Measurement of the mode II intralaminar fracture toughness and R-curve of polymer composites using a modified Iosipescu specimen and the size effect law


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Measurement of the mode II intralaminar fracture toughness and R-curve of polymer composites using a modified Iosipescu specimen and the size effect law

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Abstract

A modified Iosipescu specimen is proposed to measure the mode II intralaminar fracture toughness and the corresponding crack resistance curve of fibre reinforced composites. Due to the impossibility of scaling the specimen, a modification of the classical size effect method is proposed. The classical Iosipescu shear feature was used and tests were coupled with digital image correlation to support the proposed approach. Experiments were performed on IM7/8552 material system and the R-curve was obtained. The steady–state value of the fracture toughness of the ply is found to be equal to $R_{0ss} = 34.4 \text{ kJ/m}^2$.

Key words:
A. Polymer-matrix composites (PMCs), B. Fracture toughness, C. Analytical modelling, D. Mechanical testing

1 Introduction

The intralaminar fracture toughness is a key parameter used to screen and to qualify material systems, and is an input parameter for strength analysis models. Some of those, as recent progressive damage models, use softening laws

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that can be completely defined once the R-curve of the material is known [1–4]. Others, as Finite Fracture Mechanics (FFMs) models [5–8], require the fracture toughness (together with the strength of the laminate) as input parameters.

Despite the importance of knowing the intra-laminar fracture toughness of composite laminates, very few tests methods were developed in the recent past. Generally the R-curve is measured using Compact Tension or Compact Compression specimens [9], the last if the R-curve associated with the propagation of a kink band is required. Several difficulties arise in the execution of the experimental tests, in particular the measurement of the crack extension and the computation of the J-Integral around the crack tip [10]. Moreover, the orthotropy of the material complicates the tests and the data post-processing.

Those difficulties drastically increase when measuring the R-curve in mode II. In this case no tests method exists and, to the authors’ best knowledge, no relevant work has been done using fibre-reinforced plastics. However, the measurement of the R-curve in mode II could be very valuable because it would enable the generalization of several analysis models developed for mode I propagation (e.g., FFMs and progressive damage models).

The authors recently proposed the use of the size-effect methods developed by Bažant (mainly for rocks and ceramics) to measure the R-curve of fibre-reinforced composites [11,12]. It was demonstrated that size effect method can be effectively used to measure the R-curve of a laminate (or a ply), drastically simplifying the experimental tests. Indeed, using the size effect law it is not required to measure the crack extension, a parameter whose experimental measurement is quite difficult when using heterogeneous materials, particularly under compression, under shear, and in the presence of bridging effects.

In this paper the size effect method is applied to a modified Iosipescu specimen. This experimental method is commonly used to measure the elastic and strength parameters of fibre-reinforced plastic in shear and its ability to apply a uniform shear stress field in the gauge section of the specimen has been demonstrated [13]. The details of the method are given in [14]. Therefore, the Iosipescu test method was chosen to apply the shear stress to a cracked specimen. The main inconvenient of this method is that the experimental device is standardized and it is not possible to scale specimens of the same geometry to use the classical size effects method [15]. To overcome this limitation, a modification of the size effects method is proposed. Digital image correlation was coupled to the Iospiescu tests to provide the strain field at the centre of the specimens.
2 Analytical Model

2.1 Linear-elastic material

Let $x_1$ and $x_2$ be the material axes of a two-dimensional orthotropic solid. The energy release rate in mode II for a crack propagating in the $x_1$ direction reads:

$$ G_{II} = \frac{1}{\dot{E}} K_{II}^2 $$  \hspace{1cm} (1)

where $\dot{E}$ is the equivalent modulus and $K_{II}$ is the stress intensity factor in mode II. The equivalent modulus reads [16]:

$$ \dot{E} = \left( s_{11} s_{22} \frac{1 + \rho}{2} \right)^{-1/2} \lambda^{-1/4} $$  \hspace{1cm} (2)

where $s_{lm}$ are components of the compliance tensor computed in the $x_1$-$x_2$ coordinate system, and $\lambda$ and $\rho$ are two dimensionless elastic parameters given as [16]:

$$ \lambda = \frac{s_{11}}{s_{22}} \quad \rho = \frac{2 s_{12} + s_{66}}{2 \sqrt{s_{11} s_{22}}} $$  \hspace{1cm} (3)

The measurement of the ply fracture toughness is based on balanced cross ply laminates [9]. The plies that are perpendicular to the direction of the crack propagation ($0^\circ$ plies) have a fracture toughness that is two or three orders of magnitude higher than that of the plies parallel to the crack propagation ($90^\circ$ plies). This is due to the fact that the former plies experience a fibre dominated failure whilst the latter experience a matrix dominated failure. Neglecting the possible small interactions between these two different failure modes, an energy balance shows that the fracture toughness of the laminate along the $x_1$ direction is simply one half of the fracture toughness of the $0^\circ$ ply for the same direction.

If a balanced cross ply is used ($\lambda = 1$), equation (2) yields:

$$ \dot{E} = E \left( \frac{1 + \rho}{2} \right)^{-1/2} $$  \hspace{1cm} (4)

where $E$ is the Young’s modulus along a preferred direction of the material.
Consider now a modified Iosipescu specimen as shown in Figure 1. The width of the specimen is $2W = 20\text{mm}$ while the length is $80\text{mm}$. The specimen has a gauge section of width $2w$, created with two symmetric notches, and two symmetric cracks on the gauge section of length $a$. Two dimensionless parameters are defined: (i) the ratio between the crack length and the width of the gauge section, $\alpha = a/w$; and (ii) the ratio between the width of the gauge section and the width of the specimen, $\beta = w/W$. Figures 1(a) and 1(b) shows the specimen when $0 < \beta < 1$ and $\beta = 1$, respectively.

\[ \text{Fig. 1 about here.} \]

The stress intensity factor of equation (1) depends on the dimension of the specimen (a characteristic size, $w$), on the geometry ($\alpha$ and $\beta$), and on the remote applied load $P$ or stress $\tau$:

\[ K_{II} = \tau \sqrt{w \phi (\alpha, \rho, \beta)} = \frac{P}{2t \sqrt{w}} \phi (\alpha, \rho, \beta) \]  
\[ \text{(5)} \]

where $\tau = P/(2wt)$.

Since $\beta = w/W$, the previous equation can be rewritten as:

\[ K_{II} = \tau \sqrt{\beta W \phi (\alpha, \rho, \beta)} \]  
\[ \text{(6)} \]

Using (1), the energy release rate reads:

\[ G_{II} = \frac{W}{E} \beta \tau^2 \phi^2 (\alpha, \rho, \beta) . \]  
\[ \text{(7)} \]

Equation (7) yields the energy release rate for a given applied stress $\tau$. At the peak load, when the maximum stress is $\tau_u$, the energy release rate equals the mode II fracture toughness of the material:

\[ R (\Delta a) = \frac{W}{E} \beta \tau_u^2 \phi^2 \left( \alpha_0 + \frac{\Delta a}{\beta W}, \rho, \beta \right) \]  
\[ \text{(8)} \]

where $\Delta a$ is the crack increment and $\alpha_0 = a_0/w$, where $a_0$ is the initial crack length. The R-curve is a material parameter that does not depend on the shape or dimension of the specimen. In particular it does not depend on the parameter $\beta$. Therefore, taking the derivative of $R$ with respect to $\beta$, it follows that $\partial_\beta R = 0$. Deriving equation (8) with respect to $\beta$ and imposing $\partial_\beta R = 0$ yields:
\[
\partial_\beta R = \frac{W}{E} \partial_\beta \left( \beta \tau^2 \phi^2 \right) = 0. \tag{9}
\]

If \( \tau = \tau(\beta) \) (for a given \( W \)) is known and under the hypothesis that \( \alpha_0 \) is a constant, equation (9) can be solved for \( \beta = \beta(\Delta a) \). Substituting \( \beta(\Delta a) \) in equation (8) yields the R-curve for \( 0 < \beta < 1 \) or, in other words, for \( 0 < w < W \).

It should be noted that the procedure described enables the determination of part of the R-curve (in the range \( 0 < \beta < 1 \)) but not the entire R-curve. This is due to the fact that it is not possible to extrapolate the region of the R-curve when \( w \rightarrow +\infty \) because \( 0 < w < W \). To obtain the entire R-curve the size effect law must be obtained with a characteristic length that ranges in the interval \([0, +\infty)\). In the impossibility of scaling the specimens, the size effect law can be obtained by mapping the part of the R-curve already defined. Consider a specimen with a constant \( \beta \), for example \( \beta = 1 \) as shown in Figure 2.

The crack driving force for this specimen reads:

\[
G_{II} = \frac{1}{E} \tau^2 \psi^2 \tag{10}
\]

where \( \psi = \phi|_{\beta=1} \). At the peak load the following system of equations is satisfied:

\[
\begin{cases}
G_{II} = R \\
\partial_{\Delta a} G_{II} \geq \partial_{\Delta a} R
\end{cases}
\]

where \( \partial_{\Delta a} G_{II} = \partial_{\Delta a} R \) represents the condition for unstable crack propagation.

Equation (11) represents the tangency condition between the crack driving force and the R-curve, as shown in Figure 3. Therefore, for a given \( w^* \), the value of \( \tau^* \) tangent to the R-curve can be found. The strategy used here is quite simple: for a given characteristic dimension \( w^* \) the stress \( \tau \) for which there is just an intersection point (\( * \) in Figure 3) is the stress \( \tau^* \). This is illustrated in Figure 3 in which, for a given point of the R-curve (indicated with \( * \)), only two crack-driving force curves are defined (assuming \( \alpha_0 \) constant). The known portion of the R-curve (in red in Figure 3) extends between two points that are tangent to the driving forces curve identified with \( \beta_{\min} \) and \( \beta_{\max} \).
\( \beta_{\text{min}} \) and \( \beta_{\text{max}} \) are the limiting values of \( \beta \). Since the numerical model described in the following section is defined for \( \beta \in [0.2, 1.0] \), \( \beta_{\text{min}} \) and \( \beta_{\text{max}} \) are taken as 0.2 and 1.0, respectively.

By repeating this procedure several times, the calculation of several pairs \((w^*, \tau^*)\) are obtained. By fitting these points it is possible to define the size effect law (see Table 1 for suggested expressions of the size effect laws), \( \tau = \tau(w) \), that is necessary to calculate the entire R-curve, \( R(\Delta a) \).

To do that it should be noted that the R-curve is a material parameter and it does not depend on \( w \). Therefore, \( \partial_w R = 0 \). Substituting (10) in the first (11) and deriving with respect to \( w \) yields:

\[
\partial_w R = \frac{1}{E} \partial_w \left( w \tau_w^2 \psi^2 \right) = 0. \tag{12}
\]

Knowing the size effect law, \( \tau = \tau(w) \), under the hypothesis that similar specimens are considered (\( \alpha_0 \) is constant) equation (12) can be solved for \( w = w(\Delta a) \). Substituting \( w = w(\Delta a) \) in the first (11) yields the entire R-curve. In this case the steady state value can be obtained simply taking the limit, for \( w \to +\infty \), of \( R \) in the first of (11):

\[
R_{\text{ss}} = \lim_{w \to +\infty} R(\Delta a) = \frac{1}{E} w \tau_w^2 \psi_0^2 \tag{13}
\]

where \( \psi_0 = \psi(\alpha = \alpha_0) \).

2.2 Elasto-plastic materials

The method proposed in the previous section is rigorous for an elastic body and it may be used when the plastic region is negligible when compared to the characteristic crack length. This is not the case when dealing with cross ply laminates loaded in shear where the material behaviour is non-linear. Therefore, the R-curve obtained using the method proposed in the previous section may substantially differ from the real one. However, this discrepancy should affect only the rising part of the R-curve whilst it should not influence its plateau. In fact, the steady–state value of the R-curve, \( R_{\text{ss}} \), is obtained for an infinitely large specimen (see equation (13)) for which LEFM is valid. Suppose, for example, that the material exhibits a Ramberg–Osgood relationship.
between stress and strain in shear:

\[ \gamma = \frac{\tau}{G} + m\frac{\tau_0}{G} \left( \frac{\tau}{\tau_0} \right)^n \]  

(14)

where \( G, \tau_0, \gamma, m, \) and \( n, \) are the shear modulus, the yield stress, the shear strain, and the two constants of the Ramberg–Osgood equation, respectively.

Under linear elastic conditions, the J-integral equals the energy release rate:

\[ J_e = \frac{1}{E} w \tau^2 \psi^2 \]  

(15)

For a fully plastic cracked body the J-integral reads [17]:

\[ J_p = m \gamma_0 \tau_0 w g(\alpha) h(\alpha, n) \left( \frac{\tau}{\tau_0} \right)^{n+1} \]  

(16)

where \( \gamma_0 \) is the yield strain of the material, and \( g \) and \( h \) are two correction factors. For an elasto-plastic cracked body the J-integral is normally computed superposing the elastic and the fully plastic solutions [18–20]:

\[ J = \frac{1}{E} w \tau^2 \psi^2 (\alpha + \alpha_{corr}) + m \gamma_0 \tau_0 w g(\alpha) h(\alpha) \left( \frac{\tau}{\tau_0} \right)^{n+1} \]  

(17)

where \( \alpha_{corr} \) is the correction for strain hardening. At the peak stress, \( \tau_u, \) the elasto-plastic J-integral equals the material fracture toughness:

\[ R = \frac{1}{E} w \tau_u^2 \psi^2 (\alpha + \alpha_{corr}) + m \gamma_0 \tau_0 w g(\alpha) h(\alpha) \left( \frac{\tau_u}{\tau_0} \right)^{n+1} \]  

(18)

It is straightforward to calculate the steady state value of \( R_{ss} \) as the limit of (18) for \( w \to +\infty \). Taking into account that the correction factor \( \alpha_{corr} = r_p/w, \) with \( r_p = r_p(n, \tau_0) \) being Irwin’s correction, it follows that \( \lim_{w \to +\infty} \alpha_{corr} = 0. \)

Also, the limit of the second term in (18), \( J_p, \) is equal to zero. Without loss of generality, suppose that the size effect law can be fitted using a linear regression II fit (see Table 1):

\[ \frac{1}{w \tau_u^2} = \hat{A} \frac{1}{w} + \hat{C} \]  

(19)

The limit of \( J_p, \) when \( w \to +\infty, \) reads:
\[ \lim_{w \to +\infty} J_p = \lim_{w \to +\infty} m \gamma_0 \tau_0^{-n} g(\alpha) h(\alpha, n) \frac{w}{(\bar{A} + \bar{C}w)^{\frac{n+1}{2}}} = 0 \quad (20) \]

The previous limit is equal to zero because \( n > 1 \). Therefore the steady state value of the R-curve, \( R_{ss} \), computed as the limit, for \( w \to +\infty \), of the elasto-plastic J-integral is exactly that computed in equation (13) using LEFM.

3 Numerical Model

The dimensionless functions \( \phi \) and \( \psi \) are calculated for the problem under analysis, using the Finite Element Method (FEM). For this purpose a parametric Finite Element model was created using Python [21] together with the Abaqus 6.8-3 Finite Element code [22].

The characteristic distance \( w \) is taken as constant and equal to the unity, while the variables are: the shape parameters or, in other words, the crack length \( 0 < \alpha < 1 \) and the notch length \( 0.2 < \beta < 1 \); and the dimensionless parameter \( \rho \) that takes into account the effect of the orthotropy of the material \( (0 \leq \rho \leq 20) \). Figure 4(a) shows the mesh of the finite element model while figures 4(b)–4(c) show the deformed shape overall the specimen and, in detail, in the crack.

[Fig. 4 about here.]

The finite element used is the 4-node quadratic, reduced integration element, CPS4R. The Virtual Crack Closure Technique (VCCT) [23] is used to compute the energy release rates, \( G_I \) and \( G_{II} \). For this purpose, a Python script was written to be used into Abaqus. Both energy release rates are calculated and compared to ensure that the crack propagation occurs in mode II. In addition, the total energy release rate computed was compared with that obtained from the Abaqus built-in method to calculate the J-Integral. This redundant measure was used to validate the Python script. Displacements are applied on the model and frictionless contact is assumed. The applied load \( P \) is calculated summing the forces at the reaction nodes.

Using the results obtained in the FE analysis the correction factor \( \phi \) is approximated by the following polynomial:

\[
\phi = \frac{\alpha}{1 - \alpha} \sum_i \sum_j \sum_k \Phi_{ijk} \alpha^{i-1} \rho^{j-1} \beta^{k-1}
\]

(21)
where $\Phi_{ijk}$ is the element of the matrix $\Phi$ with indexes $i$, $j$, and $k$. The matrix $\Phi$ is shown in Table 2.

The fitting surface $\phi$ is shown, together with the numerical points, in Figure 5.

The maximum error of equation (21) in fitting the numerical points is 5%, with an average error of 2%. Function $\psi$ can trivially obtained posing $\beta = 1$ in equation (21). However, in order to reduce the error a new fit was calculated. The correction factor $\psi$ is computed using the following polynomial:

$$\psi = \frac{\alpha}{1 - \alpha} \sum_i \sum_j \Psi_{ij} \alpha^{i-1} \rho^{j-1}$$

(22)

where $\Psi_{ij}$ is the element, with the indexes $i$ and $j$ of the matrix $\Psi$ defined as:

$$\Psi = \begin{bmatrix}
-0.03644 & 0.22363 & -0.00437 & 2.21749E-5 \\
5.23616 & 1.44575 & -0.16601 & 0.00449 \\
-7.39060 & -6.89756 & 0.63003 & -0.01629 \\
1.55840 & 9.24044 & -0.79486 & 0.02025 \\
0.84958 & -4.05275 & 0.33844 & -0.00855
\end{bmatrix}$$

(23)

The average error obtained using the expression of equation (22) is 1% while the maximum error is less than 2%. Figure 6 shows the comparison between the fitting surface $\psi$ and the numerical points obtained using the aforementioned finite element model.

4 Experiments

IM7/8552 16-ply cross-ply ([90/0]$_{8s}$) laminate with a nominal thickness of 4 mm is used here. The nominal ply thickness is 0.125 mm, and the details of the manufacturing process given in [11]. The pre-crack was machined in a CNC machine equipped with a 1 mm drill bit. The Young’s modulus (along the orthotropic axes of the material) and the dimensionless parameter $\rho$ of the cross-ply are $E = 90649$MPa and $\rho = 8.5$, respectively.
The tests were carried out on a universal testing machine with a displacement rate of 2mm/min. The load was measured with a load cell of 100kN. A Iosipescu shear fixture was used to transfer the vertical movement of the cross-head of the testing machine in predominant shear at the specimen central section [24]. In particular, this set-up was used to apply mode II loading to the modified Iosipescu specimen. A torque wrench was used to tighten the sliding wedges of the fixture to a torque of 1Nm. Specimens were centered within the fixed and movable parts of the fixture.

The fracture tests were coupled with digital image correlation (DIC). The photo-mechanical set-up is shown in Figure 7. In this work, the ARAMIS DIC-2D system by GOM was used [25,26]. A 8-bit Charged Coupled Device (CCD) Baumer Optronic FWX20 camera coupled with an Opto-Engineering telecentric lens TC 23 36 was selected (Table 3). With this type of lens, the magnification of the optical system can be kept constant over a defined working-distance range. The measurement are therefore less sensitive to any parasitic out-of-plane movement of the specimen. The gauge section at the specimen centre was painted using an airbrush in order to guarantee the refiled speckled surface for DIC measurements. The illumination and shutter time were then adjusted in order to enhance the contrast of the target image. Based on rigid-body translation tests carried out systematically before the fracture tests [26], an estimation of the displacement and strain resolution was obtained in the range of $2 \times 10^{-2}$ pixel (0.36 µm) and 0.02-0.04%, respectively.

Six different geometries were tested, indicated with labels from A to F, corresponding to a variation of $\beta$ from 0.5 to 1.0. Table 4 shows the relevant geometrical parameters together with the experimental results obtained. Three specimens were tested for each geometry. The dimensionless parameter $\alpha_0$ was taken as $\alpha_0 = 0.5$.

Figure 8 shows the specimen after testing. The specimens failed by mode II crack propagation.

The digital image correlation is used to verify the validity of the tests performed. Figure 9 shows the contour plot of the shear strain, $\gamma_{12}$. Any misalignment in the load condition are checked in this way to ensure a predominant mode II crack propagation.
Fitting the experimental results shown in Table 4, the law $\tau_u(\beta)$ is obtained. It should be noted that $\tau_u(\beta)$ is not, strictly speaking, a size effect law because it is not obtained scaling all the specimen’s dimension. This law is valid for the specimen shown in Figure 1 (with the width $W=20$ mm). Nevertheless, as we showed in the previous section, this specific size effect law can be used to obtain part of the R-curve. Due to the fact that $\tau_u(\beta)$ is not a size effect law, no particular caution should be taken in choosing the fitting function to use. However, it should be ensured that the strength decreases monotonically with $\beta$. In other words, it is not necessary to use the fitting formulas suggested in Table 1 because some characteristic of those formula are not required (for example, to provide a finite value of the strength when the characteristic size tends to infinity). The best fitting of the experimental data reported in Table 4 was obtained with the linear regression II fit:

$$\frac{1}{\beta \tau_u^2} = \hat{A}_\beta \frac{1}{\beta} + \hat{C}_\beta$$

(24)

The best fitting ($R^2=0.91$) is obtained for $\hat{A}_\beta = 6.666 \times 10^{-5}$ MPa$^{-2}$ and $\hat{C}_\beta = 4.084 \times 10^{-5}$ MPa$^{-2}$. The experimental results and the fitted curve are shown in Figure 10.

Having obtained the law $\tau_u(\beta)$, part of the R-curve can be calculated solving the systems of equations (8)–(9) as shown in Figure 11.

To obtain the entire R-curve the actual size effect law should be estimated as explained in the previous section (see Figure 3).

A very simple algorithm was written in Matlab for this purpose allowing the definition of the points shown in Figure 12. The numerical points obtained were fitted again (together with the experimental point for $\beta = 1.0$ because this point belongs to this size effect law), using the fitting formulas reported in Table 4. The best fitting ($R^2=0.99$) was obtained with the linear regression II:

$$\frac{1}{w \tau_u^2} = \hat{A} \frac{1}{w} + \hat{C}$$

(25)

where $\hat{A} = 6.964 \times 10^{-5}$ MPa$^{-2}$ and $\hat{C} = 3.46 \times 10^{-6}$ MPa$^{-2}$ mm$^{-1}$. The corre-
sponding points and the fitted curve are shown in Figure 12.

Using the size-effect law obtained is possible to extrapolate the entire R-curve solving the equations (12) and (11).

The steady state value of the R-curve and the length of fracture process zone are calculated as [15]:

\[
\begin{align*}
R_{ss} &= \psi_0^2 \frac{1}{E} \frac{1}{C} = 17.92 \text{ kJ/m}^2 \\
l_{fpz} &= \frac{\psi_0 \dot{A}}{2 \psi_0} \frac{1}{C} = 4.95 \text{ mm}
\end{align*}
\]  

(26)

where \( \psi_0 = \psi|_{\alpha=\alpha_0} \) and \( \dot{\psi}_0 = (\partial \psi / \partial \alpha)|_{\alpha=\alpha_0} \). As previously mentioned, the fracture toughness of the 0° ply is obtained as twice the fracture toughness of the cross ply laminate, \( R_0 (\Delta a) = 2 R (\Delta a) \). Therefore the steady state value of the ply fracture toughness is \( R_{0ss} = 34.4 \text{ kJ/m}^2 \).

It is worth to find a simple analytical expression of the R-curve to use in strength prediction models. A formula generally used is [11,12,15]:

\[
\begin{align*}
\mathcal{R} &= \mathcal{R}_{ss} \left(1 - (1 - \Delta a / l_{fpz})^\kappa\right) \quad \text{if} \quad 0 \leq \Delta a \leq l_{fpz} < 0 \\
\mathcal{R} &= \mathcal{R}_{ss} \quad \text{if} \quad \Delta a \geq l_{fpz}
\end{align*}
\]  

(27)

For the material system used here the best fit (R²=0.997) is \( \kappa = 2.36 \).

5 Conclusion

The main conclusions of this work can be summarized as follows:

- A new cracked Iosipescu specimens was proposed to measure the intralaminar fracture toughness and R-curve in mode II of fibre reinforced composites.
- In the impossibility of scaling the specimen a different formulation of the classical size effect method was proposed. The new formulation enables the measurement of the part of the R-curve that can be calculated once that \( \tau_u = \tau_u (\beta) \) is known. Differently from the classical size effect method \( \beta \) is
not a characteristic size but a dimensionless parameter that depends on the shape of the specimen.

- The entire R-curve can be calculated using the classical size effect method after determining the size effect law $\tau_u = \tau_u(w)$. This is calculated mapping the known portion of the R-curve.
- The ply steady state value of the fracture toughness for the material system investigated was found to be $R_{0ss} = 34.4 \text{ kJ/m}^2$ and the corresponding length of the fracture process zone is 4.95mm.

The drawbacks of the proposed methods are (i) that the rising part of the R-curve may substantially differs from the real one for cross ply laminates because the plastic behaviour of the material is ignored, and (ii) the maximum crack propagation obtained is limited by the gauge section of the specimen, therefore, for material that exhibit very large length of fracture process zone, the size effect obtained using this methodology may not be reliable. In this case the specimen should be scaled and a different test method should be proposed.

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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Size effect law fits [15].</td>
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<td>Elements of $\Phi$ matrix.</td>
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<td>33</td>
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<td>4</td>
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<td>34</td>
</tr>
</tbody>
</table>
Table 1
Size effect law fits [15].

<table>
<thead>
<tr>
<th>Regressions fit</th>
<th>Formula</th>
<th>Fitting parameters</th>
<th>$R_{ss}$</th>
<th>$l_{fpz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilogarithmic</td>
<td>$\ln \sigma_u = \ln \frac{M}{\sqrt{N + w}}$</td>
<td>$M, N$</td>
<td>$\phi_0^2 M^2$</td>
<td>$\phi_0 N$</td>
</tr>
<tr>
<td>Linear regression I</td>
<td>$\frac{1}{\sigma_u^2} = A w + C$</td>
<td>$A, C$</td>
<td>$\phi_0^2 \frac{1}{E A}$</td>
<td>$\phi_0 \frac{C}{2 \phi_0 A}$</td>
</tr>
<tr>
<td>Linear regression II</td>
<td>$\frac{1}{w \sigma_u^2} = \hat{A} \frac{1}{w} + \hat{C}$</td>
<td>$\hat{A}, \hat{C}$</td>
<td>$\phi_0^2 \frac{1}{E \hat{C}}$</td>
<td>$\phi_0 \frac{\hat{A}}{2 \phi_0 \hat{C}}$</td>
</tr>
</tbody>
</table>
Table 2
Elements of $\Phi$ matrix.

<table>
<thead>
<tr>
<th>$(i, j)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>1.794</td>
<td>-1.308</td>
<td>0.1716</td>
<td>-0.01097</td>
<td>0.0002523</td>
</tr>
<tr>
<td>(1,2)</td>
<td>-10.43</td>
<td>13.21</td>
<td>-1.36</td>
<td>0.0792</td>
<td>-0.001765</td>
</tr>
<tr>
<td>(1,3)</td>
<td>25.35</td>
<td>-32.6</td>
<td>2.888</td>
<td>-0.1572</td>
<td>0.003473</td>
</tr>
<tr>
<td>(1,4)</td>
<td>-16.95</td>
<td>38.9</td>
<td>-3.18</td>
<td>0.1614</td>
<td>-0.003451</td>
</tr>
<tr>
<td>(1,5)</td>
<td>0.1882</td>
<td>-17.92</td>
<td>1.474</td>
<td>-0.07248</td>
<td>0.001496</td>
</tr>
<tr>
<td>(2,1)</td>
<td>-0.3254</td>
<td>6.094</td>
<td>-0.6752</td>
<td>0.04665</td>
<td>-0.001181</td>
</tr>
<tr>
<td>(2,2)</td>
<td>34.16</td>
<td>-36.52</td>
<td>1.377</td>
<td>-0.08447</td>
<td>0.002929</td>
</tr>
<tr>
<td>(2,3)</td>
<td>-38.05</td>
<td>64.97</td>
<td>3.783</td>
<td>-0.2963</td>
<td>0.003905</td>
</tr>
<tr>
<td>(2,4)</td>
<td>-5.217</td>
<td>-85.65</td>
<td>-5.751</td>
<td>0.5185</td>
<td>-0.009382</td>
</tr>
<tr>
<td>(2,5)</td>
<td>13.79</td>
<td>53.24</td>
<td>0.8606</td>
<td>-0.1592</td>
<td>0.003198</td>
</tr>
<tr>
<td>(3,1)</td>
<td>-2.122</td>
<td>-11.88</td>
<td>1.238</td>
<td>-0.0982</td>
<td>0.002733</td>
</tr>
<tr>
<td>(3,2)</td>
<td>-55.28</td>
<td>34.68</td>
<td>3.842</td>
<td>-0.1465</td>
<td>-0.001238</td>
</tr>
<tr>
<td>(3,3)</td>
<td>-5.196</td>
<td>-2.537</td>
<td>-30.47</td>
<td>1.794</td>
<td>-0.02811</td>
</tr>
<tr>
<td>(3,4)</td>
<td>63.33</td>
<td>34.79</td>
<td>38.99</td>
<td>-2.568</td>
<td>0.04627</td>
</tr>
<tr>
<td>(3,5)</td>
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<td>-64.79</td>
<td>-12.1</td>
<td>0.93</td>
<td>-0.01781</td>
</tr>
<tr>
<td>(4,1)</td>
<td>-0.1686</td>
<td>12.14</td>
<td>-1.385</td>
<td>0.1204</td>
<td>-0.003441</td>
</tr>
<tr>
<td>(4,2)</td>
<td>60.16</td>
<td>-18.51</td>
<td>-5.287</td>
<td>0.1265</td>
<td>0.003617</td>
</tr>
<tr>
<td>(4,3)</td>
<td>-3.963</td>
<td>-45.88</td>
<td>35.77</td>
<td>-1.841</td>
<td>0.024</td>
</tr>
<tr>
<td>(4,4)</td>
<td>-5.052</td>
<td>14.51</td>
<td>-45.77</td>
<td>2.713</td>
<td>-0.04409</td>
</tr>
<tr>
<td>(4,5)</td>
<td>-54.11</td>
<td>50.67</td>
<td>14.8</td>
<td>-1.009</td>
<td>0.01765</td>
</tr>
<tr>
<td>(5,1)</td>
<td>1.106</td>
<td>-5.201</td>
<td>0.6829</td>
<td>-0.06038</td>
<td>0.001703</td>
</tr>
<tr>
<td>(5,2)</td>
<td>-29.87</td>
<td>8.16</td>
<td>1.231</td>
<td>0.04213</td>
<td>-0.003985</td>
</tr>
<tr>
<td>(5,3)</td>
<td>25.31</td>
<td>13.52</td>
<td>-11.51</td>
<td>0.4595</td>
<td>-0.00217</td>
</tr>
<tr>
<td>(5,4)</td>
<td>-40.86</td>
<td>0.4896</td>
<td>15.21</td>
<td>-0.782</td>
<td>0.009515</td>
</tr>
<tr>
<td>(5,5)</td>
<td>47.24</td>
<td>-22.61</td>
<td>-4.832</td>
<td>0.2949</td>
<td>-0.004118</td>
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</tbody>
</table>
Table 3
Optical system components and measurement parameters.

<table>
<thead>
<tr>
<th>Camera-lens optical system</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CCD camera</td>
<td>Baumer Optronic FWX20</td>
</tr>
<tr>
<td></td>
<td>8 bit, 1624×1236 pixels</td>
</tr>
<tr>
<td>lens</td>
<td>TC 23 36 Telecentric lens</td>
</tr>
<tr>
<td></td>
<td>Magnification: 0.243 ± 3 %</td>
</tr>
<tr>
<td></td>
<td>Field Of View: 29.31 × 22.1 mm</td>
</tr>
<tr>
<td></td>
<td>Working Distance: 103.5 ± 3 mm</td>
</tr>
<tr>
<td></td>
<td>Working F-number: 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image recording</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Acquisition frequency</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Exposure time</td>
<td>0.7 ms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Project parameter – Subset</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Subset size</td>
<td>15×15 pixels² (0.270×0.270 mm²)</td>
</tr>
<tr>
<td>Subset step</td>
<td>13×13 pixels² (0.234×0.234 mm²)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Project parameter – Strain</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain length</td>
<td>5×5 subsets² (1.17×1.17 mm²)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Resolution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>2×10⁻² pixel (0.36 µm)</td>
</tr>
<tr>
<td>Strain</td>
<td>0.02-0.04%</td>
</tr>
</tbody>
</table>
Table 4
Experimental results.

<table>
<thead>
<tr>
<th>specimen</th>
<th>$\beta$ [-]</th>
<th>$w$ [mm]</th>
<th>$\tau$ [MPa]</th>
<th>SD [MPa]</th>
<th>IC 95% [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>5</td>
<td>107.0</td>
<td>3.0</td>
<td>3.3</td>
</tr>
<tr>
<td>B</td>
<td>0.6</td>
<td>6</td>
<td>104.3</td>
<td>4.9</td>
<td>5.5</td>
</tr>
<tr>
<td>C</td>
<td>0.7</td>
<td>7</td>
<td>103.3</td>
<td>2.3</td>
<td>2.6</td>
</tr>
<tr>
<td>D</td>
<td>0.8</td>
<td>8</td>
<td>102.3</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>E</td>
<td>0.9</td>
<td>9</td>
<td>98.0</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>F</td>
<td>1.0</td>
<td>10</td>
<td>95.0</td>
<td>2.6</td>
<td>2.9</td>
</tr>
</tbody>
</table>