been established. Moreover, the DEA+OLS approach shows better performance than Simar and Wilson's (2007) approach when contextual variables significantly affect the output. Even in cases where contextual variables have no impact on the output, the DEA+OLS approach performs similarly or occasionally better than Simar and Wilson's (2007) approach. Consequently, the DEA+OLS approach is recommended for evaluating the influence of contextual variables.

In addition to the conventional DEA model, several extended models are used for performance evaluation in the first stage of analysis. Examples include the DEA model with undesirable outputs (Bandyopadhyay, 2011) and two-stage network DEA (Tan et al., 2021). Giménez et al. (2024) applied the directional Benefit-of-the-Doubt model[1] in the first stage of analysis to evaluate countries' pandemic management.

We also found several studies that extend the analysis of contextual variables under the DEA framework. Ramalho et al. (2010) suggested using fractional regression models as a second-stage measure. By combining fractional regression models with the dynamic generalised method of moments, e Souza and Gomes (2015) proposed a cross-sectional approach to measure the dynamic effects caused by contextual variables. Karagiannis (2015) extended the work of Banker and Natarajan (2008) to decompose the efficiency effect into simultaneous effects on technical efficiency and capacity utilisation. Johnson and Kuosmanen (2012) stated that DEA can be formulated as a constrained special case of the CNLS model and suggested a semi-nonparametric approach to estimate the coefficients of contextual variables through one-stage computation. Yu et al. (2024) proposed a centralised resource allocation model to assess the effect of contextual variables through one-stage computation. Shi et al. (2025) applied Shapley Additive Explanations to scrutinise the impacts of contextual variables. To the best of our knowledge, only Johnson and Kuosmanen (2012) built a simulation framework[2] to compare the performance of their proposed model with the DEA+OLS approach. We suggest developing a more comprehensive simulation framework that incorporates varying levels of sample size and noise to better compare the performance of the aforementioned approaches.

5 Authorship network and organised conferences

To provide a comprehensive overview of Banker's scholarly contributions, a bibliometric analysis was conducted based on 203 of his publications. The Scopus database served as the primary source[3] for extracting relevant data collected in December 2024. Table 1 presents a chronological history of Banker's affiliations with various universities and business schools across different states in the United States from 1980 to 2023. His academic journey began in 1980 at Harvard Business School in Massachusetts. Over the years, he made significant contributions to various universities, while spending much of his later academic career from 2005 to 2023 at Temple University's Fox School of Business in Philadelphia. This extensive history showcases Banker's dedication to academia and his contributions across multiple educational institutions in the United States.

Table 2 outlines Banker's publications by type, totaling 203. It highlights that articles represent the majority with 133 (65.5%), followed by conference papers at 36 (17.7%). Reviews,

[1] From a definitional perspective, the applied directional Benefit-of-the-Doubt model can be regarded as a DDF model without explicit inputs. However, from a computational perspective, it can be treated as a conventional DDF model due to the presence of undesirable outputs and the absence of inputs.

[2] The sample size for the simulation tests in Johnson and Kuosmanen's (2012) study is set at 100, which is categorised as a 'medium-sized' sample in Banker et al. (2019). Their proposed model demonstrated better performance than the DEA + OLS approach in simulation testing.

[3] In addition to Scopus, Google Scholar is used to extract seven conference proceedings co-edited by Banker.

Table 1 Banker's university affiliations (1980–2023)

Year	Affiliation	State
2005–2023	Temple University/Fox School of Business	Philadelphia
2006	Arizona State University/W. P. Carey School of Business	Arizona
2003–2005	University of California, Riverside	California
2005	University of California/Anderson School of Management	California
1992–2004	University of Texas at Dallas	Texas
1989–1997	University of Minnesota/Carlson School of Management	Minneapolis
1984–1996	Carnegie Mellon University	Pittsburgh
1994	University of Minnesota/Carlson School of Management	Minnesota
1980	Harvard University/ Harvard Business School	Massachusetts

Table 2 Banker's publications by type

Type	Number of publications	Percentage
Article	133	65.5
Conference Paper	36	17.7
Review	11	5.4
Edited Book of Conference Proceedings	7	3.4
Book Chapter	8	3.9
Editorial	7	3.4
Note	1	0.5
Total	203	100

conference proceedings, book chapters, editorials, and notes make up smaller portions of his works, demonstrating his diverse scholarly contributions across various platforms.

Table 3 presents Banker's distribution of authorship positions in his publications. It shows that he was the first author in 159 papers (78.3%), indicating his primary contribution as the lead researcher. Additionally, he was listed as the second author in 28 papers (13.8%) and the third author in 14 papers (6.9%). For publications where he was listed as the fourth author or beyond, there were only two instances, representing 1% of his total publications. This breakdown illustrates the prominence of his role as the primary contributor or lead author in the majority of his scholarly works.

Table 4 outlines Banker's top 10 co-authors based on the number of co-authorships they shared in scholarly works. It presents the respective co-authors' names, the count of collaborative papers with Banker, their titles, affiliated universities or institutions, specific schools

Table 3 Banker's authorship order in publications

	First author	Second author	Third author	Fourth^{+} author	Total
Number of publications	159	28	14	2	203
Percentage	78.3	13.8	6.9	1.0	100

Table 4 Top 10 scholars who collaborated with Banker

Co-author	No. of co-authorships	Title	University/Affiliation	School/ Faculty	Country
Chang, Hsihui	24	Professor of Accounting	Drexel University	Drexel University	United States
Natarajan, Ramachandran Nat	13	Professor of Management	University of Texas at Dallas	The Naveen Jindal School of Management	United States
Emrouznejad, A	11	Professor of Business Analytics	University of Surrey	Surrey Business School	UK
Kauffman, Robert J	11	Professor of Information Systems	Singapore Management University	Singapore Management University	Singapore
Cooper, William W	11	Professor of Operations Research	University of Texas at Austin	McCombs School of Business	United States
Datar, Srikant M	10	Professor of Administration	Harvard University	Harvard Business School	United States
Anderson, Mark C	10	Associate Professor of Accounting	University of Calgary	Haskayne School of Business	Canada
Riedl, René	9	Professor of Digital Business & Innovation	University of Applied Sciences Upper	School of Management	Austria
Pavlou, Paul A	9	Professor of Information Sciences	University of Houston	C. T. Bauer College of Business	United States
Davis, Fred D.D	9	Professor of Information Technology	Texas Tech University	Rawls College of Business	United States

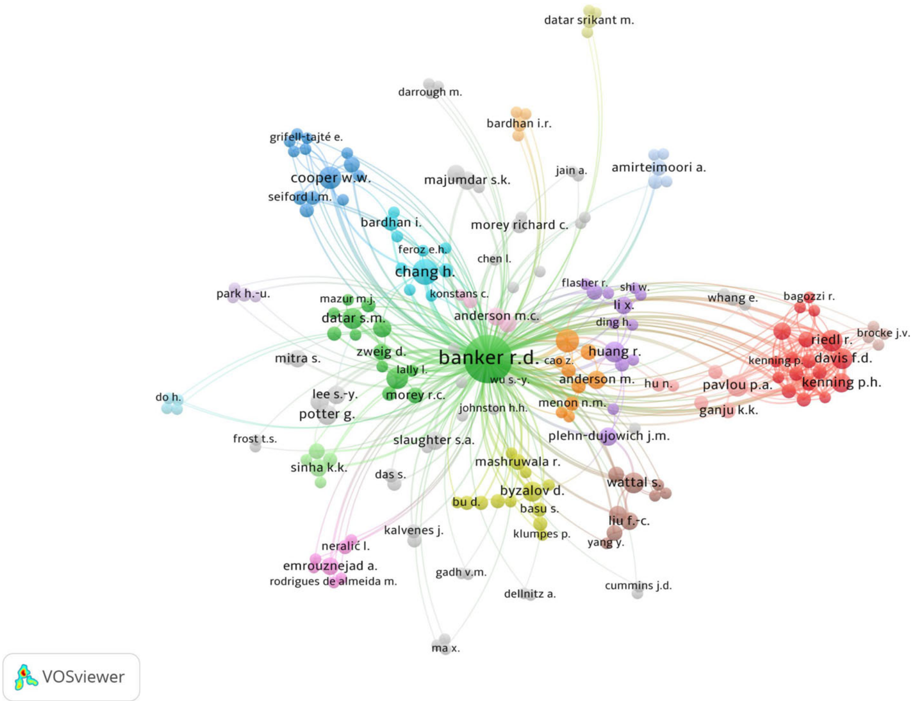

Fig. 8 Banker's co-authors network

or faculties within those institutions, and the countries where they are located. This table demonstrates the collaborative relationships that Banker established with these individuals across various universities, showcasing his partnerships within the academic community, particularly in the fields of accounting, management, business analytics, information systems, and operations research.

Figure 8 shows a network of co-authorships with Banker, where nodes represent co-authors, and distinct colours correspond to different clusters of authors. We utilised VOSviewer, a software tool developed by Van Eck and Waltman (2010), to create and visualise maps based on network data. The size of the nodes represents the number of co-authored publications. Banker is the central node in the network, and he has co-authored papers with many other researchers in several clusters. The clusters of authors, shown in different colours, represent researchers with common interests, locations, and/or expertise. For example, the navy-blue cluster contains scholars such as Cooper and Charnes, who made significant contributions to the introduction and development of DEA, while the phosphorus blue cluster includes researchers mainly in the accounting field. The fact that Banker is connected to researchers in multiple clusters suggests that he had a broad range of global collaborations and research interests. This is not surprising, given that he is a highly respected scholar who made significant contributions to multiple fields, including applied mathematics, accounting, and economics.

Table 5 Journal editorials co-authored by Banker

Title	Journal	Reference
Advances in Data Envelopment Analysis: Celebrating the 40th anniversary of DEA and the 100th anniversary of Professor Abraham Charnes' birthday	European Journal of Operational Research	Emrouznejad et al. (2019)
Recent developments on the use of DEA in the Public Sector	Socio-Economic Planning Sciences	Ahn et al. (2018)
Business performance management under uncertain environments- II	Journal of Centrum Cathedra: The Business and Economics Research Journal	Charles and Banker (2017)
Business performance management under uncertain environments - I	Journal of Centrum Cathedra: The Business and Economics Research Journal	Charles and Banker (2016)
Data Envelopment Analysis in the public sector	Socio-Economic Planning Sciences	Emrouznejad et al., (2014a, b)
Efficiency and productivity: Theory and applications	Annals of Operations Research	Emrouznejad and Banker (2010)

Table 5 presents a compilation of journal editorials co-authored by Banker, including their titles, respective journals, and their references. Banker has made significant contributions to editorial pieces in esteemed journals, focusing primarily on DEA and its applications across various sectors. In collaboration with co-authors like Emrouznejad, Charles, Ahn, and others, he authored editorials celebrating milestones in DEA history, exploring recent developments, particularly in the public sector, and examining the theoretical aspects of efficiency and productivity.

Table 6 provides a list of conference proceedings co-edited by Banker across various international conferences on DEA, including the titles of the proceedings, the respective conference titles and references. These proceedings, spanning from 2012 to 2017, highlight the diverse applications and advancements in DEA methodologies discussed during significant academic gatherings worldwide. The titles reflect a wide range of topics, including the recent applications of DEA, its role in sustainable development, performance measurement, and theoretical explorations.

Table 7 represents Banker's publication record across academic journals where he contributed three or more publications. This table shows his substantial contributions to various fields of study. It begins with the European Journal of Operational Research, which has the highest number of publications at 17, followed closely by Management Science with 15 publications. Table 7 also highlights the journals where Banker published more than half of his scholarly works (55.1%), demonstrating his significant impact through these distinguished journals.

This data illustrates Banker's diverse and extensive scholarly output, highlighting his influence across multiple academic fields and his active participation in advancing the knowledge and application of DEA and related methodologies.

Table 6 Conference proceedings co-edited by Banker*

Title	Conference	Authors/Editors	ISBN
Recent applications of Data Envelopment Analysis	16th International Conference of DEA, June 2017, University of Economics, Prague, Czech Republic (DEA2017)	Emrouznejad, Banker et al. (2017)	978 1 85449 433 7
Recent applications of Data Envelopment Analysis	14th International Conference of DEA, May 2016, Jianghan University, Wuhan, China (DEA2016)	Emrouznejad, Banker et al. (2016)	978 1 85449 413 9
Data Envelopment Analysis and its applications	13th International Conference of DEA, August 2015, Braunschweig, Germany (DEA2015)	Emrouznejad, Banker et al. (2015)	978 1 85449 497 9
Sustainable development and performance measurement	International DEA Workshop, September 17–19, 2014, Hermosillo, Sonora, Mexico	Banker et al. (2014)	978 1 85449 482 5
Theory and applications of Data Envelopment Analysis	12th International Conference of DEA, April 2014, University of Malaya, Kuala Lumpur, Malaysia (DEA2014)	Emrouznejad et al. (2014a, b)	978 1 85449 487 0
Data Envelopment Analysis and performance measurement	11th International Conference of DEA, June 2013, Samsun, Turkey (DEA2013)	Banker, Emrouznejad et al. (2013)	978 1 85449 477 1
Data Envelopment Analysis: theory and applications	10th International Conference on DEA, Natal, Brazil (DEA2012)	Banker, Emrouznejad et al. (2012)	978 1 85449 437 5

*Banker also co-organised or served as an invited speaker at several other DEA conferences, including: DEA2002 (invited speaker; Moscow, Russia), DEA2004 (invited speaker; Birmingham, UK), DEA2007 (co-organiser; Hyderabad, India), DEA2009 (organiser; Philadelphia, USA), DEA2010 (invited speaker; Beirut, Lebanon), DEA2011 (invited speaker; Thessaloniki, Greece), DEA workshop 2013 (co-organiser; Hebrew University, Israel), DEA2018 (co-organiser; Wuhan, China), DEA2019 (co-organiser; Calgary, Canada)

Table 7 Journals with three or more publications authored by Banker

Journal/Source	No. of publications	% Publications	% Cumulative
European Journal of Operational Research	17	8.7	8.7
Management Science	15	7.7	16.3
Annals of Operations Research	9	4.6	20.9
Accounting Review	7	3.6	24.5
Contemporary Accounting Research	7	3.6	28.1
Journal of Accounting and Economics	7	3.6	31.6
Information Systems Research	6	3.1	34.7
MIS Quarterly: Management Information Systems	6	3.1	37.8
Lecture Notes in Information Systems and Organisation	5	2.6	40.3
Journal of Accounting Research	4	2.0	42.3
Journal of Accounting, Auditing & Finance	4	2.0	44.4
Journal of Management Information Systems	4	2.0	46.4
Journal of Productivity Analysis	4	2.0	48.5
Proceedings of the Annual Hawaii International Conference on System Sciences	4	2.0	50.5
Communications of the ACM	3	1.5	52.0
IEEE Transactions on Software Engineering	3	1.5	53.6
Journal of Management Accounting Research	3	1.5	55.1

Table 8 Top ten most cited papers authored by Banker

Publication	Journal	No. of Citations
Banker et al. (1984)	Management Science	11,547
Banker and Morey (1986)	Operations Research	863
Banker (1984)	European Journal of Operational Research	696
Anderson et al. (2003)	Journal of Accounting Research	639
Banker (1993)	Management Science	616
Banker and Natarajan (2008)	Operations Research	559
Banker and Thrall (1992)	European Journal of Operational Research	517
Banker et al. (2000)	Accounting Review	490
Banker et al. (1998)	Management Science	451
Banker and Morey (1986)	Management Science	418

6 Bibliometric analysis

6.1 Macro-level

Figure 9 illustrates the network of keywords used in Banker's publications, each appearing at least three times, showcasing the interconnected themes within his research. The larger nodes within the network signify keywords used more frequently, indicating their prominence in his research. The clustering of keywords represents cohesive themes and subfields within his work, such as DEA, linear programming, and productivity in the earlier years, and innovation, finance, and commerce in more recent times. This shift in keywords over time reflects the

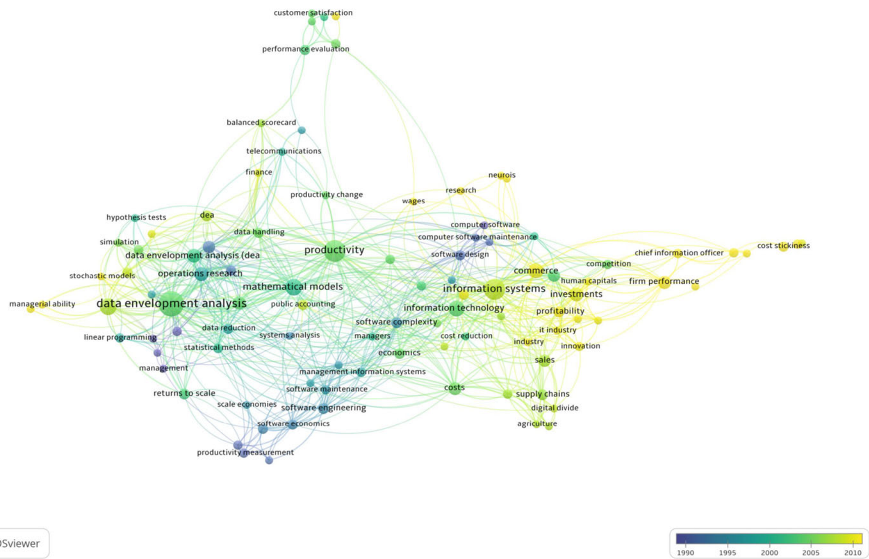

Fig. 9 Network of keywords used by Banker

evolving focus of his research, from traditional topics to more contemporary subjects within the field.

Table 8 presents the ten most cited papers authored or co-authored by Banker. It includes the titles of the publications, the respective journals where they were published, and the number of citations each paper has received. These papers, published across various prestigious academic journals such as Management Science, Operations Research, European Journal of Operational Research, Journal of Accounting Research, and Accounting Review, highlight the significant impact of Banker in advancing DEA models and applications.

6.2 Micro-level

In this section, we provide a detailed bibliometric analysis outcome of the three main research clusters identified in Banker's work.

6.2.1 Analysis of the returns-to-scale topic

Figure 10 illustrates the citation trends of Banker's key papers on RTS and MPSS. Banker et al. (1984) stands out as the most cited paper, demonstrating a continuous upward trend over nearly three decades, underscoring its foundational role in DEA literature. Banker (1984) is the second most cited paper, showing an upward trend, although with significantly fewer citations per year compared to Banker et al. (1984). While other papers in Fig. 10 (apart from those published in 1984) have fewer citations, the subjects of RTS and MPSS still represent a substantial segment of the DEA literature.

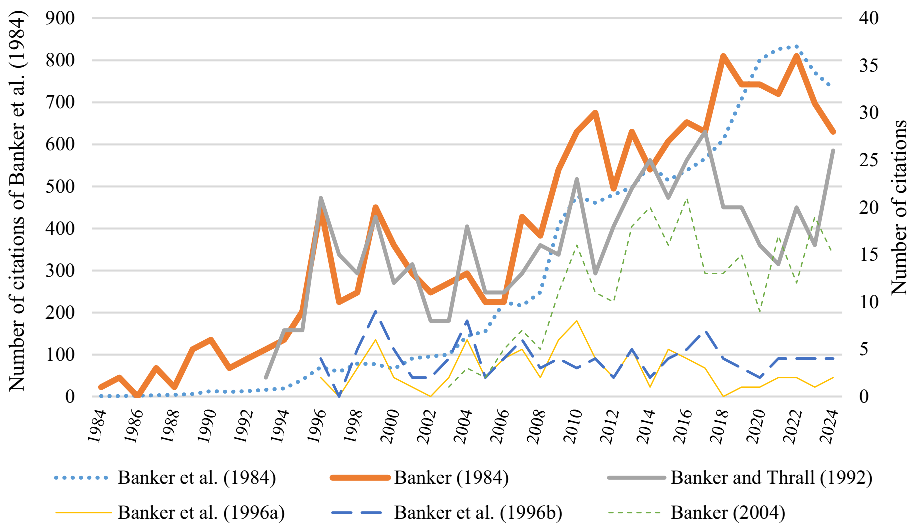

Fig. 10 Citation trends for RTS and MPSS related papers

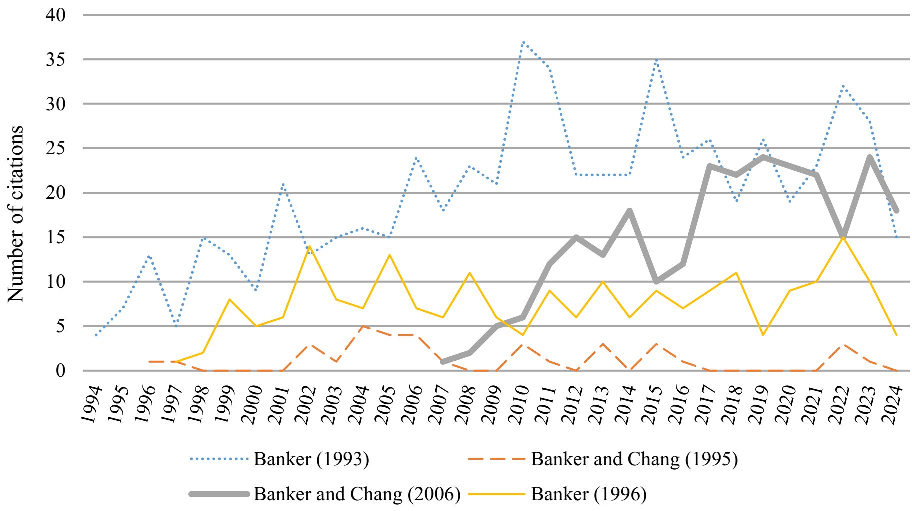

Fig. 11 Citation trends for statistical analysis related papers

6.2.2 Analysis of the statistical analysis in DEA topic

Figure 11 depicts the citation trends of Banker's key papers on statistical analysis in DEA. The most cited paper in this cluster is Banker (1993), which has shown a relatively stable citation trend over the last decade. Banker and Chang (2006) follow closely, with an upward trend in its initial years but stagnating in recent years. Overall, it appears that Banker's contributions to statistical analysis in DEA have not achieved the same level of prominence as his papers on RTS.

These analyses highlight the impact and ongoing relevance of Banker's research within the academic community, particularly in the areas of RTS, MPSS, and statistical methodologies in DEA. His work continues to influence and shape the direction of research in these areas, as evidenced by the sustained citation trends and evolving research focus over time.

6.2.3 Analysis of the contextual analysis topic

Figure 12 illustrates the contrasting trajectories in citations between two of Banker's papers on second-stage analysis in DEA. Banker and Natarajan's paper, published in 2008, initially experienced a surge in citations during its early years but has shown a decline in momentum in recent years. Conversely, Banker's 2019 paper has enjoyed a consistent climb in citations, indicating sustained interest and relevance in the field. However, when comparing the number of citations overall, Banker's paper on the second-stage analysis shows relatively fewer citations than the alternative approach proposed by Simar and Wilson (2007). For a comprehensive discussion on the alternative method and its bibliometric analysis, refer to Moradi-Motlagh and Emrouznejad (2022).

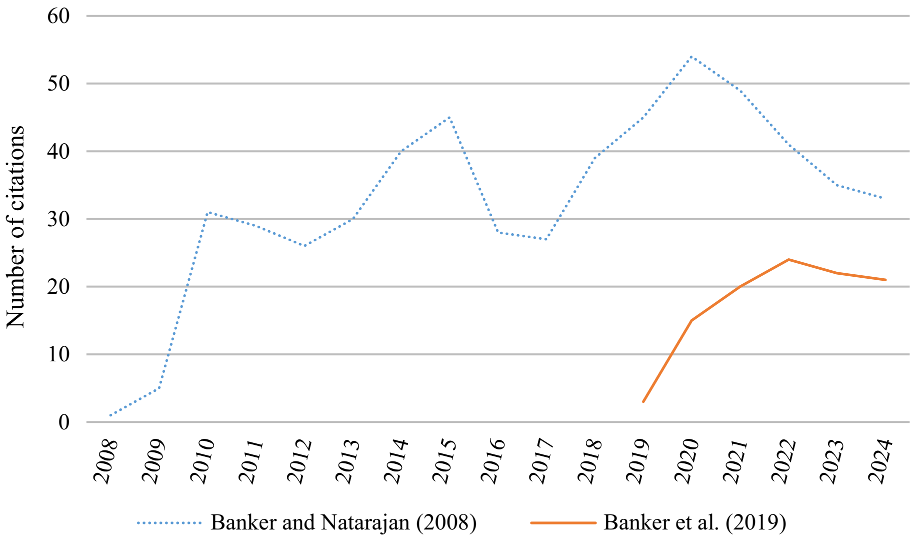

Fig. 12 Citation trends for second-stage efficiency analysis related papers

7 Conclusion

This paper explored the significant and enduring impact of Professor Rajiv Banker on the field of Data Envelopment Analysis (DEA). Throughout his esteemed career, Banker has been instrumental in shaping the trajectory of DEA, leaving a legacy that continues to inspire scholars, researchers, and practitioners alike.

Our comprehensive tribute to Banker's lifetime contributions revealed a distinct and pioneering approach to DEA, focusing on three key clusters: Returns-to-Scale (RTS) and Most Productive Scale Size (MPSS) in DEA, Statistical Inference of DEA, and Contextual Analysis. Each of these clusters represents a paradigm shift within the DEA landscape, catalysed by Banker's groundbreaking research and unwavering commitment to advancing the field.

In honouring Banker's lifelong dedication to DEA, we recognise not only the intellectual depth of his contributions but also the inspiration he has provided to countless scholars. As the DEA community progresses, it does so on the solid foundation laid by Banker. Through this paper, we celebrate his enduring legacy and express gratitude for his transformative influence on Data Envelopment Analysis.

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Declarations

Conflict of interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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